Valuing the strategic option to sell life insurance business: Theory and evidence

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Received 24 September 1998; accepted 20 March 2000

Abstract

We present a simple put option pricing procedure within an asset–liability valuation model that can be used to estimate the incentives facing stock-based life insurance firms to voluntarily sell their businesses under various operating and regulatory conditions. Estimates are derived for samples of 11 sold firms and 24 continuing Australian life insurance companies over a period of industry consolidation. The put option values interact with other actuarial and accounting components of the fair value of these life insurance firms and are used to assess the effectiveness of accounting and actuarial measures of capital, under static or dynamic based solvency testing models. © 2000 Elsevier Science B.V. All rights reserved.

JEL classification: G22

Keywords: Termination put option; Life insurance; Solvency testing

An earlier version of this paper was presented at a joint Lancaster–Manchester seminar.

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PII: S 0 3 7 8 - 4 2 6 6 ( 0 0 ) 0 0 0 9 7 - 2
1. Introduction

Recent years have witnessed a large-scale increase in the world-wide consolidation of financial services (Berger et al., 1999). Merton and Perold (1993) claim that these trends highlight the importance of management’s strategic ability to exit from existing businesses, particularly for life insurance firms since they have long-term fixed liabilities. It has also led to increased demands by capital market participants for the public disclosure of the fair value of the assets and liabilities of life insurance firms. Barth and Landsman (1995) define fair value as the amount at which an asset could be exchanged in a current transaction between willing parties. Vanderhoof (1998) notes that although accountants, actuaries, regulators and analysts generally agree that the fair value of assets can be taken at market value, estimating the corresponding fair value of liabilities is far more problematic.

In the actual marketplace, which is characterized by change, uncertainty and competitive interactions, management may have valuable flexibility to alter its initial operating strategy in order to capitalize on favourable opportunities or to react so as to mitigate losses. Trigeorgis (1996) links this managerial operating flexibility to financial options. Thus, viewing life insurance funds as containing a put option, the liabilities play the role of the exercise price while the fund assets plus shareholder expectations of future profits on selling policies (the spread) play the role of the underlying asset or stock price. Upton (1997) claims that option values are also potentially relevant to the fair valuation of life insurance firms.

This paper estimates management’s put option to sell life insurance business as a separate component of life insurance firm fair value in Babbel and Staking’s (1995) asset–liability model that distinguishes accounting and actuarial valuation elements. This calculation is potentially value-relevant to various decision-makers since an asset–liability valuation approach has recently been endorsed by the International Accounting Standard Committee (1999) as a basis for developing a uniform set of accounting principles for life insurance that is acceptable to both regulators and investors world-wide. The put value is based on a standard Merton (1973) two-variable, put option pricing model which Marcus (1985) extended to include a pension firm’s termination option. We show that this asset–liability valuation model yields an estimate of the put option value of a life insurance business, which is terminated only when that action is optimal for the firm. The put option values derived allow for the calculation of fair value-based measures of capital in which the termination

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2 The issue of life insurance policies gives firms the right to ‘sell’ the assets of the insurance policy plus a percentage of intermediary spread return to shareholders to the policyholders at a ‘price’ equal to the present value of life insurance liabilities. The gain to the firm equals the life insurance liabilities it transfers to the policyholders less the asset value the policyholders acquire.
decision of a stock-based life insurance firm is determined endogenously under various operating conditions. 3

The form and content implications of the put option prices for the strategic decision to sell life insurance business is empirically investigated by utilising liability, asset and return data of a sample of Australian life insurance companies, during a period of significant industry consolidation in 1987–1990. Consistent with our expectations, the calculated put option values are strongly associated with the termination decision facing the sample of firms, even after controlling for other explanatory variables. The intrinsic value of life insurance business, calculated as the market value of assets adjusted for the voluntary termination put option value, is also used to evaluate the relative effectiveness of accounting-based and actuarial-based capital measures under alternative static and dynamic-based models of solvency for continuing firms. Consistent with our predictions, we find that accounting-based measures of capital are more strongly associated with intrinsic value under a static solvency testing model, while actuarial-based measures of capital are more strongly associated with intrinsic value under a dynamic solvency testing model. These results lend empirical support to the asset–liability valuation model as a valid basis for developing international accounting principles for life insurance firms.

The rest of this paper is organised as follows. Section 2 provides the theoretical and institutional background to the study. Section 3 describes the empirical framework for the study. Section 4 describes the sample, key variables and empirical estimates of the put option values for a sample of Australian life insurance companies. Section 5 discusses the results of tests of the strength of association between the put option value and other (accounting and actuarial) elements that comprise the fair value of the life insurance firm. Finally, Section 6 provides summary and conclusion.

2. Theoretical and institutional background

This section briefly outlines the theoretical antecedents (Section 2.1) and overviews the institutional background (Section 2.2) required to understand the valuation methodology used to value a life insurance fund.

