ELSEVIER Journal of Banking & Finance 24 (2000) 203–227

Journal of BANKING & FINANCE

www.elsevier.com/locate/econbase

Stability of rating transitions ☆

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Abstract

The distribution of ratings changes plays a crucial role in many credit risk models. As is well-known, these distributions vary across time and different issuer types. Ignoring such dependencies may lead to inaccurate assessments of credit risk. In this paper, we quantify the dependence of rating transition probabilities on the industry and domicile of the obligor, and on the stage of the business cycle. Employing ordered probit models, we identify the incremental impact of these factors. Our approach gives a clearer picture of which conditioning factors are important than comparing transition matrices estimated from different sub-samples.

JEL classification: C25; G21; G33

Keywords: Bond rating; Credit risk; Markov chain

1. Introduction

In the new generation of credit risk models, agency ratings of credit quality play an important role. For illiquid bonds or non-marketed loans, mark-to-market prices are not observable and hence measuring risk is very difficult. A

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common approach (see JP Morgan, 1997; Credit Suisse Financial Products, 1997) is to link value changes to transitions in ratings. Risk may then be measured by looking at the joint distribution of rating transitions for the loans and bonds which make up the portfolio of interest.

A crucial element in such calculations is the matrix of different rating transition probabilities. A typical element in this matrix is the probability that a bond of a given rating (say Aa) moves to some other rating (for example Baa), over a given period of time. Of course, knowing the rating transition matrix applicable to a group of loans is only the first step in credit risk modelling since this matrix contains no information about correlations in the rating transitions of different loans. Nevertheless, it is a vital ingredient in many modelling approaches.

A number of studies have documented the fact that rating transition matrices vary according to the stage of the business cycle, the industry of the obligor and the length of time that has elapsed since the issuance of the bond. These studies have either been performed by the ratings agencies themselves (see Lucas and Lonski, 1992; Carty and Fons, 1993; Carty, 1997) for summaries of research carried out by staff of Moody's and reports by Standard and Poor's, such as Standard and Poor's (1998a,b) for research by that agency), or by academics. Prominent among the latter are Altman and Kao (1992a,b). Altman (1997) critically compares the results of these three sets of research papers.

A notable feature of the studies just mentioned is that they examine the stability of rating transition matrices (across different time periods, type of obligor or stage of the business cycle) in what one might term a *univariate* manner. In other words, rating transition matrices are estimated and compared, for example, for two different industries without holding constant other sources of variation such as the obligor's domicile. Though informative, studies which take such a univariate approach do not directly reveal the *ceteris paribus* significance of different conditioning variables. For an analyst designing a credit risk model and wondering whether to allow for particular dependencies, it is the incremental or ceteris paribus impact of conditioning variables on rating transitions which is important.

In the present paper, we study the distribution of rating transitions using the universe of Moody's long-term corporate and sovereign bond ratings in the period December 1970–1997. (This dataset excludes Moody's municipal bond and short-term bond and commercial paper ratings.) Like the studies by the ratings agencies cited above (and in contrast to those by Altman and his co-authors), we employ obligor-specific, senior, unsecured ratings. (When an obligor has not issued senior, unsecured debt, these are inferred by Moody's from ratings of other kinds of issue.)

In the first part of our analysis (see Section 2), we up-date and extend the existing literature by comparing simple, non-parametric estimates of rating transition matrices for different sets of conditioning variables. We focus in particular on the stability of transition matrices for different industries and domicile of obligor and for different stages of the business cycle.

In Section 3 of the paper, we gauge the ceteris paribus impact of different conditioning variables on rating transitions, by formulating and estimating ordered probit models of rating transitions. Given a particular initial rating, whether or not an obligor has switched to another rating one year later is a simple discrete choice problem. Conditioning factors may be introduced through dummy variables. The fact that different possible end-of-period ratings are naturally ordered from high to low credit quality makes it natural to employ ordered, discrete choice modelling methods.

Throughout our study, we attempt to draw out the implications of results for credit risk modelling applications. To take a simple example, we note that highly rated Japanese obligors have unusually large rating volatility while lowly rated Japanese obligors have unusually small rating volatility. Applying credit risk models based on non-specific, unconditional rating transition matrices to loan books of high- or low-quality Japanese loans is likely to under- or over-estimate the risk respectively.

2. Multinomial modelling of rating changes

2.1. The data

Our dataset covers all long-term bonds rated by Moody's in the period December 1970–1997 with the exception of municipals [1]. Our sample contained 6,534 obligor ratings histories and the total number of obligor-years excluding withdrawn ratings (and hence observations in our sample) was 50,831. The ratings we employ are notional senior unsecured ratings created by Moody's for all obligors who possess Moody's rated long bonds at a given moment in time. Lucas and Lonski (1992) mention that in their dataset, which is close to ours, 56% of ratings are based on directly observed senior, unsecured ratings. The remainder are derived from Moody's ratings of subordinated or secured bonds. The approach taken by Moody's is described in Carty (1997).

Our use of notional, obligor-specific, senior, unsecured ratings has immediate implications for the type of analysis one may perform. For example, the rating histories in our sample have no associated maturity or issue dates. It is, therefore, not possible to examine the impact on default risk and other non-

[1] Some data prior to 1970 were available to us but the coverage was only complete (with the exception of municipals) from that date on, so we restricted the sample to examine the post-1970 experience.

default transitions probabilities of the time which has elapsed since the bond's initial issue or the period remaining until maturity. In contrast, the studies by Altman and Kao (1992a,b) track over time ratings for individual bond issues. These authors stress the impact on risk of the length of time since issue. They find that default risk is increasing in the first three or four years of an issue's life although the effect disappears thereafter [2].

An issue that arises in estimating rating transition matrices is the appropriate treatment of withdrawn ratings. Ratings are withdrawn for a variety of reasons, for example because the bond is called or because the obligor ceases to continue paying Moody's the required annual fee. Typically, Aaa borrowers have an annual risk of rating withdrawal of 4% while for B-rated issuers the risk is just over 10%. Carty (1997) argues that few rating withdrawals (around 13%) are possibly correlated with changes in credit standing and hence that one should calculate rating transition probabilities simply leaving the withdrawn ratings aside. This is the approach we take in the present study.

The coverage of the Moody's data we employ has evolved significantly over time. In particular, the geographical coverage has changed from an overwhelming bias towards obligors domiciled in the US to a more even geographical spread. In December 1970, 98.0% of Moody's-rated obligors were United States-domiciled. A negligible fraction were Japanese, while European-domiciled issuers amounted to just 0.3%. In December 1997, only 66.0% of obligors were US-domiciled while 4.7% and 5.4% of issuers were domiciled respectively in the UK and Japan. European-domiciled issuers amounted to 20.0% at the end of 1997 [3].

A clear evolution has also occurred in the spread of rated obligors across different industries. In December 1970, utilities and industrials made up, respectively, 27.8% and 57.9% of rated issuers. Banks constituted a negligible fraction of rated obligors. By the end of 1997, utilities, industrials and banks contributed 9.1%, 59.5% and 15.8% of long-term-bond obligors rated by Moody's.

The fact that the predominant types of issuers have evolved over time (with US-domiciled utilities in decline and European and Japanese borrowers, especially banks playing a larger role) means that transition matrices estimated unconditionally based on all the entities rated at a given moment in time will change even if the underlying approach taken by Moody's is constant.

[2] Ageing effects are also looked at by Jonsson and Fridson (1996) and Helwege and Kleiman (1996).

[3] Growth in the proportion of non-US obligors has accelerated considerably in the 1990s. At the close of 1989, the fractions of issuers from the US, Japan, the UK and other European countries were 84.7%, 2.1%, 2.3% and 4.3%, respectively.

Finally, we employ in our study the coarser rating categories Aaa, Aa, A, Baa, Ba, B, Caa, and C/Ca used by Moody's prior to 1982. After that date, Moody's split the upper six categories into numbered sub-categories. Thus, for example, Aaa was split into Aaa1, Aaa2 and Aaa3, with Aaa1 being the highest credit quality.

