This paper investigates parole decisions when the offender may commit a second crime after having been set free. A convicted person is discharged earlier if the cost of the crime declines or the cost of the imprisonment increases. More dangerous offenders will be dismissed later unless the second penalty has a stronger deterrence effect on them. Other results require an insignificant deterrence effect of the second punishment to overcome their general ambiguity. If this condition holds, the prison term actually served will increase with a more distant time horizon, more severe sentences, and a higher apprehension probability. © 1999 by Elsevier Science Inc.

I. Introduction

Parole rulings are quite usual in the enforcement of criminal laws. A convicted criminal is commonly set free before his regular term in prison ends. The remaining punishment will then be added to a new punishment should the convicted commit a second crime within a certain period. This paper addresses the question how the optimal parole ruling is determined.

In contrast to the mainstream logic in the economics of crime literature since Becker’s (1968) seminal article, the criminal is not assumed to behave fully rationally, i.e., the criminal choice does not reflect a maximization of expected utility. Although a punishment may have a deterrent effect, offenses mainly occur due to an urge to commit a crime. This view seems appropriate for nonproperty offenses like rape or murder. In such cases it is not even unusual that the criminal turns himself over to the police—the offender himself thus recognizing the threat imposed on society. Hence,
the cost of a prolonged imprisonment has to be balanced against the risk of an additional crime if the convicted is set free “too early.”

Avio (1973) and Lewis (1983) report that parole boards mainly base their decision to discharge a prisoner on an estimation of the probability that additional crimes will be committed by the paroled. Because the estimation of the probability of recidivism poses no problem in our model, we do not follow the argument of Avio (1973) that parole rulings are used to correct punishments that are excessive, given the updated estimation of this probability. Contrasting further, parole rulings do not encourage the prisoner to behave in a manner that increases his opportunity cost of recidivism. When addressing the question of the optimal parole ruling, the sentence lengths for first-time and repeat offenders are taken for granted. Thus, a realized first-time conviction constitutes the starting point of the current analysis. Effects of the parole ruling on the decision to commit the first crime, which have been discussed by Miceli (1994), are therefore also ignored in the following.

On receiving parole, the offender may either stay honest or commit a second crime. In the latter case, he will either be arrested and sentenced immediately with a given apprehension probability or remain free. No third crime will occur afterward. The only decision to be taken determines the point of time when the criminal is discharged on parole. It is then shown that the criminal will be set free earlier if the cost of imprisonment rises or the social cost of crime decreases. These results are quite plausible and are in line with Miceli’s, although derived in a quite different framework. While individuals with a higher propensity to commit a crime will usually be discharged later, the opposite may result if the deterrence by a second punishment is increasing with the risk imposed on society. Given an insignificant deterrence effect of the repeated punishment, both an increase in the time horizon and in the length of the prison term spelled out in either of the two sentences will also lead to a later discharge from prison. The same outcome arises if the apprehension probability rises. However, with strong deterrence the three latter results may be reversed.

II. The Model

A serious crime has been committed, and the criminal has been caught. Assume that the social cost of a crime is equal to \( M \), and the social cost of imprisonment is \( Z \) per period of time, i.e., a year. In contrast to Miceli (1994), the latter does not vary with the prisoner’s efforts. The criminal is initially sent to prison for \( S_1 \) years at time \( t = 0 \). He will be set free after \( D \) years with \( D \leq S_1 \). If he commits a second crime after \( D \), he will be caught and sentenced with the apprehension probability \( p \in (0, 1] \), but stays free with probability \((1 - p)\). If he is convicted, he must serve \( S_2 + S_1 - D \) years. In either case, the criminal does not commit a third offense. No crime is committed after the time horizon \( T \).