2.1. Theoretical antecedents

In a typical life insurance contract, a customer pays a cash premium to the life insurer in return for a contract promising a stream of state-contingent cash

3 Of course, certain specialised types of life insurance policies are often more complex than this, and typically include a bonus element. Wilkie (1987) analyses these types of policies by utilising an option pricing framework.
payments in the future. Fama and Jensen (1983) claim that issuing this contract creates another liability for the life insurer, the vast bulk of which are held by their customers rather than their investors. Berger et al. (1991) report that, if returns are certain, then there is widespread agreement that fair value is the present discounted ‘embedded’ value or market value equivalent. Commentators have noted that this actuarial calculation does not necessarily provide a sufficient understanding of the risk dynamics of the liability structure which affects management’s decision to sell life insurance business (Hogan, 1995; O’Brien, 1995).

An alternative options-based valuation methodology is based on the observation that the strategic management of a life insurance firm, together with the fixed nature of its contingent obligations, differentiates it from other organisations (Brennan, 1993). Modern finance theory recognises the role of contingent claims analysis in facilitating the reduction in variability of consumption, production or investment opportunities. Briys and De Varenne (1994) claim that the advantage of a contingent claims approach is that it captures the shareholders’ option to walk away if things go wrong. Previous researchers have recognised the formal correspondence between put options and term insurance policies by using contingent claims analysis to value insurance-based funds in other contexts (e.g. Wilkie, 1987; Briys and De Varenne, 1994).

However, while the analogy between put options and the option to sell life insurance business appears straightforward, the correspondence between the two is not at all clear regarding the effective time to maturity. Brennan (1995) notes that a standard option pricing approach to value real assets and liabilities using stochastic interest rate methodologies is also limited by the need to find a traded security whose return is perfectly correlated with the underlying cash flow claim. Since life insurance firms specialise in markets that insure individual, rather than market-wide risks, they cannot freely trade their assets and liabilities in transparent markets.

This fact leads Babbel (1997) to claim that standard stochastic interest rate option pricing valuation methodologies that are now coming into greater use (e.g. the deductive valuation approaches proposed by Doll et al., 1997) are in fact rather ill-suited to derive fair values for these financial institutions. Babbel and Staking (1995) propose an asset–liability valuation model that takes account of the specialized nature of a life insurance firm’s assets and liabilities, the major features of which is introduced in Section 3.

2.2. Institutional setting

The Australian life insurance industry provides an interesting setting to test Babbel and Staking’s (1995) valuation model because: (i) de-regulation in the late 1980s subsequently led to product competition from other financial ser-
vices firms; (ii) the Australian economy was in a deep recession and credit crisis which placed severe strains on life insurance firms in meeting capital adequacy requirements; (iii) until the 1990s, no significant barriers were placed on entry to or exit from the industry, which experienced growth and turnover in registrations and (iv) reporting life insurance business profitability to shareholders was not mandatory before 1997 (Klumpes, 1995).

The legal method of determining the solvency (or capital adequacy) of Australian life insurance firms was also unique and subject to change over this study period, which has a significant bearing on the calculation of the option to sell life insurance business. Approaches used to test the solvency of life insurance firms in Anglo-American countries may be characterised as being either ‘static’ or ‘dynamic’. Static tests focus on industry-wide conservative requirements as to the assumptions and methods by which reserves are to be set, and are currently used by regulators in UK and USA, based on Regulatory Accounting Principles (RAP). By contrast, dynamic tests imply firm-specific actuarial cash flow testing of liabilities and assets, including projections of new business to be sold in the future. Brender (1997) observes that these tests are currently required in Canada to both demonstrate solvency and report profitability to shareholders, while Freeman and Vincent (1991) claim that they are also used internally by firm management. In 1997 Australian regulators adopted dynamic solvency tests in order to ensure uniformity of regulatory and shareholder reporting.

Previously, the adoption of more stringent static-based solvency requirements imposed Australian regulatory authorities in 1992 significantly increased the incentives of many foreign-owned firms to sell their life insurance businesses. Solvency requirements for Australian life insurance firms had been relatively relaxed until the insolvency of two life insurance firms in November 1990, following a fraudulent take-over attempt. New legislation (a) increased capital requirements for life insurance firms (from A$2 to A$10 million, plus $5 million in restricted assets – Insurance Laws Amendment Act, 1991) and (b) restricted foreign ownership of insurance businesses to 15% of the insurers assets (Insurance Acquisitions and Takeovers Act, 1991).

These legislative changes increased barriers of entry to the insurance industry and resulted in a number of foreign owned firms divesting the ownership of Australian life insurance firms. During the period 1991–1993; (1) six proposals for the sale of life insurance businesses or changes in life company ownership were approved; (2) no new life insurance firms were registered and (3) total registered life insurance firms drop from 58 to 48. The new Life Insurance Act, 1995, effective in 1997, prohibited registration of foreign-owned life insurance firms other than via domestic incorporation. According to the OECD (1999), Australia is currently the only OECD country that restricts international life insurance business operations in this way.
These changes accelerated the consolidation of the Australian life insurance industry. From 1987 to 1996, one-half of all firms merged, sold or otherwise became insolvent. Commentators (e.g. Salmon and Fine, 1990; Miles and Fraser, 1990; Friedman, 1990) note that these issues were also of major strategic concern to the management of many Australian life insurance firms at this time.

