Settling multidefendant lawsuits under incomplete information

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Abstract

We develop a model that integrates incomplete information in the analysis of multidefendant settlements, and we compare the proportionate rule (P), the unconditional pro tanto rule (T) and the conditional pro tanto rule (C). We show that the well-known results derived under full information do not hold under incomplete information. Under full information, settlements occur under P and C but are discouraged under rule T. We show that this advantage of P and C disappears under incomplete information. Though the plaintiff never prefers to directly litigate both defendants under P, she might offer higher settlement amounts than under T, and this can lead to a higher frequency of litigation. The same holds for the comparison of T and C. © 2000 Elsevier Science Inc. All rights reserved.

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1. Introduction

Since many legal disputes are resolved by settlements, the comparison of different settlement rules is important when analyzing multidefendant lawsuits. While the pro tanto setoff rule (T) seems to still dominate in the United States, several states have adopted the proportionate rule (P).\textsuperscript{1} Recently, the Private Securities Litigation Reform Act of 1995
replaced rule $T$ with rule $P$ to reduce accountant’s settlement payments that were judged as being excessively high.\textsuperscript{2} Under $T$, a nonsettling party pays the difference between total damages and the other party’s settlement. Conversely, under $P$, each party is liable for her original share independently of whether the other party settled or not.\textsuperscript{3} Besides $T$ and $P$, a second important distinction refers to conditional and unconditional rules.\textsuperscript{4} Under the unconditional rule, each defendant’s payment is independent of whether a settling party would have been liable or not. Under the conditional rule, a setoff is only granted if the settling party would have been liable.

The main result of the preceding literature is that unconditional setoff rules discourage settlements if the probabilities of prevailing against both defendants are uncorrelated.\textsuperscript{5} This is due to the possibility of receiving a setoff even in cases where a settling party is nonliable. It follows that the maximum settlement a defendant is willing to accept is lower than the plaintiff’s expected reward from litigation, so that the plaintiff prefers to directly litigate if litigation costs are sufficiently low. The problem disappears if conditional setoff rules are applied instead.\textsuperscript{6}

Though there are many interesting extensions since the seminal paper by Kornhauser & Revesz,\textsuperscript{7} the present article is to the best of our knowledge the first one that integrates incomplete information in the analysis of multidefendant settlements.\textsuperscript{8} We demonstrate that the main result mentioned above does not hold under incomplete information: the frequency of litigation does not necessarily decrease if unconditional rules are substituted by conditional rules. Following Korn-
hauser & Revesz and Klerman, we assume that the plaintiff can simultaneously make take-it-or-leave-it offers to the defendants. To capture the problem of asymmetric information as easily as possible, we assume that there are only two types of defendants: a “bad” type, who is certain to be found liable, and a “good” type, who is liable with a probability below one. Whereas each defendant knows his own type, the plaintiff and the other defendant only know the probability of meeting a good or a bad type. In this setting, we compare the three settlement rules commonly used: the (conditional) proportionate rule \((P)\), the unconditional pro tanto rule \((T)\) and the conditional pro tanto rule \((C)\).

It is difficult to present the intuition behind our finding that the settlement frequency can be highest under rule \(T\) before the game is presented, and before the plaintiff’s optimal strategies under either rule are understood (see Section 4.5 below for a detailed intuition). First, as under full information, it is possible that the proportionate rule \((P)\) is superior because the plaintiff never prefers to directly litigate both defendants. But on the other hand, there are plausible constellations where the settlement offers, and hence the frequency of rejection, are lowest under the unconditional pro tanto rule \((T)\). This leads to a higher quota of rejections and, thus, to a higher percentage of cases going to trial. A similar result holds for the comparison between the conditional and the unconditional pro tanto setoff rules.

Generally spoken, the plaintiff’s strategy under each rule depends on the difference between her expected revenues from settlement and litigation, and this difference is influenced by all parameters playing a role within our model. We analyze the Perfect Bayesian Equilibrium (PBE) under either rule arising under the assumption that the defendants accept a pair of offers whenever this constitutes a Bayesian Nash Equilibrium (BE) in Stage 2. Then we discuss the impacts of the probability distribution over types, their respective probabilities of being held liable and the defendants’ litigation costs.

We present our model in Section 2. Section 3 briefly reconsiders the case with perfect information in the framework of our model. Section 4 analyzes the three settlement rules mentioned above under incomplete information. Section 5 concludes.

### 2. The Model

There are two defendants named \(A\) and \(B\). Both defendants have two possible types: a “good” type with probability \(p\), and a “bad” type with probability \(1 - p\). The type determines the probability of being held liable in court. A bad type is surely held liable, whereas the probability for a good type is given by \(0 < L < 1\). The probabilities for the two defendants are uncorrelated. All players are risk-neutral.

Total harm is normalized to \(D = 1\). If both defendants are found liable, each defendant’s share is \(0.5\). If only one defendant is liable, she has to bear total damages \(D = 1\). The plaintiff can

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9 As does Klerman, we do not consider an unconditional proportionate rule. First, neither Klerman nor we were able to find cases where the unconditional proportionate rule was applied. Second, we want to proceed along the lines of Klerman’s paper to clearly point out the consequence of incomplete information. Third, the reader would have to consider an additional case without changing the paper’s main argument.
simultaneously make take-it-or-leave-it offers to the defendants. Each defendant’s litigation costs are $0 < t < 1$, while the plaintiff’s costs are zero as to guarantee a credible threat to sue.\(^{10}\)

With $d^i$ as defendant $i$’s payment if she is found liable after defendant $j$’s settlement $s^j$, the three different settlement rules explained in the introduction can be defined as follows:

- proportionate rule ($P$): $d^i = \begin{cases} 0.5 & \text{if } j \text{ would have been liable} \\ 1 & \text{otherwise} \end{cases}$
- unconditional pro tanto rule ($T$): $d^i = 1 - s^i$
- conditional pro tanto rule ($C$): $d^i = \begin{cases} 1 - s^i & \text{if } j \text{ would have been liable} \\ 1 & \text{otherwise} \end{cases}$

3. Full information

3.1. The equilibrium concept under full information

Under full information, we assume $p = 1$, so that both defendants are certainly good types.\(^{11}\) Note that the plaintiff’s expected recovery $R$ is

$$R = 1 - (1 - L)^2 = 2L - L^2$$ (1)

if she directly litigates both defendants because she gets total damages $D = 1$ if at least one defendant is found liable.

