Punishing repeat offenders more severely

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Abstract

That repeat offenders are punished more severely than first-time offenders is a generally accepted practice of almost all penal codes or sentencing guidelines. So far, what has been established in the literature is that sometimes it is better to penalize repeat offenders more severely than first-time offenders. This qualified result seems to be incompatible with the unqualified practices followed in most penal codes. The contribution of this article is to point out why the optimal-deterrence framework adopted in the previous literature cannot generate an unqualified result and to provide an unconditional justification for the practice of punishing repeat offenders more severely, based on an argument that is different from those used in the recent literature.

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1. Introduction

1.1. Background

That repeat offenders are punished more severely than first-time offenders is a generally accepted practice of almost all penal codes or sentencing guidelines. Some recent researchers have tried to justify such a practice theoretically, and much of the discussion so far has focused on finding an efficiency-based rationale for such a practice. The specific question addressed is: To minimize the (expected) \textit{ex post} social costs, should the \textit{ex ante} penalty structure be designed in such a way that repeat offenders are punished more severely? So far, what has been established in the literature is that sometimes it is better to penalize repeat

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offenders more severely than first-time offenders. This qualified result seems to be incompatible with the unqualified practices followed in most penal codes. The contribution of this article is to provide an unconditional answer to the above question based on an argument different from those used in the recent literature. In the rest of this introductory section we shall first review the literature and then explain why the existing literature cannot generate an unconditional answer to the question raised above.

1.2. Previous literature

Rubinstein (1979) provided the first formal analysis of the optimal-penalty structure when the criminal history of an offender is taken into consideration. The social losses in Rubinstein’s model include the harm caused by crimes as well as the costs of mistakenly punishing the innocent. Given a set of specific utility values assigned to various possible outcomes, he showed that sometimes it is optimal not to punish first-time offenders. In a subsequent article, Rubinstein (1980) showed that there exists a utility function under which the policy of punishing repeat offenders more harshly (than first-time offenders) can improve deterrence. These seriously qualified arguments are clearly not enough to justify the unqualified general practices observed.

Polinsky & Rubinfeld (1991) introduced the notion of “offense propensity” and provided another possible justification for the practice of punishing repeat offenders more severely. The offense propensity is characterized by a person’s level of illicit gain, which cannot be observed by the court. Repeat offenses by a person are signals of his high offense propensity, and, hence, imposing more severe penalties on this person is an indirect way of deterring people with higher illicit gains. Using an algebraic example, Polinsky and Rubinfeld showed that within some parametric range, imposing more severe penalties on repeat offenders is superior to a uniform penalty structure. Polinsky and Rubinfeld’s explanation would be more comprehensive if the specific parametric range in which their results hold could be shown to be compatible with reality.

Recently, Polinsky & Shavell (1996) proposed a third explanation for severely punishing repeat offenders. They showed that in a two-period model without discounting, allowing a wedge between the penalties on first-time and second-time offenses can enhance overall deterrence. However, the so-called severe sanctions on repeat offenders in their model are slightly different from common practice. Specifically, let $s_1$, $s_2$, and $s_r$ be, respectively, the sanctions on first-time offenders in the first period, first-time offenders in the second period, and repeat offenders in the second period. Polinsky and Shavell proved that it may be optimal to have $s_2 < s_r = s_1$. But this result says that repeat offenders are not punished more severely in the second period than in the first period (as they were first-time offenders); what it says is that senior (second-period of life) first-time offenders should be punished less severely than repeat offenders and junior (first-period of life) first-time offenders.

Burnovski & Safra (1994) considered a model in which the decision maker tries to determine ex ante the optimal number of times the crime (such as tax evasion) may be committed. The probability of being caught is fixed, but the fine inflicted is a function of the number of times that their past offenses have been detected. They showed that if the probability of detection is small enough, reducing the penalties on subsequent crimes while
increasing the penalties on previous crimes results in a decrease in the overall crime activity. This result is in contrast with existing practice. Moreover, the focus of Burnovski and Safra’s study is actually on the penalty structure of multiple offenses (detected all at once) instead of repeat offenses (detected and penalized one at a time).