3. Empirical framework

This section develops predictions concerning the strength of association between various components of life insurance firm value, conditioned by static versus dynamic tests of solvency. Babbel and Staking (1995) propose an asset–liability based ‘financial institution’ valuation model of the market value of a life insurance firm, comprising three components: (1) the franchise value of capital (defined as the present value of future earnings, based on actuarial-based ‘embedded value’); (2) the liquidation value of capital (market value of tangible assets less the present value of liabilities calculated in accordance with RAP) and (3) the put option value arising from regulatory guarantees.

In this study we examine the effectiveness of the Babbel and Staking (1995) valuation model by developing predictions about the strength of association between each of these components of life insurance firm value, for a sample of Australian life insurance firms, under both static and solvency testing conditions. For empirical testing purposes, the intrinsic value of life insurance business, calculated as the market value of assets adjusted for the voluntary termination put option value, is used to evaluate the relative effectiveness of accounting-based (RAP) and actuarial-based capital measures under alternative static and dynamic-based models of solvency for continuing firms. We predict that the RAP-based capital measure is more strongly associated with intrinsic value under a static solvency testing model, while the actuarial-based capital measure is more strongly associated with intrinsic value under a dynamic solvency testing model.

For empirical testing purposes, we therefore specify the fair value of a stock-based life insurance firm as comprising three elements: The put option value (\(PUT\)), the franchise value (\(FRANCHV\)) and the liquidation value (\(RESERVES\)). The difference between the market value of equity and the put option value equals the intrinsic value of capital, which Barth et al. (1992) claim is the theoretical measure that should be utilised in any termination value calculation. Intrinsic values are benchmark performance measures used to assess the adequacy of alternative risk-based capital measures for banks (e.g. Coulombe et al., 1997). The intrinsic value of life insurance firms (\(INTV\)), calculated as the market value of equity less the put option value estimates, can
be used as a benchmark to assess the effectiveness of alternative RAP-based and actuarial-based capital measures.

According to Babbel and Staking (1995), the fair value of life insurance firms comprises the liquidation value (in this case RESERVES plus INTV), the franchise value (FRANCHV) and the put option value (PUT). For empirical testing purposes, we assume that there is a linear relationship among these variables and thus assume that PUT for any firm i at time t can be represented by an OLS regression model as set out in Eq. (1):

\[
PUT_{it} = \gamma_1 + \gamma_2 RESERVES_{it} + \gamma_3 FRANCHV_{it} + \gamma_4 INTV_{it} + \epsilon_{it}.
\]

We would generally expect \( \gamma_3 \) to be negative and statistically significant. This is because an increase in the riskiness of the firm, as proxied by a higher put option value, is predicted to be negatively associated with both the franchise value and liquidation value. However, while an increase in risk increases the ratio of intrinsic value to liquidation value, a decrease in risk again results in a higher ratio. Thus the franchise value and the put option value are offsetting in their impact on liquidation value, which mitigates the strength of association between PUT and both RESERVES and INTV. Babbel and Staking (1995) claim that their theoretical prediction is corroborated by empirical evidence from the US multi-line insurance industry.

To assess the relative effectiveness of the valuation model to the alternative static and dynamic testing solvency models, we first examine the strength of association between the intrinsic value of equity and RAP-based and actuarial-based capital measures. However prior theory does not provide guidance as to the predicted form of the relationship between these values. Accordingly the following alternative cross-sectional OLS regression models are estimated:

\[
INTV_{it} = \delta_1 + \delta_2 RAPCAP_{it} + \epsilon_{it},
\]

or

\[
INTV_{it} = \delta_1 + \delta_2 RESERVES_{it} + \delta_2 RAA_{it} + \epsilon_{it},
\]

where INTV is the intrinsic value and RAPCAP is the traditional RAP-based capital measure that is used for periodic regulatory periodic reporting of performance to the Insurance and Superannuation Commission (the book value of shareholders’ equity, retained earnings and surplus) in the year ended 31 December 1990.

The actuarial-based capital measure (which actuaries claim is useful for both management planning and control and for profit reporting purposes) by contrast involves complex actuarial estimates of profits in excess of liabilities emerging from existing business into the future. This measure is therefore not ordinarily acceptable under Australian RAP and during the study period and was not reported by all firms. However it is possible to approximate the actuarial-based capital measure by examining the association of INTV with both
RESERVE, the actuarially-determined level of reserves available to support life insurance businesses, and total firm risk-adjusted tangible assets (RAA).

Consistent with our earlier predictions, $\beta_2$ is expected to be positive for the RAP-based capital measure, while $\beta_3$ is expected to be negative and $\beta_2$ is expected to be positive for the actuarial-based capital measure. The rest of this study examines the form and content implications of this model by developing and testing measurable proxies for each of the key variables defined in Eqs. (1)–(2b).