The reasons we focus on the coarser categories in this study are, first, that we wished to include data from 1970 onwards and to have full data-comparability throughout our sample period, and, second, that one may doubt whether it is really useful to employ the finer categorisation in credit risk modelling. The credit spread data (which are employed in conjunction with rating transition matrices in JP Morgan's CreditMetrics, for example) are not that reliable for finer ratings and the added complexity of having three times as many categories is probably not worthwhile.

2.2. Unconditional transition matrices

Before turning to multivariate modelling of the rating data, we calculate unconditional transition matrices for the sample as a whole (recall that our sample runs from 31/12/70 to 31/12/97) and for various sub-samples. This permits us to relate our results to earlier studies which have performed similar exercises. The basic assumption behind this approach is that, for a given sample, the probability of a transition from rating i to j, say, is a constant parameter, p_{ij}. This amounts to saying that, for a given initial rating, transitions to different possible future ratings follow a constant parameter, temporally independent multinomial process. Estimation may then be performed by taking the fraction of occasions in the sample (or sub-sample) on which an obligor starts the year in state i and ends it in j.

Table 1 shows the basic unconditional rating 1-year transition matrix for our sample. Each entry represents the sample frequency of transitions from the initial rating (given on the left-hand side of the matrix) to a given terminal rating (given along the top of the matrix) divided by the total number of issuer years for issuers which began in the initial rating category in question. The numbers of issuer years for different ratings are given in the right hand column of Table 1. Entries in the matrix shown as a dash correspond to cases in which the sample contained no observations which made the rating transition in question.

As one may observe, the volatility of rating transitions increases sharply as credit quality declines. Thus, for Aaa- or Aa-rated obligors, the probability that the rating is unchanged a year later is 90%. In contrast, speculative Ba, B and Caa rated issuers maintain their initial ratings with probabilities of just 85.7%, 83.0% and 66.6%, respectively.

The precision with which one may estimate rating transition probabilities is shown by the standard errors provided in brackets under each probability

Table 1
Unconditional transition matrices[a]

Initial rating	Terminal rating									Number of issuer years
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def	
Moody's ratings 12/70–12/97										
Aaa	90.4	8.7	0.8	–	0.0	–	–	–	–	2514
	(0.6)	(0.6)	(0.2)	–	(0.0)	–	–	–	–	
Aa	1.1	89.5	8.9	0.4	0.1	0.0	–	–	–	6402
	(0.1)	(0.4)	(0.4)	(0.1)	(0.0)	(0.0)	–	–	–	
A	0.1	2.3	92.1	5.0	0.5	0.1	0.0	–	0.0	13605
	(0.0)	(0.1)	(0.2)	(0.2)	(0.1)	(0.0)	(0.0)	–	(0.0)	
Baa	0.0	0.2	5.4	89.1	4.4	0.6	0.1	–	0.1	10225
	(0.0)	(0.0)	(0.2)	(0.3)	(0.2)	(0.1)	(0.0)	–	(0.0)	
Ba	0.0	0.0	0.5	5.4	85.7	6.7	0.2	0.0	1.3	8027
	(0.0)	(0.0)	(0.1)	(0.3)	(0.4)	(0.3)	(0.1)	(0.0)	(0.1)	
B	0.0	0.1	0.2	0.7	6.8	83.0	1.9	0.5	6.9	4436
	(0.0)	(0.0)	(0.1)	(0.1)	(0.4)	(0.6)	(0.2)	(0.1)	(0.4)	
Caa	–	–	–	0.9	2.5	8.0	66.6	3.7	18.4	326
	–	–	–	(0.5)	(0.9)	(1.5)	(2.6)	(1.0)	(2.1)	
Ca/C	–	–	–	–	0.9	5.6	15.0	57.9	20.6	107
	–	–	–	–	(0.9)	(2.2)	(3.4)	(4.8)	(3.9)	
Default	–	–	–	–	–	–	–	–	100.0	5190
	–	–	–	–	–	–	–	–	(0.0)	

Initial rating	Terminal rating							
	AAA Aaa	AA Aa	A A	BBB Baa	BB Ba	B B	CCC Caa	Def C/D
Previous studies								
AAA (A/K)	94.3	5.5	0.1	0.0	0.0	0.0	0.0	–
Aaa (M)	91.9	7.4	0.7	0.0	0.0	0.0	0.0	0.0
AAA (S&P)	90.8	8.3	0.7	0.1	0.1	0.0	0.0	0.0
AA (A/K)	0.7	92.6	6.4	0.2	0.1	0.1	0.0	–
Aa (M)	1.1	91.4	7.1	0.3	0.2	0.0	0.0	0.0
AA (S&P)	0.1	90.7	7.8	0.6	0.1	0.1	0.0	0.0
A (A/K)	0.0	2.6	92.1	4.7	0.3	0.2	0.0	–
A (M)	0.1	2.6	91.3	5.3	0.6	0.2	0.0	0.0
A (S&P)	0.9	2.4	91.0	5.5	0.7	0.3	0.1	0.1
BBB (A/K)	0.0	0.0	5.5	90.1	2.9	1.1	0.1	–
Baa (M)	0.0	0.2	5.4	87.9	5.5	0.8	0.1	0.1
BBB (S&P)	0.0	0.3	5.9	87.0	5.3	1.2	0.1	0.2
BB (A/K)	0.0	0.0	0.0	6.8	86.1	6.3	0.9	–
Ba (M)	0.0	0.1	0.4	5.0	85.0	7.3	0.4	1.6
BB (S&P)	0.0	0.1	0.7	7.7	80.5	8.8	1.0	1.2
B (A/K)	0.0	0.0	0.2	1.6	1.7	94.0	1.7	–
B (M)	0.0	0.1	0.1	0.5	6.0	82.1	2.2	8.9
B (S&P)	0.0	0.1	0.2	0.5	6.5	82.8	4.1	5.9
CCC (A/K)	–	–	–	–	–	–	–	–
Caa (M)	0.0	0.4	0.4	0.9	2.5	5.9	67.8	22.2
CCC (S&P)	0.2	0.0	0.2	1.3	2.3	13.2	62.0	23.1

[a] *Sources*: A/K = Altman and Kao (1992a,b) (sample = 1971–1989, newly issued bonds); M = Carty (1993) (sample = 1970-1993, Moody's static bond pool); S&P = S&P (1996) (sample = 1981–1995, S&P static bond pool). Data for upper block of results are notional unsecured bond ratings between 31/12/70 and 31/12/97 measured on 31st December each year.

entry in Table 1 [4]. As the Table shows, the sharp decline in the number of issuer years and the greater volatility of rating transitions for lower ratings combine to reduce the precision with which probabilities can be estimated in the lower right hand portion of the transition matrix. This is a problem for credit risk modelling applications since the transitions which substantially affect portfolio value are likely to involve the lower rating categories.

In the lower part of Table 1, for comparative purposes, we follow Altman (1997) in summarising results from three past studies. These are Altman and Kao (1992a), Carty and Fons (1993) and Standard and Poor's (1996). The estimates given in the last two of these studies include an additional category for the terminal state, namely 'withdrawn rating'. To make their figures comparable to ours and those of Altman and Kao, we eliminate the column corresponding to withdrawn rating and divide other columns by unity minus the entries in the withdrawn rating column. Given our focus on the use of transition matrices in credit risk modelling, we prefer to calculate transition probabilities conditional on the rating not being withdrawn.

It is noticeable that our estimates differ somewhat from those reported by the Moody's study completed by Carty and Fons and that the discrepancies appear to be statistically significant. When we restricted our sample period to coincide with theirs (which was 1970–1993), we were able to replicate their results with reasonably high accuracy so one may conclude that the differences are largely due to the inclusion of more recent data. As noted above, the geographical spread and industry of issuers has been changing rapidly in recent years as Moody's have broadened their international coverage and a greater number of banks have sought ratings on their debt issues. If one does not control for such changes in the obligor pool, transition matrix estimates will exhibit apparent changes.