We wish to explain how a Subgame Perfect Equilibrium (SPE) for each rule can be found (we refer to this when explaining the incomplete information case). In Stage 1, the plaintiff decides on the settlement amounts $s^A$ and $s^B$. In Stage 2, the defendants simultaneously decide whether to accept or not. The game ends if both defendants settle. If neither or only one of them settles, nonsettling parties are litigated in Stage 3. There are no strategic decisions in Stage 3, because the plaintiff has a credible threat to sue. Hence, for each pair of settlement offers $s^A$ and $s^B$, and for each behavior of her counterpart (i.e., whether she settles or not), a defendant can calculate if it is better to settle or not. In Stage 2, we have to consider a Nash Equilibrium (NE), because each defendant anticipates whether the other defendant accepts or not by definition of equilibrium. There are four possible NEs in Stage 2: both settle, neither settles, only $A$ settles, or only $B$ settles. For each pair of settlement offers, $s^A$ and $s^B$, the plaintiff anticipates which of the four possible NEs is played in Stage 2 by definition of subgame perfectness. Hence, she calculates her maximum payoff via backwards induction.

\(^{10}\) Otherwise it could be favorable for a bad type to reject an offer to imitate a good type, and to deter the plaintiff from litigation.

\(^{11}\) The only other possibility would be to set $p = 0$, i.e., to assume that both defendants are certainly bad types. It is not possible to assume a $p$ in between, because this is exactly the case with asymmetric information. Note that $p = 1$ does not imply that the defendants are certainly liable, because the probability of being held liable is given by $L$. 
To solve the full-information game, we proceed as follows. First note that, under full information, nothing can be gained by settling only with one party: suppose that the plaintiff settles with one party while she litigates the other. Then she could always increase her expected payment by also settling with the other party if the defendant’s litigation costs are positive. Moreover, for each particular settlement equilibrium, the plaintiff always tries to extract the maximum settlement amount that a defendant accepts. This means that the settlement offers in Stage 1 are increased to the point where it is only just a NE in Stage 2 that both parties settle. It follows that, for each settlement rule considered above, one SPE can be found by calculating the maximum settlement offers accepted by both defendants, and by comparing the plaintiff’s payoff to the expected reward from litigation. As will be shown below, this SPE is unique under rules P and T, but not necessarily under rule C.

3.2. Proportionate rule (rule P)

Under rule P, each party’s expected liability payments are independent of the other defendant’s behavior. Note that this automatically guarantees that an equilibrium is unique because each defendant has a dominant strategy.

The maximum settlement amount accepted by A is

\[ s^A_P = L[(1 - L) + 0.5L] + t = L - 0.5L^2 + t. \]  

(2)

A is found liable with probability L. Independently of whether B accepts a settlement or not, A pays 0.5 if B would have been liable too, which happens with probability L. If B is exonerated (this happens with probability 1 - L), A pays total damages of 1. Since the same holds for B due to the symmetric structure of the game, the maximum the plaintiff can get through settlements is

\[ R_P = s^A_P + s^B_P = 2L - L^2 + 2t. \]  

(3)

It follows that the plaintiff prefers to settle with both defendants, since she gets the defendants’ costs of litigation.

3.3. The unconditional pro tanto rule (rule T)

The maximum settlement amount accepted by A is

\[ s^A_T = L(1 - s^B_T) + t \]  

(4)

because each defendant pays the difference between total damages and the other defendant’s settlement amount if she is held liable. Since the same holds for B, it follows

\[ s^A = s^B. \]

 Though obvious, this might be helpful to understand the incomplete information case.

To avoid misunderstandings, we wish to emphasize that the settlement conditions are actually \( s^A \leq L(1 - s^B) + t \) and \( s^B \leq L(1 - s^A) + t \), which allows for many different solutions. However, the plaintiff’s payoff from settlements is maximized if both inequalities are binding. This leads to \( s^A = s^B \).

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\[ 12 \text{ Though obvious, this might be helpful to understand the incomplete information case.} \]

\[ 13 \text{ To avoid misunderstandings, we wish to emphasize that the settlement conditions are actually } s^A \leq L(1 - s^B) + t \text{ and } s^B \leq L(1 - s^A) + t, \text{ which allows for many different solutions. However, the plaintiff’s payoff from settlements is maximized if both inequalities are binding. This leads to } s^A = s^B. \]
which leads to

$$R_T = s_T^A + s_T^B = 2 \cdot \frac{L + t}{1 + L}. \quad (6)$$

The plaintiff prefers to settle if and only if $R_T \geq R$, i.e., if

$$t \geq \frac{L^2 - L^3}{2}, \quad (7)$$

implying that the plaintiff always directly litigates if $t = 0$. This is the main result of the preceding literature analyzed by Kornhauser & Revesz and explained in the introduction: settlements are discouraged by rule $T$.

To prove whether the SPE described above is unique, we have to check whether there is another NE at Stage 2. Clearly, NEs in which only one defendant rejects are impossible as this would destroy the NE in which both defendants accept, and the existence of this NE has already been shown. Hence, the only remaining other equilibrium at Stage 2 is that both defendants reject. In this case, they are litigated simultaneously and have expected payments of $L[(1 - L) + 0.5L] + t$ as explained under rule $P$. If only one accepts, her payment is $(L + t)/(1 + L)$. Hence, reject/reject ($r/r$) cannot be an equilibrium if

$$\frac{L + t}{1 + L} \leq L[(1 - L) + 0.5L] + t. \quad (8)$$

This is equivalent to

$$0.5L^2(1 - L) + Lt \geq 0, \quad (9)$$

which is obviously fulfilled as $0 < L, t < 1$. Hence, the equilibrium described above is unique.

