Dielectric constant determination using ground-penetrating radar reflection coefficients

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Abstract

A method to determine ground-penetrating radar (GPR) velocities, which utilize Brewster angles, is presented. The method determines the relative dielectric constant ratio at interface boundaries where the radar wave is traveling from a low-velocity to a high-velocity medium. Using Brewster angle analysis is currently the only means to determine the velocity of the medium below the deepest detectable reflector. Data are presented for water-saturated clean sand with a known velocity of 0.52 m/ns, which overlays a sandy silt with a known velocity of 0.13 m/ns. Brewster angle analysis of a common midpoint CMP survey gives a relative dielectric constant ratio of 33/4.77. The Brewster angle relative dielectric constant ratio is in good agreement with the relative dielectric constant ratio calculated from the known velocities. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Ground-penetrating radar (GPR) has gained popularity in recent years owing to its speed and ease of use. Two of its primary uses are anomaly detection and electromagnetic (EM) velocity determination of the shallow subsurface. Velocity analysis has been limited to common midpoint (CMP) surveys, which give the velocity structure above a reflector at a single location. This study presents and demonstrates a method that uses Brewster angles for determining the velocity directly below the deepest detectable reflector.

Knowing the EM velocity structure of the shallow subsurface is important in identifying electrical properties of different reflectors. The electrical properties are related to the composition of the reflectors. Sometimes, a situation is encountered when a low-velocity layer is located above a high-velocity layer, where no reflections can be obtained below the high-velocity layer. Consequently, the velocity of the high-velocity layer cannot be obtained using traditional techniques. The remainder of this paper will describe the theory for determining
the velocity of the high-velocity layer and demonstrate its effectiveness with an example.

2. Background

GPR is most frequently used to perform reflection profiles, which are commonly referred to as zero offset profiles. Such profiles are acquired by keeping the two GPR antennas an equal distance apart while taking measurements at equal spacing along a traverse (Davis and Annan, 1989). When viewing a zero offset profile, one is often looking at the topography of a layer or for anomalies in the reflection signal, such as hyperbolas associated with small buried objects. Due to the interpretation non-unique-ness of radar images, errors may occur in interpreting the source of the buried layer or anomaly. Is the hyperbola caused by a buried barrel or by a boulder? If one is looking at a horizontal reflector, how does one determine whether the anomaly is a water-table boundary or clay-layer boundary? The difficulty in interpreting GPR data is due to many factors such as attenuation, dispersion, scattering and radiation patterns (Annan, 1996). Using travel time analysis in conjunction with a zero offset profile can reduce some of the problems associated with these factors.

Velocity travel-time analysis used in conjunction with the CMP data acquisition method is the traditional technique for determining the composition of a reflector (Annan and Cosway, 1992). However, there are limitations associated with this method. To use this technique, one must look at radar wave velocities using travel times for the reflected waves. Consequently, the velocity of the medium below the lowest reflector cannot be determined.

3. Theory

The theoretical basis for GPR is found in Maxwell’s equations. However, it is not the intent of this paper to provide an in-depth mathematical derivation of the EM wave equation from Maxwell’s equations. For more detailed information on the derivation, Keller and Zhdanov (1994) and Wait (1982) contain thorough discussions of the subject. The success of GPR is based on EM waves operating in the frequency range where displacement currents dominate and losses associated with conduction currents are minimal (Annan, 1996). For the purposes of this paper, the assumption is made that we are dealing only with displacement currents and that the medium is lossless. The justification for this assumption will be discussed later.

The wave equation, in the propagation regime for electric displacement currents, is given in Eq. (1):

$$\nabla^2 \mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2},$$  

(1)

where $\mathbf{E}$ is the electric field, $\mu_0$ is the magnetic permeability and $\varepsilon$ is the permittivity. The permittivity can be defined as $\varepsilon = \varepsilon_0 \varepsilon_r$, where $\varepsilon_0$ is the permittivity of free space and $\varepsilon_r$ is the relative dielectric constant. Using phasor notation, Eq. (1) can be represented as shown in Eq. (2), where $\omega$ is the angular frequency:

$$\nabla^2 \mathbf{E} = -\omega^2 \mu_0 \varepsilon \mathbf{E}.$$  

(2)

The velocity for an EM wave in a dielectric is given by:

$$v = \frac{1}{\sqrt{\frac{\mu_0 \varepsilon}{2}} \left(1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} + 1\right)^{1/2}},$$  

(3)

where $\sigma$ represents conductivity. At high frequencies and/or very low conductivity, Eq. (3) reduces to:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon}}.$$  

(4)

It is obvious for lower radar frequencies that the dielectric properties and conductivity play a dominant role in determining the velocity of a
medium. For insulating materials such as dry rocks, dielectric properties alone determine the velocity of the EM wave. The effect that dielectric properties can have is seen in Figs. 1 and 2. Fig. 1 plots the velocity of an EM wave as a function of conductivity and frequency with a relative dielectric constant of 4. It can be seen in Fig. 1 that for frequencies greater than 100 MHz, Eq. (4) is a good approximation of the velocity. For frequencies below 100 MHz, the use of Eq. (4) will depend on the conductivity of the medium. Fig. 2 plots frequency vs. relative dielectric constant with a constant resistivity of 50 Ω m. It can be determined from Fig. 2 and Eq. (4) that for frequencies above 100 MHz, velocity is essentially independent of frequency and dependent only on the dielectric constant and the magnetic permeability.