In the above-mentioned articles, leaving aside possibly different interpretations of the term repeat offenders in Polinsky and Shavell (1996) and Burnovski and Safra (1994), what most authors have established is that sometimes it is better to penalize repeat offenders more severely than first-time offenders; their conclusions hold under a specific range of either parameters or numerical values. Because those specified parameter ranges or numerical values have not been given much empirical justification, there appears to be a gap between these qualified results and the unqualified practices followed in most penal codes. It is reasonable, therefore, to search for a theory with more general validity.

Concerning the above-mentioned literature, we have two additional remarks. First, despite the fact that the practice of punishing repeat offenders more severely applies to all kinds of crimes, some of the discussion in the previous literature applies only to crimes with acceptable criminal gains. Whether the gains of some crimes should be included in the social welfare function is controversial (see Stigler, 1970, p. 527), but this is not our focus here. However, the existence of some (socially acceptable) criminal gains logically implies an upper bound of sanction beyond which more severe penalties will cause “over-deterrence.” Such an upper bound on sanctions sometimes has limited the scope of allowable variations in the design of the penalty structure. For instance, if it turns out to be optimal to penalize the first-time offenders with the maximum sanction (as in the model of Polinsky & Shavell, 1996), then by definition it is impossible to punish any repeat offenders more severely. Or conversely, if we want to sustain the practice of punishing repeat offenders more severely, then the penalty on first-time offenders must always be less than the maximum sanction. In this article, we shall focus on the discussion of crimes without any social gains (either because the crime in question is not socially acceptable as Stigler 1970, p. 527) mentioned, or because the gains from crimes, such as theft, to the offender offset with the losses to the victim), so that if a justification for penalizing repeat offenders heavily can be obtained, the argument does not involve other irrelevant tradeoffs.¹

Second, most previous researchers tried to justify the practice of punishing repeat offenders more severely in a model of optimal deterrence. The general setting is to consider a society composed of potential criminals with unobservable characteristics, and they decide whether or not to commit crimes given the penalty structure they face. The state then is assumed to search for a penalty structure that can minimize crime-related net social costs. It is expected that observed repeat offenses may be a signal related to potential criminals’ unobservable characteristics, and more severe penalties on repeat offenders are needed to deter people having these unobservable characteristics. But, as we shall explain below, it is unlikely to generate an unqualified result along this line. Notice that the scenario just described can be treated as a standard principal-agent frame-work. The state, as the principal, cannot observe the agents’ (potential criminals’) characteristics and try to use an incentive

¹ More discussion along this line will be provided in the last section.
scheme to induce them to choose some socially desirable activities. Thus, the penalty for crimes as disincentives in a deterrence model is just like a payoff for output as rewards in a regular principal-agent model. It is well-known (see Kreps, 1990, pp. 595–596) that only under strong (monotone likelihood ratio) conditions can we derive that the agent’s payoff is an increasing function of his output. By analogy, in a crime-deterrence model we should not expect to find (unconditionally) that the penalty structure is an increasing function of the number of crimes committed. The qualified conclusion in the previous literature, thus, should not come as a surprise.

1.3. Motivating an alternative theory

The alternative theory we shall present is in fact motivated by early observations made by Stigler (1970). Stigler provided the first (informal) explanation of why first-time offenders should be punished less severely than repeat offenders. He argued that (pp. 528–529) “the first-time offender may have committed the offense almost accidentally and (given any punishment) with negligible probability of repetition, so heavy penalties (which have substantial costs to the state) are unnecessary.” The insight in the article by Rubinstein (1979) seems similar to that of Stigler, but evidently Rubinstein did not notice the earlier contribution by Stigler. Rubinstein tried to distinguish deliberate offenses from those committed by accident and to design optimal penalties that not only are lenient to those who commit offenses by accident but also deter deliberate offenders. Although, theoretically, accidental offenders without mens rea are not truly criminals and should not be convicted, under most legal systems it is unavoidable that these people sometimes are somehow convicted and punished. As a result, the society as well as the innocent convict both incur losses. To avoid this undesirable result, Rubinstein showed that it may be optimal to punish repeat offenders only. Rubinstein did not touch on the problem of the level of penalty, and he demonstrated his point using only a numerical example. In this article we will prove analytically that it is always better to punish repeat offenders more severely than first-time offenders, and, therefore, we include Rubinstein’s numerical analysis as a special case. Our results are valid without any parametric restrictions and, hence, serve as a better justification for the general practices observed in the real world.

