The Effect of Liability-Sharing Rules in Delegating Hazardous Activities

AKIHIRO WATABE

Department of Economics, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku
Yokohama 221 Japan
E-mail: awatabe@cc.kanagawa-u.ac.jp

This paper examines the effects of liability-sharing rules on social welfare and risk reduction when one firm (the principal) delegates indivisible hazardous activities to one of the potential firms (the agents). The problem is posed as providing incentives from the principal to the agents, through the contract, to reduce the level of accident probability under a liability rule in force. Our main findings are twofold: (1) when the agents are free from the potential of bankruptcy, strict liability of the principal achieves the highest level of social welfare, and (2) under the potential of the agent’s bankruptcy, a liability-sharing rule that achieves the highest level of social welfare depends on the agents’ asset levels and the costs of risk reduction. Regardless of the potential of bankruptcy, risk minimization results in social welfare maximization. © 1999 by Elsevier Science Inc.

I. Introduction

This paper examines the effects of liability-sharing rules on social welfare and risk reduction when one firm (the principal) delegates indivisible hazardous activities to one of the potential firms (the agents) under contractual agreements. In carrying out hazardous activities, damages caused by accidents would generate significant effects on neighbors, or the nearby environment, and thus create financial burdens upon the principal and agents as a form of accident liability. Financial responsibility imposed on the principal and the agents depends on the liability-sharing rule in force. Therefore, the principal and the agents would be responsible for their liability shares as specified by the liability-sharing rule. Regardless of liability-sharing rules, however, it is assumed throughout the paper that the principal possesses enough assets to fully indemnify her liability in the event of an accident, while the agents may or may not possess enough assets.
assets to do so relative to their assigned share under the liability sharing rule.\textsuperscript{1} Thus, the agents may face the possibility of bankruptcy if an accident actually occurs.

Under these circumstances, we imagine situations where the principal negotiates a contract with \( n \) different agents, possibly with different cost and safety characteristics for conducting hazardous activities.\textsuperscript{2} In the bargaining process between the principal and the agents, we focus on the uncertainties regarding safety characteristics of the agents, the resulting choices of the agents’ effort levels to reduce the magnitude of risks through the contract and liability rule in force, and the economic incentives the agents face with negotiation processes. More specifically, at the time the contract is negotiated, the principal faces a couple of uncertainties, such as the agents’ present levels of accident probabilities and prospective efforts to mitigate these levels. In this economic environment, the principal attempts to design the contract, which determines the selection of an agent and the resulting size of transfer payments to the agents, while the selected agent chooses the level of effort to reduce the present accident probability, based on the contract offered by the principal. In designing the contract, therefore, the principal would like to select the safest agent with the lowest payment, and furthermore, induce the agent to make the second best level of effort. On the other hand, the agents would like to extract the highest possible payment. For the agents, however, they face competition from other agents. Because every agent, like the principal, does not know other agents’ present accident probabilities, and if the agent conceives that his present accident probability is relatively lower than others, he will be interested in making the principal aware of this fact during the bargaining process. If the principal knew that the level of risk was high, she would forgo the activities. Then no contracts would be given to the agents. Thus, to win the contracts, the agents will have to compete against each other during the bargaining process.

In addition, the contract design is influenced by liability-sharing rules. For example, if strict liability of the agents applies, the principal bears no risk in the event of an accident. Hence, accident liability will not be an issue for the principal and neither will the agents’ levels of accident probabilities. It is, therefore, possible that the agents are unwilling to carry out the activities, possibly due to the potential risk of bankruptcy. Otherwise, the contract may require higher transfer payments to the agents to ensure their participation in the activities. Thus, the principal’s revenue earned from the activities minus the transfer payments may become negative, and then the activities will be forgone by the principal. If strict liability of the principal applies, she will be particularly concerned with the precise knowledge of accident probabilities as well as the agents’ effort levels to control risks occurring from the activities. On the other hand, if both the principal and the agents are jointly responsible for liability when designing the contract, the principal must take into account her share of the expected liability in determining transfer payments to the agents. Meanwhile, the agents must take into account their share of the expected liability in carrying out the activities as reflected by the level of accident probability and also the actual undertaking of the activities. Furthermore, the potential bankruptcy of the agents arising from the liability sharing rule affects the agents’ efforts, thereby, the level of accident probability. Accordingly,

\textsuperscript{1}We assume that there are no legal and scientific uncertainties on accidents (i.e., the magnitude of accident liability is known, and suit is always brought forth in the case of an accident).

\textsuperscript{2}We implicitly assume that the contracts will comply with existing current regulations, laws, and available insurance coverage, and such conditions will remain unchanged at least until the bargaining is finished. Furthermore, victims are unable to avoid or reduce accidental loss and the likelihood of accidents.
the interactions between the principal and the agents reflecting their incentives and liability-sharing rules affect the level of risks to be taken through contractual agreements and, therefore, the social welfare of the activities. Consequently, the costs and benefits of the principal knowing the agents’ accident probabilities and the resulting social welfare depend on the liability-sharing rules. In short, the principal needs to design a contract that provides proper incentives to the agents to reveal correct information about their risk levels and to implement the second-best level of effort, which would be chosen after the contract is offered.\(^3\)

The purpose of the paper, therefore, is to analyze the impact of liability-sharing rules on accident probabilities as well as on the social welfare of hazardous activities delegated from the principal to an agent. In the analysis, we separate the case where the agents do not go bankrupt in the event of an accident from the case where they do go bankrupt. In both cases, jointly shared liability between the principal and the agents and strict liability of the principal will be examined.

The rest of the paper is organized as follows. In Section II; we formulate the model for the delegation contract. Section III compares the level of induced accident probability associated with liability sharing rules and the possibility of the agents’ bankruptcy. Social welfare effects of liability rules are analyzed in Section IV. In Section V, we discuss the result of the analysis with relation to policy implications, and the final section concludes the paper. Proofs of all lemmas are provided in the Appendix.

II. Model

This section considers a model where the principal engages in negotiations with \(n\) different agents with different cost and safety characteristics to delegate the indivisible hazardous activities to one selected agent. It is assumed that both the principal and the agents are risk neutral, and the principal possesses enough assets to compensate fully for liability in the event of an accident. Meanwhile, the agents may or may not possess enough assets to compensate for their share of liability. Liability shares for the principal and the agents are specified by a liability-sharing rule that is given exogenously.

We denote total liability for accidents by \(L\), the liability share assigned for the agents by \(u\) \((0, 1)\), and the principal’s revenue earned from the activities by \(R\). \(L\), \(\theta\), and \(R\) are given exogenously. We also denote \(C(t)\) as the amount that must be spent on safety or protective activities by agent \(i\) to achieve \(t_i\) level of accident probability. \(C(t)\) is assumed to be twice continuously differentiable where \(C' < 0\) and \(C'' > 0\). That is, the more spent on safety, the less the accident probability is, and the marginal cost to mitigate the level of accident probability increases.

Now, agent \(i\) realizes the \textit{ex post} expected cost of the activities as \(\theta L p_i + C(p_i) = \theta L t_i + C(t_i) - \epsilon_i\), where \(\epsilon_i(\geq 0)\) is the \textit{ex post} effort level to reduce the expected cost of the activities, and \(p_i\) is the \textit{ex post} probability of accidents.\(^4\) \(\theta L t_i\) is the \textit{ex ante} expected share of accident liability for agent \(i\), and \(\theta L p_i\) is the \textit{ex post} expected share of accident liability...

---

\(^3\)A principal-agent relationship with the presence of both adverse selection and moral hazard is employed in this paper. For literature on this relationship, see, for example, Laffont and Tirole (1986, 1987) and McAfee and McMillan (1986).