Earth materials rarely have a magnetic permeability appreciably different from unity except for a few magnetic minerals (see Table 1). Magnetic permeability generally has a noticeable effect only when large quantities of Fe₂O₃ are present (Telford et al., 1990). Therefore, changes in velocity must be due to changes in dielectric constant or changes in resistivity of the medium. Consequently, for many earth materials, at high frequencies or high resistivity, the velocity of an EM wave is determined only by the relative dielectric constant of the medium. Table 2 gives a list of relative dielectric constants and conductivities for a variety of earth materials encountered when using GPR.

When EM waves are obliquely incident on an interface between two media, it is necessary to consider two different cases. The first case exists when the electric field vector is perpendicular to the plane of incidence. This is commonly referred to as perpendicular polarization. The plane of incidence is the plane containing the incident ray and is normal to the surface. The second case exists when the electric field vector is parallel to the plane of incidence. This case is commonly referred to as parallel polarization. For schematic representations of these two cases, refer to Fig. 3a and b.

<p>| Table 1 |
| List of relative magnetic permeabilities of various minerals (adapted from Telford et al., 1990) |</p>
<table>
<thead>
<tr>
<th>Mineral</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetite</td>
<td>5</td>
</tr>
<tr>
<td>Pyrrhotite</td>
<td>2.55</td>
</tr>
<tr>
<td>Hematite</td>
<td>1.05</td>
</tr>
<tr>
<td>Rutile</td>
<td>1.0000035</td>
</tr>
<tr>
<td>Calcite</td>
<td>0.999987</td>
</tr>
<tr>
<td>Quartz</td>
<td>0.999985</td>
</tr>
</tbody>
</table>
Table 2
List of relative dielectric constants and velocities for some typical earth materials (adapted from Annan and Cosway, 1992)

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative dielectric constant</th>
<th>Velocity (m/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
<td>0.30</td>
</tr>
<tr>
<td>Sea water</td>
<td>80</td>
<td>0.01</td>
</tr>
<tr>
<td>Dry sand</td>
<td>3–5</td>
<td>0.15</td>
</tr>
<tr>
<td>Saturated sand</td>
<td>20–30</td>
<td>0.06</td>
</tr>
<tr>
<td>Limestone</td>
<td>4–8</td>
<td>0.12</td>
</tr>
<tr>
<td>Silts</td>
<td>5–30</td>
<td>0.07</td>
</tr>
<tr>
<td>Granite</td>
<td>4–6</td>
<td>0.13</td>
</tr>
<tr>
<td>Ice</td>
<td>3–4</td>
<td>0.16</td>
</tr>
</tbody>
</table>

EM waves may become polarized when radiated from an antenna. The antenna design determines the type of polarization, which can be linear, circular, or elliptical. Most GPR antennas emit waves that are linearly polarized, which are further subdivided into perpendicular or parallel polarized. By changing the orientation of the antennas relative to each other and relative to the traverse, the polarization detected by the radar will change. Polarization type will affect how the wave is reflected from a planar surface or a buried object. The reflection differences show up in the reflection coefficients of the two cases. For a thorough discussion on the subject of polarization of radar waves, refer to Roberts and Daniels (1996).

The perpendicular polarized reflection coefficient equation is:

$$\frac{E_r}{E_i} = \sqrt{\varepsilon_1 \cos \theta - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}}$$  \hspace{1cm} (5)

and the parallel polarized reflection coefficient equation, Eq. (6), gives the ratio of reflected signal to incident signal at a horizontal planar boundary:

$$\frac{E_r}{E_i} = \left( \frac{\varepsilon_2}{\varepsilon_1} \cos \theta - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2 \theta} \right)$$

For details on the derivation of these equations, please refer to any textbook on EM theory.

Fig. 3. Schematic diagram of perpendicular and parallel polarized EM waves. (a) shows a perpendicular polarized EM wave where the E field points out of the page perpendicular to the plane of incidence and the H field is parallel to the plane of incidence. (b) shows a parallel polarized EM wave with the E field parallel to the plane of incidence and the H field perpendicular to the plane of incidence.
such as Electromagnetic Waves and Radiating Systems (Jordan and Balmain, 1968). Plots of the reflection coefficients vs. angle of incidence for both the parallel and perpendicular polarized waves are shown in Fig. 4. This figure shows the magnitude and phase portion of the reflection coefficients.