4. Sample selection and key variables

This section discusses the sample and develops measurable proxies for each of the key variables of the valuation model that are defined in Section 3. Section 4.1 describes the data selection procedure and data sources. Section 4.2 presents descriptive statistics for key variables used in the empirical tests. Section 4.3 presents estimates of the value of the put option value under alternative solvency testing scenarios of interest.

4.1. Sample selection and data sources

A total of 54 federally-registered Australian life insurance firms supplied statistics to the Australian life insurance industry regulator, the Insurance and Superannuation Commission (ISC), in June 1988. For empirical testing purposes firms were divided into three sub-sample groups: A control, take-over and insolvency group. For inclusion in the control sample, firms had to be continually operating life insurance funds for 18 months before and after the study period (1989–1996). Life insurance firms could not be mutuals or owned by mutual firms (10 firms), subsidiaries of other registered Australian life insurance firms (6 firms) or substantially owned by governments (4 firms). This left a final sample of 35 firms. Of these, 2 life insurance funds subsequently became technically insolvent under the provisions of the Life Insurance Act, 1945 (Occidental and Regal) in 1991. Another 9 were voluntarily terminated as a result of take-over or merger in 1991–1992 (Adriatic, Australian Eagle, Fidelity Life Insurance, Friends Provident, Guardian Royal Exchange, Investors Life, Liberty, NZI and Victory). This left a final take-over sample of 11 firms and a control sample of 24 continuing firms.

The ISC maintains semi-annual statistics on the Australian life insurance industry. This includes annualised data on life insurance fund assets, liabilities, fund surplus, shareholders’ equity and the assumed net and gross interest rates earned on policyholder fund liabilities. Data were obtained from the half-yearly Statistical Bulletins issued from December 1987 to September 1996 (ISC,
4.2. Development of key variables

This section describes the development of measurable proxies for key variables of the asset–liability fair valuation model, based on Australian life insurance firm data, under two scenarios of interest. First, ‘static-based’ estimates of the put option value are calculated separately for three discrete 3-year sub-periods: (1) an initial period of industry rationalisation, culminating in the insolvency of the two insurance firms and before the enactment of new restrictive legislation (December 1988–December 1990), (2) a further period of consolidation immediately following the insolvencies, when political threats were initially made to enact legislation placing new restrictions on capital and change of ownership (January 1991–December 1993) and (3) a subsequent period of recovery following the replacement of the Life Insurance Act, 1945 with new legislation (Life Insurance Act, 1995) that formally restricted foreign ownership of Australian life insurance firms (January 1994–September 1996). Second, we also examine the sensitivity of the put option values, when calculated over the entire sample period 1988–1996, under alternative ‘static’ versus ‘dynamic’ solvency tests.

Empirical proxies used for each variable are outlined below.

\( INTV \) is the intrinsic value of equity, defined as the book value of equity \( (RAPCAP) \) less the put option value. \(^4\) It should be noted that both \( INTV \) and \( RAPCAP \) are defined from regulatory sources (ISC, 1990). As might be expected, \( RAPCAP \) is considerably higher for continuing firms than it is for terminating firms, and increases in subsequent periods as might be expected following the introduction of new capital adequacy requirements.

\( RAA \) is the total value of risk adjusted assets supporting the life insurance funds, while \( RESERVES \) are total reserves for the life insurance firm. Risk-adjusted assets include all derivative instruments and other speculative investments. Consistent with \( INTV \), both \( RAA \) and \( RESERVES \) are significantly higher for continuing firms than for terminating firms, and increases in subsequent periods.

\(^4\) These were based on actuarial estimates of ‘appraisal value’ or ‘embedded values’ of a ‘going concern’ life insurance business, without explicitly considering the option to terminate.

\(^5\) During the study period there were no professional accounting standards governing the principles of accounting to be used by life insurance firms. However, Klumpes (1995) finds that the Australian life insurance industry developed professional accounting standards in 1985 which effectively required life insurance firms submitting annual reports to the ISC to use market value accounting.
FRANCHV is a proxy for franchise value, defined as the present value of future profits on existing business. Although some Australian life insurance firms estimated the profits emerging from existing businesses into the future during this period (using ‘Margin on Services’ methodology) this value is not observed for all sample firms. The difference between the periodic, actuarial estimated surplus of the fund, less the accounting-based profit or loss reported for that fund, is calculated and proxied for FRANCHV. FRANCHV is higher for terminating firms and is negative for the 1993 sample of continuing firms.

PUT is the value of the put option facing proprietary Australian life insurance firms to voluntarily terminate life insurance business under a static solvency testing regulatory regime. The methodology used to calculate the fair values of these put termination options is described in Section 4.3.