2.3. Industry and domicile effects

We now focus more narrowly on the impact on rating transition matrices of the obligor's industry and domicile. In a subsequent section, we will identify the incremental effects of these variables holding other factors constant, but,

[4] These are calculated under the simplifying assumption that rating transitions are temporally and cross-sectionally independent. Let p_{ij} and $\hat{p}_{ij}$ denote the population and sample probabilities of a transition from rating i to j. If one considers the binomial variable: starting from i, either go to j or to $k = 1, \ldots, N$ where $k \neq j$, it is clear that the standard error for $\hat{p}_{ij}$ can be calculated as a standard binomial standard error: $\sqrt{\hat{p}_{ij}(1 - \hat{p}_{ij})/n}$ where n is the number of issuer years starting in rating i.

for the moment, we study the dependence by simply calculating different multinomial models in the way described in the last section.

The upper part of Table 2 shows transition matrices for banks and for industrials over the period 12/31/70 to 31/12/97. The volatility of rating transitions is clearly higher for banks than for industrials in that the probabilities of remaining in the same rating are consistently lower for banks whatever the initial rating category. On the other hand, it is noticeable that large movements in ratings (for example from Aa to Ba) are just as likely or more likely for industrials than for banks. This amounts to saying that the distribution of changes in credit-standing is relatively fat-tailed for industrials (in that, relative to volatility, the fourth moment is high).

When transition probabilities differ in a statistically significant way (at a 5% level) from the unconditional probabilities shown in Table 1, they appear in Table 2 in bold type. To perform these tests, we calculate *t*-statistics equal to the difference between corresponding entries in the sub-sample and the total sample transition probabilities divided by standard errors (see footnote 4) for the sub-sample estimate. (The calculation is therefore conditional on the 'whole-sample' probabilities which, for the purpose of the exercise, are taken to be non-stochastic.[5])

As one may see, about half of the probability entries in the upper part of Table 2 (which relates to banks) are significantly different from those in Table 1 at a 5% level. Transition probabilities for more highly rated banks tend to differ more significantly from the unconditional transition matrix than for the lower rating categories although this is largely attributable to the lack of observations for those categories. There are scarcely any banks in the speculative rating categories, confirming the fact that running a bank when market confidence in the institution's credit standing is low is almost impossible.

The rating transition probabilities for industrials shown in the lower part of Table 2 are in fact very similar to those for the sample as a whole so relatively few entries are picked out in bold. This is particularly true for the more highly rated obligors. Only for some transition probabilities in the B to Baa range do we observe statistically significant discrepancies.

In Table 3, we report transition matrices for obligors domiciled in different countries. Again, we pick out in bold transition probabilities which differ in a statistically significant way (at a 5% level) from the corresponding 'whole sample' probability. As one might expect, the matrix for US-domiciled obligors closely resembles that for the sample as a whole. For UK-domiciled issuers,

[5] Allowing for the stochastic nature of the whole-sample estimates in the calculation of *t*-statistics is complicated by the fact that the larger sample includes the sub-sample whereas both models are estimated presuming that transition probabilities are constant parameter multinomial processes. We prefer therefore to perform the tests conditional on the whole-sample results.

Table 2
Conditional transition matrix[a]

Initial rating	Terminal rating									Number of issuer years
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def	
Banking										
Aaa	**84.7**	**15.0**	**0.3**	–	–	–	–	–	–	694
Aa	**0.4**	**87.8**	**11.5**	**0.3**	–	–	–	–	–	1591
A	–	2.7	**90.0**	**6.4**	0.7	0.2	–	–	–	1826
Baa	–	0.9	**16.4**	**75.1**	5.8	1.8	–	–	–	434
Ba	–	–	**4.3**	**10.3**	**76.2**	5.9	0.5	–	2.7	185
B	–	–	–	2.7	**13.4**	78.6	0.9	–	4.5	112
Caa	–	–	–	–	50.0	–	–	–	50.0	2
Ca/C	–	–	–	–	–	–	–	–	–	0
Industrial										
Aaa	91.6	7.8	0.7	–	–	–	–	–	–	876
Aa	1.1	89.3	9.1	0.3	0.2	0.0	–	–	–	2525
A	0.1	1.9	92.4	4.8	0.6	0.2	–	–	0.0	6728
Baa	0.0	**0.1**	**3.9**	89.9	4.9	0.8	0.1	–	0.2	5353
Ba	0.0	0.1	0.4	**3.4**	**87.0**	**7.4**	0.2	0.0	1.5	5995
B	0.0	0.1	0.2	0.5	6.2	84.0	1.9	0.4	6.8	3751
Caa	–	–	–	0.8	2.1	7.5	68.2	3.8	17.6	239
Ca/C	–	–	–	–	1.4	6.8	20.5	56.2	15.1	73

[a] *Notes*: Data are notional unsecured Moody's long-term corporate and sovereign bond ratings between 31/12/70 and 31/12/97 measured on 31st December each year.

transition probabilities also look similar to the whole-sample results. Where differences occur, in the lower rated categories, the discrepancies are not statistically significant because of the paucity of observations. Most striking of all, no defaults occurred within our sample period and no Caa or C/Ca rated obligors were present.

Japanese-domiciled entities on the other hand differ substantially from the whole sample results. In particular, relatively lowly rated Japanese obligors (in the Baa to B range) exhibit strikingly little volatility compared to US-domiciled issuers. Highly rated Japanese issuers on the other hand possess somewhat more volatile ratings than their US counterparts in that down-grades are more likely. Similar to the UK, Japan had no obligors which defaulted in our sample period and the number of issuers in the more highly speculative categories was negligible.

2.4. Business cycle effects

From a credit risk modelling perspective, variation in ratings transition matrices attributable to the business cycle is potentially very important.

Table 3
Conditional transition matrix[a]

Initial rating	Terminal rating									Number of issuer years
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def	
United States										
Aaa	**91.9**	**6.9**	1.1	–	0.1	–	–	–	–	1523
Aa	1.2	89.3	8.8	0.5	0.2	0.0	–	–	–	4129
A	0.1	2.3	92.0	4.9	0.6	0.2	0.0	–	0.0	11282
Baa	0.0	0.2	5.5	88.9	4.5	0.6	0.1	–	0.1	9277
Ba	0.0	0.1	0.5	5.4	85.5	6.9	0.3	0.0	1.4	7452
B	0.0	0.1	0.2	0.7	6.5	82.9	1.9	0.5	7.2	4128
Caa	–	–	–	1.0	2.5	7.6	67.3	3.5	18.1	315
Ca/C	–	–	–	–	1.0	5.7	14.3	58.1	21.0	105
United Kingdom										
Aaa	90.4	8.9	0.7	–	–	–	–	–	–	135
Aa	**0.3**	88.2	11.0	0.5	–	–	–	–	–	390
A	–	3.4	94.1	**2.5**	–	–	–	–	–	444
Baa	–	–	11.9	86.4	1.7	–	–	–	–	59
Ba	–	–	–	16.0	76.0	8.0	–	–	–	25
B	–	–	–	11.1	5.6	83.3	–	–	–	18
Caa	–	–	–	–	–	–	–	–	–	0
Ca/C	–	–	–	–	–	–	–	–	–	0
Japan										
Aaa	86.9	12.1	1.0	–	–	–	–	–	–	99
Aa	**0.3**	88.9	10.5	0.3	–	–	–	–	–	306
A	–	**0.8**	**95.2**	4.0	–	–	–	–	–	396
Baa	–	–	**1.2**	**96.9**	**1.6**	–	0.3	–	–	322
Ba	–	–	–	3.5	**94.4**	**2.1**	–	–	–	142
B	–	–	–	–	9.5	90.5	–	–	–	21
Caa	–	–	–	–	–	–	–	–	–	0
Ca/C	–	–	–	–	–	–	–	–	–	0

[a] *Notes*: Data are notional unsecured Moody's long-term corporate and sovereign bond ratings between 31/12/70 and 31/12/97 measured on 31st December each year.