### 3.4 The conditional pro tanto rule (rule C)

Under rule $C$, a setoff for the nonsettling defendant is only granted if the settling defendant would have been liable. Thus, $A$’s expected payment depends not only on whether $B$ settles, but also on whether $B$ would have been liable. The maximum amount accepted is thus

$$s_C^A = L[L(1 - s_C^B) + (1 - L)] + t = L(1 - Ls_C^B) + t. \quad (10)$$

If $A$ is found liable (this happens with probability $L$), and $B$ settles, $A$ pays only $1 - s_C^B$ if $B$ would have been liable. This happens with probability $L$. Otherwise, she pays total damages of 1 (this occurs with probability $1 - L$). Since $B$’s situation is identical, it follows

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14 Recall that all settlement rules are identical if both defendants are litigated.
\[ s_C^A = s_C^B = \frac{L + t}{1 + L^2}. \]  

(11)

\( s_C^A \) is strictly higher than \( s_T^A \), because the expected setoff is smaller, and hence the accepted settlement higher. The plaintiff gets

\[ R_C = s_C^A + s_C^B = 2 \cdot \frac{L + t}{1 + L^2}. \]

(12)

It follows that the plaintiff always prefers to settle, because

\[ 2 \cdot \frac{L + t}{1 + L^2} > 2L - L^2. \]

(13)

However, conversely to rules \( P \) and \( T \), the SPE found above is not unique for all possible parameters. If both defendants reject, their expected payments are again \( L[(1 - L) + 0.5L] + t \) as under rules \( P \) and \( T \). If only one accepts, her payment is \( (L + t)/(1 + L^2) \). Hence, reject/reject \((r/r)\) cannot be an equilibrium if

\[ \frac{L + t}{1 + L^2} \leq L[(1 - L) + 0.5L] + t. \]

(14)

It can be shown that this is fulfilled only for

\[ t \geq 0.5 + L(0.5L - 1). \]

(15)

The reason is that a setoff is only granted under rule \( C \) if the other defendant would also have been liable, and this leads to higher settlements if both accept. Hence, rejection becomes superior if the codefendant also rejects. To avoid misunderstandings, we wish to emphasize that this does not mean that there are two SPEs where the plaintiff offers \( s_C^A = s_C^B = (L + t)/(1 + L^2) \), one in which both defendants accept and one in which they reject. By definition of SPE, the plaintiff anticipates if the defendants reject in Stage 2, and in this case it would not be an equilibrium strategy to offer \((L + t)/(1 + L^2)\) because she knows that she would have to litigate both defendants. It follows that we get infinitely many SPEs: suppose, for instance, that the defendants play \( r/r \) for \( s_C^A = s_C^B = (L + t)/(1 + L^2) \), but accept/accept \((a/a)\) for \( s_C^A = s_C^B = (L + t)/(1 + L^2) - \varepsilon \) for an arbitrarily small \( \varepsilon \), then the plaintiff would clearly offer \( s_C^A = s_C^B = (L + t)/(1 + L^2) - \varepsilon \). However, it is also possible that the defendants play \( r/r \) whenever this is a NE in Stage 2. In this case, the unique SPE has the plaintiff reducing her offers to the point where \( a/a \) is the only NE in Stage 2, and it can be shown that this point is reached if \( s_C^A = s_C^B = L - 0.5L^2 + t \) as under rule \( P \). Note that even in this case, full settlement occurs in all SPEs, so that only the distribution of wealth, but not the settlement frequency is altered by the problem of multiple equilibria.

It follows that we can summarize as follows: independently of the problem that multiple

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15 \( 2 \cdot (L + t)/(1 + L^2) > 2L - L^2 \) is equivalent to \( 2t > L^2(2L - 1 - L^2) \). Hence, a sufficient condition is \( 2L - 1 - L^2 < 0 \), i.e., \( 2L - L^2 < 1 \). This holds for \( L < 1 \).
equilibria arise under rule $C$, we can confirm the results of Kornhauser & Revesz, and Klerman in our model: The plaintiff always settles with both defendants under rules $P$ and $C$, but not always under rule $T$.

4. Incomplete information

4.1. The equilibrium concept under incomplete information

Note that all setoff rules are again identical if the plaintiff directly litigates. The plaintiff gets total damages $D = 1$ whenever at least one defendant is found liable. Since a bad type is certainly found liable, the plaintiff’s expected rewards are

$$R_L = 1 - p^2(1 - L)^2,$$

(16)

where $R_L$ denotes expected rewards under “litigation.” $R_L$ is strictly increasing in $L$, and strictly decreasing in $p$.

Under incomplete information, we have to apply the concept of Perfect Bayesian Equilibrium (PBE). This means that the plaintiff must form beliefs on how the different types of defendants act in equilibrium and that these beliefs are anticipated by all players. Then one has to check whether these beliefs are consistent with equilibrium behavior. Of course, each defendant must also form beliefs over the behavior of her codefendant.

Under full information, it could never be favorable for the plaintiff to offer settlements that are only accepted by one defendant. Now the situation is different, because a bad type (who is always liable by assumption) accepts higher settlements than a good type (who is only liable with $L < 1$). For each pair of settlement offers, there are generally four possible BEs in Stage 2: both defendants accept, neither accepts, or solely $A$ or $B$ settles. It can no longer be excluded that the plaintiff prefers partial settlement.

For reasons similar to the situation under rule $C$ with full information, we get always exactly one or infinitely many PBEs. When comparing the different strategies and rules, we assume that a pair of settlement offers is accepted whenever it constitutes a BE in Stage 2. This seems to be reasonable as it is the best PBE for the plaintiff who has full the bargaining power. Moreover, it would be impossible to compare rule $C$ to the other rules as there are infinitely many PBEs under rule $C$ (it turns out that the PBEs are unique under rules $P$ and $T$). Nevertheless, we carefully analyze whether a PBE is unique or not.