Table 1 reports the descriptive statistics for the samples of continuing and terminating firms for the three years ended 31 December 1990 (Panels A and B) and for continuing firms only for the two subsequent study periods (Panels C and D). Although Table 1 indicates that most individual continuing firms derive little value from the option to terminate their businesses, a small number of ‘problem firms’ derive considerable value. These tend to be the larger firms where the weighted average of the termination option values are substantially greater than the means. Looking across the sub-samples, consistent with our expectations, terminating firms (Panel A) derive considerably more value from the put value than the continuing firms (Panel B). Further, the put option values are considerably higher for continuing firms in the second period of consolidation (Panel C) than in the subsequent period of recovery (Panel D) as we would expect. Panel E reports summary statistics of the empirical estimates of the put option value for the 24 continuing firms, based on both the static and dynamic solvency testing model assumptions, over the entire period 1988–1996. As expected the dynamic put option values are generally lower and less volatile.

4.3. Estimating the put option value

The valuation methodology employed to derive the empirical estimates of the put option value is consistent with the geometric model of Marcus (1985), and is outlined in Appendix A. The estimates are based on parameter estimates from a small number of observations and assumptions, and are therefore subject to error since standard deviations and covariances for the processes concerned were calculated directly from each insurance firm time series. While these features of this model are required to satisfy the uniformity and conservatism requirements of static-based solvency testing, they do not shed light

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6 The analytic solutions presented in Appendix A on a static model of solvency testing, which imposes impose passive behaviour on the life insurance firm in that shareholders’ funding of accrued life insurance fund liabilities is always equal to a fixed fraction of current assets, S.
Table 1
Descriptive statistics (in $Am): continuing firms (n = 24) and terminating firms (n = 11)*

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Continuing firms – 3 years ended 31 December 1990</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PUT</td>
<td>0.5</td>
<td>1.6</td>
<td>0</td>
<td>7.7</td>
</tr>
<tr>
<td>INTV</td>
<td>40.2</td>
<td>121.1</td>
<td>-3.7</td>
<td>598.2</td>
</tr>
<tr>
<td>RAA</td>
<td>1012.5</td>
<td>2154.4</td>
<td>10.9</td>
<td>9923.8</td>
</tr>
<tr>
<td>RESERVES</td>
<td>72.1</td>
<td>207.2</td>
<td>0</td>
<td>1011.8</td>
</tr>
<tr>
<td>RAPCAP</td>
<td>37.7</td>
<td>121.0</td>
<td>1.2</td>
<td>598.2</td>
</tr>
<tr>
<td>FRANCHV</td>
<td>8.6</td>
<td>39.9</td>
<td>0</td>
<td>171.1</td>
</tr>
<tr>
<td><strong>Panel B: Terminating firms – 3 years ended 31 December 1990</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PUT</td>
<td>2.2</td>
<td>3.4</td>
<td>0</td>
<td>10.2</td>
</tr>
<tr>
<td>INTV</td>
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<td>43.7</td>
<td>-6.8</td>
<td>146.3</td>
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<tr>
<td>RAA</td>
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<td>354.4</td>
<td>15.4</td>
<td>1160.5</td>
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<tr>
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<td>146.3</td>
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<tr>
<td>FRANCHV</td>
<td>21.3</td>
<td>101.9</td>
<td>-72.6</td>
<td>190.7</td>
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<tr>
<td><strong>Panel C: Continuing firms – 3 years ended 31 December 1993</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>PUT</td>
<td>31.6</td>
<td>147.9</td>
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<tr>
<td>INTV</td>
<td>25.5</td>
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<td>843.7</td>
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<tr>
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<td>RAPCAP</td>
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<td>169.8</td>
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<td>843.7</td>
</tr>
<tr>
<td>FRANCHV</td>
<td>2.8</td>
<td>72.4</td>
<td>-231.7</td>
<td>167.2</td>
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<tr>
<td><strong>Panel D: Continuing firms – 3 years ended 31 December 1996</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>PUT</td>
<td>14.5</td>
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<td>3941.3</td>
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<td>353.1</td>
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<td>8.3</td>
<td>1021.0</td>
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<tr>
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<td>50.9</td>
<td>161.0</td>
<td>-48.1</td>
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<tr>
<td><strong>Panel E: Continuing firms – 9 years ended 31 December 1996</strong></td>
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<td></td>
<td></td>
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<tr>
<td>PUT (static model)</td>
<td>405.7</td>
<td>1020.9</td>
<td>0</td>
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<tr>
<td>PUT (dynamic model)</td>
<td>209.8</td>
<td>536.0</td>
<td>0</td>
<td>2318.0</td>
</tr>
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</table>

*This table reports the summary descriptive statistics for the variables used in the study. PUT is the value of put option to voluntarily terminate life insurance business, calculated to accord with Marcus (1985). INTV is the intrinsic value of equity, defined as GAAPCAP less option value. RAA is the risk-adjusted assets, obtained from Insurance and Superannuation Commission Statistics, Table 13. RESERVES is the total reserves, obtained from Insurance and Superannuation Commission Statistics, Table 13. RAPCAP is the accounting-based measure of capital, comprising book value of shareholders’ equity, retained earnings and surplus, obtained from Insurance and Superannuation Commission Statistics, Table 13. FRANCHV is a proxy for franchise value, representing the average difference between economic surplus and accounting revenue accounts for the last three years, obtained from Insurance and Superannuation Commission Statistics, Table 5.
into the unique net cash-flow characteristics or operating circumstances facing individual life insurance firms.