Notice that to avoid penalizing an offender without mens rea is essentially to avoid the type-II error associated with trials. Type-II errors may appear to be an issue of “fairness” instead of efficiency, but, as Stigler (1970), Posner (1973), and Png (1986) pointed out, there are indeed costs associated with type-II errors. Taking into account such type-II error social costs, the following intuitive reasoning helps us formalize our ideas: If there is a probability of convicting innocent individuals, then there may be guilty and innocent people among those people who are convicted. A more severe sanction has two different effects on social costs: It enhances deterrence; and it also increases type-II error costs (by punishing innocent

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2 “In any real enforcement system, there will in fact be conviction and punishment of some innocent parties, and these miscarriages of justice impose costs of both resources and loss of confidence in the enforcement machinery” (Stigler, 1970, p 528). More detailed analysis is given in section 4.
people more severely). But under a legal system of any reasonable degree of reliability, the probability of convicting an innocent offender twice is relatively low. Suppose first-time offenders and repeat offenders are punished uniformly under the original regime. To maintain the same deterrence effect as in the original regime, the state can reduce the penalty on first-time offenders a bit and correspondingly increase the penalty on repeat offenders. By doing so, we assign a larger (smaller) cost coefficient to the term less (more) likely to contain mistakes. The type-II error costs are, therefore, reduced with the net deterrence effect unchanged. It will be shown in later sections that this intuition is indeed correct.

Our argument is, in fact, a formal characterization of some recent informal arguments in the literature of law and economics. Posner (1992) took possibly erroneous convictions as one of the reasons to punish repeat offenders more severely. He mentions that (p. 233) "... the fact the defendant has committed previous crimes makes us more confident that he really is guilty of the crime with which he currently is charged; the risk of error if a heavy sentence is imposed is therefore less." Cooter & Ulen (1997) also point out that convicting an innocent person is more costly than failing to convict a guilty person, and the criminal law strikes the balance between costs associated with type-I and type-II errors in favor of the defendant. In the next section, we shall formalize these arguments and provide an unconditional answer to the question of whether we should penalize repeat offenders more severely.

The model and the main result will be described, respectively, in sections 2 and 3. Section 4 presents conclusions and extensions.

2. The model

2.1. Individual decision making

Our analytical framework adopts the two-period model proposed by Polinsky & Rubinfeld (1991) and Polinsky & Shavell (1996). Each individual in the society lives two periods, and the discount rate is zero (no time preferences). During each period, one offense at most can be committed. All individuals are assumed to be risk neutral. If they choose to commit the crime, they obtain a benefit \( b \). The objective of each individual is to maximize his expected payoff, which is the sum of the payoffs in both periods.

Let \( F(b) \) be the distribution of benefit \( b \) among individuals. If we consider the crime in question to be theft, then offenders with larger \( b \) can be interpreted as more "efficient" thieves who can steal more things than other thieves with smaller \( b \). The legal system can be characterized by \( (p, q, s, s_r) \), where \( p \) is the probability of convicting guilty offenders, and \( q (< p) \) is the probability of convicting innocent offenders. These probabilities are independent of one's record and are the same in both periods.\(^3\) \( s \) is the sanction for first-time offenders and is the same in both periods, and \( s_r \) is the sanction for repeat offenders. Those who commit crimes in period 1 but are not convicted have no record and are treated as

\(^3\) For the case when the conviction probability is subject to state control, see the discussion in section 4.
first-time offenders. We allow the penalty on repeat offenders to be higher than first-time offenders: \( s_r \geq s \). If \( s_r = s \), the penalty structure is uniform and repeat offenders face the same sanctions as first-time offenders. Given \((p, q, s, s_r)\), individuals choose whether to commit crimes or not in both periods.

In period 2, if one has an offense record and decides to commit a crime, his expected payoff is \( b - ps_r \). If one has an offense record and decides to be law-abiding, his expected payoff is \(-qs_r\). Thus, one who has a record will commit a crime in period 2 if \( b - ps_r > -qs_r \); otherwise he will be law-abiding. As such, in period 2 the expected payoff for one who has an offense record, denoted \( \pi_{2r} \), is

\[
\pi_{2r} = \max\{b - ps_r, -qs_r\}.
\]

Similarly, the expected payoff for one who has no offense record, denoted \( \pi_{2f} \), is

\[
\pi_{2f} = \max\{b - ps, -qs\}.
\]

In period 1, a rational individual will take the expected payoff in period 2 into account in making his decision. The expected payoff of committing a crime in period 1, denoted \( \pi_{1f} \), is

\[
\pi_{1f} = b - ps + p \pi_{2r} + (1 - p) \pi_{2f}.
\]

The expected payoff of being law abiding in period 1, denoted \( \pi_{1n} \), is

\[
\pi_{1n} = -qs + q \pi_{2r} + (1 - q) \pi_{2f}.
\]

### 2.2. Individual Optimal Decisions

Depending on the size of \( b \), individual decision patterns are separated into three types. Because the analysis involved is straightforward algebra, we present the result below and leave the details to Appendix A.