For parallel polarized waves, there exists an angle where no reflected wave exists. This angle is referred to as the Brewster angle and given by Eq. (7). The Brewster angle occurs when the numerator in Eq. (6) goes to zero:

$$\tan \theta_1 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}. \quad (7)$$

The Brewster angle is more easily understood when one looks at the magnitude and phase portions of Fig. 4. The Brewster angle occurs in Fig. 4b and d when a step change of 90° in phase occurs prior to reaching the critical angle where all energies are reflected from the interface. When a step change of 90° in phase occurs prior to all energies being reflected, the reflected energy goes to zero. When viewing Fig. 4c and d, other phase changes are evident and these occur after all energies have been reflected. It should be noted that these phase changes are not step changes in phase.

4. Application of theory

If the Brewster angle or phase changes can be detected in radar data, it should be possible to determine the relative dielectric constant of the materials on either side of the boundary. This is accomplished by utilizing the reflection coefficient properties of the boundary. The Brewster angle can be determined from the radar data by determining the event of interest and looking for a 90° phase change occurring in the CMP data of that event. Depending on the Brewster angle, a null in the amplitude may be detectable at the same location. The data are easiest to read if color contour plots are used as opposed to using radar wiggle plots. Once the
Fig. 5. Simplified cross-section of the drain field survey area.

Because the velocity (relative dielectric constant) can be determined for the medium above the reflector, the relative dielectric constant and velocity can be determined for the medium below the reflector using the Brewster angle and Eq. (7).

5. Data collection

A field study was conducted to test whether the Brewster angle could be detected and used to determine relative dielectric constants and velocities. Data were collected over a recently constructed drain field in Ashby, MA. Bedrock in the area consists of a light gray, medium-grained, weakly foliated, weakly metamorphosed granite that is of the Fitchburg Complex. The overburden of the drain field consists of clean sand overlying sandy silt, which contains various-sized cobbles which overlie the hardpan. Hardpan is a term used to refer to glacial till that has been compacted to the point where pneumatic hammers are required to excavate the material.

The drain field data consist of the layers that are shown in Fig. 5. These layers are: Layer 1, air; Layer 2, 1.8 m of clean sand; Layer 3, 0.2 m of water-saturated clean sand; Layer 4, 1.9 m of sandy silt; Layer 5, hardpan. Initial drain field stratification data were obtained from a deep-hole survey conducted prior to the construction of the drain field. Radar calculation of the depths of the strata is in agreement with the deep hole data except for the location of the water saturation zone. However, the water saturation zone moved due to seasonal fluctuations in the water table.

Fig. 6. Reflection profile over the test site with a velocity profile shown alongside the CMP.
The data were collected using a Pulse Ekko IV radar with 200 MHz antennas. A reflection profile of the survey line, using a perpendicular broadside antenna configuration, is shown in Fig. 6 with the center point of the CMP located 2 m on the reflection profile. The CMP is shown in Fig. 7 with the events of interest marked. Event (a) is the air wave, (b) is the ground wave, (c) is the reflection from the top of the water-saturated sand, (d) is the bottom of the water-saturated sand and the top of the sandy silt, and (e) is the top of the hardpan. The CMP was performed using the parallel endfire antenna configuration. The parallel endfire configuration is shown in Fig. 8 along with other antenna configurations.

Traditional $X^2-T^2$ velocity analysis was done on the CMP data, giving the air velocity as 0.3 m/ns. The average velocity of the sand, Layer 2, is approximately 0.114 m/ns. The average velocity of the water-saturated sand is approximately 0.052 m/ns and the velocity of the sandy silt is 0.13 m/ns. A velocity profile is shown in Fig. 6 along with the reflection profile.

Based on the discussions in Section 3, the Brewster angle will be found on the interface that is bounded by a low-velocity layer above the interface and a high-velocity layer below the interface. Therefore, from the previous velocity analysis, the Brewster angle should be found on the event made by the interface between the water-saturated sand and the sandy silt, event (d) shown in Fig. 7. Following event (d) in Fig. 9, which starts at approximately 42 ns, a phase change can be seen at approximately 4.5 m. Using travel time curves constructed from the traditional velocity analysis, the incident angle for each offset distance is calculated for event (d). A horizontal distance of 4.5 m gives an angle of 20.77° for the incident ray contacting the interface. A relative dielectric constant of 33 for the water-saturated clean sand was determined using the $X^2-T^2$ velocity of 0.052 m/ns. Using this relative dielectric constant, the angle determined above, and the Brewster angle equation, Eq. (7), a relative dielectric constant of 4.78 was calculated for the sandy silt. In turn, a relative dielectric constant of 4.78 gives a velocity of 0.1369 m/ns, which is comparable to a velocity of 0.13 m/ns that was determined using traditional $X^2-T^2$ velocity analysis. Comparisons of the velocities determined using $X^2-T^2$ analysis and the Brewster angle method are given in Table 3.