Some credit risk modelling approaches (see, for example, Credit Suisse Financial Products, 1997; Wilson, 1997) suppose that transition probabilities change over time as the state of the economy evolves and that these changes drive correlations between changes in the credit quality of different obligors. Though other approaches (see, for example, JP Morgan, 1997) employ an unconditional transition matrix, one could in principle introduce transition matrices specific to the current stage of the business cycle without difficulty.

To investigate the dependence of rating transition probabilities on the state of the economy, we define different levels of economic activity as follows. For each G7 country, we allocate our set of sample years (1970–1997) into three categories, 'peak', 'normal times' and 'trough', depending on whether real GDP growth in the country in question was in the upper, middle or lower third of the growth rates recorded in the sample period. For non-G7 countries, we subtract from a world, real GDP series the real GDP of the G7 and then calculate growth rates and categorise years as 'peak', 'normal times' or 'trough' in the way described above.

In Table 4, we present estimates of simple, multinomial-model, transition matrices for issuer years which fall, respectively, into periods of business cycle peak or trough. Again, we pick out in bold entries in the matrices which differ

Table 4
Conditional transition matrix[a]

Initial rating	Terminal rating									Number of issuer years
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def	
Business cycle trough										
Aaa	89.6	10.0	0.4	–	–	–	–	–	–	930
Aa	0.9	88.3	**10.7**	**0.1**	0.0	–	–	–	–	2195
A	0.1	2.7	**91.1**	5.6	0.4	0.0	–	–	0.0	4591
Baa	0.0	0.3	**6.6**	**86.8**	**5.6**	0.4	0.2	–	0.1	3656
Ba	–	0.1	0.5	5.9	**83.1**	**8.4**	0.3	0.0	1.7	2715
B	–	0.1	0.2	0.8	6.6	**79.6**	2.2	1.0	**9.4**	1459
Caa	–	–	–	0.9	1.9	9.3	63.0	1.9	23.1	108
Ca/C	–	–	–	–	–	5.9	**5.9**	64.7	23.5	34
Business cycle normal										
Aaa	92.2	7.4	**0.3**	–	0.1	–	–	–	–	757
Aa	1.5	**87.5**	10.1	0.7	0.2	–	–	–	–	2256
A	0.0	**1.8**	91.7	5.4	**0.8**	0.2	0.0	–	–	4420
Baa	0.1	0.2	5.2	88.1	4.9	**1.2**	0.0	–	0.2	2825
Ba	0.1	0.0	0.3	5.4	85.7	6.7	0.2	0.0	1.5	2615
B	0.1	0.1	0.4	0.8	6.6	83.6	1.6	0.3	6.6	1548
Caa	–	–	–	–	2.8	9.3	59.8	8.4	19.6	107
Ca/C	–	–	–	–	–	8.3	8.3	70.8	12.5	24
Business cycle peak										
Aaa	89.7	8.5	**1.8**	–	–	–	–	–	–	827
Aa	0.8	**93.2**	**5.6**	0.3	0.1	0.1	–	–	–	1951
A	0.0	2.3	**93.4**	**3.9**	**0.3**	0.1	–	–	–	4594
Baa	–	0.2	**4.4**	**92.2**	**2.8**	0.3	0.1	–	0.1	3744
Ba	–	0.0	0.6	4.8	**88.5**	**5.0**	0.3	–	**0.7**	2697
B	–	–	0.1	0.3	7.2	**85.8**	2.0	**0.1**	**4.5**	1429
Caa	–	–	–	1.8	2.7	5.4	**76.6**	**0.9**	12.6	111
Ca/C	–	–	–	–	2.0	4.1	24.5	46.9	22.4	49

[a] *Notes*: Data are notional unsecured Moody's long-term corporate and sovereign bond ratings between 31/12/70 and 31/12/97 measured on 31st December each year.

in a statistically significant way (at a 5% level) from corresponding entries in the 'whole sample' transition matrix. [6]

In business cycle peaks, low-rated bonds have much less ratings volatility and, in particular, are less prone to down-grades. Default probabilities are especially sensitive to business cycle. This is interesting since default is the one rating category which is based on a clear objectively observable event rather than on the subjective assessment of ratings agencies. Some non-default transition probabilities have counter-intuitive values. For example, the Caa to Ca/C probability is highest in normal times. But, generally the results for low-rated obligors are intuitively convincing.

The general finding for investment grade bonds (Baa and above) is that volatility falls sharply in business cycle peak years and rises in business cycle troughs. It is noticeable that the effect of the cycle on such highly rated obligors is more to raise volatility than to shift ratings systematically down. Thus, for example, for an A-rated obligor, the probability of an up-grade to Aa may even be marginally higher in troughs than in peaks but this is balanced by a rise in down-grade probabilities and therefore overall volatility rises.

3. An ordered probit model of ratings changes

3.1. Analysis of ceteris paribus effects

In this section, we describe an ordered, discrete choice model of rating transitions. Using this we will be able to calculate fitted transition probability matrices for quite specific obligor categories. The advantage of being able to do this is that one may evaluate the ceteris paribus significance of different borrower characteristics or stages of the business cycle. In principle, if one had enough data, one could perform this kind of comparison with simple, multinomial transition matrix estimates by dividing the sample into fine enough sub-samples and separately estimating transition matrices on the different sub-samples.

As should be clear from the results of the last section, even though we have tens of thousands of issuer years in our sample, there is insufficient information to estimate models based on finely differentiated sub-samples with any precision. One may regard our parametric model of rating transitions as a way of partially pooling information from different sub-samples, while nevertheless identifying ceteris paribus effects. If we estimated our model with a full set of

[6] Once again, the standard error used in calculating the *t*-statistic is that of the sub-sample estimate, i.e., we hold the unconditional matrix constant.

interaction effects between the different dummy variables, we would effectively be estimating multinominal models on finely differentiated sub-samples of the total dataset.

3.2. Statistical techniques

The statistical techniques we employ are those of ordered probit analysis. This approach explicitly allows for the discreteness of possible rating transitions but also for the fact that ratings possess a natural ordering from high to low credit quality. Greene (1997, Chapter 9) provides a straightforward exposition. These techniques have been widely employed in a variety of contexts. Cheung (1996) uses ordered discrete choice modelling on provincial, Canadian credit ratings but her focus is on employing data, for example, on indebtedness to explain rating levels rather than to examine the probability of different rating transitions.

We briefly describe the ordered probit approach we employ. Consider a sample of obligor ratings observed at t and $t+1$. Suppose the initial ratings at t are identical but that at $t+1$, a given obligor may be in any one of n different terminal states corresponding to default (state 1) and $n-1$ non-default ratings categories. Suppose that an obligor's credit standing at $t+1$ is determined by the realisation of a unobserved, latent random variable Z_{t+1} where

$$Z_{t+1} + \beta' X_t = \epsilon_{t+1}. \tag{1}$$

Here, X_t is an M-dimensional vector comprising cross-sectional borrower characteristics (independent of t) and time series data such as the state of the business cycle at t or in lagged periods. β is an M-dimensional column vector of regression parameters to be estimated.

The rating at $t+1$, denoted Y_{t+1} is determined in the following way:

$$\begin{aligned}
Y_{t+1} &= 0 \quad \text{if} \quad Z_{t+1} \leqslant 0,\\
Y_{t+1} &= 1 \quad \text{if} \quad 0 < Z_{t+1} \leqslant \mu_1,\\
Y_{t+1} &= 2 \quad \text{if} \quad \mu_1 < Z_{t+1} \leqslant \mu_2,\\
&\vdots \quad\ \ \vdots \qquad \vdots\\
Y_{t+1} &= N \quad \text{if} \quad \mu_{N-1} < Z_{t+1}.
\end{aligned} \tag{2}$$

Here, the μ_i's are unknown parameters which collectively define a series of ranges into which the latent variable may fall. Like the β_j's, the μ_i's are to be estimated.