**Theorem 1:** 1) Individuals with \( b < (p - q)s \equiv b_2 \) will be completely deterred and will not commit crimes in either period. 2) Individuals with \( b > (p - q)s_r \equiv b_1 \) can never be deterred and will commit crimes in both periods. 3) Individuals with \( b \in [(p - q)s, (p - q)s_r] \) will commit an offense in period 1 if and only if

\[
b > \frac{(p - q)[(1 - p)s + qs_r]}{1 - p + q} \equiv b_0,
\]

and they will commit an offense in period 2 if and only if they are not convicted in the first period.

Fig. 1 illustrates the conclusion in Theorem 1.

### 2.3. The Composition of Social Costs

Our objective is to minimize total social costs. In view of the remark in section 1.2, we consider crimes without social gains. The harm from committing a crime (including the
direct harm and the indirect costs suffered by society) is denoted by \( h \), which is assumed to be a constant.

The total social cost, \( C \), is composed of two parts: \( C = H + kE \), where \( H \) is the direct harm from crimes, \( kE \) is the cost of erroneously convicting innocent offenders, and \( k \) characterizes the weight the society puts on erroneous conviction. The gains of the offender are not counted in the calculation of the social cost either because the crime in question is not socially acceptable or because the gains of offenders from crimes (such as theft or other zero-sum crimes) offset with the losses of victims. In view of Fig. 1, we see that the two types of social costs in period 1 are as follows:

\[
H_1 = [1 - F(b_0)]h, \quad kE_1 = kF(b_0)qs.
\]  

Those with \( b > b_0 \) (with probability \( 1 - F(b_0) \)) will commit crimes that cause direct harm. Others (with proportion \( F(b_0) \)) will not commit a crime, but \( q \) proportion of them will be erroneously convicted.

The social costs in period 2 are:

\[
H_2 = [F(b_0) - F(b_2)](1 - q)h + [F(b_1) - F(b_2)](1 - p)h + [1 - F(b_1)]h
\]

\[
kE_2 = kF(b_2)q[(1 - q)s + (qs_r)]
\]

\[
+ k[F(b_0) - F(b_2)]q(qs_r) + k[F(b_1) - F(b_0)]p(qs_r).
\]  

Eqs. (4) and (5) will be interpreted below. Those with \( b < b_2 \) will not commit a crime in period 2, but \( q \) proportion of them will be erroneously convicted. Among those who are erroneously convicted in the second period, part of them (with proportion \( q \)) were also erroneously convicted in period 1 and have a criminal record; these people will receive the penalty \( s_r \). Among those who are erroneously convicted in the second period, \( 1 - q \) of them do not have any criminal record, and they will receive the penalty \( s \). This explains the first term of \( kE_2 \).

Those with \( b_2 < b < b_0 \) will not commit a crime in period 1. However, \( q \) proportion of them were erroneously convicted in period 1 and have criminal records. These people will not commit a crime in period 2, but they still face probability \( q \) of being erroneously convicted again and punished by \( s_r \). This explains the second term of \( kE_2 \). The other \( (1 - q) \) will commit crimes in period 2 that cause direct harm to society, characterized by the first term of \( H_2 \).

Those with \( b_0 < b < b_1 \) will commit crimes in period 1. Part of them (\( p \)) were convicted in period 1 and have a criminal record. These people will not commit a crime in period 2, but they still face probability \( q \) of being erroneously convicted and punished by \( s_r \). This is the third term of \( kE_2 \). The others \( (1 - p) \) will commit crimes in period 2 that cause direct harm to society, constituting the second term of \( H_2 \).

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\(^4\) The punishment involved can be thought of as fines, and we assume that there is no social costs associated with it.