An important point to note is that it is impossible that a good type settles and a bad type refuses, because a bad type’s expected payments from litigation are higher. Hence, the only possible partial settlement BE in Stage 2 is that only bad types settle. For each possible combination of types, and for each possible BE in Stage 2, the plaintiff then can count the maximum settlement offers accepted by specific types of defendants, thus maximizing her reward.

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16 We are grateful to an anonymous referee for insisting on this point.

17 As in the full-information case, partial settlement with identical defendants can never be favorable to the plaintiff.
for each BE. This given, she chooses the settlement offers as to maximize her expected payoff in the PBE. Hence, our assumption that types accept offers whenever this is a BE in Stage 2 guarantees that we have to consider exactly three possible PBE:

1. The plaintiff offers settlement amounts so low that full settlement is achieved even if both types are good (we call this low offers). In this case, the plaintiff knows her payoff when offering settlements, because her payoff is independent of the defendants’ types.
2. The plaintiff offers settlement amounts so high that full settlement is achieved only if both types are bad (we refer to this strategy as high offers). With high offers, the plaintiff can only calculate her expected payoff, because bad types accept, while good types are litigated.
3. The plaintiff offers a pair of settlements that leads to partial settlement whenever only one defendant is a good type. In other words, she offers one amount that is certainly accepted and another amount that is only accepted if the respective defendant is a bad type (we call this strategy mixed offers). Again, the plaintiff knows only her expected payoff, because the defendant who gets the high offer might either settle or not.18

First, we separately calculate the plaintiff’s maximum payoff for each strategy. Second, the plaintiff chooses the strategy that maximizes her expected payoff. Third, this allows us to compare the different rules.

4.2. Proportionate rule (P)

4.2.1. Low offers (Pl)

As explained in Section 4.1, there are always three possible strategies. The first is to offer amounts accepted even by good types.19 The maximum amount accepted by a good type is20

\[
s_{pl}^A = L[0.5(1 - p) + 0.5pL + p(1 - L)] + t = L[0.5 + 0.5p(1 - L)] + t. \tag{17}
\]

First, A is found liable with probability L. Independently of whether B settles or not, A pays 0.5 if B would have been liable too, and total damages otherwise. With probability 1 − p, B is a bad type, so that A pays only half of the damages. The same occurs if B is a good type, but is nevertheless found liable (0.5pL). Finally, A has to compensate the victim’s total loss if she is found solely liable (p(1 − L)).

Since the model is symmetric, the plaintiff’s total recovery R is

\[
R_{pl} = s_{pl}^A + s_{pl}^B = L[1 + p(1 - L)] + 2t. \tag{18}
\]

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18 In expected terms, it does not matter which offer is made to A and which to B, because their ex ante probabilities of being good types are identical.

19 “Pl” denotes “proportionate rule” and “low offers.”

20 We have to calculate the maximum amount, because if the plaintiff wants to make sure that both types accept with certainty, she gets her maximum payment when good types are indifferent between settling and rejecting.
Obviously, \( R \) is strictly increasing in \( L, p \) and \( t \). It might be somewhat surprising that \( R \) is increasing in \( p \), because a high probability for good types should reduce the plaintiff’s expected payoff. However, since the plaintiff offers low amounts anyhow, it does not matter for her whether she meets good or bad types (both types accept the settlement offer). But the offer accepted by \( A \) (for instance) is the higher the higher the probability that \( B \) is a good type, since this increases the probability that \( A \) is found solely liable if she refuses the offer.

We do not have to check formally whether \( r/r \) is also a BE in Stage 2 as, under rule \( P \), each defendant’s behavior is independent of whether the other defendant accepts or not. Hence, each BE in Stage 2 is unique under rule \( P \).

### 4.2.2. High offers (Ph)

Next, we consider offers that are accepted by bad types, but not by good types. Again, the amount accepted by a bad type \( A \) can be calculated independently of \( B \)’s behavior, and is given by:

\[
A_{Ph} = s_{Ph} = 0.5(1 - p) + 0.5pL + p(1 - L) + t.
\]  

(19)

If \( B \) would have been liable, \( A \) pays 0.5. This happens with probability \((1 - p) + pL\). Otherwise she pays total damages, which has probability \(p(1 - L)\). With probability \((1 - p)^2\), the plaintiff meets two bad types and gets \(2s_{Ph}^A\). If she meets two good types (this happens with probability \(p^2\)), her expected remedies from litigation are \(1 - (1 - L)^2 = 2L - L^2\).

With probability \(2p(1 - p)\) she meets one good type and one bad type, and receives \(s_{Ph}\) from the bad type, and \(0.5L\) from litigating the good type.

Adding up and simplifying yields

\[
R_{Ph} = 1 - p^2 + 2p^2L - p^2L^2 + 2t - 2pt
\]

\[
= R_L + 2t(1 - p).
\]  

(20)

Obviously, \( R_{Ph} \) is increasing in \( t \), because each defendant’s incentive to avoid litigation is the higher \( t \). Also, \( R_{Ph} \) is increasing in \( L \) and decreasing in \( p \). The partial derivatives with respect to \( L \) and \( p \) are unambiguous, although there are opposite effects: the higher \( L \), the higher the plaintiff’s expected reward from litigation if she meets a good type. On the other hand, the amount accepted by a bad type decreases if the liability risk of good types increases (i.e., if \( L \) increases), so that the risk of being held liable alone is decreasing. However, the first effect dominates so that \( R_{Ph} \) is increasing in \( L \). Analogously, the higher \( p \), the higher the probability that a defendant must be litigated, thus reducing \( R_{Ph} \). But the higher \( p \), the higher the settlement that is accepted by a bad type, because it is more likely that the other defendant would not have been liable. Again, the first effect dominates so that \( R_{Ph} \) is decreasing in \( p \).

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21 “Ph” denotes “proportionate rule” and “high offers.”