\(^4\)It is implicitly assumed that the liability share of the agents \(\theta\) is strictly greater than zero. When \(\theta = 0\), \(C(p_i) = C(t_i) - \epsilon_i\). Because \(\epsilon_i \geq 0\) and \(C' < 0, t_i < p_i\), so that the principal does not need to provide any incentives to the agents for the second-best level of efforts. The case where \(\theta = 0\), such as strict liability of the principal, will be analyzed in Section II, Bankruptcy.
for agent $i$ with the *ex post* effort level at $e_i$. Disutility of agent $i$’s effort is characterized by $h(e_i)$, where $h(0) = 0$, $h’ > 0$, $h” > 0$ and $h”’ > 0$.\(^5\)

When the principal makes a decision on whether or not to delegate the activities to the agents and during the selection of a particular agent, she does not know the present safety level of every agent, or more succinctly, the present accident probability. We denote agent $i$’s present accident probability as $t_i$, where $t_i \in T_i = [\underline{t}, \overline{t}]$ and $0 < \underline{t} < \overline{t} < 1$. While $t_i$ is not observed, it is assumed that the uncertainty about $t_i$ is randomly drawn from a continuous and differentiable density function $f(t_i)$, where $f(t_i) > 0$ for $t_i \in T_i$ and is common knowledge. It is also assumed that $t_i$ is an independent random variable so that each agent is uncertain about the types of other agents and, thus, the safety level of each agent is determined independently.

As defined above, the function $C$ gives the agents’ expenditures to maintain accident probabilities at achieved levels. Now uncertainties regarding the expected cost of the activities we face emerge from the fact that neither the level of *ex ante* accident probability nor associated effort level are observed by the principal. Thus, once the level of the accident probability is revealed and, *ex post*, accident probability is observed, then the principal knows how much each agent spends for safety expenditures as well as additional expenditures such as agents’ effort to further lessen accident probability.

**Nonbankruptcy**

In this section, we exclude the case in which the agents will go bankrupt under any liability-sharing rules in force. For notational convenience, we denote $T = \times^n [\underline{t}, \overline{t}]$, $T_{-i} = \times^{n-1} [\underline{t}, \overline{t}]$, $f(t_i) = \Pi^n f(t_i)$, $f_{-i}(t_{-i}) = \Pi^{n-1} f(t_i)$, $t_{-i} = (t_1, t_2, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$, $dt = dt_1 \ldots dt_n$ and $dt_{-i} = dt_1 \ldots dt_{i-1} dt_{i+1} \ldots dt_n$. The principal’s objective is to design a contract that maximizes her expected payoff of the activities. In doing so, the principal attempts to provide proper incentives to the agents for revealing correct information about the levels of accident probabilities and for implementing the second-best level of efforts that would be chosen after the contract is offered. Thus, the principal determines a triad of functions $[p_i(t), x_i(t), y_i(t)]$, where $t = [t_1, \ldots, t_n]$, based on the reports of the agents about their types. $p_i(t)$, $x_i(t)$, and $y_i(t)$ respectively represent the probability of accidents agent $i$ is motivated to select in carrying out the activities under the contract, the expected payment from the principal to agent $i$, and the probability that the activities will actually be delegated to (or conducted by) agent $i$. Therefore, the contract specifies which agent will be selected, how much the agents will be paid, and the level of safety effort, which the selected agent is motivated to undertake given the transfer payment scheme associated with the contract and liability rules in force.

Consider the expected payoffs of the agents under the contract $[p, x, y]$. Because every agent knows his own accident probability and, therefore, the associated expected cost of the activities but other agents’ accident probabilities, the expected payoff of agent $i$ under the contract $[p, x, y]$, given that the announced values of all the agents other than agent $i$ are truthfully revealed, is expressed as:\(^6\)

---

\(^5\) $h”’ > 0$ means that the marginal disutility is convex, and also ensures that there is no need for using random incentive schemes [Laffont and Tirole (1986)].

\(^6\) $p_i(t)$, $x_i(t)$, and $y_i(t)$ are assumed to be differentiable almost everywhere and $p_i(t) : T \to [0, \overline{t}]$, $x_i(t) : T \to \mathbb{R}^n$ and $y_i(t) : T \to [0, 1]$. 
level of care taken by agent $i$. As above, the ex post effort level of agent $i$ is characterized by $e_i(t) = \theta L_i + C(t_i) - \theta L_i - C(t_i)$. Thus, by substituting $e_i(t)$ into (1), $U_i$ depends on $p_i(t)$, $x(t)$, and $y(t)$. Following the direct revelation mechanism, the designed contract $\{p_i(t), x(t), y(t)\}$ must induce the true reports of the agents (i.e., incentive compatibility) and also provide the agents with payoffs no less than their status quo for them to participate in the activities (i.e., individual rationality).\footnote{Without loss of generality, we assume that a direct mechanism [e.g., Myerson (1979)] is implemented for truth telling, and furthermore, it will induce the optimal level of efforts by the agents. The status quo of the agent requires that the agents are provided with at least a non-negative expected payoff by the principal.} Let $s_i$ be the announcement of agent $i$ about his present accident probability to the principal. Then the necessary conditions for incentive compatibility are obtained by:

$$\frac{\partial U(s_i, t_x)}{\partial s_i} = 0 \text{ at } s_i = t_x$$

Thus, it is easily shown that the necessary conditions for incentive compatibility are characterized by:\footnote{The proof of the sufficiency of incentive compatibility is provided by the author.}

$$\frac{\partial U_i}{\partial t} = -\int_{T-x} \left[ C'(t) + \theta L \right] h'(C(t) - C(p(t)) + \theta L(t - p(t))) y(t) f_{-x}(t_x) dt_x$$

We shall assume here that the ex ante marginal cost of the activities with respect to accident probability $t_x$ is greater than zero, i.e., $C'(t) + \theta L > 0$. It means that the ex ante level of care taken by agent $i$ is below due care. And $h' > 0$ and $y(t) \geq 0$, so the right-hand side of (3) is nonpositive. Thus $U_i(t_x)$ decreases with $t_x$. Hence, the individual rationality conditions are satisfied if the expected payoffs for the agents at the highest type are greater than zero, i.e., $U_i(t_x) \geq 0$.

Next, the expected payoff for the principal $V$ is expressed as:

$$V(p(t), x(t), y(t)) = \int_{T-x} \sum_{i=1}^{n} [R - (1 - \theta) L_p(t) - x(t)] y(t) f(t) dt$$

where $(1 - \theta) L_p(t)$ is the expected liability share for the principal. The principal’s objective is to design the contract $\{p(t), x(t), y(t)\}$ so as to maximize her expected payoff subject to the incentive compatibility and the individual rationality conditions of the agents. Using (1), (4) can be expressed as the objective function of the optimal control problem right below.\footnote{See Appendix for the derivation.} Because the principal’s expected payoff is decreasing in $U_i(t_x)$, inequality of the individual rationality conditions can be replaced by equality without loss of generality. Hence, the optimal contract of the principal is characterized by the following optimal control problem.
Liability-sharing rules in hazardous activities

\[
\text{Max}_{\mathcal{U}, \mathcal{P}} \sum_{i=1}^{n} \int_{t_i}^{T} \left[ R - L p_i(t) - C(p_i(t)) - h(C(t)) - C(p_i(t)) + \theta L(t_i - p(t_i)) \right] y_i(t) f(t) dt - \sum_{i=1}^{n} \int_{T}^{t_i} U_i(t) f(t) dt_i
\]

subject to:

\[
\frac{\partial U_i}{\partial t_i} = -\int_{t_i}^{T} \left( C'(t) + \theta L h'(C(t)) - C(p_i(t)) \right. \\
+ \left. \theta L(t_i - p(t_i)) y_i(t) f(t) \right) dt_i \\
U_i(t) = 0 \\
0 \leq p_i(t) \leq \bar{t} \\
0 \leq y_i(t) \leq 1
\]