The point of time \( t \) when the criminal receives an impulse from his urge to commit a second offense constitutes a random variable. If the impulse occurs while the criminal is imprisoned or after \( T \), it is meaningless and cannot be observed. Otherwise, he commits a second crime at this very moment. Let \( f(t; S_2 + S_1 - D, p, \theta) \) denote the density function of the random variable. Assume strictly positive density for \( t \in [0, T] \). Although \( f_1 < 0 \) for \( t \in [0, T] \) seems plausible, i.e., the propensity to commit a crime decreases with age, no assumption with respect to the sign of \( f_1 \) is needed to derive the results below. However, we impose \( f_2 \approx 0 \) and \( f_3 \approx 0 \) for \( t \in [0, T] \) to capture a possibly existing deterrence effect of the punishment associated with repeated conviction.
Further, $f_d > 0$ for $t \in [0, T]$ holds where the shift parameter $\theta$ represents the criminal energy of the offender. The probability that he commits a second crime is given by

$$
\Pi = F(T; S_2 + S_1 - D, p, \theta) - F(D; S_2 + S_1 - D, p, \theta)
$$

$$
= \int_D^T f(t; S_2 + S_1 - D, p, \theta) \, dt,
$$

with $F$ denoting the distribution function of the random variable. It is assumed that $F(T; S_2 + S_1 - D, p, \theta) < 1$. This implies that there always exists a positive probability of never committing a second offense. Differentiating $\Pi$ with respect to $D$ yields

$$
\frac{\partial \Pi}{\partial D} = -f(D; S_2 + S_1 - D, p, \theta) - \int_D^T f_2(t; S_2 + S_1 - D, p, \theta) \, dt,
$$

$$
\frac{\partial^2 \Pi}{\partial D^2} = f_1(D; S_2 + S_1 - D, p, \theta) + 2 \int_D^T f_2(t; S_2 + S_1 - D, p, \theta) \, dt.
$$

The two terms of the right-hand side (RHS) of equation (2) can be interpreted as follows: A later date of discharge reduces the potential threat for society by simply precluding crime possibilities during this period. However, the decreased deterrence yields a counteracting effect on this probability.

The judge minimizes expected social cost

$$
V = ZD + \Pi \left[ M + p(S_2 + S_1 - D) Z \right]
$$

subject to $0 \leq D \leq S_1$ by choice of $D$. For simplicity, there is no discounting. While the first-order condition for an interior solution is

$$
\frac{\partial V}{\partial D} = Z(1 - p \Pi(D^*)) + [M + p(S_2 + S_1 - D^*) Z] \frac{\partial \Pi(D^*)}{\partial D} = 0,
$$

the sufficient second-order condition can be obtained as

$$
\frac{\partial^2 V}{\partial D^2} = -2pZ \frac{\partial \Pi(D^*)}{\partial D} + [M + p(S_2 + S_1 - D^*) Z] \frac{\partial^2 \Pi(D^*)}{\partial D^2} > 0.
$$

Increasing the first punishment implies a higher cost of the first imprisonment. At the same time, the expected cost of the second imprisonment decreases. Yet, it can be seen from equation (4) that the first effect always dominates because $\Pi < 1$. In addition, the second term of $\frac{\partial V}{\partial D}$ indicates that increasing the first punishment induces the two counteracting effects on the probability of a second crime mentioned above. If there exists an interior solution for the minimization problem, the net impact of an increase of the first punishment on the probability of a second crime must be negative at the optimum. Equation (2) then implies that the marginal deterrence effect of the second punishment must be overcompensated by the crime precluding effect of a longer first stay in prison.
The boundary solution $D^* = 0$ can occur if the probability of a second crime is close to zero and changes in the parole rule have no significant impact on the probability of a second crime. The second boundary solution $D^* = S_1$ may arise if the probability of a second crime is close to unity, the cost of the crime is high, and the second punishment has no significant deterring effect.

III. Comparative Statics

Assume now that there exists a unique interior solution $D^*$ that satisfies the sufficient second-order condition. Proposition 1 deals with the effects of changes in the relative prices on the optimal parole ruling:

**Proposition 1:** The criminal is discharged earlier if the cost of imprisonment increases or the cost of the crime decreases.