\(^5\) If the sanctions refer to imprisonment, then \( k \) is the cost of per unit length of wrongful imprisonment. See also the discussion in section 4.
Finally, those with $b > b_1$ will commit crimes in both periods. The harm they caused in period 2 is the third term of $H_2$. We now complete the interpretation of Eqs. (4) and (5).

2.4. Social Cost Minimization

With zero interperiod discount, the total social costs are the sum of $H$ and $kE$ in both periods. Straightforward algebra gives us the following formula:

$$
H = H_1 + H_2 = 2h - F(b_2)(1 - q)h - F(b_0)(1 - p + q)h - F(b_1)ph
$$

$$
kE = kE_1 + kE_2 = kF(b_2)(1 - q)qs + kF(b_0)[qs + (q - p)qs] + kF(b_1)pqs.
$$

The objective of the society is to minimize

$$
L = H + kE
$$

by choosing $s$ and $s_r$.

An alternative interpretation of Eq. (6) is as follows. The solution to the minimization problem in Eq. (6) will be the same as the solution to the following constrained minimization problem.

$$
\begin{align*}
\min & H \\
\text{s.t.} & E \leq \bar{E}
\end{align*}
$$

where $\bar{E}$ is the upper bound of erroneous conviction that the society can endure. The above problem says that we minimize the social harm subject to a tolerable type-II error, and the $k$ in Eq. (6) in fact can be interpreted as the Lagrange multiplier, representing the shadow price of type-II errors.

3. The rationale for differential penalties

In this section, we shall prove that, regardless of the size of $k$, the solution to the problem of minimizing $L$ defined in Eq. (6) must be such that $s_r > s$, meaning that we should punish repeat offenders more severely. Let $\tilde{s}^*$ be the optimal uniform penalty; that is, $\tilde{s}^*$ is the solution to the problem of minimizing Eq. (6) subject to the constraint $s = s_r = \tilde{s}$. Let us define

$$
\begin{align*}
s_r &= \tilde{s}^* + \epsilon, \\
s &= \tilde{s}^* + \alpha \epsilon
\end{align*}
$$

where $\epsilon \geq 0$ and $\alpha < 1$. If $\epsilon = 0$, the sanctions for first-time offenders ($s$) and repeat offenders ($s_r$) are the same as $\tilde{s}^*$, which degenerates to the uniform penalty solution.

In Eq. (7), as long as $\epsilon > 0$ the penalty on repeat offenders ($s_r$) is clearly more severe than $\tilde{s}^*$; but because $\alpha$ can be positive or negative, the penalty on first-time offenders may be higher or lower than $\tilde{s}^*$: if $\alpha > 0$ ($\alpha < 0$), it implies that the first-time offender is punished more (less) severely under ($s, s_r$). Therefore, the $\epsilon$ in Eq. (7) characterizes the degree of change, and $\alpha$ characterizes the direction of change. Thus, with a given $\tilde{s}^*$, from Eq. (7) we
transform our problem of choosing \( s \) and \( s_r \) (to minimize Eq. [6]) to choosing \( \alpha \) and \( \epsilon \). Let these optimally chosen \( \alpha \) and \( \epsilon \) be denoted \( \alpha^* \) and \( \epsilon^* \).

Now, we can present the main theorem of this paper.

**Theorem 2:** Punishing repeat offenders more severely than first-time offenders incurs lower social costs than a uniform penalty structure.

The proof of Theorem 2 is given in Appendix B; here we only provide a sketch. To prove that the optimally chosen \((\epsilon^*, \alpha^*)\) incurs lower social costs than a uniform penalty structure, it suffices to prove that the social costs corresponding to a specific direction \(\alpha_0\) and a specific degree \(\epsilon\) are indeed lower. In Appendix B, we show that the particular direction we choose is

\[
\alpha = \alpha_0 \equiv -\frac{(p + q)}{2 - p - q} < 0.
\]