The optimal contract of the principal \( \{p_i^n(t), x_i^n(t), y_i^n(t)\} \) is given as follows:\textsuperscript{10}

\[
-(C(p_i^n(t)) + L) (C(p_i^n(t)) + \theta L h'(C(t)) + \psi(t) (C(t) + \theta L) (C(p_i^n(t))) \right. \\
+ \left. \theta L h'(\phi_1(t)) = 0 \quad (5)
\]

\[
X_i^n(t) = \left[ \theta L p_i^n(t) + C(p_i^n(t)) + h(\phi_1(t)) \right] y_i^n(t) + \int_{t_i}^{t} \left( C'(s) + \theta L h'(\phi_1(s)) y_i^n(s) \right) ds_i \\
\]

\[
y_i^n(t) = 1 \text{ if } R \geq L p_i^n(t) + C(p_i^n(t)) + h(\phi_1(t)) + \psi(t) (C(t) + \theta L) h'(\phi_1(t)) \\
\text{ and } t_i = \min_j \{ t_j \} \\
= 0 \text{ otherwise} \quad (7)
\]

where \( \phi_1(t) = C(t) - C(p_i^n(t)) + \theta L(t_i - p_i^n(t)) \), \( \psi(t) = F(t) / f(t) \), and \( X_i^n(t) = x_i^n(t) y_i^n(t) \).

Condition (5) determines the level of the induced probability of accidents under the optimal contract. Condition (6) gives the expected payment from the principal to agent \( i \) under the optimal contract, and (7) characterizes the probability that the activities are actually delegated to agent \( i \). Applying the implicit function theorem into (5), \( dp_i^n(t) / d\theta > 0 \). Thus, the smaller the liability share of the agents, the less risky it is to carry out the activities. The rationale behind this assertion is as follows. As the agents are held less liable, the principal is held more liable, because the principal and the agents are jointly responsible for the entire liability. Therefore, in response to increases in liability share, the principal would like to induce the agents with higher incentives to

\textsuperscript{10}The derivation of the solution is provided in Appendix.
increase their \textit{ex post} effort level, which reduces the \textit{ex post} expected cost of the activities by making the level of the induced probability of accidents decrease.\textsuperscript{11} The reverse is true for the case in which the agents will be held more liable.

\textbf{Bankruptcy}

This section analyzes the case where the agents face bankruptcy in the event of an accident. In fact, bankruptcy of the agents will depend on how much they are held liable for compensating liability with the principal. Consequently, the agents go bankrupt if the assigned liability exceeds the level of their assets; in other words, they are unable to finance the amount equivalent to the portion of liability for which they are responsible. In such a case, there exists uncompensated liability that cannot be indemnified unless either a liability-sharing rule is modified or a third party makes compensation to cover the remaining liability.\textsuperscript{12} To eliminate uncompensated liability, or internalize the entire liability, we consider a couple of ways in which liability is fully indemnified by the principal and the agents. One is joint liability, and the other is vicarious liability. By joint liability, we mean that both the principal and the agents are jointly responsible for liability.\textsuperscript{13} Because the agents go bankrupt, the principal is required to compensate for residual liability after the agents make as much compensation as they can, which is equivalent to the asset levels. By vicarious liability, we mean that the principal will be responsible for the entire liability in the event of an accident so that the agents are free from compensating any liability at all, that is, there is strict liability of the principal.

First, consider the case of joint liability between the principal and the agents. The liability share of the principal is $L - A_i$, and that of agent $i$ is $A_i$, where $A_i$ denotes the agent $i$'s asset level. Like the case of nonbankruptcy, the expected cost of the activities agent $i$ realizes is equal to $A_i p_i + C(p_i) = A_i t_i + C(t_i) - c$. Thus, an analysis of the joint liability case immediately follows from the nonbankruptcy case by replacing $u$ with $A_i$.

The optimal contract of the principal under joint liability with the potential bankruptcy of the agents is given by:

\begin{equation}
-(C'(p_i(t_i)) + L) + (C'(p_i(t_i)) + A_i)h'(\phi_2(t_i)) + \psi(t_i)(C'(t_i) + A_i)(C'(p_i(t_i)) + A_i)h'(\phi_2(t_i)) = 0 \tag{8}
\end{equation}

\begin{equation}
X_i(t_i) = [A_i p_i(t_i) + C(p_i(t_i)) + h(\phi_2(t_i))]y_i(t_i) + \int_{t_i}^T (C'(s_i) + A_i)h'(\phi_2(s_i))y_i(s_i) ds_i \tag{9}
\end{equation}

\textsuperscript{11}As above, the \textit{ex post} effort level is characterized by the difference between the \textit{ex ante} expected cost of the activities and the \textit{ex post} expected cost of the activities. Because the \textit{ex ante} expected cost is characterized by agents’ type $t_i$ and the \textit{ex post} expected cost is characterized by the induced probability of accidents $p_i(t_i)$, increases in the effort level result in decreases in the induced probability of accidents.

\textsuperscript{12}Possible third parties are insurance companies or “public pays principle.” The analysis of third parties is beyond the scope of this paper.

\textsuperscript{13}It should be noted that deep pockets of the firm results in the strict liability of the firm. The difference between our analysis and deep pockets, however, is that for deep pockets of the firm, there is an uncertainty regarding the firm’s responsibility for residual liability. In other words, deep pockets of the firm are \textit{ex post} in the sense of a court’s decision on shifting residual liability to the firm after an accident actually occurs, while our analysis assumes the firm to be surely held liable for residual liability.
where \( \phi_2(t_i) = C(t_i) - C(p^*_i(t_i)) + A_i(t_i - p^*_i(t_i)) \).

As in the nonbankruptcy case, it immediately follows from (8) that \( d\tilde{p}^*_i(t_i) / dA_i > 0 \). Thus, the agents’ asset levels adversely affect the induced probability of accidents. The rationale behind the fact is similar to the nonbankruptcy case. That is, decreases in the agents’ asset levels increase the level of shared liability for which the principal is responsible. Therefore, the principal likes to induce the selected agent with higher incentives so as to increase his \( \text{ex post} \) effort level to reduce the level of the induced probability of accidents.