22 This can be calculated by subtracting \( t \) from \( s_{Ph}^A \), because under the proportionate rule, it does not matter for a good defendant if the other injurer has settled or not.
4.2.3. Mixed offers (Pm)

Now suppose that the plaintiff offers a low settlement to A and a high settlement to B,\(^{23}\) so that A always settles, whereas B rejects if she is a good type.\(^{24}\) Since each defendant’s expected payment is still independent from the other defendant’s strategy, the maximum offer that even good types accept is again

\[
s^A_{Pm} = s^i_{Pl} = L[0.5 + 0.5p(1 - L)] + t. \tag{21}\]

Similarly, the high offer only accepted by bad types is still

\[
s^B_{Pm} = s^i_{Ph} = 0.5(1 - p) + 0.5pL + p(1 - L) + t. \tag{22}\]

Since B accepts only with probability \(1 - p\), the plaintiff must litigate her with probability \(p\), and her expected rewards form litigation are

\[
L[0.5(1 - p) + 0.5pL + p(1 - L)], \tag{23}\]

since B’s payment depends on whether A would have been liable or not.

\[
R_{Pm} = 0.5 + 0.5L + 0.5pL - 0.5pL^2 + p^2L - 0.5p^2L^2 - 0.5p^2 + 2t - pt
= 0.5R_{Pl} + 0.5R_{Ph}, \tag{24}\]

As before, \(R_{Pm}\) is increasing in \(t\) and \(L\). Since it is a linear combination of \(R_{Pl}\) and \(R_{Ph}\), the derivative with respect to \(p\) is ambiguous because \(R_{Pl}\) is increasing in \(p\) and \(R_{Ph}\) is decreasing in \(p\). Which effect dominates depends on the parameter values.

4.2.4. The plaintiff’s optimal strategy

Given the expected rewards of the four strategies, it follows

**Proposition 1:** Depending on the parameters of the model, the plaintiff chooses either **Pl** or **Ph**.

**Proof:** see Appendix.

Proposition 1 implies that the plaintiff neither prefers to directly litigate both defendants (L) nor to suggest mixed offers (Pm). First, direct litigation is never chosen because it is dominated by Ph. This is so because the plaintiff can only gain when offering high offers instead of directly litigating: if both defendants refuse to settle, then her reward is identical to litigation, and otherwise it is higher because of the litigation costs. Moreover, Pm is neither dominated by Pl nor by Ph alone but can never be favorable compared to Pl and to Ph. Pm is simply a linear combination of Pl and Ph since, under the proportionate rule, it is of no importance for one defendant if the other settles or not. For low values of \(t\), Pm is dominated by Ph, for high values of \(t\) by Pl.

\(^{23}\) “Pm” denotes “proportionate rule” and “mixed offers.”

\(^{24}\) Of course, it does not matter whether the low offer is made to A or B, because the probability distribution over types is identical.
That $Pl$ is chosen is more likely the higher the probability for good types ($p$), the higher the litigation costs ($t$), and the higher the probability that a good type is found liable ($L$). All these effects are intuitive (but nevertheless important for the differences to the other rules considered below): the higher $p$, the more likely that the plaintiff must litigate if she chooses $Ph$. The higher $t$, the more attractive it is to avoid litigation. And the higher $L$, the higher the settlement offer accepted by a good type, so that the loss from ignoring the possibility of bad types decreases.

4.3. The unconditional pro tanto rule ($T$)

Under $T$, a liable nonsettling defendant pays the difference between total damages and the other defendant’s settlement, independently of whether the settling defendant would have been liable or not. The situation is more complicated than under rule $P$ because we have to check whether multiple BE might arise in Stage 2.

4.3.1. Low offers ($Tl$)

With low offers, the amounts are chosen as to guarantee that both types accept in a BE in Stage 2, so that each defendant $i$ knows that she pays $1 - s^i$ if she refuses and is found liable in court. Hence, the maximum amount accepted by a good type is

$$s^A_{Tl} = L(1 - s^B_{Tl}) + t$$  \hspace{1cm} (25)

since she pays $1 - s^B_{Tl}$ with probability $L$ if $B$ settles. The same holds for $B$, and solving for $s^A$ and $s^B$ yields

$$s^A_{Tl} = s^B_{Tl} = \frac{L + t}{1 + L}. \hspace{1cm} (26)$$

The plaintiff’s total rewards are

$$R_{Tl} = s^A_{Tl} + s^B_{Tl} = 2 \cdot \frac{L + t}{1 + L}. \hspace{1cm} (27)$$

It is straightforward that $R_{Tl}$ is increasing in $t$ and is independent of $p$, because all types accept the offer with certainty. Moreover, $R_{Tl}$ increases in $L$, because the maximum settlement amount accepted by a good type is increasing in her risk of being found liable ($L$).

Now we analyze whether there is another BE in Stage 2 in which both good types reject the offer $s^i = (L + t)/(1 + L)$. If so, we certainly have infinitely many PBEs because the plaintiff reduces her pair of offers to the point where she believes that good types accept. Hence, many different PBEs are then supported by different beliefs. To show whether the BE calculated above is unique or not, we have to analyze whether there are beliefs that constitute other Bayesian equilibria for Stage 2. The relevant belief is “both bad types accept, but both good types reject.”

However, it can be shown that these beliefs do not form a BE as it would be better for good types to accept: Suppose that $A$ is a good type and assumes that only bad types accept. Her expected payment from refusing is then

$$(1 - p)L(1 - s^B_{Tl}) + pL(0.5L + (1 - L)) + t. \hspace{1cm} (28)$$
Since the settlement offer considered is $s_{T_1} = (L + t)/(1 + L)$, it is better for her to accept if
\[
\frac{L + t}{1 + L} < (1 - p)\left(1 - \left(\frac{L + t}{1 + L}\right)\right) + pL(0.5L + (1 - L)) + t. \tag{29}
\]
This can be simplified to yield the condition
\[
t > -0.5L(1 - L). \tag{30}
\]
and this is always fulfilled since the right-hand side is negative. It follows that the settlement offers $s_{T_1}^A = s_{T_1}^B = (L + t)/(1 + L)$ indeed support the unique PBE if the plaintiff prefers to suggest low offers.