An alternative dynamic-based solvency testing approach implies instead that the put option value should be based on modelling the net cash flow as an Arithmetic Process. Just as the optimal put level and resultant put price can be calculated in a geometric framework assuming a penalty that is proportional to $E$, the processes in question can be modelled under Arithmetic Processes if a fixed $ penalty were applicable. Considering the legislation in effect when the data was taken (effectively an $15 million penalty for default as oppose to a 30% of $E$ charge) this may prove more realistic. Consequently, the analysis was repeated assuming Arithmetic, not Geometric processes and a fixed rather than proportional penalty yielding a second set of put values. This so-called dynamic model of insolvency, is compared to the static model with proportional costs.

This alternative model specification has a number of advantages over the static-based model. First, it more reasonably approximates an actuarial ‘best estimate’ valuation of an entire given life insurance fund portfolio. Second, it is also more consistent with the asset–liability modelling approach used by holding companies of financial firms to value similar types of financial products, such as banking deposits (Brender, 1997).

Further insight into the effect of the differing regulatory assumptions about solvency on managerial propensity to sell life insurance business can be obtained by plotting firm-by-firm the static and dynamic-based put option values (Fig. 1). The diagram shows that, while both put option values are consistent for a majority of the sample firms, the static-based value is relatively higher for some firms. This implies that the strategic option to sell life insurance business,
at least for some firms, is significantly affected by regulatory discretion over the method used to determine solvency.

5. Empirical tests of effectiveness

This section first discusses various cross-sectional empirical tests of the association between the put option values estimated in Section 4 with the termination decision for the sample firms, after controlling for with other variables that may affect the termination decision. We then evaluate the relative effectiveness of the asset–liability valuation model under alternative static and dynamic-based solvency modelling assumptions for the sample of continuing firms.

5.1. Sensitivity tests of the put option value

We first conducted sensitivity tests to examine the robustness of the empirical estimates of $PUT$ as reported in Table 1. One robustness check on the put option estimates is to examine whether the put values can be used to distinguish between the samples of continuing and terminating Australian life insurance firms during the first three-year sub-period of industry rationalisation (i.e. in 1988–1990). Table 2 reports a logistic regression of the firms’ decision whether or not (i.e. where 1 = termination; 0 = continuation) to terminate life insurance business during the initial sample period 1988–1990, following the 11 firms which terminated their life insurance businesses. Consistent with our expectations and casual mean empirical differences observed in Table 1, $PUT$ is significantly higher for terminating firms at the 7% level. These results hold even after controlling for other financial characteristics related to solvency, reserves and franchise values usually associated with the termination decision. The overall chi-squared statistic is also significant at the 2% level.

Table 2
Logistic model of decision to sell life insurance business$^a,b$

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>$PUT$</th>
<th>$FRANCHV$</th>
<th>$RESERVES$</th>
<th>$RAPCAP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected sign</td>
<td>?</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Coefficient</td>
<td>–0.844</td>
<td>0.490</td>
<td>0.004</td>
<td>–0.057</td>
<td>0.012</td>
</tr>
<tr>
<td>Significance</td>
<td>0.177</td>
<td>0.042</td>
<td>0.579</td>
<td>0.220</td>
<td>0.360</td>
</tr>
</tbody>
</table>

Chi-squared statistic (H0: all parameters except intercept are zero): 9.797 ($p = 0.044$)

$^a$This table presents a logistic multivariate test of the association between the decision to terminate life insurance business in the three years ended 31 December 1990 and put option values and other components of life insurance firm fair valuation. The sample comprises 24 continuing terminated their life insurance businesses within 18 months of this period. Number of observations = 35; Dependent variable = 1 if terminated firm ($n = 11$) and Dependent variable = 0 if continuing firm ($n = 24$).

$^b$Variables are defined in Table 1.
As a final sensitivity check, we examine the empirical association of the estimated put option values with other components of life insurance fair value for the sample of continuing firms. Empirical tests of the strength of association between $PUT$ and each of the other components of life insurance firm fair value, as specified by empirical model (1), for the sample of continuing firms in each of the three sub-periods, is reported in Table 3.