A plot of theoretical amplitudes for the relative dielectric constant ratio of $33/4.77$ is shown in Fig. 10. The theoretical parallel polarized
reflection coefficient, calculated from Eq. (6), was converted to amplitudes by multiplying the theoretical reflection coefficients by a constant. The same equation was used to generate the model for Fig. 4 where a parallel polarized wave goes from high permittivity to low permittivity. A simple exponential attenuation was then applied to the theoretical amplitudes. As can be seen in Fig. 10, there is good similarity between the data amplitudes from event (d) and the theoretical amplitudes. It can also be seen in Fig. 10 that two changes in phase occur for this relative dielectric constant ratio, one at 20.77° and the other at 23.75°. An incident angle of 23.75° on event (d) gives a surface distance of approximately 7.25 m. When the event is examined, another phase change is apparent at 7.25 m. On first examination, it seems unusual that a small change in angle of the incident ray, 3°,

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Brewster angle velocity (m/ns)</th>
<th>$X^2 - T^2$ velocity (m/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>air</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>clean sand</td>
<td>0.114</td>
<td>0.114</td>
</tr>
<tr>
<td>3</td>
<td>water-saturated sand</td>
<td>0.052</td>
<td>0.052</td>
</tr>
<tr>
<td>4</td>
<td>sandy silt</td>
<td>0.137</td>
<td>0.137</td>
</tr>
<tr>
<td>5</td>
<td>hardpan</td>
<td>0.153–0.17</td>
<td>0.153–0.17</td>
</tr>
</tbody>
</table>

Fig. 9. Color plot of the data shown in Fig. 7. The Brewster angle location is highlighted by the circled area.

Fig. 10. Real portion of theoretical amplitudes for a relative dielectric constant ratio of 33/4.8. The Brewster angle occurs at 20.77°. The amplitudes were obtained by multiplying the reflection coefficients by a constant and then multiplying by a simple exponential attenuation. The theory is compared to data obtained from event (d).
should occur over such a large surface offset, 2.5 m, when the depth of the reflector is only 2 m deep. However, due to the phenomenon of ray bending at interface boundaries, small changes in incident angle can occur with large surface offset distances.

For completeness, there are other anomalies in Fig. 9, which must be addressed. At position 2 m and 18 ns, 3.8 m and 30 ns, and 6 m and 45 ns, these anomalies resemble some time of phase change. It is apparent from the plot that these anomalies are caused by the critically refracted air wave. This conclusion can be confirmed by the slope of the events as well as by constructing travel time curves. The anomalies at positions 2 m and 30 ns and 3 m and 40 ns are caused by the first reflector merging into the direct wave (ground wave). Again, this can be confirmed by construction of travel time curves. It should also be noted that none of the above anomalies has the same characteristics as the anomaly identified as the Brewster angle.

Below event (d), it is not possible to determine velocities to use to confirm any value determined using the Brewster angle method. Positions 1 m and 2 m at 60 ns appear to be weak reflectors. This conclusion was arrived at by looking at the wave forms and amplitudes of individual events. From the velocity analysis and the deep hole data, event (e) is on the interface of the sandy silt and hardpan. At the positions of 2.4 m and 4.2 m on event (e), the anomaly appears to be similar to the Brewster angle on event (d). Using the same type of analysis as above, angles of 41° and 54° for the first and second phase change, respectively, are obtained. These angles do not give a perfect fit to a reflection coefficient curve but are within expected measurement errors. They give a range of relative dielectric constant ratios for the sandy silt/hardpan interface of 4.77/3.4 to 4.77/3, which gives a range of velocities of 0.153–0.17 m/ns for the hardpan. The first velocity is not unrealistic based on the velocity of the sandy silt, but this velocity cannot be substantiated at the present time.

6. Summary

Brewster angles can be used to determine relative dielectric constants and velocities. Preliminary results appear to support the theory. This method has the advantage of being useful in determining the velocity of the medium below the deepest detectable reflector. The use of Brewster angles is restricted to horizontal planar targets compared to the wavelength of the antenna. If the dipping angle of planar target is known, it may be possible to use this method on nonhorizontal targets. Also, this method is limited by the velocity contrasts in the subsurface. Certain combinations of velocity contrasts may prevent the Brewster angle from being reached before the GPR signal is lost. Future work in the area will include the construction of a test pit to better understand Brewster angle reflections in a variety of controlled conditions.

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References