### 4.3.2. High offers ($T_h$)

From now on, we simply state whether a BE in Stage 2 is unique or not, and we no longer prove it formally for two reasons. First, the argumentation is straightforward but tedious, and we do not want to bother our readers. Second, when analyzing which pair of offers the plaintiff chooses, and when comparing the three settlement rules, we compare the PBE that lead to the highest payoffs for the plaintiff anyway. Hence, uniqueness is not that important in our model. However, all proofs on uniqueness are available on request.

The maximum settlement offer accepted by a bad type is
\[
s_{T_h}^A = (1 - p)(1 - s_{T_h}^B) + p(1 - 0.5L) + t. \tag{31}
\]

The first part is $A$’s expected payment $(1 - s_{T_h}^B)$ if $B$ is a bad type and settles, which happens with probability $1 - p$.\(^{25}\) If $B$ is a good type and rejects, $A$ pays total damages with probability $1 - L$, and half of the damages with probability $L$. This adds up to $1 - 0.5L$ and occurs with probability $p$. It follows
\[
s_{T_h}^A = s_{T_h}^B = \frac{1 + t - 0.5pL}{2 - p}. \tag{32}
\]

When calculating $P$’s expected payoff, we have to take into account that (a) with probability $(1 - p)^2$ both types are bad and settle, (b) with $p^2$ both types are good and must be litigated and (c) with $2p(1 - p)$ one defendant accepts, while the other must be litigated. Adding up leads to
\[
R_{T_h} = \frac{2[1 - p + t(1 - p)] + pL[1 + p(1 - L) + 2t(p - 1)]}{2 - p}. \tag{33}
\]

As usual, $R_{T_h}$ increases in $t$. However, the derivatives with respect to $L$ and $p$ are ambiguous. It can be shown that the BE is unique.

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\(^{25}\) Note that $A$ must be a bad type to settle, by the definition of high offers.
4.3.3. Mixed offers (Tm)

Now suppose that the plaintiff offers a low settlement to A and a high settlement to B. By the definition of low and high offers, this means that A always settles, whereas B rejects if she is a good type. Since B knows that A certainly settles by anticipating the BPE, her expected costs from litigation if she is a bad type are independent of A’s type given by

\[ s_{Tm}^B = 1 - s_{Tm}^A + t. \] (34)

The maximum amount accepted by A depends on her expectations on B’s type:

\[ s_{Tm}^A = (1 - p)[L(1 - s_{Tm}^B)] + p(L - 0.5L^2) + t. \] (35)

The first part is A’s expected payment if B is a bad type and settles, and the second part is A’s expected payment if both defendants are litigated. Solving for \( s_{Tm}^A \) and \( s_{Tm}^B \) yields

\[ s_{Tm}^A = 1 + t - \frac{1 - L + 0.5pL^2}{1 - L + pL}, \] (36)

and

\[ s_{Tm}^B = \frac{1 - L + 0.5pL^2}{1 - L + pL}. \] (37)

Since the probability that both defendants agree is \( 1 - p \) (one defendant certainly settles), the plaintiff has to litigate against one defendant with probability \( p \). Taking this into account, \( P \)'s total expected recovery from settlement and litigation can be written as

\[ R_{Tm} = \frac{1 - L - p + pL[3 - L + 0.5pL(L - 1) + Lt(1 - p)] + t(1 - L)}{1 - L + pL}. \] (38)

Whereas the derivatives with respect to \( t \) and \( L \) are the same as under high offers, we get \( \partial R_{Tm}/\partial p < 0 \). There are two effects if \( p \) is increasing. First, the probability that the high offer is accepted is decreasing in \( p \). Second, a bad type accepts higher settlements because she is afraid that the other type is good and that she pays total damages when litigated. Conversely to high offers, it can now be shown that the first effect always dominates. Again, it can be shown that the BE is unique.

4.3.4. The plaintiff’s optimal strategy

The main result for rule T is that all strategies may be supported in a PBE. As the PBEs for each offer are unique, so is the PBE for the total game. For \( p \) and \( L \) given, either strategy can become favorable if \( t \) changes. Assume, for example \( p = L = 0.3 \). First, as under full information, the plaintiff always prefers to directly litigate both defendants if \( t \approx 0.09 \). For \( 0.09 \leq t \leq 0.48 \), the best strategy is \( Th \). There are two reasons for this. First, the higher

\[ \text{To avoid misunderstandings: we do not know yet whether mixed offers support a PBE. But if so, the strategies are anticipated, and since A certainly settles, B does not have to build expectations on A’s type.} \]
the litigation costs, the higher the amount wasted through litigation. Hence, high offers are better compared to litigation if $t$ increases. Second, litigation costs must not be too high, because the probability of litigation under high offers is clearly higher than under mixed or low offers. The same argument explains that $Th$ becomes inferior to $Tm$, and $Tm$ inferior to $Th$ if litigation costs are further increasing. In our example, $Tm$ is favorable for $0.48 \leq t \leq 0.55$, and $Tl$ for $t \geq 0.55$. Hence, no strategy can be excluded.

**Proposition 2:** Depending on the parameters of the model, all strategies can be favorable for the plaintiff.

**Proof:** By construction; see example above.

### 4.4. The conditional pro tanto rule (C)

#### 4.4.1. Low offers (Cl)

Under the conditional rule, each defendant’s payment in litigation depends on whether a settling party would have been liable in court. Hence, the maximum amount accepted by a good type is

$$s^A_{cl} = L[(1 - p + pL)(1 - s^B_{cl}) - p(1 - L)] + t.$$  

(39)

If $A$ is found liable (this happens with probability $L$), and $B$ settles, $A$ pays only $1 - s^B_{cl}$ if $B$ would have been liable. This happens with probability $1 - p + pL$. Otherwise, she pays total damages. Since $B$’s situation is identical, it follows

$$s^A_{cl} = s^B_{cl} = \frac{L + t}{1 + L - pL + pL^2}.$$  

(40)

Since both types certainly settle, the plaintiff’s payoff is

$$R_{cl} = s^A_{cl} + s^B_{cl} = \frac{L + t}{1 + L - pL + pL^2}.$$  

(41)

Conversely to rule $T$, $R_{cl}$ is not independent of $p$, though all types still accept the offer with certainty, the probability that a setoff is granted decreases in $p$. It follows that $R_{cl}$ is increasing in $p$. Again, $\partial R_{cl}/\partial t > 0$, but, conversely to low offers under rule $T$, $\partial R_{cl}/\partial L$ is now ambiguous. This follows from the fact that high $L$’s increase the probability that a setoff is granted, hence, ceteris paribus, reducing the maximum settlement accepted by good types. If this effect is strong enough, then $R_{cl}$ is decreasing in $L$.