**Proof:** By the implicit function theorem, it follows that

$$\frac{\partial D^*}{\partial M} = -\frac{V_{DM}}{V_{DD}},$$

$$\frac{\partial D^*}{\partial Z} = -\frac{V_{DZ}}{V_{DD}},$$

where

$$V_{DM} = \frac{\partial^2 V}{\partial D \partial M} = \frac{\partial \Pi(D^*)}{\partial D},$$

$$V_{DZ} = \frac{\partial^2 V}{\partial D \partial Z} = 1 - \rho \Pi(D^*) + \rho(S_2 + S_1 - D^*) \frac{\partial \Pi(D^*)}{\partial D}.$$ 

Due to (5), $V_{DD} > 0$ holds. The first-order condition (4) implies that $\frac{\partial \Pi(D^*)}{\partial D} < 0$. Hence, $\frac{\partial D^*}{\partial M} > 0$. Moreover, dividing (4) by $Z$ yields

$$1 - \rho \Pi(D^*) + \rho(S_2 + S_1 - D^*) \frac{\partial \Pi(D^*)}{\partial D} + \frac{M \partial \Pi(D^*)}{Z \partial D} = 0.$$ 

Because $\frac{M \partial \Pi(D^*)}{Z \partial D} < 0$, it follows that $V_{DZ} > 0$ and $\frac{\partial D^*}{\partial Z} < 0$. 

Increasing the social cost of a second crime $M$ unambiguously leads to a higher level of the first punishment. It pays to keep the offender in prison for a longer time to avoid the higher cost of a second crime. The first two terms on the RHS of (9) reflect that increasing $Z$ raises the present cost of imprisonment per year by more than the expected future cost of imprisonment per year. Yet, this also increases the benefit from crime reduction by raising the cost of a second crime—as captured by the third term on the RHS of (9). Only the first effect provides a tendency towards earlier dismissal.
However, the first effect always dominates the second. Hence, a higher level of $Z$ reduces $D^*$. Proposition 1 shows that relative price variations affect the parole decision with the expected signs. Similar results are derived in Miceli (1994) assuming that the loss of deterrence of the first crime is compensated by conditioning an early discharge on costly good behavior of the prisoner.

Interestingly, the effects of increases in the threat potential associated with a particular offender or the time horizon are generally ambiguous.

**PROPOSITION 2:** An increase in the propensity to commit a crime induces a later discharge date if the deterrence imposed by the second punishment is nonincreasing with the individual’s crime propensity. An increase in the time horizon unambiguously implies later dismissal if the deterrence effect is insignificant.

**Proof:** Note that

\[
\frac{\partial^2 V}{\partial D \partial \theta} = -pZ \int_{T}^{T} f_3(t, S_2 + S_1 - D^*, p, \theta) dt - \left[ M + p(S_2 + S_1 - D^*)Z \right]
\]

\[
\cdot \left[ f_3(D^*, S_2 + S_1 - D^*, p, \theta) + \int_{T}^{T} f_2(t, S_2 + S_1 - D^*, p, \theta) dt \right],
\]

(10)

\[
\frac{\partial^2 V}{\partial D \partial T} = -pZ [f_2(T; S_2 + S_1 - D^*, p, \theta) - \left[ M + p(S_2 + S_1 - D^*)Z \right]
\]

\[
\cdot f_2(T; S_2 + S_1 - D^*, p, \theta).
\]

(11)

If the deterrence of the second penalty is nonincreasing in $\theta$, i.e., $f_2 \equiv 0$, it follows that $V_{D^*} < 0$ and $\frac{\partial D^*}{\partial \theta} > 0$. Assuming an insignificant deterrence effect of the second punishment is associated with $f_2 \approx 0$. This implies that $V_{DT} < 0$ and $\frac{\partial D^*}{\partial T} > 0$. □

Increasing either the time horizon $T$ or the individual’s propensity to commit a crime both raise the probability of a second crime. The respective first terms on the RHS of (10) and (11) show that the gain in expected cost of imprisonment associated with an earlier dismissal is reduced, which implies a tendency to increase $D^*$. However, a more distant time horizon also prolongs the deterring effect of the parole decision, captured by the second term on the RHS of (11). This induces a counteracting negative impact on $D^*$. The ambiguity can be overcome by assuming an insignificant deterrence of the punishment. Focusing on an increase in the propensity to commit a crime, the impact of variations of the discharge date on the probability of a second crime needs to be additionally considered. We can see from the remaining parts of the RHS of (10) that a higher $\theta$ strengthens the crime-precluding effect of a longer imprisonment, but also affects the deterrence effect of the second punishment via $f_2$. Because the impact of the latter on $D^*$ cannot be signed unambiguously, the plausible proposition that a more dangerous criminal will be discharged later does not hold generally. However, if an increase in the propensity to commit a crime is not accompanied by a stronger deterrence effect, this conclusion follows.
Hence, dismissal on parole will occur earlier if the probability of recidivism is lower and the second punishment has no significant deterrence effect. More distant time horizons should plausibly be associated with younger criminals. If the urge to commit a crime decreases with age, both parts of Proposition 2 suggest that older criminals should be discharged earlier than younger offenders if the latter of these condition holds.