Next, consider the case of vicarious liability. Because the agents are free from any financial burdens in the event of an accident, the expected cost of the activities the agent \( i \) realizes is how much he will spend on maintaining the level of the induced probability of accidents at \( C(p_i) \). The induced probability of accidents, with agent \( i \)'s effort, needs to be at least as low as the \( \text{ex ante} \) accident probability such that \( p_i \equiv t_i \), and thus, \( C(t_i) \equiv C(p_i) \). Therefore, the \( \text{ex post} \) effort level at which agent \( i \) is motivated to take under the contract is given by \( t_i = C(p_i) - C(t_i) \). Then, the expected payoff for the principal is expressed by:

\[
V(p(t), x(t), y(t)) = \int \sum_{T = 1}^{n} [R - Lp(t)] y(t) f(t) dt \tag{11}
\]

and the expected payoff for agent \( i \) is expressed by:

\[
U_i(p_i(t), x_i(t), y_i(t)) = \int [x_i(t) - C(p_i(t)) - h(\epsilon_i(t))] y_i(t) f_{-i}(t_{-i}) dt_{-i} \tag{12}
\]

where \( \epsilon_i(t) = C(p_i) - C(t_i) \). Using (12), the incentive compatibility conditions [see (2)] are characterized by:

\[
\frac{\partial U_i}{\partial t_i} = -\int_{T_{-i}} C(t_i) h'(C(p_i(t_i) - C(t_i)) y_i(t) f_{-i}(t_{-i}) dt_{-i} \tag{13}
\]

where the right-hand side of (13) is nonpositive, because \( C' < 0, h' > 0 \) and \( y_i(t) \geq 0 \). Given (11)–(13), following the previous two cases, the optimal contract of the principal under vicarious liability with the potential bankruptcy of the agents is characterized by:

\[
-(C'(p^*_i(t_i)) + L) - C'(p^*_i(t_i)) h'(\phi_3(t_i)) + \psi(t_i) C'(t_i) C(p^*_i(t_i)) h'(\phi_3(t_i)) = 0 \tag{14}
\]

\[
X^\pi_i(t_i) = [C(p^*_i(t_i)) + h(\phi_3(t_i))] y^\pi_i(t_i) - \int_{t_i}^{T} C'(s_i) h'(\phi_3(s_i)) y^\pi_i(s_i) ds_i \tag{15}
\]

\[
y^\pi_i(t_i) = 1 \text{ if } R \geq L p^*_i(t_i) + C(p^*_i(t_i)) + h(\phi_3(t_i)) - \psi(t_i) C'(t_i) h'(\phi_3(t_i))
\]

and \( t_i = \min_j \{t_j\} \)

\[
= 0 \text{ otherwise} \tag{16}
\]
where \( \phi_3(t_i) = C(p_3^*(t_i)) - C(t_i) \).

It should be noted that the induced probability of accidents characterized by (14) is identical to that for the case of strict liability under nonbankruptcy (also see footnote 4). That is because, regardless of the agents’ asset levels relative to liability, the agents can escape from bankruptcy if the principal is held responsible for the entire liability.

III. Effects of Liability Rules on the Induced Probabilities of Accidents

In this section, we compare the levels of the induced probability of accidents associated with each liability-sharing rule to examine the impacts of liability-sharing rules on the risks of the activities. The induced probabilities of accidents for each case are characterized by (5), (8), and (14), respectively. As analyzed above, when there is no likelihood of the agents going bankrupt, the liability share for the agents adversely affects the level of the induced probability of accidents (i.e., \( dp_3^*(t_i)/d\theta > 0 \)). Thus, it is obvious that changes in liability sharing between the principal and the agents affect the induced probability of accidents. Under bankruptcy of the agents, we examined jointly shared liability between the principal and the agents with \( L - A_i \) and \( A_i \), respectively, as one of internalizing liability. Then the induced probability of accidents is adversely affected by the agents’ asset levels (i.e., \( dp_3^*(t_i)/dA_i > 0 \)). The agents’ bankruptcy case, therefore, shows how changes in the agents’ asset levels, rather than liability-sharing rules, affect the induced probability of accidents. In contrast to joint liability cases, it is not obvious what affects the induced probability of accidents for the case of strict liability of the principal because condition (14) is neutral to both \( \theta \) and \( A_i \). Upon considering all liability-sharing rules examined above, general reasoning suggests that the induced probability of accidents under nonbankruptcy is lower than that under bankruptcy. Nevertheless, given the three conditions (5), (8), and (14), it is not quite clear—particularly when it is possible for agents to become bankrupt—which liability-sharing rule minimizes the induced probability of accidents, i.e., provides the safest environment for the activities.

Using \( p^* \) to denote the induced probabilities of accidents determined by the optimal contracts under nonbankruptcy, \( p^*_J \) to denote joint liability under bankruptcy of the agents, and \( p^*_S \) to denote strict liability of the principal, we obtain the following lemmas.\(^{14}\)

**Lemma 1:** \( p^* < p^*_J \)

**Lemma 2:** \( p^* > p^*_S \) if \( \theta L/2 \leq |C'(t_i)| < |C'(p^*)| \)
\[ p^* < p^*_S \] if \( |C'(t_i)| < |C'(p^*)| \approx \theta L/2 \)

**Lemma 3:** \( p^*_S < p^*_J \) if \( A_i/2 \leq |C'(t_i)| < |C'(p^*_S)| \)
\[ p^*_S > p^*_J \] if \( |C'(t_i)| < |C'(p^*_S)| \approx A_i/2 \)

Lemma 1 implies that the induced probability of accidents under nonbankruptcy is always lower than that under bankruptcy when the principal and the agents are jointly responsible for compensating liability. Therefore, in the case that the agents face bankruptcy, setting liability share as \( \theta \approx A_i/L \) enables the agents to stay out of potential bankruptcy; thus, the induced probability of accidents will be lower. The essence of this

\(^{14}\)To distinguish the induced probabilities of accidents for the three different cases, a subscript for agent \( i \) is omitted. Instead, we use subscripts for joint and strict liability cases.
lemma is to imply that an appropriate choice of a liability sharing rule not only prevents
the agents from going bankrupt but also lowers the induced probability of accidents.

Lemma 2 indicates that the induced probability of accidents when liability is shared
by the principal and the agents for \((1 - \theta)L\) and \(\theta L\), respectively, is lower (res. higher)
\textit{than when the principal is held strictly liable, if the marginal cost of risk reduction is less
(res. greater) than half of the agents’ liability share}. As above, the induced probability
of accidents decreases with the agent’s liability share if there is no likelihood of
bankruptcy. When strict liability of the principal applies, \(\theta = 0\). And the condition
\(\theta L/2 \leq |C(t_i)| < |C'(p^*)|\) always holds for \(\theta = 0\). Thus, \(p^* > p^*_C\) holds for \(\theta = 0\). As a
result of Lemmas 1 and 2, therefore, strict liability of the principal provides the safest
environment for conducting activities when there is no likelihood of bankruptcy.

Lemma 3 signifies that, given the possibility of the agents’ becoming bankrupt, the
induced probability of accidents when liability is shared by the principal and the agents
for \(L - A_i\) and \(A_i\), respectively, is lower (res. higher) than when the principal is held
strictly liable, if the marginal cost of risk reduction is less (res. greater) than half of
the agents’ asset levels. Unlike the case of nonbankruptcy, strict liability of the principal may
not necessarily provide the safest environment for activities.

To examine Lemma 3 in detail, consider the case in which the marginal cost of risk
reduction exceeds half of the agents’ assets. In this case, strict liability of the principal is
preferred to joint liability. Strict liability of the principal is equivalent to
\(u \geq |C'(p^*)|\). Hence, when there is a likelihood of bankruptcy for the agents, strict liability
of the principal provides the safest environment for the activities if \(A_i/2 \leq |C(t_i)|\).
Meanwhile, if half of the agents’ assets exceeds the marginal cost of risk reduction, joint
liability is preferred to strict liability of the principal, i.e., \(p^*_J < p^*_C\). Suppose that \(\theta\) is set
sufficiently small but strictly above zero. Then, the agents’ bankruptcy is eliminated and
from Lemma 1, \(p^* < p^*_J\). Thus, if the condition for \(p^* < p^*_J\) always holds whenever the
condition for \(p^*_J < p^*_C\) holds, setting \(\theta < A_i/L\) results in \(p^* < p^*_J < p^*_C\). This, however,
never happens. To see this, when \(|C(t_i)| \leq A_i/2\) holds, \(p^*_J < p^*_C\), and when \(|C(t_i)| < \theta L/2\)
holds, \(p^* < p^*_J\). For the condition \(|C(t_i)| < \theta L/2\) to be satisfied whenever the condition
\(|C(t_i)| \leq A_i/2\) holds, liability share \(\theta\) must be greater than \(A_i/L\). But, this is
contradictory when \(\theta > A_i/L\). Consequently, when there is a likelihood of
bankruptcy for the agents and \(|C(t_i)| \leq A_i/2\), liability sharing between the principal
and the agents such as \(L - A_i\), and \(A_i\) provides the safest environment for activities.