Conversely to rule $T$, it can be shown that the settlement offers $s^A_{cl} = s^B_{cl} = (L + t)/(1 + L - pL + pL^2)$ also allow for a second Bayesian equilibrium in Stage 2 where only bad types accept, whereas all good types reject (see Appendix). When comparing the different strategies, we restrict our attention to the PBE analyzed above.

#### 4.4.2. High offers (Ch)

Conditional and unconditional setoff rules are identical if the plaintiff suggests high offers: only bad types accept high offers, so that each settling party would certainly have been liable. Thus, $R_{ch} = R_{Th}$. 
4.4.3. Mixed offers (Cm)

Since only a bad type B accepts, the situation for A is the same as under rule T:

\[ s_{Cm}^A = (1 - p)[L(1 - s_{Cm}^B)] + p(L - 0.5L^2) + t. \] (42)

However, the situation changes for a bad type B: though she still knows that A settles, she does not know whether she gets a setoff. It follows

\[ s_{Cm}^B = (1 - p)(1 - s_{Cm}^A) + pL(1 - s_{Cm}^A) + p(1 - L) + t. \] (43)

Solving for \( s^A \) and \( s^B \) yields

\[ s_{Cm}^A = \frac{pL(1 - 0.5L) + t(1 - L + pL)}{1 - L(1 - p)(1 - p + pL)} \] (44)

and

\[ s_{Cm}^B = 1 + t + \frac{(1 - p + pL)[pL(0.5L - 1) - t(1 - L + pL)]}{1 - L(1 - p)(1 - p + pL)}. \] (45)

Taking the different possibilities into account, the plaintiff’s expected payoff adds up to \(^{27}\)

\[ R_{Cm} = \frac{1 - L - p + t(1 - L + p + p^2) + pL[t(L - 1 + p - pL)]}{(1 - L + 2pL - p^2L + p^2L^2)} \\
+ \frac{pL[4 - 2L + p(L - 1 - 0.5pL + 0.5pL^2)]}{(1 - L + 2pL - p^2L - p^2L + p^2L^2)}. \] (46)

Though \( R_{Cm} \) is ugly, it is relatively easy to show that the partial derivatives of \( R_{Cm} \) with respect to \( t, L \) and \( p \) are the same as with mixed offers under rule T for similar reasons.

As under low offers, the equilibrium derived above might not be unique, depending on the parameter values. However, as already explained, we will again restrict attention to the PBE which yields the highest payoff for the plaintiff.

4.4.4. The plaintiff’s optimal strategy

As under the unconditional pro tanto rule (rule T), all four strategies can be favorable to the plaintiff. This means that, in contrast to the full-information case, litigation cannot be excluded under rule C. However, this does not mean that rules T and C are identical with respect to the plaintiff’s strategies, because the ranges where either strategy is favorable are different.\(^{28}\) In our example with \( p = L = 0.3 \), the plaintiff chooses \( L \) if \( t \leq 0.09 \), \( Ch \) for \( 0.09 \leq t \leq 0.22 \), \( Cm \) for \( 0.22 \leq t \leq 0.58 \) and \( Cl \) for \( t \geq 0.58 \). This becomes important when comparing the different rules in Section 4.5.

\[ \text{PROPOSITION 3: Depending on the parameters of the model, all strategies can be favorable for the plaintiff.} \]

\(^{27}\) Sorry, but we were not able to simplify in a more intuitive way.

\(^{28}\) Note that the optimal strategy depends not only on \( t \), but also on \( L \) and \( p \).
4.5. Comparison of the different setoff rules

Recall that the preceding literature showed that (a) rules $P$ and $C$ are always favorable to the plaintiff compared to Rule $T$; (b) litigation is certainly avoided under rules $P$ and $C$, but not under $T$, and (c) whether rule $P$ or $C$ is favorable to the plaintiff depends on the parameters of the model. Though (a) and (c) are valid even under incomplete information, (b) has always been judged as being the most important result: it seems to be a strong argument against rule $T$ if it leads to a higher frequency of litigation, thus wasting socially valuable resources.

Our main result is that (b) cannot be upheld under incomplete information. First, litigation can also occur under rules $P$ and $C$, since low offers are not always the best strategies, and litigation is possible under high offers and under mixed offers. Admittedly, this is not surprising, since it is intuitive that settlements are less likely under asymmetric information. Hence, the straightforward analogue to result (b) would be that, though litigation is possible under either rule, $T$ is still dominated by $P$ and $C$ in the sense that both defendants always settle under $P$ and $C$ if they settle under $T$. But this is not the case: it cannot be excluded that rules $P$ and $C$ lead to litigation, whereas a settlement takes place under rule $T$.

This result can be proven simply by constructing examples where the frequency of litigation is lowest under the unconditional pro tanto rule (rule $T$). Consider again our example with $p = L = 0.3$, i.e., an example where neither the probability for good types ($p$) nor the probability that a good type is liable ($L$) is extreme. With $p = L = 0.3$, it can easily be shown that it is always (i.e., for $0 \leq t \leq 1$) best to offer high amounts under the proportionate rule ($Ph$). Now suppose $t = 0.57$. Under the conditional pro tanto rule, it is best to make mixed offers ($Cm$), where the best strategy under the conditional pro tanto rule are low offers ($Tl$). Hence, in this example, the settlement probability is lowest under the proportionate rule ($P$), followed by the conditional pro tanto rule ($C$) and the unconditional pro tanto rule ($T$). More precisely, the probability of full settlement is one under rule $T$, $1 - p = 0.7$ under rule $C$, and $(1 - p)^2 = 0.49$ under rule $P$.