We now turn to influences from the political sphere on the parole decision. Changes in the apprehension probability may not only arise due to technical progress, but also if the expenditures on combating crime vary [see Becker (1968)]. Similarly, the initial sentence length may reflect politically induced, discrete changes in criminal law, whereas decisions on parole will rather be based on judicial experience.

**Proposition 3:** An increase in either of the two legal punishment levels leads to a later discharge if the deterrence effect of the second punishment is insignificant. If the apprehension probability rises and the deterrence effect is negligible, the dismissal occurs later.

**Proof:** In this case

\[
\frac{\partial^2 V}{\partial D \partial S_i} = pZ \frac{\partial \Pi(D^*)}{\partial D} - pZ \frac{\partial \Pi(D^*)}{\partial S_i} + \left[ M + p(S_2 + S_1 - D^*) \right] \frac{\partial^2 \Pi(D^*)}{\partial S_i \partial D} \\
= pZ \left[ -f(D^*; S_2 + S_1 - D^*, p, \theta) - 2 \int_{t^T}^T f_2(t, S_2 + S_1 - D^*, p, \theta) \, dt \right] \\
+ \left[ M + p(S_2 + S_1 - D^*) \right] \left[ -f_2(D^*, S_2 + S_1 - D^*, p, \theta) \right] \\
- \int_{t^T}^T f_{22}(t, S_2 + S_1 - D^*, p, \theta) \, dt, \\
\frac{\partial^2 V}{\partial D \partial \theta} = - \Pi(D^*) Z \left[ S_2 + S_1 - D^* \right] \frac{\partial \Pi(D^*)}{\partial D} - pZ \frac{\partial \Pi(D^*)}{\partial \theta} \\
+ \left[ M + p(S_2 + S_1 - D) \right] \frac{\partial^2 \Pi(D^*)}{\partial D \partial \theta} \\
= -Z \int_{t^T}^T f(t; S_2 + S_1 - D^*, p, \theta) \, dt - (S_2 + S_1 - D^*) \left[ f(D^*; S_2 + S_1 - D^*) \right] \\
- D^*, p, \theta) + \left[ M + p(S_2 + S_1 - D^*) \right] \left[ f_2(D^*; S_2 + S_1 - D^*, p, \theta) \right] \\
- D^*, p, \theta) \, dt - \left[ M + p(S_2 + S_1 - D^*) \right] \left[ f_3(D^*; S_2 + S_1 - D^*, p, \theta) \right] \\
+ \left[ M + p(S_2 + S_1 - D^*) \right] \left[ f_4(D^*; S_2 + S_1 - D^*, p, \theta) \right],
\]

(12)

Optimal parole decisions
If the deterrence effect is negligible, then \( f_2 \approx 0 \) and \( f_3 \approx 0 \), implying that \( f_{22} \approx 0 \) and \( f_{23} \approx 0 \). It then follows that \( \frac{\partial^2 V}{\partial D \partial S_i} < 0 \) and \( \frac{\partial^2 V}{\partial D \partial p} < 0 \), which proves the two claims. □