As we just mentioned, \(\alpha = \alpha_0\) being negative means that the sanction for first-time offenders is lower than \(\tilde{s}^*\) and that the sanction for repeat offenders is higher than \(\tilde{s}^*\). By choosing \(\epsilon > 0\) and \(\alpha = \alpha_0 < 0\), we are able to make the marginal change in costs from committing crimes just offset with the marginal change of social loss from erroneous convictions. The reason that this is achievable is explained as follows: When \(\alpha\) is positive, then as \(\epsilon\) increases both \(s\) and \(s_r\) increase. Hence, the change in total number of crimes committed will be negative, and the change in total number of erroneous punishments will be positive (because only those who are deterred could possibly be erroneously punished). When \(\alpha \rightarrow -\tilde{s}^*/\epsilon\), then \(s = \tilde{s}^* - \tilde{s}^* = 0\) and there is no penalty for first-time offense at all. In this case, the change in total number of crimes committed will be positive and the change in total number of erroneous punishments will be negative. Because the population size is fixed, as \(\alpha\) changes, the marginal increase in the group of people who do commit crimes must equal the marginal decrease in the group who do not. Therefore, for any given \(\epsilon\), as we change \(\alpha\) we can always find a critical \(\alpha_0\) somewhere in \((-\tilde{s}^*/\epsilon, 0)\) such that the change in marginal cost due to the change in criminality is equal to the change in marginal cost due to erroneous penalties.

The total social cost will be lowered under this specified \(\alpha_0\), however, because there will be a net reduction in the costs associated with type-II errors. With \(\epsilon > 0\) and \(\alpha = \alpha_0 < 0\), the first-time erroneously punished offenders are punished less severely, and the twice erroneously punished (repeat) offenders are punished more severely. Because the latter event has smaller probability than the former event, the total social cost is lowered as a result. This is the intuition behind Theorem 2.

### 4. Extensions and remarks

We have shown that the optimal sanctions should be lenient to first-time offenders but severe to repeat offenders if we consider the possibility of the erroneous conviction of innocent offenders. It is important to note that our results hold without any restrictions on
parametric specifications. The basic idea is simple: the probability of erroneously convicting repeat offenders is lower than that of convicting first-time offenders. The total social cost is the sum of various terms representing costs from different events. As far as type-II errors are concerned, the sanctions are social costs inflicted on erroneously punished people. It is obvious that we should impose more severe sanctions on suspects with criminal records, because the probability of erroneously convicting a person twice is relatively low.

There are several simplifying assumptions underlying our results. For instance, the cost coefficients associated with $H$ and $E$ are constant, and the probability of detecting an offense is independent of the criminal record of a person. Here, we present some brief discussion as to the robustness of our result with respect to changes in these assumptions.

The conviction probabilities $p$ and $q$ are assumed to be constant solely for the purpose of simplifying our analysis; our conclusions will not be affected even if these probabilities are control variables by the state. The traditional analysis tells us that, when $p$ and $q$ are variable, it is always more efficient to increase the punishment and, correspondingly to decrease the probability of conviction (until the highest possible punishment, say $\hat{s}$, is reached). In that case, it can be shown (see Polinsky & Shavell, 1996) that it is always optimal to set $s = \hat{s}$, and, therefore, it becomes unlikely that we can possibly punish repeat offenders “more severely” ($s_r > \hat{s}$). But this reasoning is not necessarily true when $q$ is positive. Notice that as long as there is some positive probability of type-II errors, which is indeed the case in reality, a more severe sanction must imply a harsher penalty on some innocent people, and, hence, a higher type-II error social cost. For instance, when the punishment on first-time offenders is set at the highest possible level, then potential offenders with $b > b_0$ may be effectively deterred, but many innocent people with $b < b_0$ may face the cost of type-II errors. Under weak assumptions of the cost structure and the distribution function of $b$, it is easy to see that such a maximum-penalty design is often suboptimal. Thus, the scope of allowable variation in the design of the penalty structure is not restricted in our analysis, even if the probability of conviction is allowed to vary.

Another assumption we made, as mentioned in section 1.2, is that there is no social gain associated with the crime in question. While this assumption is valid for crimes such as theft (where the private gain to the thief offsets with the loss to the victim), it may not be valid for other cases. If there is some legitimate gain to the offender that is counted as part of the social benefit, then the concern of “overdeterrence” will arise. This concern, as is well known, implies an upper bound of sanctions. In this case, the optimal penalty for the first-time offender may well reach this upper bound, and, hence, the possibility of inflicting “more” severe penalty on repeated offenders is limited. Our analysis evidently cannot go through in this case, and the reader is referred to Polinsky & Shavell (1996) for a related discussion.