If one supposes that, conditional on X_t, ϵ_{t+1} is standard normally distributed (therefore having zero mean and unit variance), the probabilities that Y_{t+1} takes values $1, 2, \ldots, N$ are

$$
\begin{aligned}
\text{Prob}\{Y_{t+1} = 0\} &= \Phi(\beta' X_t), \\
\text{Prob}\{Y_{t+1} = 1\} &= \Phi(\mu_1 + \beta' X_t) - \Phi(\beta' X_t), \\
\text{Prob}\{Y_{t+1} = 2\} &= \Phi(\mu_2 + \beta' X_t) - \Phi(\mu_1 + \beta' X_t), \\
\vdots \quad & \vdots \quad \vdots \\
\text{Prob}\{Y_{t+1} = N\} &= 1 - \Phi(\mu_{N-1} + \beta' X_t).
\end{aligned}
\tag{3}
$$

To obtain positive probabilities, one must impose the restriction that $0 < \mu_1 < \mu_1 < \cdots < \mu_{N-1}$ when estimating the model. To carry out estimation, we supposed that, conditional on realisations of X_t, rating transitions for different obligors are independent both cross-sectionally and through time. This enabled us to form a likelihood made up of probability terms like those shown in Eq. (3) for each obligor year in the sample.

The assumption that rating transitions are cross-sectionally independent might be questioned. Approaches to credit risk modelling such as JP Morgan's CreditMetrics® (see JP Morgan, 1997) stress contemporaneous rating transition correlations. On the other hand, Wilson (1997) and Credit Suisse Financial Products (1997) assume that, conditional on the business cycle, transitions for different obligors are independent. Over, say, a two year horizon, the random evolution of the business cycle induces correlations by shifting transition probabilities but over a one-year period transitions for different obligors are independent. The assumption that underlies our Maximum Likelihood estimation is similar to that of Credit Suisse Financial Products' CreditRisk+®.

3.3. Model estimates

Table 5 shows the parameter estimates obtained when we estimate ordered probit models on sub-samples of issuer years starting in each of the eight possible non-default ratings. The X variables that appear in the models include, first, dummy variables for four different domiciles (US, UK, Japan, Europe excluding the UK), with a fifth category (other) serving as the reference category. Second, they include dummies for ten industry categories with financial institutes acting as reference category. Third, dummies for the current and (1 year) lagged business cycle state (normal times or trough) are included with peak being the reference category.

Depending on the initial rating, there may or may not be sufficient observations in the sample to identify statistically all the coefficients for the above list of dummies. In cases in which parameters are not identified, we merge categories with the reference category. Where a dummy does not appear in a particular initial rating model for this reason, we indicate this in Table 5 with a dash. For example, for the Aaa-initial-rating model, 'Other non-bank' and 'Thrifts' categories are merged into the 'Financial institutes' category.

Table 5
Parameter estimates[a]

	Initial rating							
	Aaa	Aa	A	Baa	Ba	B	Caa	Ca/C
Constant	1.70	−2.20	−3.25	−2.82	−2.42	−2.65	−1.78	−2.06
	(0.28)	(0.13)	(0.13)	(0.12)	(0.12)	(0.16)	(0.41)	(0.38)
United States	0.17	−0.13	0.05	0.06	−0.02	−0.19	0.25	–
	(0.12)	(0.07)	(0.06)	(0.08)	(0.08)	(0.10)	(0.36)	(–)
United Kingdom	0.15	−0.21	0.39	0.44	0.37	0.53	–	–
	(0.17)	(0.10)	(0.10)	(0.20)	(0.28)	(0.32)	(–)	(–)
Japan	−0.05	−0.09	0.10	0.04	0.24	0.22	–	–
	(0.18)	(0.11)	(0.10)	(0.12)	(0.15)	(0.31)	(–)	(–)
Europe excl. UK	0.17	0.04	0.22	0.24	0.06	0.08	–	–
	(0.12)	(0.08)	(0.09)	(0.14)	(0.20)	(0.25)	(–)	(–)
Banking	−0.71	−0.03	−0.13	0.27	0.16	0.33	–	–
	(0.26)	(0.11)	(0.07)	(0.08)	(0.12)	(0.15)	(–)	(–)
Finance	−0.42	0.22	−0.00	0.06	0.44	−0.53	–	–
	(0.30)	(0.14)	(0.10)	(0.12)	(0.18)	(0.41)	(–)	(–)
Industrial	−0.44	0.13	−0.06	−0.21	−0.27	−0.01	−0.14	0.71
	(0.25)	(0.10)	(0.06)	(0.06)	(0.07)	(0.08)	(0.17)	(0.24)
Insurance	−0.15	0.23	−0.03	0.19	0.15	−0.07	–	–
	(0.30)	(0.14)	(0.11)	(0.12)	(0.12)	(0.23)	(–)	(–)
Other non-bank	–	0.02	−0.14	−0.00	−0.08	0.27	–	–
	(–)	(0.20)	(0.13)	(0.13)	(0.17)	(0.22)	(–)	(–)
Public utility	−0.40	0.32	0.05	−0.00	0.35	0.71	−0.30	–
	(0.26)	(0.11)	(0.07)	(0.06)	(0.08)	(0.14)	(0.37)	(–)
Securities	–	−0.31	−0.04	0.03	−0.04	–	–	–
	(–)	(0.25)	(0.15)	(0.22)	(0.42)	(–)	(–)	(–)
Sovereign	−0.18	0.99	0.55	0.18	−0.05	–	–	–
	(0.30)	(0.19)	(0.23)	(0.28)	(0.24)	(–)	(–)	(–)
Thrifts	–	–	−0.16	−0.12	−0.62	−0.44	−1.45	–
	(–)	(–)	(0.23)	(0.19)	(0.13)	(0.16)	(0.47)	(–)
Business cycle: trough	0.13	−0.25	−0.08	−0.05	−0.15	−0.20	−0.29	−0.36
	(0.08)	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)	(0.17)	(0.29)
Business cycle: normal	0.27	−0.24	−0.18	−0.14	−0.07	−0.06	−0.27	0.10
	(0.09)	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)	(0.17)	(0.35)
Business cycle (1 lag):	−0.33	−0.05	0.08	0.06	0.03	0.08	−0.06	−0.05
trough	(0.09)	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)	(0.18)	(0.32)
Business cycle (1 lag):	−0.08	−0.03	−0.00	−0.03	0.01	−0.04	−0.33	−0.01
normal	(0.10)	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)	(0.19)	(0.33)
Aaa–Aa	0.00	0.00	0.00	–	–	–	–	–
	(0.00)	(0.00)	(0.00)	–	–	–	–	–
Aa–A	1.12	3.70	1.26	0.00	–	–	–	–
	(0.08)	(0.05)	(0.10)	(0.00)	–	–	–	–
A–Baa	–	5.00	4.87	1.28	0.00	0.00	–	–
	–	(0.08)	(0.10)	(0.07)	(0.00)	(0.00)	–	–

Table 5 (Continued)

	Initial rating							
	Aaa	Aa	A	Baa	Ba	B	Caa	Ca/C
Baa–Ba	–	5.48	5.79	4.52	1.04	0.47	–	–
	–	(0.12)	(0.11)	(0.07)	(0.05)	(0.09)	–	–
Ba–B	–	–	6.32	5.30	4.08	1.43	0.00	–
	–	–	(0.13)	(0.08)	(0.06)	(0.10)	(0.00)	–
B–Caa	–	–	–	5.78	4.85	4.24	0.72	0.00
	–	–	–	(0.10)	(0.07)	(0.10)	(0.14)	(0.00)
Caa–Ca/C	–	–	–	5.95	4.92	4.36	2.78	0.81
	–	–	–	(0.11)	(0.07)	(0.11)	(0.17)	(0.19)
Ca/C–Default	–	–	–	–	–	4.40	2.92	2.56
	–	–	–	–	–	(0.11)	(0.17)	(0.24)

[a] *Note*: Reference categories for dummies are: (1) other countries; (2) financial institutes; (3) business cycle peak. Omitted categories are merged with reference categories. *t*-statistics are shown in brackets under the parameters.

