Market structure and multiperiod hedging

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Abstract

This paper develops a multiperiod hedging model for a competitive risk-averse international firm. We study the optimal sequential hedging strategy and analyze the impact of the structure of available risk sharing markets on the firm’s export decision. As a main result, we find that the number of risk sharing markets critically affects the export level while the timing of these markets is inconsequential.

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1. Introduction

Most of the literature which links risk aversion and exchange rate uncertainty to allocation and hedging decisions by international firms is based on the assumption that the firm is concerned with a single period. There is only a single period during which hedging and investment take place (e.g., Danthine, 1978; Kawai & Zilcha, 1986; Broll & Eckwert, 1999). However, in many circumstances the distribution of profits over time and the structure of available risk sharing markets is important to the management of the firm. In this case, a multiperiod framework for decision making is needed (e.g., Benninga, Eldor, & Zilcha, 1985; Zilcha & Eldor, 1991; Froot, Scharfstein, & Stein, 1993; Moschini & Hennessy, 2000).

In the literature about international firms under uncertainty, hedging policies in the static case have been discussed by Benninga, Eldor, and Zilcha (1985); Kawai and Zilcha (1986); and Zilcha and Broll (1992), to name just a few. They show that

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introducing unbiased currency futures markets results in a separation and a full-hedge property. An intertemporal hedging model has been introduced by Zilcha and Eldor (1991). They study risk sharing arrangements in the form of currency futures markets in each period. It is shown that introducing unbiased currency futures markets results in a decline of the capital-labor ratio in all periods, and in some cases that production increases at all dates (i.e., the separation property does not hold). Contrary to Zilcha and Eldor (1991), the purpose of our paper is to analyze a multiperiod hedging model of an exporting firm in which the hedge policy can be updated over time. We consider an international firm which produces and exports a commodity at some fixed date in the future. The spot exchange rate at the date when the commodity will be sold is uncertain. Hence, the profits from the firm’s production and export activity are stochastic. The firm takes a position in the futures market at the initial date, and may adjust its portfolio at various intermediate dates. At the final date, the firm offsets the futures position and sells the commodity on the world market.

As a main result, we find that an optimal hedging policy requires future commitments at all dates, although the risky sales revenues accrue only in the last period. The intuition for this result lies in the observation that the firm faces an uncertain spot exchange rate not only at the time when the output is sold. Rather, at the time when the output decision is made, the forward rates at the intermediate trading dates are also uncertain. A sequential hedging strategy allows the firm to hedge both the exchange rate risk on the spot market at the time when the output is sold and the forward rate risk at the intermediate trading dates.

We also demonstrate that the structure of available risk sharing markets critically affects the export decision of the firm. If the market structure is incomplete (i.e., if some futures markets are missing), then the firm exports less, as compared to a situation where the firm has access to a complete system of risk sharing markets. Surprisingly, with an incomplete market structure the export level does not depend on the specific dates at which the futures markets are organized. Only the degree of incompleteness of the market structure (i.e., the number of missing futures markets) matters.

The paper is organized as follows. Section 2 introduces a model of export production and intertemporal hedging under random spot exchange rates and a complete set of currency futures markets. The firm chooses an export level and a sequential hedging strategy. Section 3 analyzes the firm’s production and hedging policy when the system of futures markets is incomplete. Section 4 contains some concluding remarks.

2. Hedging with a complete set of futures markets

We develop an intertemporal framework with three trading dates. Consider a firm which produces a commodity \( x \) domestically at cost \( C(x) \) and exports its output to a foreign country. The cost function is increasing and strictly convex: \( C'(x) > 0, C''(x) > 0 \). The firm makes the production decision at date 0. Date 1 is the production period. At date 2, the output will be ready for sale on the foreign country’s market at a known price \( p \). The exchange rates \( \hat{e}_1 \) (at \( t = 1 \)) and \( \hat{e}_2 \) (at \( t = 2 \)) are random variables with
known joint distribution. The firm maximizes expected utility of random profits at date 2.