We summarize the results of the analysis shown in Lemmas 1–3 in the following
propositions:

**PROPOSITION 1:** Under the optimal contract, if there is no likelihood of bankruptcy for the agents,
the induced probability of accidents becomes lowest given strict liability of the principal.

**PROPOSITION 2:** Under the optimal contract, if there is a likelihood of bankruptcy for the agents,
the induced probability of accidents becomes lowest for strict liability of the principal if \(A_i/2 \leq |C(t_i)|\).
Otherwise, the induced probability of accidents becomes lowest for joint liability compensating for \(L - A_i\), and \(A_i\),
by the principal and the agents, respectively.

### IV. Social Welfare Effects of Liability Rules

The previous section examined the impacts of liability rules on the induced probabilities
of accidents under the optimal contract. The resulting impacts depend on the possibility of the agents’ bankruptcy as well as the marginal cost of risk reduction.
relative to the agents’ liability shares or asset levels. It is obvious that achieving the safest environment for carrying out hazardous activities is essential to managing risks, and is also important due to growing public concerns for such activities. At the same time, it is clear that the activities yield economic surplus. Along with controlling the risks of the activities, therefore, as in previous sections, we need to examine the impacts of liability rules on social welfare, particularly when agents may become bankrupt.

Currently, regardless of the possibility of agents’ becoming bankrupt, the three liability rules we have examined exclude the situation in which accident liability is left uncompensated by the principal and the agents. Because accident liability is always internalized under any liability rules in question, therefore, we consider the expected total surplus of the principal and the agents as an appropriate criterion of social welfare here. To examine the social welfare effects of liability rules, it is assumed that an utilitarian government will set liability rules so as to maximize the expected total surplus of the activities under the optimal contracts. That is, we shall maximize

\[ V^* = \sum_{i=1}^{n} \left\{ \int_{T_i} \left[ R - Lp^*_i - C(p^*_i) - \psi(t_i)(C'(t_i) + \theta L)h^* \right] y^*_if(t_i) dt_i \right\}. \]  

(17)

Now, for the agents, the utilitarian government does not know \( t_i \) but \( f(t_i) \) is common knowledge. Thus, from the government’s perspective, the maximized expected payoff of agent \( i \) \( U^*_i(p^*, x^*, y^*, t_i) \) is expressed by [see also (A9)]:

\[ U^*_i = \int_{T_i} U^*_i(t_i) f(t_i) dt_i = \int_{T_i} \left( \int_{T} (C'(s_i) + \theta L)h'(s_i) \phi_i(s_i) y^*_if(s_i) ds \right) f(t_i) dt_i \]

\[ = \int_{T_i} \psi(t_i)(C'(t_i) + \theta L)h'(\phi_i(t_i)) y^*_if(t_i) dt_i \]  

(18)

Thus, from (17) and (18), the maximized expected total surplus from the perspective of the government is obtained by:

\[ V^* + \sum_{i=1}^{n} U^*_i = \sum_{i=1}^{n} \left\{ \int_{T_i} \left[ R - Lp^*_i - C(p^*_i) - \psi(t_i)(C'(t_i) + \theta L)h^* \right] y^*_if(t_i) dt_i \right\} \]  

(19)

From (19), maximizing the expected total surplus results in minimizing the expected social cost subject to the incentive compatibility and the individual rationality of the agents. It is also straightforward to show that the maximized expected total surplus given the possibility of bankruptcy for the agents will result in similar formulations as in (19), and equivalently in minimizing the expected social cost. From Lemmas 1–3, the levels of the induced probability of accidents depend on both liability rules and the possibility of bankruptcy. Therefore, the levels of the maximized expected total surplus will vary with liability rules. As a result, the selection of a liability rule for the consideration of
social welfare should be determined by how the level of the maximized expected total surplus will alter across liability rules.

When the principal decides not to delegate the activities to the agents [i.e., \( y^*(t) = 0 \)], no economic surplus is yielded, and the maximized expected total surplus is zero. Meanwhile, whenever activities are delegated to the agents [i.e., \( y^*(t) = 1 \)], they yield positive surplus. This arises because the principal’s decision to delegate is made if the principal’s revenue exceeds the expected social costs plus the additional costs incurred by asymmetric information between the principal and the agents [see (7), (10) and (16)].

In fact, the probability that agent \( i \) is selected for each liability rule [i.e., \( y^*(t) \)] affects the maximized expected total surplus. Regardless of liability rules, however, when the activities are delegated to the agents, the most efficient agent will be selected under the optimal contract, and the probability that agent \( i \) is selected will be smaller as he becomes less efficient because \( dy^*(t)/dt \) \( \leq 0 \). Furthermore, the probability that agent \( i \) is selected is unrelated to the induced probability of accidents. Therefore, when making social welfare comparison on liability rules, we restrict attention to the situations in which activities will be delegated to the agents.

We denote the level of social welfare under nonbankruptcy by \( SW \), under joint liability under bankruptcy of the agents by \( SW_J \) and under strict liability of the principal by \( SW_S \). Then, under the optimal contracts, the following lemmas are obtained.

**Lemma 4:** \( SW_J < SW \)

**Lemma 5:**

\[
SW_J < SW \text{ if } p^* < p^*_S \\
SW_J > SW \text{ if } p^* > p^*_S
\]

**Lemma 6:**

\[
SW_J < SW_S \text{ if } p^*_S < p^*_L \\
SW_J > SW_S \text{ if } p^*_S > p^*_L
\]

Lemmas 4–6 are closely related to Lemmas 1–3 in the previous section, and essentially imply that the safer the risks of conducting the activities, the higher the levels of social welfare.

Lemma 4 implies that social welfare under nonbankruptcy is always higher than under bankruptcy when the principal and the agents are jointly held responsible for liability. Accordingly, along with Lemma 1, when agents may become bankrupt, allocating small liability shares to agents so as to avoid bankruptcy not only decreases the induced probability of accidents but also increases social welfare.

Lemma 5 compares the level of social welfare when the principal and the agents share liability for \( (1 - \theta)L \) and \( \theta L \), respectively, versus strict liability of the principal. The level of social welfare will be higher if the induced probability of accidents becomes lower by selecting one of the liability rules. Recalling Proposition 1, if there is no likelihood of bankruptcy for agents, the induced probability of accidents becomes lowest under strict liability of the principal. As a result, strict liability of the principal is desirable for social welfare.

Lemma 6 means that, given the possibility of agents’ bankruptcy, the level of social welfare when liability is shared by the principal and the agents at \( L - A_i \) and \( A_S \), respectively, is lower (res. higher) than when the principal is held strictly liable, if the
induced probability of accidents under joint liability is greater (res. less) than under strict liability of the principal.

Let us examine the case of bankruptcy in detail. First, consider the case in which \( p^*_S < p^*_J \). Then, strict liability of the principal is preferable to joint liability. From Lemma 4, \( SW_J < SW \). Therefore, if \( p^*_S < p^*_J \) holds when \( p^*_S > p^*_J \) is satisfied, strict liability of the principal is more desirable than joint liability. From Proposition 2, if \( A_i/2 \leq |C'(t_i)| \), then \( p^*_S < p^*_J \). Consequently, if \( A_i/2 \leq |C'(t_i)| \) holds, \( SW_J < SW < SW_S \).