Of course, our result does not mean that settlements are more likely under rule $T$, as is the case in our example. Even for our example with $L = p = 0.3$, the frequency of litigation could be higher under rule $T$ than under rule $C$ if $t \leq 0.55$. For higher values of $L$ and $p$, the frequency of litigation is highest under rule $T$ for a broad range of litigation costs $t$. Though the effects of $L$, $p$ and $t$ cannot, unfortunately, be isolated within a comparative statics analysis, the intuition behind our results can be sharpened. First, compare rules $T$ and $C$. The crucial point is that the plaintiff’s expected rewards under litigation and high offers are identical, whereas the expected rewards under mixed offers are strictly higher under rule $C$. This means that, for $p$ and $L$ given, the plaintiff switches earlier from high offers to mixed offers under rule $C$. Standing alone, this confirms the result of the full-information case that litigation is more likely under rule $T$, because settlements are more likely under mixed offers. But on the other hand, a higher $t$ might be required to switch from mixed offers to low offers, so that the overall result is ambiguous. The

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29 Under mixed offers, one defendant settles certainly, but the other only if she is a bad type.
main reason is that mixed offers are more favorable under rule \( C \), and either low offers or high offers might be chosen under rule \( T \) when mixed offers are chosen under rule \( C \).

The main point for understanding that the ranking between proportionate and pro tanto is ambiguous with respect to the frequency of litigation is that neither “litigation” nor “mixed offers” are chosen under the proportionate rule \( P \). Clearly, \( P \) is favorable if \( t \) is so low that the plaintiff prefers to directly litigate under rules \( T \) and \( C \). On the other hand, there are many constellations where high offers are chosen under rule \( P \), but low offers or mixed offers under rules \( T \) and \( C \). Intuitively if a high settlement offer is accepted under the pro tanto rule, a high setoff must also be granted, while this is not the case under rule \( P \). This makes high offers less favorable under rule \( T \) as compared to rule \( P \).

Summing up, it is not possible to order the three rules with respect to their frequency of litigation, so that the advantage of conditional rules (rules \( P \) and \( C \)) emphasized in the literature disappears if the implausible assumption of full information is dropped. The main result of the paper is summarized in the following proposition.

**Proposition 4**: Conversely to the full information case, rule \( T \) can lead to the lowest frequency of litigation.

**Proof**: see example above.

## 5. Conclusion

We integrated incomplete information about the defendants’ probabilities of being held liable into the basic model of multidefendant settlements developed by Kornhauser & Revesz, and Klerman. We demonstrated that the main results of these studies, that the frequency of litigation is highest under the unconditional pro tanto setoff rule, does not hold under incomplete information. Though the plaintiff never prefers to directly litigate both defendants under the (conditional) proportionate rule, she might offer higher settlement amounts that lead to a higher frequency of litigation.

There are at least three extensions of our model that might be worth considering. First, the model can be extended relatively easily to potentially insolvent parties. Second, one might wish to allow for more than two types. Third, if one drops the assumption that the plaintiff has a credible threat to sue, then strategic effects arise, and one has to search for PBEs in mixed strategies. Though interesting, this analysis turned out to be difficult, and we have not been able to derive clear results yet. Moreover, it would be interesting to experimentally test the model to get further information on the different settlement frequencies of the rules.

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Appendix

Proof of proposition 1

To see that $R_L$ is dominated by $R_{Ph}$ in the relevant range, subtract $R_L$ from $R_{Ph}$, which yields $2t(1 - p)$. Clearly, this term is positive for $t > 0$.

Similarly, note that $R_{Pm} = 0.5R_{Pl} + 0.5R_{Ph}$ can never be better than $R_{Pl}$ and $R_{Ph}$, since it is a linear combination of $R_{Pl}$ and $R_{Ph}$. Thus, it is (weakly) dominated by $R_{Ph}$ if $R_{Ph} \geq R_{Pl}$ and (weakly) dominated by $R_{Pl}$ if $R_{Pl} \geq R_{Ph}$.

Now we have to show that either $R_{Pl}$ and $R_{Ph}$ are optimal. First, note that $R_{Pl}$ and $R_{Ph}$ are linear in $t$. Second, for $t = 0$, we have $R_{Pl} - R_{Ph} = 1 - L - pL + pL^2 - p^2 + 2p^2L - p^2L^2 > 0$. Third, for the derivatives with respect to $t$, we have $\partial R_{Pl}/\partial t < \partial R_{Ph}/\partial t < \partial R_{Ph}/\partial t$. Finally, note that $R_{Pl}$ and $R_{Ph}$ (and $R_{Pm}$) have a common intersect at $t^* = 0.5 [L^2 - L + 1/p - L/p - p - 2pL - pL^2]$. It follows that we have $R_{Ph} > R_{Pm} > R_{Pl}$ for $t < t^*$ and $R_{Ph} < R_{Pm} < R_{Pl}$ for $t > t^*$.

Multiplicity of equilibria in stage 2 under rule C

We confine our attention to low offers. The respective proofs for high and mixed offers are similar and are available on request. To see that there might be multiple BEs in Stage 2 under rule C, consider the beliefs “both good types reject, both bad types accept.” Again, the good type is the critical one. Given these beliefs, when rejecting the low offer $s^A_{Cl}$ as a good type, A’s expected payments are

$$ (1 - p) L (1 - s^B_{Cl}) + pL (0.5L + (1 - L)) + t. $$

This has to be compared with the settlement offer $s^A_{Cl}$. Taking into account that $s^A_{Cl} = s^B_{Cl} = (L + t)/(1 + L - pL + pL^2)$ and simplifying yields the following condition for uniqueness:

$$ t \geq 0.5 [1 - L (1 + p (1 - L))]. $$

Clearly, this condition does not hold for all parameter values. Thus, given these beliefs, it might be the best response for a good type to reject, and thus we can have reject/reject as a BE in Stage 2.

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30 Note that, given these beliefs, only bad types accept, so that a setoff is always granted.