As predicted, there is a statistically significant negative association with $FRANCHV$ and $INTV$ for continuing firms, but only for the 1993 sub-period of industry consolidation ($\text{Adjusted } R^2 = 0.954; F \text{ value} = 159.87$). This is to be expected, since the magnitude of the put option values were lower for continuing firms in both the earlier and subsequent sub-periods as reported in Table 1. The results for the relationship between $PUT$ and $RESERVES$ and $INTV$ are more equivocal, perhaps reflecting the fact that some continuing firms as well as terminating firms have high put values. The low explanatory power of the model for the 1988–1990 sub-period is fully in line with our expectations, since only those firms that subsequently sold their businesses would be expected to show significant relationships between all components of value. However our results for the subsequent period 1994–1996 are more equivocal. The non-significant relationships between $PUT$ and both $RESERVES$ and $FRANCHV$ are expected, since most firms during this recovery period did not

Table 3
Estimation of the association of termination option value with other components of life insurance firm fair value for continuing firms ($n = 24$)$^{a,b}$

<table>
<thead>
<tr>
<th>Component</th>
<th>1990 Coefficient</th>
<th>1993 Coefficient</th>
<th>1996 Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$\gamma_1$</td>
<td>0.544 ns</td>
<td>15.344 0.03**</td>
</tr>
<tr>
<td>RESERVES</td>
<td>$\gamma_2$</td>
<td>0.003 ns</td>
<td>0.126 0.00***</td>
</tr>
<tr>
<td>FRANCHV</td>
<td>$\gamma_3$</td>
<td>-0.003 ns</td>
<td>-0.182 0.10*</td>
</tr>
<tr>
<td>INTV</td>
<td>$\gamma_4$</td>
<td>-0.006 ns</td>
<td>-0.976 0.00***</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td></td>
<td>-0.14</td>
<td>0.953</td>
</tr>
<tr>
<td>Model $F$</td>
<td></td>
<td>0.09</td>
<td>156.29</td>
</tr>
<tr>
<td>Prob &gt; $F$</td>
<td></td>
<td>ns</td>
<td>0.00***</td>
</tr>
</tbody>
</table>

---

$a$ This table presents estimates of Eq. (1), measuring the association between the termination option, franchise and liquidation components of life insurance firm fair value. The sample comprises 24 continuing Australian life insurance firms and the tests are conducted over three year periods: 1988–1990; 1991–1993 and 1994–1996.

$b$ Variables are defined in Table 2.

ns: Not statistically different from zero.

$^*$ Statistically different from zero at 0.10.

$^{**}$ Statistically different from zero at 0.05.

$^{***}$ Statistically different from zero at 0.01.
exhibit signs of weak capital. However we did not expect the significant relationship between INTV and PUT to continue across all periods. The overall model is also statistically significant (Adjusted $R^2 = 0.528$; $F$ value $= 9.57$).

5.2. Effectiveness tests

The above tests suggest that the static solvency testing model-based put option values are reasonably robust to alternative explanations for the termination decision. We now examine the effectiveness of the asset–liability model as a basis for providing the fair valuation, for the sample of 24 continuing firms, by examining the strength of association between intrinsic value and RAP-based and actuarial-based capital measures. Results from the tests of the strength of association between these key variables, as specified by the empirical model (2), appear in Table 4.

Table 4
Estimation of the association of intrinsic value with alternative measures of capital for continuing firms ($n = 24$)\textsuperscript{a,b}

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$INTV_t = \delta_1 + \delta_2 RAPCAP_{it} + \varepsilon_{it}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>$\delta_1$</td>
<td>$-0.524$</td>
<td>$-33.438$</td>
<td>ns</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>$RAPCAP$</td>
<td>$\delta_2$</td>
<td>$1.000$</td>
<td>$1.032$</td>
<td>$0.00^{***}$</td>
<td>$0.839$</td>
<td>$0.00^{***}$</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td></td>
<td>$0.999$</td>
<td>$0.565$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model $F$</td>
<td></td>
<td>$123054$</td>
<td>$30.91$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob $&gt; F$</td>
<td></td>
<td>$0.00^{***}$</td>
<td>$0.00^{***}$</td>
<td></td>
<td></td>
<td>$0.00^{***}$</td>
</tr>
</tbody>
</table>

\textbf{For three years ended 31 December}

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$INTV_t = \delta_1 + \delta_2 RESERVES_{it} + \delta_3 RAA_{it} + \varepsilon_{it}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>$\delta_1$</td>
<td>$1.238$</td>
<td>$-13.467$</td>
<td>ns</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>$RESERVES$</td>
<td>$\delta_2$</td>
<td>$0.740$</td>
<td>$0.136$</td>
<td>$0.01^{***}$</td>
<td>$-0.138$</td>
<td>$0.04^{**}$</td>
</tr>
<tr>
<td>$RAA$</td>
<td>$\delta_3$</td>
<td>$-0.017$</td>
<td>$-0.003$</td>
<td>ns</td>
<td>$0.041$</td>
<td>$0.00^{***}$</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td></td>
<td>$0.954$</td>
<td>$0.538$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model $F$</td>
<td></td>
<td>$243.53$</td>
<td>$14.39$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob $&gt; F$</td>
<td></td>
<td>$0.00^{***}$</td>
<td>$0.00^{***}$</td>
<td></td>
<td></td>
<td>$0.00^{***}$</td>
</tr>
</tbody>
</table>

\textsuperscript{a}This table presents estimates of Eqs. (2a) and (2b), measuring the association between the intrinsic value and either RAP-based or actuarial-based (comprising capital reserves within the fund together with a measure of risk-adjusted assets) measures of capital. The sample comprises 24 continuing Australian life insurance firms and the tests are conducted over three year periods: 1988–1990; 1991–1993 and 1994–1996.