Next, consider the case in which \( p^*_J < p^*_S \). Then, joint liability is preferable to strict liability of the principal. Because \( SW_J < SW \), if \( p^*_S < p^*_J \) holds whenever \( p^*_S < p^*_J \) holds, choosing liability shares as \( 0 < A_i/L \) results in \( p^*_S < p^*_J < p^*_S \) such that \( SW_S < SW < SW_J \). Nonetheless, as was analyzed in the previous section, the inequality \( p^*_S < p^*_J < p^*_S \) never holds. Consequently, if the induced probabilities of accidents under bankruptcy are such that \( p^*_J < p^*_S \), liability sharing between the principal and the agents at \( L - A_i \) and \( A_i \), respectively, attains the highest social welfare.

From Lemmas 4–6, the social welfare effects of the selection of liability rules by the government are summarized in the following propositions:

**Proposition 3:** Under the optimal contract, if there is no likelihood of bankruptcy for agents, strict liability of the principal attains the highest social welfare.

**Proposition 4:** Under the optimal contract, if there is the possibility of the agents becoming bankrupt, strict liability of the principal attains the highest social welfare if \( A_i/2 \leq |C'(t_i)| \). Meanwhile, joint liability compensating for \( L - A_i \) and \( A_i \) by the principal and the agents respectively attains the highest social welfare if \( A_i/2 > |C'(t_i)| \).

V. Policy Implications

When hazardous activities are delegated from the principal to the agents, the possibility of bankruptcy and the financial allocation of accident liability play crucial roles in determining social welfare as well as the risks of the activities. The result of the analysis finds that the lower the induced probability of accidents, the higher the level of social welfare. Thus, risk minimization is equivalent to social welfare maximization, and this is influenced by the potential bankruptcy of the agents as well as liability rules. Indeed, the level of risk reduction and its associated cost reflect agents’ technology for controlling risks.

As in Propositions 1–4, the costs of risk reduction relative to agents’ asset levels or liability shares affect risk levels, and thus social welfare through liability rules in force.\(^{17}\) The effects of liability rules and technology on social welfare and risks, through the likelihood of agents’ bankruptcy, emerge from delegation contracts. When the agents are free from the potential of bankruptcy, the result of the analysis provides straightforward policy implementations for liability rules. Namely, when the principal and the agents engage in negotiations of delegation contracts of the hazardous activities, strict liability of the principal provides the safest environment in carrying out the activities and achieves the highest social welfare. Consequently, if the principal and the agents possess large asset bases and are unlikely to go bankrupt, the optimal policy option of liability rules is the strict liability of the principal.

When the agents face the possibility of bankruptcy, however, the policy option of

\(^{17}\)Because \( C(t_i) \) is the amount that must be spent to achieve accident probability \( t_i, |C'(t_i)| \) implies the cost of risk reduction at risk level \( t_i \).
liability rules is more complex. By Proposition 4, a socially desirable liability rule depends on the *ex ante* costs of risk reduction comparative to the agents’ asset levels. To interpret this proposition in practical perspectives, suppose that the agents possess advanced risk reduction technology. Then, risk reduction would be less costly than the agents with less advanced risk reduction technology. Moreover, the levels of technology rely on financial investments for research and development, and thus the asset levels reflect the agents’ abilities to induce better technology. In other words, the levels of risk reduction technology would be proportional to the asset levels unless the agents have incentives to improve the technology. Because the agents face competition against other agents, they would invest higher expenditures on risk reduction to lower their levels of risk and risk reduction costs to win the contract. Therefore, the costs of risk reduction comparative to the asset levels will be generally less for the agents of the advanced technology than for the agents of the less advanced technology. Accordingly, it can be interpreted that an agent with the condition \( A_i/2 \leq |C(t_i)| \) has the less advanced risk reduction technology than one with the condition \( A_i/2 > |C(t_i)| \). Consequently, if the level of the agent’s technology is considerably low and it incurs costs more than half of the asset level to reduce the level of risk, strict liability of the principal is preferred to joint liability like nonbankruptcy case. Otherwise, joint liability is socially desirable.

General insights into the agents’ asset levels and liability rules suggest that under competition, the level of risk reduction technology is enhanced with asset levels, and it influences the choice of a liability rule. In parallel to the results of the analysis, we would like to discuss two points. First, regarding risk reduction technology, we assumed that the technology is static. Technical innovation will affect both risk reduction and its associated costs, and it is crucial to environmental risks [Cooter (1986)]. If the agents with less advanced technology invest in the technology and their levels of the technology get advanced over time to meet the condition \( A_i/2 > |C(t_i)| \), it will be socially desirable for a bankruptcy case where accident liability is shared jointly between the agents and the principal. Second, suppose that there is an uncertainty about the magnitude of accident damages, and the expected damages are underestimated. Then, the agents with advanced technology will go bankrupt, and joint liability is socially desirable. The underestimates of accident damages, therefore, have policy makers choose an incorrect liability rule such as strict liability of the principal.

For a liability system in the latent hazard setting, firms often segregate hazardous activities in small firms to avoid liability payments. The advantage of such segregation is that the claimants are restricted to the assets of the small firms to compensate liability and a long latency period poses enforcement problems for the liability system [Ringleb and Wiggins (1990)]. In this paper, we ruled out incentives for such segregation. Although we considered the case of the agents’ bankruptcy resulting from insufficient assets to pay liability, liability is fully internalized by which the principal will finance either residual or entire liability. Thus, our analysis is relevant to short-term property damage or injury-based damage and the case where there are essentially no problems associated with the evaluation of liability as well as the enforcement of liability.

Last, one of the examples related to our analysis is the transportation of hazardous materials. For hazardous materials transportation, strict liability of carriers is common practice in many countries today. Carriers, such as trucking companies, are often at small asset bases, so that they will face a high possibility of bankruptcy in case of an
accident. Consequently, policy implications drawn from the results of the analysis lead to the reconsideration of a common liability-sharing rule in hazardous materials transportation.

VI. Conclusions

This paper studied the effects of liability-sharing rules on the level of social welfare and risks when the principal delegates indivisible hazardous activities to one selected agent under the presence of both adverse selection and moral hazard. An auction mechanism was used for the analysis. The impacts of liability-sharing rules depend on whether or not the agents will face the possibility of bankruptcy in the event of an accident.

Given nonbankruptcy of the agent, strict liability of the principal attains the highest social welfare level. Meanwhile, given the likelihood of agents' bankruptcy, whether or not strict liability of the principal results in the highest social welfare depends on the agents' asset levels relative to the costs of risk reduction, which reflect the levels of risk reduction technology of the agents. If half of agents' asset levels is less than the costs of risk reduction, implicating that the agents' technology levels are considerably low, then strict liability of the principal attains the highest social welfare level. Otherwise, joint liability between the principal and the agents such as $L - A_i$ and $A_i$ should be applied. Regardless of the possibility of agents' bankruptcy, however, risk minimization implies social welfare maximization. Hence, risk management is essential to the activities.

The rationale behind the finding is based on the nature of negotiation processes and the risk neutrality of both the principal and the agents. The principal designs a contract while satisfying incentive compatibility and individual rationality conditions for the agents so that whenever the contract is feasible, it is binding. In other words, the principal offers the contract to the agents with a take-it or leave-it option. Therefore, the level of the agents' efforts induced by the contract will depend on how the maximized expected payoff of the principal varies with a change in the liability-sharing rule. Although our study is relevant to short-term property damage or injury-based damage rather than long-term latent hazards that contain numerous problems, the result of the analysis can substantiate policy recommendations for liability-sharing rules in some areas of hazardous activities such as hazardous materials transportation.