In the lower part of Table 5, we show the cut-off points $\mu_1, \mu_2, \ldots, \mu_N$ for the latent variable which determines the rating transition. Recall that the lowest of these (corresponding to the cutoff between Aaa and Aa) is normalised to zero and that the subsequent ones are restricted to be monotonically increasing.

Again, for some of the sub-models for particular initial ratings, the sample did not include enough observations to identify cut-off point parameters. To cope with this, we deleted from the sample used in the estimation (for that particular initial rating) observations for which the terminal ratings fell into a category for which we had fewer than five transitions. For example, for the Aaa-initial-rating model, there were only enough observations to identify μ_2, the cut-off between Aa and A (μ_1, the cut-off between Aaa and Aa being given by the normalisation). Hence, we dropped issuer years for which the terminal rating was A or below when we estimated this Aaa-initial-rating model. In Table 5, when a particular cut-off point, μ_i, is not estimated, we indicate this with a dash.

3.4. Parameter values

The parameter values in Table 5 allow one to compare a large number of different individual categories. We shall focus here on the same comparisons that we discussed above which seem to us the most important, i.e., bank versus industrial, UK and Japan versus US, and business cycle trough versus peak.

On the first comparison, it is apparent that, relative to industrials, bank ratings may be thought of as reverting to some low investment-grade mean in that highly rated banks are consistently more subject to down-grades than industrials while low-rated banks are relatively more subject to up-grades. The differences appear to be statistically significant for most of the initial-rating-specific models, especially for the Baa and Ba categories.

On country effects, these are present. For example, lowly rated Japanese and UK obligors are much more likely to experience an up-grade but the results are not very significant statistically. The statistically strong findings for Japanese obligors referred to above in the section on multinomial model estimations thus may reflect an interaction with the results on banks versus industrials commented on in the last paragraph.

Business cycle effects are clear in our parameter estimates. The parameters for 'trough' and 'normal times' are the most statistically significant of all our conditioning variables. For investment grade but non-Aaa rated obligors, down-grades seem to be just as likely in normal times as in troughs, but in both cases are clearly more likely than in peak years. For sub-investment-grade obligors, trough years are associated with large down-grade probabilities.

To gauge the magnitudes of the effects implied by our dummy parameter values, one may examine the μ_i cut-off parameters shown in the lower part of Table 5. The distance between successive μ's corresponds to the distance the latent variable (which is standard normal distributed and hence has unit standard deviation) has to go to cross from one rating to the next.

To take an example, for an Aa-rated entity to remain in the Aa category requires that the latent variable end in the range 0–3.70. Being in a business cycle trough reduces the latent variable by 25% of a standard deviation. For an obligor which starts half-way through the category (i.e., at $X\beta = 1.85$), the chance of a down-grade goes from the P-level associated with 1.85 standard deviations to that associated with 1.6 standard deviations.

3.5. Transition matrices

Tables 6 and 7 show fitted, 1-year transition matrices implied for our models. To calculate these, we take a particular borrower type (industry and domicile) and suppose that the economy has been in a given business cycle state for the last two years, and then we evaluate the transition probabilities implied by our models for different initial ratings. [7]

We report in the lower third of Tables 7–9 t-statistics for the differences between the two sets of probabilities in the blocks of numbers immediately above. Thus, for example, the lower part of Table 6 shows t-statistics for the differences between US banks in business cycle troughs and peaks. The standard errors for these t-statistics are the roots of the sum of the squared stan-

[7] When, for a given initial rating, a category we are considering has been merged with the reference category, we then report, for that initial rating, transition probabilities appropriate to that reference category.

Table 6
Model-based transition matrix (US: Banking)[a]

Initial rating	Terminal rating								
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def
Business cycle trough									
Aaa	83.1	15.0	1.9	–	–	–	–	–	–
Aa	0.4	84.6	14.1	0.7	0.2	–	–	–	–
A	0.0	2.0	92.0	5.3	0.5	0.1	–	–	–
Baa	–	0.7	11.1	86.3	1.7	0.2	0.0	–	0.0
Ba	–	–	0.8	7.9	86.6	4.0	0.1	–	0.6
B	–	–	0.4	1.1	10.2	82.9	1.3	0.3	3.8
Caa	–	–	–	–	1.7	6.2	66.5	4.3	21.3
Ca/C	–	–	–	–	–	0.7	4.2	48.8	46.3
Business cycle peak									
Aaa	87.7	11.1	1.1	–	–	–	–	–	–
Aa	0.9	90.0	8.7	0.3	0.1	–	–	–	–
A	0.0	2.0	91.9	5.4	0.5	0.1	–	–	–
Baa	–	0.7	10.8	86.5	1.8	0.2	0.0	–	0.0
Ba	–	–	1.1	9.7	85.6	3.1	0.1	–	0.4
B	–	–	0.6	1.5	12.1	81.7	1.0	0.2	2.9
Caa	–	–	–	–	3.7	10.6	69.8	3.1	12.7
Ca/C	–	–	–	–	–	2.0	8.5	58.6	30.9
t-Statistics									
Aaa	−1.3	1.3	1.1	–	–	–	–	–	–
Aa	**−2.4**	**−3.1**	**3.1**	2.0	1.4	–	–	–	–
A	0.1	0.1	0.1	−0.1	−0.1	−0.1	–	–	–
Baa	–	0.1	0.2	−0.2	−0.1	−0.1	−0.1	–	−0.1
Ba	–	–	−0.7	−0.8	0.7	0.8	0.6	–	0.7
B	–	–	−0.5	−0.5	−0.6	0.5	0.5	0.4	0.6
Caa	–	–	–	–	−0.6	−0.9	−0.8	0.8	1.5
Ca/C	–	–	–	–	–	−0.6	−0.8	−0.8	0.9

[a] *Notes*: Data are derived from ordered probit model based on Moody's corporate and sovereign bond ratings between 31/12/70 and 31/12/97.

dard deviations for the two probabilities being compared. The latter are worked out by regarding the probabilities as non-linear functions of the estimated parameters and applying the delta method which yields asymptotically valid standard errors.

If one compares US banks to US industrials the results show that, in a trough (see the upper blocks of Tables 6 and 7), highly rated banks are much more subject to down-grades than industrials. There appears to be less of a difference for lower credit-quality bank and industrial obligors.

Comparing US- to UK-domiciled banks as we do in Table 8, we see statistically significant differences only for Aa-rated banks, UK obligors being less prone to downgrades than their US counterparts. US and UK-domiciled in-

Table 7
Model-based transition matrix (US: Industrial)[a]

Initial rating	Terminal rating								
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def
Business cycle trough									
Aaa	89.0	10.0	0.9	–	–	–	–	–	–
Aa	0.6	87.8	10.9	0.5	0.1	–	–	–	–
A	0.1	2.3	92.4	4.7	0.4	0.1	–	–	–
Baa	–	0.2	4.6	89.5	4.8	0.7	0.1	–	0.1
Ba	–	–	0.2	3.5	85.7	8.5	0.3	–	1.8
B	–	–	0.2	0.5	5.7	83.5	2.1	0.5	7.5
Caa	–	–	–	–	2.2	7.5	68.1	3.9	18.3
Ca/C	–	–	–	–	–	3.9	13.1	61.8	21.2
Business cycle peak									
Aaa	92.4	7.1	0.5	–	–	–	–	–	–
Aa	1.4	91.9	6.5	0.2	0.1	–	–	–	–
A	0.1	2.3	92.3	4.8	0.5	0.1	–	–	–
Baa	–	0.2	4.5	89.5	4.9	0.7	0.1	–	0.1
Ba	–	–	0.3	4.4	86.7	7.0	0.2	–	1.3
B	–	–	0.2	0.6	7.0	83.9	1.8	0.4	6.0
Caa	–	–	–	–	4.8	12.3	69.7	2.8	10.5
Ca/C	–	–	–	–	–	8.8	20.5	59.4	11.4
t-Statistics									
Aaa	−1.6	1.6	1.2	–	–	–	–	–	–
Aa	**−2.9**	**−3.7**	**3.9**	**2.2**	1.5	–	–	–	–
A	0.1	0.2	0.1	−0.2	−0.1	−0.1	–	–	–
Baa	–	0.2	0.3	0.1	−0.3	−0.2	−0.1	–	−0.1
Ba	–	–	−1.3	−2.0	−1.4	**2.1**	0.8	–	1.7
B	–	–	−0.7	−0.9	−1.4	−0.4	0.9	0.5	1.5
Caa	–	–	–	–	−1.0	−1.2	−0.4	0.7	1.4
Ca/C	–	–	–	–	–	−0.8	−0.8	0.3	0.9

[a] *Notes*: Data are derived from ordered probit model based on Moody's corporate and sovereign bond ratings between 31/12/70 and 31/12/97.

dustrials also differ somewhat from each other as one may see from Table 9. Here, the statistically significant discrepancies arise primarily in the lower-rated industrial grades such as A and Baa.