At dates 0 and 1, futures markets open. On these markets, contracts for future delivery of foreign currency can be traded. A futures contract signed at date $t$ ($t = 0, 1$) falls due at date $t + 1$ and trades at a price $e^{f}_{t+1}$. At the initial date 0, the firm takes a futures position denoted by $z_1$. At $t = 1$, the gain or loss on the futures position is determined and is reflected in the firm’s margin account. The futures position of period 0 is then closed and a new futures position is taken, denoted by $z_2$. At $t = 2$, the firm offsets $z_2$ and sells its output on the commodity market. The sequential structure of the decision process is illustrated in Fig. 1.

Without loss of generality, let us assume that the risk-free interest rate is equal to zero. Then the firm’s random profit at $t = 2$ is given by Eq. (1):

$$\Pi = \bar{e}_2 px - C(x) + z_1(e^f_1 - \bar{e}_1) + z_2(\bar{e}_2 - \bar{e}_1).$$

The futures price $e^f_1$ for delivery at trading date $t$ is quoted at date $t - 1$. The futures position $z_1$ consists of the number of foreign currency units sold at date $t$. $px$ is the cash position in foreign currency at $t = 2$, and $C(x)$ is the cost function for production of $x$ units of exports. At the initial date 0, the forward rate $e^f_1$ is known while $e^f_2$ and $e_2$ are unknown.

**Assumption (A.1)** *The foreign spot exchange rate is a random walk: $\bar{e}_t = \bar{e}_{t-1} + \bar{e}$, $t = 1, 2$, $\bar{E}\bar{e} = 0$, $\bar{e}_{t-1}$ and $\bar{e}$ stochastically independent for $t = 1, 2$. Furthermore, futures markets are unbiased [i.e., $e^f_t = E(\bar{e}_t | \bar{e}_{t-1}) = e_{t-1}, \ t = 1, 2$].*

The hedger’s decision problem may then be written as shown in Eq. (2),

$$\max_{x, z_1, z_2} EU(\Pi),$$

where $U$ is a strictly concave and twice continuously differentiable utility function. The objective function in Eq. (2) is maximized with respect to exports $x$ and future commitments $z_1, z_2$. The firm chooses $x$ and $z_1$ at date 0, where $z_2$ is chosen at $t = 1$ (i.e., after the forward rate $e^f_2$ has become known).

To explore the impact of hedging on the optimal export decision, we solve the
model backwards, starting with the firm’s hedging decision in period 1. To find the optimal hedge, $z_1$, at $t = 0$ the firm needs to know the optimal hedge, $z_2$, at $t = 1$. Once $z_2$ is determined, it is relatively easy to solve for $z_1$ and the optimal export production, $x$. Using the specification in Assumption (A.1), the firm enters period 1 with the profit function $\Pi$ conditional on $e_1$,

$$ EU'(\Pi^*)|e_1)(e_1 - \bar{e}) = 0. \quad (3) $$

From Eq. (3), we derive our first result:

**Proposition 1.** Under assumption (A.1), for any realization of the spot exchange rate, $e_1$, the optimal futures contract at $t = 1$ is a full hedge of the cash flow in period 2 (i.e., $z^*_2 = px$).

**Proof.** Observe that, due to the strict concavity of $U$, if Eq. (3) has a solution, it is unique. Using the assumption of unbiasedness, condition (3) is equivalent to Eq. (4),

$$ \text{cov}(U'[\hat{\epsilon}_1 px - C(x) + z_1(e_1' - e_1) + \hat{\epsilon}_2(e_1' - \hat{e}_2)], \hat{\epsilon}_2) = 0, \quad (4) $$

for all $e_1$ and $x$. Obviously, Eq. (4) is satisfied for $z^*_2 = px$ which is also the unique solution to Eq. (3).