Last, in this paper, it is assumed that only the agents can affect the level of accident probability and face the possibility of bankruptcy in the event of an accident. The probability of accidents for joint torts is often dependent upon the actions of two parties and victims [e.g., Landes and Posner (1980); Miceli and Segerson (1991)]. And also, the principal may face the possibility of bankruptcy. We will leave these for future research. In addition to these, uncertainties regarding accident liability will have to be incorporated into the future studies as well.

Appendix

Principal's Expected Payoff:

We rewrite the principal's expected payoff function given in (4) as follows.
Liability-sharing rules in hazardous activities

\[ V = \int \sum_{i=1}^{n} \left[ R - (1 - \theta) L \frac{x_i(t)}{y_i(t)} - x_i(t) \right] y_i(t) f(t) \, dt \]

\[ = \int \sum_{i=1}^{n} \left[ R - L \frac{x_i(t)}{y_i(t)} + \theta L \frac{x_i(t)}{y_i(t)} - x_i(t) \right] y_i(t) f(t) \, dt \]

\[ = \sum_{i=1}^{n} \int_{T_i} \left[ R - L x_i(t) - C - h \right] y_i(t) f(t) \, dt \]

\[ - \sum_{i=1}^{n} \int_{T_i} \left[ x_i(t) - \theta L x_i(t) - C - h \right] y_i(t) f(t) \, dt \]

(using (1))

\[ = \sum_{i=1}^{n} \left\{ \int_{T_i} \left[ R - L x_i(t) - C - h \right] y_i(t) f(t) \, dt - \int_{T_i} U_i(t_i) f(t_i) \, dt \right\} \quad (A1) \]

**Solutions of the Optimal Contracts:**

The deviation of the solution essentially follows Laffont and Tirole (1987). We have assumed that the agents are risk neutral and their types are independent. Hence, the expected accident liability that depends on the reports of other agents can be replaced by one with the same expected accident liability. Furthermore, the agents’ cost function \( C \) and disutility function \( h \) are strictly convex so that the principal does not like to randomize in effort. We thus replace \( p_i(t) \) by \( \tilde{p}_i(t) \). Then, given \( \gamma_i(t) = \int_{T_{-i}} \gamma_i(t) f(t) \, dt \), and integrating the first integral in (A1) with respect to \( T_{-i} \), the optimal control problem with respect to \( p_i(t) \) can be decomposed into \( n \) different problems for each agent. Therefore, the principal’s designing problem is described by the following optimal control.

Max \[ \int_{T_i} \left\{ \left[ R - L x_i(t) - C(\tilde{p}_i(t)) - h(C(t_i) - C(\tilde{p}_i(t)) + \theta L(t_i - \tilde{p}_i(t))) \right] y_i(t) \right. \]

\[ \left. - U_i(t_i) \right\} f(t_i) \, dt_i \quad (A1') \]

subject to:

\[ dU_i/\, dt_i = -(C'(t_i) + \theta L) h'(C(t_i) - C(\tilde{p}_i(t_i)) + \theta L(t_i - \tilde{p}_i(t_i))) y_i(t_i) \quad (A2) \]

\[ U_i(t_0) = 0 \quad (A3) \]

The Hamiltonian is:

\[ H = \left\{ \left[ R - L x_i(t) - C(\tilde{p}_i(t)) - h(C(t_i) - C(\tilde{p}_i(t)) + \theta L(t_i - \tilde{p}_i(t))) \right] y_i(t) \right. \]

\[ \left. - U_i(t_i) \right\} f(t_i) - \lambda_i(t_i) \left( C'(t_i) + \theta L h'(C(t_i) - C(\tilde{p}_i(t_i)) + \theta L(t_i - \tilde{p}_i(t_i))) \right) y_i(t_i) \quad (A4) \]

where \( \lambda_i(t_i) \) is a costate variable associated with \( U_i(t_i) \). The first-order necessary conditions (FOC) are:
\[ \begin{align*}
\frac{\partial H}{\partial p_i} &= \left[ -L - C'(p_i) + (C'(p_i) + \theta L) h'(\phi_i(t_i)) \gamma_i(t_i) f(t_i) \right. \\
&\left. + \lambda_i(t_i)(C'(t_i) + \theta L)(C'(p_i) + \theta L) h''(\phi_i(t_i)) \gamma_i(t_i) \right] = 0 \\
(\text{A5})
\end{align*} \]

\[ \lambda_i(t_i) = -\frac{\partial H}{\partial U_i} = f(t_i) \]

(A6)

and by transversality condition,

\[ \lambda_i(t_i) = 0. \]

(A7)

The FOC (A5) gives the optimal \( \hat{p}_i^*(t_i) \). Here, we assume that \( C'(p_i) + \theta L \geq 0 \) such that corner solutions are ruled out. This assumption implies that care taken by agent \( i \) is at most due care. Also, the second-order conditions (SOC) are assumed to be satisfied. From (A6) and (A7), \( \lambda_i(t_i) = F(t_i) \). Thus, given \( \gamma_i(t_i) \), \( p_i^*(t_i) \) is characterized by (5).

To solve \( \gamma_i(t) \), we substitute \( p_i^*(t_i) \) into the principal’s expected payoff (A1), because the problem cannot be decomposed into \( n \) different problems with respect to \( \gamma_i(t) \).

Then,

\[ V = \sum_{i=1}^{n} \left\{ \int_{T} \left[ R - L p_i^*(t_i) - C - h \right] \gamma_i(t_i) f(t_i) dt - \int_{T_i} U_i(t_i) f(t_i) dt \right\} \]

(A8)

Integrating (A2) from \( t_i \) to \( \bar{t} \) and using (A2),

\[ U_i(t_i) = \int_{t_i}^{\bar{t}} (C'(s_i) + \theta L) h'(\phi_i(s_i)) \gamma_i(s_i) ds_i \]

(A9)

Substituting (A9) into (A8),

\[ V = \sum_{i=1}^{n} \left\{ \int_{T} \left[ R - L p_i^*(t_i) - C - h \right] \gamma_i(t_i) f(t_i) dt - \int_{T_i} \int_{t_i}^{\bar{t}} (C'(s_i)) \\
+ \theta L) h'(\phi_i(s_i)) \gamma_i(s_i) ds_i f(t_i) dt \right\} \]

(\text{using Fubini’s theorem for the second term inside the bracket and collecting terms})

\[ = \sum_{i=1}^{n} \left\{ \int_{T} \left[ R - L p_i^*(t_i) - C - h \right] \gamma_i(t_i) f(t_i) dt - \int_{T_i} F(s_i)(C'(s_i)) \\
+ \theta L) h'(\phi_i(s_i)) \gamma_i(s_i) ds_i, \right\} \]

\[ = \sum_{i=1}^{n} \left\{ \int_{T} \left[ R - L p_i^*(t_i) - C - h \right] \gamma_i(t_i) f(t_i) dt - \int_{T_i} F(t_i)(C'(t_i)) \\
+ \theta L) h'(\phi_i(t_i)) \gamma_i(t_i) dt \right\} \]

(365)
(integrating the first term inside the bracket with respect to \( T_{-i} \) and collecting terms)

\[
= \sum_{i=1}^{n} \left\{ \int_{T_i} \left[ R - Lp^n_i(t) - C - \psi(t)C_i(t) + \theta Lh_i'(\phi_1(t_i)) \right] y(t_i) f(t_i) dt_i \right\}
\]

(A10)

Employing the regularity assumption on \( \psi(t) = \bar{F}(t)/\bar{f}(t) \), that is, \( \psi(t) \) is nondecreasing in \( t \), the term inside the bracket in (A10) is nonincreasing in \( t \), given by the proof of the sufficiency of incentive compatibility conditions). Therefore, \( \hat{y}_i(t) \) is given as (7), because \( \hat{y}_i(t) = \int_{T} \gamma_i(t) f_{-i}(t) dt \).