3.6. Multi-period distributions and the business cycle

So far, we have concentrated on single period transition probabilities. For credit risk modelling purposes, one may be also interested in transitions over distinctly longer periods. To calculate fitted transition matrices for our ordered probit model, we have to allow for the fact that the business cycle is evolving stochastically over time.

Table 8
Model-based transition matrix (Banking: business cycle normal)[a]

Initial rating	Terminal rating								
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def
United States									
Aaa	91.1	8.2	0.7	–	–	–	–	–	–
Aa	0.4	85.3	13.4	0.7	0.2	–	–	–	–
A	0.0	1.3	90.2	7.4	0.9	0.2	–	–	–
Baa	–	0.4	8.1	88.5	2.6	0.3	0.0	–	0.0
Ba	–	–	1.0	8.8	86.1	3.5	0.1	–	0.5
B	–	–	0.5	1.2	10.4	82.8	1.2	0.3	3.7
Caa	–	–	–	–	0.9	4.0	60.9	5.0	29.3
Ca/C	–	–	–	–	–	2.5	9.9	60.1	27.6
United Kingdom									
Aaa	90.8	8.5	0.7	–	–	–	–	–	–
Aa	0.3	83.5	15.1	0.8	0.3	–	–	–	–
A	0.1	2.8	92.7	3.9	0.3	0.1	–	–	–
Baa	–	1.1	14.8	82.8	1.1	0.1	0.0	–	0.0
Ba	–	–	2.6	15.7	80.1	1.5	0.0	–	0.1
B	–	–	3.0	4.8	24.7	66.5	0.3	0.1	0.6
Caa	–	–	–	–	0.9	4.0	60.9	5.0	29.3
Ca/C	–	–	–	–	–	2.5	9.9	60.1	27.6
t-Statistics									
Aaa	0.1	−0.1	−0.1	–	–	–	–	–	–
Aa	0.6	0.7	−0.7	−0.5	−0.4	–	–	–	–
A	−1.5	**−2.6**	**−3.5**	**3.5**	**3.0**	**2.2**	–	–	–
Baa	–	−1.2	−1.6	1.4	**2.2**	**2.1**	1.5	–	1.7
Ba	–	–	−0.9	−1.1	0.9	1.6	1.4	–	1.6
B	–	–	−1.1	−1.3	−1.9	1.5	**2.4**	**2.1**	**2.4**
Caa	–	–	–	–	–	–	–	–	–
Ca/C	–	–	–	–	–	–	–	–	–

[a] *Notes*: Data are derived from ordered probit model based on Moody's corporate and sovereign bond ratings between 31/12/70 and 31/12/97.

We begin by making the simple assumption that changes in the business cycle between our three states of peak, normal times and trough, are themselves driven by a temporally independent Markov chain (i.e., there is a multinomial model with three possible outcomes in each period). Using real GDP growth figures (from 1965 to 1987) for each G7 country and for the world minus the G7, we estimate the parameters of this transition matrix by taking the fractions of transitions to different states observed in the sample.

Given this assumed data generating process for the evolution of the business cycle in each country, we can calculate the probability of observing say Aa in five years time given an initial rating of Aaa and that the business cycle is initially at its peak, by expanding the set of states. If there are no lags

Table 9
Model-based transition matrix (Industrial: business cycle normal)[a]

Initial rating	Terminal rating								
	Aaa	Aa	A	Baa	Ba	B	Caa	C/Ca	Def
United States									
Aaa	94.7	5.0	0.3	–	–	–	–	–	–
Aa	0.7	88.4	10.4	0.4	0.1	–	–	–	–
A	0.0	1.5	90.9	6.7	0.7	0.2	–	–	–
Baa	–	0.1	3.1	88.7	6.6	1.1	0.2	–	0.2
Ba	–	–	0.3	3.9	86.2	7.7	0.3	–	1.6
B	–	–	0.2	0.5	5.8	83.6	2.1	0.5	7.4
Caa	–	–	–	–	1.2	4.9	63.7	4.7	25.6
Ca/C	–	–	–	–	–	10.4	22.3	57.7	9.6
United Kingdom									
Aaa	94.5	5.2	0.3	–	–	–	–	–	–
Aa	0.5	87.0	11.8	0.5	0.2	–	–	–	–
A	0.1	3.3	92.8	3.5	0.3	0.1	–	–	–
Baa	–	0.3	6.8	89.1	3.3	0.4	0.1	–	0.1
Ba	–	–	0.9	8.2	86.5	3.8	0.1	–	0.6
B	–	–	1.3	2.6	17.5	76.4	0.6	0.1	1.5
Caa	–	–	–	–	0.9	4.0	60.9	5.0	29.3
Ca/C	–	–	–	–	–	2.5	9.9	60.1	27.6
t-Statistics									
Aaa	0.1	−0.1	−0.1	–	–	–	–	–	–
Aa	0.6	0.7	−0.7	−0.5	−0.4	–	–	–	–
A	−1.5	**−2.7**	**−3.8**	**4.0**	**3.3**	**2.4**	–	–	–
Baa	–	−1.1	−1.5	−0.4	**2.4**	**2.4**	1.6	–	2.0
Ba	–	–	−0.9	−1.1	−0.1	1.8	1.6	–	**2.2**
B	–	–	−1.1	−1.3	−1.8	1.0	3.2	**2.4**	**4.3**
Caa	–	–	–	–	0.1	0.2	0.5	−0.1	−0.4
Ca/C	–	–	–	–	–	0.9	1.2	−0.2	−1.9

[a] *Notes*: Data are derived from ordered probit model based on Moody's corporate and sovereign bond ratings between 31/12/70 and 31/12/97.

in the business cycle state, this is simple in that the expanded transition matrix is

$$T(h) \equiv \begin{bmatrix} \pi_{11}T_{1,h} & \pi_{12}T_{1,h} & \pi_{13}T_{1,h} \\ \pi_{21}T_{2,h} & \pi_{22}T_{2,h} & \pi_{23}T_{2,h} \\ \pi_{31}T_{3,h} & \pi_{32}T_{3,h} & \pi_{33}T_{3,h} \end{bmatrix}. \tag{4}$$

Here, $T_{k,h}$ is the unexpanded transition matrix of the kind reported in Tables 6 and 7, k is the initial stage of the business cycle, and h denotes cross-sectional characteristics of the obligor of interest. $\pi_{n,m}$ is a typical element of the transition matrix for business cycle states.

To obtain the probabilities of ending after say 5 years in a rating state j (and in business cycle states $m = 1, 2, 3$) after starting in rating i in normal times, one must multiply the expanded matrix $T(h)$ with itself five times, select the row corresponding to state i and normal times (this would be the $(N + i)$th row in this example), and then pick out the column elements for the jth rating (this would be j, $N + j$ and $2N + j$). This process yields three probabilities each specific to a different terminal business cycle state.

Finally, to obtain the probability of ending in a particular rating state integrating over the different possible terminal business cycle states, one must multiply the three probabilities just described by the corresponding three elements in the ith row of the fivefold product with itself of the transition matrix for business cycle states, $[\pi_{mn}]$, and then sum the result.