Substituting $z^*_2 = px$ into the profit Eq. (1), and using Assumption (A.1), we can rewrite the firm’s objective function at the initial trade date 0:

$$ \hat{\Pi} = \hat{\epsilon}_1 px - C(x) + z_1(e_1' - \bar{e}_1). $$

Taking the first-order conditions of the expected utility with respect to $x$ and $z_1$, we obtain in Eqs. (5) and (6):

$$ EU'(\hat{\Pi}^*)(\hat{\epsilon}_1 px - C'(x^*)) = 0, \quad (5) $$

$$ EU'(\hat{\Pi}^*)(e_1' - \bar{e}_1) = 0. \quad (6) $$

Thus, making use of the Eqs. (5) and (6), we get:

**Proposition 2.** (a) The export decision of the firm is independent of the distribution of the random exchange rates and of the firm’s attitudes towards risk (separation property). (b) Under Assumption (A.1), the optimal commitment on the futures market at $t = 0$ is a full hedge of the cash flow in period 2 (i.e., $z^*_1 = px$).

**Proof.** Combining Eqs. (5) and (6) yields the separation result shown in Eq. (7),

$$ C'(x^*) = \hat{\epsilon}_1 p, \quad (7) $$

which proves part (a) of the proposition. With the assumption of an unbiased futures market at $t = 0$, Eq. (6) implies Eq. (8),
cov\left(U^*[\hat{\theta}_t p x^* - C(x^*) + z^*_t (\hat{\epsilon}_t - \hat{\epsilon}_1)], \hat{\epsilon}_1 \right) = 0, \quad (8)

which holds for $z^*_t = px^*$. This proves part (b) of the proposition.

According to Propositions 1 and 2, the optimal intertemporal hedging strategy implies a full hedge in all periods. This holds true even though risky revenues accrue only in the last period, when the firm sells its output at a random price $\hat{\epsilon}_2 p$. By contracting on the futures market at $t = 1$, this price risk can be fully insured. Why, then, does the firm adopt a sequential hedging strategy which begins at $t = 0$ and is updated at $t = 1$? The superiority of a sequential hedging strategy which is updated in each period results from the fact that, as of time $t = 0$, the forward rate at $t = 1$ is uncertain. Contracting on the futures market at $t = 1$ allows the firm to insure its sales revenues against the exchange rate risk, $\hat{\epsilon}_2$, on the basis of the relevant forward rate. However, since the forward rate at $t = 1$ is uncertain too, the final cash position is still risky. In contrast to that, the optimal updated sequential hedging strategy eliminates the forward rate risk as well, and results in a nonrandom cash position at $t = 2$.

3. Hedging with an incomplete set of futures markets

In this section, we analyze the optimal hedging and export decision of the firm when one of the two futures markets at $t = 0$ and $t = 1$ is closed. As was demonstrated in the previous section, with a full set of futures markets, the producer chooses an intertemporal hedging strategy which replicates the payoff of a futures contract, traded at date 0, for delivery of foreign currency at date 2. If the set of futures markets is incomplete, the producer solves a cross-hedging problem: Instead of effectively buying a futures contract for period 2 foreign currency at date 0, he must buy a futures contract at date 0 (respectively date 1) which falls due at date 1 (respectively date 2).

3.1. No futures market at date 0

Again we proceed backwards and first solve the firm’s hedging problem at $t = 1$. The firm enters period 1 with the profit function given in Eq. (9),

$$\tilde{\Pi} = \tilde{\epsilon}_t p x - C(x) + z^*_t (\hat{\epsilon}_t - \hat{\epsilon}_1), \quad (9)$$

and chooses an optimal futures commitment $z^*_t$. Using Assumption (A.1), the first order condition is shown by Eq. (10):

$$\text{cov}(U^*[\tilde{\epsilon}_t p x - C(x) + z^*_t (\hat{\epsilon}_t - \hat{\epsilon}_1)], \hat{\epsilon}_1) = 0. \quad (10)$$

From Eq. (10), we obtain $z^*_t = px$. Substituting $z^*_t = px$ into Eq. (9), we may state the profit function at date 0 as is shown in Eq. (11):

$$\tilde{\Pi} = \tilde{\epsilon}_0 p x - C(x) + \hat{\epsilon}_0 p x - C(x). \quad (11)$$

The first order condition for an optimal export decision is given by Eq. (12):
Since $U'(*\Pi)$ is negatively correlated to $[\hat{e}_1 p - C'(x^*)]$, Eq. (12) implies Eq. (13):
\[
E(\hat{e}_1)p > C'(x^*).
\]
Thus the export decision depends on the distribution of the exchange rate and, hence, the separation property no longer holds. From a comparison of Eqs. (13) and (7), it is immediately apparent that the firm reduces exports if the futures market at date 0 is closed down.