An increase in the sentence length \( S_i, i \in \{1, 2\} \), induces three different effects. First, it raises the cost of a second imprisonment. Thus, the cost saving of an early discharge is reduced, which is captured in equation (12) by \( pZ[ -f(D^p) - \int_{D^p} f_2 dt ] \). Second, it decreases the probability of a second crime because the higher punishment adds to the deterrence effect. This raises the cost saving of an early discharge, described by \( -pZ \int_{D^p} f_2 dt \), and weakens the crime-precluding effect of the first punishment according to \( -[M + p(S_2 + S_1 - D^p) Z] f_2(D^p) \). Third, the impact on the deterrence effect of the second punishment via \( f_{22} \) is generally ambiguous. Hence, comparative static analysis alone cannot yield a clear-cut result with respect to \( \frac{\partial D^p}{\partial S} \). Yet, if the second punishment again does not deter crime \( (f_2 = 0) \), the discharge must occur later. Although the threat to society does not change, a more severe sentence unambiguously leads to a later discharge on parole if the second punishment only provides a negligible deterrence. Moreover, if the regular sentence length should be positively correlated with the social cost of crime, Proposition 1 would also imply that a more serious crime will normally be accompanied by a later discharge from prison.

A rise in the apprehension probability \( p \) leads to a higher expected social cost of a second imprisonment. This reduces the gain from an early dismissal, indicated by the term \( -Z \int_{D^p} f(t) dt \) in equation (13), and raises the value of precluding crimes by a longer imprisonment, being captured by \( -(S_2 + S_1 - D^p) Z[f(D^p) + \int_{D^p} f_2 dt] \). Both effects work toward a later discharge. If the deterrence effect is positive, the probability of a second crime is decreased, resulting in higher cost saving by an early dismissal, as shown by the term \( -pZ \int_{D^p} f_2 dt \). Moreover, with a significant deterrence the value of precluding a second crime is decreased, expressed by \( -[M + p(S_2 + S_1 - D^p) Z] f_2(D^p) \), again implying a tendency toward an earlier discharge. Finally, changing the apprehension probability affects the deterrence quality of the punishment level via \( f_{23} \), the sign being generally ambiguous. Because only the first two effects need to be considered if deterrence is negligible, imprisonment will be prolonged in this case.

### IV. Conclusions

Minimizing expected social cost implies that criminals are discharged earlier if the cost of the crime declines or if the cost of imprisonment increases. The dismissal will occur later if the criminal is more dangerous unless the higher propensity to commit a crime is not associated with a stronger deterrence of the second punishment. If the threat of a punishment does not deter crime, both a more distant time horizon and an increase in either of the two sentences prolong the term in prison. However, these two results need not hold, accounting for a significant deterrence effect of the second punishment. With a more distant time horizon, the prolonged deterrence by an earlier discharge can overcompensate the reduced saving in expected cost of imprisonment. Similarly, the increased deterrence by the second penalty may justify a decrease in the term served if the original prison term sentence rises. A higher apprehension probability leads to a later discharge if it does not raise crime deterrence. Again, with a significant deterrence effect the opposite result may occur.

Obviously some important determinants of the parole decision have been neglected.
For example, the current approach did not discuss parole rulings during a second prison term served in the case of repeated conviction. This would generally be subject to a time inconsistency problem. Given that the crime has already been committed, the second sentence no longer induces a deterrence effect. Therefore, the optimal second penalty is *ex post* equal to zero. However, following Boadway et al. (1996), even a punishment that solely increases social cost from an *ex post* point of view may be enforced to earn a reputation in the judicial system. Also, the model does not allow for series of crimes before the offender is arrested. Moreover, repercussions of the parole ruling on the decision to commit the initial crime have not been analyzed. Finally, the possibility of convicting innocent individuals—as in Ehrlich (1975) and Andreoni (1991), or similarly in Rubinstein (1979)—does not enter the current analysis. An integration of the latter argument can be guessed to imply more liberal parole rulings, whereas accounting for the two former aspects should support a more restrictive regime.

Our analysis assumes exogenous political decisions on the sentence lengths and the apprehension probability. In a more general framework, both will represent choice variables rather than parameters. While the trade-off between apprehension probability and punishment level has been a major subject in the economics of crime literature since Becker (1968), the issue of dynamic deterrence has received increasing attention during the current decade [Polinsky and Rubinfeld (1991); Leung (1995)]. However, a more complex model structure would be required to rule out the unreasonable result that without deterrence both the sentence for repeat offenders and the apprehension probability are set to zero. Moreover, politicians are typically not supposed to minimize social cost but to take into account the preferences of the median voter. Hence, there still seems to exist an explanatory gap between the isolated analysis of parole decisions and the investigation of the role of parole rulings within the legal system.

**References**