To obtain the expected payment from the principal to agent \( i \),

\[
X^n_i(t_i) = \int_{T_{-i}} X^n_i(t)_f f_{-i}(t) dt_{-i} = U^n_i(t_i) + [\theta Lp^n_i(t) + C + h] \hat{y}_i(t_i)
\]

(A11)

Then, substituting (A9) into (A11) gives (6).

The solutions of joint and vicarious liability cases under bankruptcy are similarly obtained by following the deviation process above.

**Proof of Lemma 1:** It suffices to show that the FOC (8) evaluated at \( p^n_i(t_i) \) determined by (5) is negative (since assuming the SOC of Hamiltonian are satisfied). Substituting (5) into (8),

\[
-(C'(p^n_i) + \theta L)h_i'(\phi_1(t_i)) - \psi(t)(C'(t_i) + \theta L)(C'(p^n_i) + \theta L)h_i''(\phi_1(t_i)) + (C'(p^n_i) + A_i)h_i''(\phi_2(t_i)) + A_i)h_i''(\phi_2(t_i))\]

\[
< 0
\]

because

\[
C'(p^n_i) + \theta L > C'(p^n_i) + A_i C'(p^n_i) + \theta L \geq 0, \quad C'(t_i) + \theta L > C'(t_i) + A_i > 0, \quad \phi_1 > \phi_2 \text{ and } h' > 0, h'' > 0, \text{ and } h''' \geq 0.
\]

**Proof of Lemma 2:** Like the proof of Lemma 1, it suffices to examine the sign of the FOC (14) with \( p^n_i(t_i) \) determined by (5). Substituting (5) into (14),

\[
-(C'(p^n_i) + \theta L)h_i'(\phi_1(t_i)) - \psi(t)(C'(t_i) + \theta L)(C'(p^n_i) + \theta L)h_i''(\phi_1(t_i)) - C'(p^n_i)h_i''(\phi_3(t_i)) + \psi(t)(C'(t_i) + \theta L)h_i''(\phi_3(t_i))
\]

(A12)

We rewrite (12) as follows:

\[
-(C'(p^n_i) + \theta L/2)h_i'(\phi_1(t_i)) - \psi(t)(C'(t_i) + C'(p^n_i) + \theta L)h_i''(\phi_1(t_i)) - \psi(t)C_i(t_i)C'(p^n_i)h_i''(\phi_3(t_i)) - C'(p^n_i) + \theta L/2)h_i''(\phi_3(t_i))
\]

- \( \theta L/2(h_i'(\phi_1(t_i)) + h_i'(\phi_3(t_i))) \) (A13)

For \( \phi_1 > \phi_3, \theta L/2 > [C(p^n_i) - C(t_i)]/[t_i - p^n_i] \). By convexity of \( C, \phi_1 > \phi_3 \) holds for \( \theta L/2 \geq |C(p^n_i) - C(t_i)|/|t_i - p^n_i| \). By convexity of \( C, \phi_1 > \phi_3 \) holds for \( \theta L/2 \geq |C(p^n_i) - C(t_i)|/|t_i - p^n_i| \). Thus, (13) is negative such that \( \hat{p}^n_i(t) < p^n_i(t) \). The same argument applies for \( \theta L/2 \leq |C'(t_i)| \).
PROOF OF LEMMA 3: Proof exactly follows from proofs of Lemmas 1 and 2 by examining the FOC (8) with \( p_0'(t_s) \) determined by (14). □

PROOF OF LEMMA 4: Because we restrict attention to the case where \( dy^0(t)/dt = 1 \), we only need to examine the expected social costs under different liability rules, given the induced probabilities of accidents specified by the optimal contracts.

Now, let

\[
SC = Lp + C(p) + h(\phi_1) \quad \text{and} \quad SC_j = Lp_j + C(p_j) + h(\phi_2).
\]

Then,

\[
SC_j - SC = L(p_j - p) + C(p_j) - C(p) + h(\phi_2) - h(\phi_1) < (p_j - p)[L + C'(p_j)] + (\phi_2 - \phi_1)h'(\phi_2) = \Delta p[L + C'(p_j) + (\Delta e/\Delta p)h'(\phi_2)]
\]

where \( \Delta p = p_j - p \) and \( \Delta e = \phi_2 - \phi_1 \). From Lemma 1 and its proof, \( \Delta p > 0 \) and \( \Delta e < 0 \). Now, suppose that \( SC_j < SC \). Then, from (8), \( L + C'(p_j) - (C'(p_j) + A_i)h'(\phi_2) > 0 \) such that

\[
-(\Delta e/\Delta p)h'(\phi_2) > L + C'(p_j).
\]

Thereby,

\[-(\Delta e/\Delta p)h'(\phi_2) > C'(p_j) + A_i\]

But,

\[-(\Delta e/\Delta p) = [(C(p_j) - C(p) + \theta L(t - p) + A_i(t - p_j))/(p_j - p) < C'(p_j) + A_i + (\theta L - A_i)(t - p)/(p_j - p) < C'(p_j) + A_i\]

because \( \theta L - A_i < 0 \) by bankruptcy of the agents. Contradiction. □

PROOF OF LEMMA 5: Like in the proof of lemma 4, let \( SC = Lp + C(p) + h(\phi_2) \) and \( SC_0 = Lp_0 + C(p_0) + h(\phi_3) \). Then, \( SC - SC_0 = L(p - p_0) + C(p - p_0) + h(\phi_1) - h(\phi_3) \). If \( p < p_0 \), then,

\[
SC - SC_0 < (p - p_0)[L + C'(p)] + (\phi_1 - \phi_3)h'(\phi_1) = \Delta p[L + C'(p) + (\Delta e/\Delta p)h'(\phi_1)]
\]

where \( \Delta p = p - p_0 < 0 \) and \( \Delta e = \phi_1 - \phi_3 > 0 \). From (5), \( L + C'(p) - (C'(p) + \theta L)h'(\phi_1) > 0 \). Now, totally differentiating \( C(p) + \theta lp + \epsilon = C(t) + \theta Lt, \) \( (C'(p) + \theta L)\Delta p + \Delta e = C'(t) + \theta L > 0 \). Then, \( -\Delta e/\Delta p < C'(p) + \theta L \). Hence, \( C'(p) + \theta L + (\Delta e/\Delta p)h'(\phi_1) > 0 \) and, thus, \( SC < SC_0 \)
For $p > p_S$

$$SC - SC_N > (p - p_N)[L + C'(p_S)] + (\phi_3 - \phi_1)h'(\phi_3)$$

$$= \Delta p[L + C'(p_S) + (\Delta e/\Delta p)h'(\phi_3)]$$

where $\Delta p = p - p_S > 0$ and $\Delta e = \phi_3 - \phi_1 < 0$. From (14), $L + C'(p_S) + C'(p_S)h'(\phi_3) > 0$. Totally differentiating $C(p) - C(t) = e, C'(p) - C'(t) = \Delta e$. Then,

$C'(p) \Delta p - \Delta e = C'(t) < 0$ and, thus, $-\Delta e/\Delta p < -C'(p_S)$. Hence, $L + C'(p_S) + (\Delta e/\Delta p)h'(\phi_3) > 0$ and, thus, $SC > SC_N$.

Proof of Lemma 6: It is similarly proved by following the proof of Lemma 5.

References