We summarize our findings in

**Proposition 3.** If the firm has access to a futures market at date 1, but not at date 0, then the optimal futures commitment is a full hedge of the cash flow in period 2 (i.e., $z_1^* = px^*$). In addition, the export level is less than under a complete set of futures markets.

### 3.2. No futures market at date 1

In this case, the firm chooses an export level and a futures commitment at date 0 such that the expected utility of profits as given by Eq. (14) is maximized.

\[
\hat{\Pi} = \hat{e}_1 px - C(x) + z_1(e_1^* - \hat{e}_1)
\]  

Using the probabilistic specification in Assumption (A.1), the first order conditions can be stated as shown in Eqs. (15) and (16):

\[
EU'(*\Pi)(\hat{e}_1 + \hat{e}) - C'(x^*) = 0, 
\]  

\[
EU'(*\Pi)(\hat{e}_1 - \hat{e}_1) = 0. 
\]

Since the futures market is unbiased, Eq. (16) implies $z_1^* = px^*$. Combining Eqs. (15) and (16), we arrive at Eq. (17):

\[
EU'(*\Pi)(\hat{e}_1 + \hat{e}) - C'(x^*) = 0. 
\]

By Assumption (A.1), the spot exchange rate at $t = 1$ can be expressed as $\hat{e}_1 = e_1^* + \hat{e}$. Using this equality, the condition at Eq. (17) reduces to Eq. (18),

\[
EU'(*\Pi)(\hat{e}_1 p - C'(x^*)) = 0, 
\]

where $*\Pi = \hat{e}_1 px^* - C(x^*)$.

Obviously, Eqs. (18) and (12) are identical, and hence the firm’s production decision is the same in both cases. Thus, while the number of available risk sharing markets matters, the timing of these markets has no impact on the export level.

**Proposition 4.** If the spot exchange rate follows a random walk, then the exports of a competitive firm are independent of the specific dates at which risk sharing markets are organized. Only the number of available risk sharing markets (i.e., the degree of incompleteness of the market structure) matters.

The finding that the timing of the futures markets is inconsequential depends
critically on our random walk specification for the spot exchange rate. Under this specification, the exchange rate risk does not change through time. Hence, the producer is indifferent between having access to a futures market at date 0 or, alternatively, at date 1. In the absence of the random walk assumption, the production decision of the firm will be responsive to the timing of futures markets.

4. Concluding remarks

Given the great volatility of foreign exchange rates, firms engaged in international operations have been highly interested in developing ways to protect themselves from exchange rate risk. Currency futures are commonly used as hedging instruments by international firms to insure against price and exchange rate risk. Our paper examines whether the separation theorem and the full-hedge theorem remain valid in a generalized multiperiod model and how the market structure (missing risk sharing markets) affects the hedging and production decisions of the firm.

Our results are as follows: Although the risky revenues accrue only in the last period of the firm’s planning horizon, the optimal hedging strategy involves futures commitments at all dates. By adapting a sequential hedging strategy, the firm is able to hedge both the exchange rate risk on the spot market at the time when the output is sold and the risk of fluctuating forward rates at intermediate trading dates. We also demonstrate that the optimal multiperiod hedge exhibits a separation property if the set of futures markets is complete. With an incomplete set of risk sharing markets, the separation theorem breaks down. While the export level depends on the number of risk sharing markets available to the firm, the timing of the markets (i.e., the specific dates at which they are organized) does not matter as long as the spot exchange rate follows a random walk.

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References