Consider, for instance, zero-sum crimes such as theft without social gains. Since there is no social gain, there does not exist concerns of “over-deterrence.” Suppose under the penalty $s = \hat{s}$ nearly everyone is deterred. Then the social costs involved are mostly costs of type-II errors. If $k$ is large and if the elasticity of crime supply with respect to $s$ is small in absolute value, then we can reduce total social costs by decreasing $s$, so that the number of offenses does not increase much but the type-II error cost reduces significantly, and the total social cost is thereby reduced.
In section 2.3, we assume that the social cost associated with erroneously convicting innocent offenders \((k)\) is an exogenously given constant. This assumption also can be relaxed without affecting our main conclusion. For instance, suppose a firm owner is considering whether to adopt a more efficient accounting system. The problem with this accounting system is that there is a positive probability that some mistakes may be made that will cause an underreporting of profit. Without persuasive explanations, this underreporting of profit may be identified by the Internal Revenue Service as an evasion. If the business owner is mistakenly punished at time \(t\), starting from period \(t + 1\) the owner may be discouraged and decide to give up this efficient accounting system. The type-II error cost here refers to the chilling of the adoption of efficient (but risky) mechanisms. In this case, \(k\) should represent the discounted efficiency difference between the original and the new accounting system starting from period \(t + 1\). This more complicated scenario can be considered the same as the analysis in Png (1986), but our argument supporting the severe punishment of repeat offenders will not be affected by such a complication.

**Appendix A: Proof of Theorem 1**

**Case 1:**
\[ b - ps < -qs \]

Notice that \(b - ps < -qs\) implies \(b - ps_r < -qs_r\). So people with \(b\) in this range will not commit a crime in period 2, whether or not they have a criminal record. The expected payoff in this case can be written as follows:

\[
\begin{align*}
\pi_{2r} &= -qs_r \\
\pi_{2f} &= -qs \\
\pi_{1f} &= b - ps + p(-qs_r) + (1 - p)(-qs) \\
\pi_{1n} &= -qs + q(-qs_r) + (1 - q)(-qs).
\end{align*}
\]

It is easy to verify that \(\pi_{1f} < \pi_{1n}\), and therefore one will not commit a crime in period 1 either. Thus, the legal system successfully deters those with \(b < (p - q)s\) in both periods.

**Case 2:**
\[ b - ps_r > -qs_r \]

Notice that \(b - ps_r > -qs_r\) implies \(b - ps > -qs\). So people with \(b\) in this range will commit a crime in period 2, whether or not they have a criminal record. The expected payoff in this case can be written as follow:

\[
\begin{align*}
\pi_{2r} &= b - ps_r \\
\pi_{2f} &= b - ps \\
\pi_{1f} &= b - ps + p(b - ps_r) + (1 - p)(b - ps) \\
\pi_{1n} &= -qs + q(b - ps_r) + (1 - q)(b - ps).
\end{align*}
\]
It is also easy to verify that $\pi_{1f} > \pi_{1n}$, which implies that these people will commit crimes in period 1. Thus, people with $b > (p - q)s_r$ cannot be deterred from committing crimes in either period.

**Case 3:**

$(p - q)s < b < (p - q)s_r$

People with $b$ in this range will not commit crimes in period 2 if they have a criminal record; and they will commit crimes in period 2 if they do not have a record. The expected payoff in this case can be written as follow:

- $\pi_{2r} = -qs_r$
- $\pi_{2f} = b - ps$
- $\pi_{1f} = b - ps + p(-qs_r) + (1 - p)(b - ps)$
- $\pi_{1n} = -qs + q(-qs_r) + (1 - q)(b - ps)$.

People with $b$ in this range will (will not) commit a crime in period 1 if $\pi_{1f} > (\leq)\pi_{1n}$.

Substituting the $\pi_{1f}$ and $\pi_{1n}$ formula back into the inequality, we have the following result:

$$b < \frac{(p - q)[(1 - p)s + qs_r]}{1 - p + q} \Rightarrow \text{commits offense in period 1}$$

$$\text{law-abiding in period 1}.$$

Let $b_2 = (p - q)s$, and $b_1 = (p - q)s_r$. According to the above analysis, we can divide the individuals with $b$ in the range $(p - q)s < b < (p - q)s_r$ into two subgroups.

**Case 3.1:**

$b_2 < b < b_0$

Individuals with $b$ in this range will not commit crimes in period 1. But if they are mistakenly convicted and have a criminal record (with probability $q$), they will not commit crimes in period 2; otherwise (with probability $1 - q$) they will commit crimes in period 2.