These calculations are slightly more complicated when the rating transition matrices depend not just on the current business cycle state but also on the lagged state. A similar approach may be taken, however, expanding the state space into $9 \times N$ states rather than the $3 \times N$ states used in the example just discussed.

Fig. 1 shows the probability of default at 1, 3, and 5 year horizons for obligors of different initial rating when the starting point is either a business

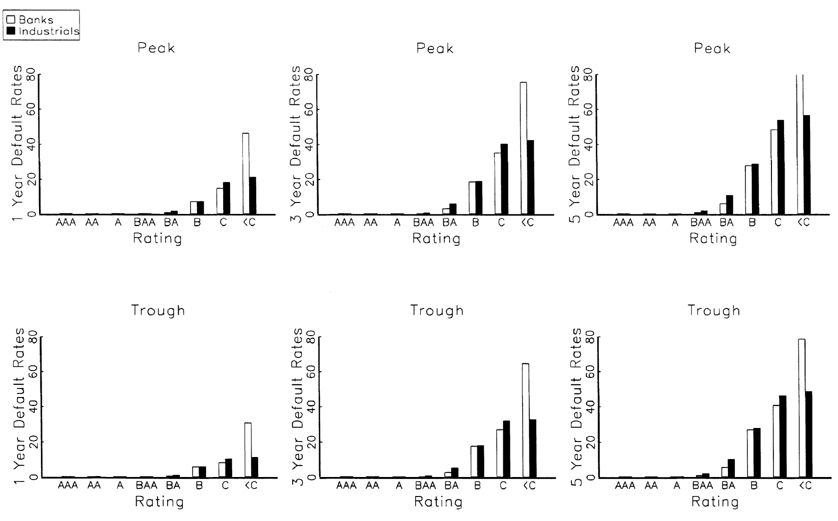

Fig. 1. Mean default rates are shown for industrial and banking obligors of different initial ratings over 1, 3 and 5 year horizons. The starting point is assumed to be either a business cycle peak or a business cycle trough. Changes in the business cycle are assumed to be driven by a temporally independent Markov chain, the transition matrix parameters of which are estimated from figures on real GDP. The fitted rating transition probabilities employed in the calculations are those implied by our ordered profit models when estimated on Moody's notional, senior, unsecured ratings reported between December 1970 and December 1997.

cycle peak (the first three panels) or a business cycle trough (the last three panels). The transition matrices employed are composed of fitted rating transition probabilities implied by our ordered probit models and the stochastic evolution of the business cycle is allowed for using a 9×9 transition matrix for current and lagged business cycle state as described above. We report results both for banks and for industrials.

The differences between the corresponding plots in the trough and peak figures diminish as the horizon grows. This is as one would expect as the importance of the initial state disappears as time goes by. Comparing banks and industrials in the plots, we find that lowly rated banks have relatively high default probabilities while the opposite is true of highly rated banks. This finding is consistent with our earlier observation that lowly graded banks have a very high probability of early default whereas for investment grade banks there is a kind of 'mean reversion effect' with up-grades relatively more likely than down-grades for low investment grade obligors and the opposite for high investment grade obligors.

4. Conclusion

When agencies like Moody's or Standard and Poor's attribute ratings to bond obligors, they are engaged in a complex judgmental process. (Details of how these judgments are made are described in Ederington and Yawitz, 1987.) The definitions of rating categories the agencies employ are explicitly non-quantitative and not directly linked to explicit probabilities of borrower delinquency. [8]

This complicates interpretation of econometric modelling of rating transitions since one may ask to what extent the results shed light on the true evolution of obligor credit standing and to what extent one is modelling the bureaucratic processes of a rating agency. This is particularly the case when one examines ratings for sub-categories like Japanese or UK obligors for which the past sample (at least since 1970) contains not a single default and scarcely even any declines into the speculative rating categories of sub-Baa.

Though difficult to interpret, the need to understand the stochastic behaviour of rating transitions has recently become a pressing practical matter given their increasing use as a key component of credit risk modelling techniques. Assessing the risk of illiquid bonds or loans for which mark-to-market values over time are not readily available is difficult and analysts have looked to

[8] House (1995) suggests the stress on non-quantitative methods is a means of excluding new entrants from the rating industry. Cantor and Packer (1994) describe the competitative pressures faced by the agencies and the way in which this affects their working practices.

ratings as an additional source of information (i) about the level of the value of a loan or bond, and perhaps more importantly (ii) about the distribution of changes in value.

It is the use of rating transition matrices to adduce the distribution of value changes which has motivated the present study. Our basic question has been: given that rating transition probabilities vary for different obligors and different stages of the business cycle, which are the most important dimensions of this variation? We examine the question (a) by calculating unconditional and conditional rating transition matrices in the standard way (supposing cross-sectional and temporal independence and that transitions are driven by simple constant-parameter multinomial models); and (b) by estimating ordered probit models in which transitions are driven by realisations of a latent variable which incorporates a series of dummies for obligor type and business cycle state.

What conclusions emerge from our study? Significant differences appear when one compares simple transition matrices estimated from rating transition data on different sub-samples of the post-1970 Moody's universe of rated entities. In particular, dimensions of variation which appear significant are banks versus industrials, US versus non-US obligors, and business cycle peaks versus troughs.

Ceteris paribus analysis of these effects using ordered probit models generally confirms their importance. The cross-country differences are confirmed for highly rated obligors but appear less important for non-investment grade issuers. Business cycle effects make an important difference especially for lowly graded issuers. Default probabilities in particular depend strongly on the stage of the business cycle.

Acknowledgements

We thank Reza Bahar and Bank of England colleagues for valuable comments and Angus Guyatt and Steve Grice for research assistance.

References

Altman, E.I., 1997. Rating migration of corporate bonds – comparative results and investor/lender implications. Mimeo. Salomon Brothers, New York.

Altman, E.I., Kao, D.L. 1992a. Rating drift of high yield bonds. Journal of Fixed Income 15–20.

Altman, E.I., Kao, D.L. 1992b. The implications of corporate bond ratings drift. Financial Analysts Journal 64–75.

Cantor, R., Packer, F., 1994. The credit rating industry. Federal Reserve Bank of New York Quarterly Review, 1–26.

Carty, L.V., 1997. Moody's Rating Migration and Credit Quality Correlation, 1920–1996. Special comment, Moody's Investors Service, New York.

Carty, L.V., Fons, J., 1993 Measuring Changes in Credit Quality. Moody's special report, Moody's Investors Service, New York.

Cheung, S., 1996. Provincial credit ratings in Canada: An ordered probit analysis. Working paper 96-6, Bank of Canada, Ottawa.

Credit Suisse Financial Products, 1997. CreditRisk+: Technical manual. Discussion paper, CSFP, New York.

Ederington, L., Yawitz, J., 1987. The bond rating process. In: Altman, E. (Ed.), Handbook of Financial Markets. Wiley, New York.

Greene, W.H., 1997. Econometric Analysis. Prentice-Hall, New York.

Helwege, J., Kleiman, P., 1996. Understanding aggregate default rates of high yield bonds. Federal Reserve Bank of New York Current Issues In Economics and Finance 2 (6), 1–6.

House, R. 1995. Rating the raters. Institutional Investor 53–66.

Jonsson, J.G., Fridson, M.S., 1996. Forecasting default rates on high-yield bonds. The Journal of Fixed Income 69–77.

JP Morgan, 1997. Creditmetrics – Technical Document. Discussion paper, JP Morgan, New York.

Lucas, D.J., Lonski, J.G., 1992. Changes in corporate credit quality 1970–1990. Journal of Fixed Income 7–14.

Standard and Poor's, 1998a. Ratings performance 1997 – stability and transition. Research report, Standard and Poor's, New York.

Standard and Poor's, 1998b. Ratings performance 1995 – stability and transition. Research report, Standard and Poor's, New York.

Wilson, T., 1997. Credit risk modelling: A new approach. Unpublished mimeo, McKinsey Inc., New York.