**Case 3.2:**

$b_0 < b < b_1$

Individuals with $b$ in this range will commit crimes in period 1. If they are convicted and have a record (with probability $p$), they will not commit crimes in period 2; otherwise (with probability $1 - p$) they will continue committing crimes in period 2. **Q.E.D**

### Appendix B: Proof of Theorem 2

Differentiating $H$ with respect to $\epsilon$, we have

$$\frac{\partial H}{\partial \epsilon} = h \cdot \left[ -(1 - q)F'(b_2) \frac{\partial b_2}{\partial \epsilon} - (1 + q - p)F'(b_0) \frac{\partial b_0}{\partial \epsilon} - pF'(b_1) \frac{\partial b_1}{\partial \epsilon} \right]. \quad (A1)$$
A marginal change in $e$ induces changes in the critical values $b_1$, $b_2$ and $b_0$, which in turn changes the deterrence effect. Recalling the formula

$$b_2 = (p - q)s, \quad b_0 = \frac{(p - q)[(1 - p)s + q s_r]}{1 - p + q}, \quad b_1 = (p - q)s,$$

we can calculate $\partial b_1 / \partial e$, $\partial b_2 / \partial e$ and $\partial b_0 / \partial e$ explicitly, and rewrite Eq. (A1) as

$$\frac{\partial H}{\partial e} = h \cdot \left[ -(1 - q) F'(b_2) (p - q) \alpha ight.
\left. - (1 + q - p) F'(b_0) \frac{(p - q)[(1 - p)\alpha + q]}{1 - p + q} - p F'(b_1) (p - q) \right].$$

Similarly, differentiating $E$ with respect to $e$ yields

$$\frac{\partial E}{\partial e} = (1 - q)qs F'(b_2) (p - q) \alpha + [qs + (q - p)qs,]
\times F'(b_0) \frac{(p - q)[(1 - p)\alpha + q]}{1 - p + q} + pqs, F'(b_1) (p - q) + q \cdot [F(b_2)(1 - q)\alpha
+ F(b_0)(\alpha + q - p) + F(b_1)p].$$

Combining Eqs. (A1) and (A2), we see that the marginal change in total social cost becomes

$$\frac{\partial H}{\partial e} + k \frac{\partial E}{\partial e} = (1 - q)(kqs - h) F'(b_2) (p - q) \alpha + [kqs - h + (q - p)(kqs, - h)]
\times F'(b_0) \frac{(p - q)[(1 - p)\alpha + q]}{1 - p + q} + p(kqs, - h) F'(b_1) (p - q)
+ kq[F(b_2)(1 - q)\alpha + F(b_0)(\alpha + q - p) + F(b_1)p].$$

When $e = 0$, we have $b_2 = b_0 = b_1 = (p - q)s \equiv \tilde{b}$. Evaluating expression (A3) at $e = 0$ and simplifying yields

$$\frac{\partial H}{\partial e} + k \frac{\partial E}{\partial e} \bigg|_{e=0} = kq F(\tilde{b})[q + (2 - q)\alpha] + F'(\tilde{b})(kq\tilde{s}^* - h)(p - q)
\cdot [(z - p - q)\alpha + (p + q)].$$

We want to show that Eq. (A4) is negative when $\alpha$ is chosen optimally. Because solving for the optimal $\alpha^*$ and substituting $\alpha^*$ back into Eq. (A4) involves tedious algebra, we adopt
the following trick: we shall show that a particular $\alpha_0 < 1$ can make Eq. (A4) negative. This certainly implies that Eq. (A4) is also negative when $\alpha$ is replaced by the optimal $\alpha^*$. Let $\alpha = -(p + q)/(2 - p - q) = \alpha_0$, then Eq. (A4) becomes

$$
\frac{\partial L}{\partial \epsilon} \bigg|_{\epsilon=0} = \frac{\partial H}{\partial \epsilon} + k \frac{\partial E}{\partial \epsilon} \bigg|_{\epsilon=0} = kqF(b) \frac{-2p}{2 - p - q} < 0.
$$

Because $\partial L/\partial \epsilon$ is negative as $\epsilon$ is in the positive interval near $\epsilon = 0$, it is certainly true that $L$ can be reduced even more when $\epsilon$ is chosen optimally. Q.E.D

References


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