\textsuperscript{b}Variables are defined in Table 2.

ns: Not statistically different from zero.

** Statistically different from zero at 0.05.

*** Statistically different from zero at 0.01.
Consistent with expectations, the intrinsic value of equity for the continuing firms is significant and positively associated with the RAP-based fair value measure of capital in all three sub-periods. The overall model is also significant at the 1% level ($R^2 = 36\%$; $F$ value 14.193). However the results are more equivocal for the actuarial-based fair value model. Both reserves and RAA are consistent with predictions in the early sub-periods, but not in the 1996 sub-period. The overall model is statistically significant at the 1% level in all three periods. This result is expected, since the estimated put option values used in the empirical tests are based on a static solvency testing model, which relies heavily upon RAP-based asset and liability data.

Table 5 reports the equivalent results presented in Table 4, but now utilises the entire period of 1988–1996 for the sample of the 24 continuing firms. This is a test of the effectiveness of put values calculated on the basis of alternative static and dynamic-based solvency testing models. The RAP-based capital measure ($RAPCAP$) appears to be most sensitive to the choice of solvency model.

Table 5
Estimation of the effectiveness of alternative put option value of intrinsic value with GAAP measure of capital for continuing firms ($n = 24$)$^{a,b}$

<table>
<thead>
<tr>
<th>Static model</th>
<th>Dynamic model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coeff.</strong></td>
<td><strong>P-value</strong></td>
</tr>
<tr>
<td>$INTV_{it}$</td>
<td>$\delta_1$</td>
</tr>
<tr>
<td></td>
<td>$\delta_2$</td>
</tr>
<tr>
<td>$RAPCAP$</td>
<td></td>
</tr>
<tr>
<td>Model $F$</td>
<td></td>
</tr>
<tr>
<td>Prob $&gt; F$</td>
<td></td>
</tr>
</tbody>
</table>

$INTV_{it} = \delta_1 + \delta_2 RAP_{it} + \delta_3 RAA_{it} + \epsilon_{it}$

(2a)

| **Coeff.**   | **P-value**  | **Coeff.**   | **P-value**  |
| $INTV_{it}$  | $\delta_1$  | -16.295      | ns           | 55.837       | ns           |
| $RESERVES$   | $\delta_2$  | 0.521        | 0.00***      | 0.920        | 0.01***      |
| $RAA$        | $\delta_3$  | 0.001        | ns           | -0.127       | 0.00*        |
| Adj $R^2$    |              | 0.902        |              | 0.837        |              |
| Model $F$    |              | 107.40       |              | 60.08        |              |
| Prob $> F$   |              | 0.00***      |              | 0.00***      |              |

$a$ This table presents estimates of the effectiveness of alternative measures of intrinsic value using either static or dynamic capital testing models to calculate the put option value. As for Table 3, the tests measure the association between intrinsic value and either RAP-based measure of fair-value or actuarial-based measure of fair value comprising capital reserves within the fund together with a measure of risk-adjusted assets. The sample comprises 24 continuing Australian life insurance firms over the entire sample period of 1987–1996.

$^{b}$ Variables are defined in Table 2.

$^{*}$ Statistically different from zero at 0.10.

$^{***}$ Statistically different from zero at 0.01.
testing model. While the results for the static model are entirely consistent with those reported in Table 4 in terms of their direction and statistical significance, the coefficient for the RAP-based capital measure (Eq. (2a)) reported in Table 5 is negative and statistically significant for the dynamic testing model. Moreover, the goodness-of-fit measure (Adjusted $R^2$) is significantly higher for the static model. Again this result is consistent with our expectations that the strength of association between $RAP_{CAP}$ and $INT_V$ is conditioned by the choice of solvency testing model.

By contrast, coefficients of both actuarial-based variables (Eq. (2b)) are consistent for both models, but $RAA$ is only statistically significant for the dynamic testing model. The statistical significance of the relationship between these variables is consistent with our prediction that actuarial-based measures of capital are more strongly associated with $INT_V$ under a dynamic solvency-testing model. However the overall goodness-of-fit (Adjusted $R^2$) for Eq. (2b) under both models is relatively high. This result is consistent with our expectation that accounting component of life insurance firm valuation are more likely to be sensitive to the choice of solvency testing model than the actuarial component.

6. Conclusion

This paper estimates components of the fair value of financial services firms that represent options to voluntarily sell their life insurance businesses. These put options were strategically valuable for managers of at least some Australian life insurance firms during the early 1990s, a period featured by insolvency, extensive competition and subsequent political threats to discriminate against foreign-owned life insurance firms. Empirical estimates of put option values and other components of life insurance firm value are calculated for a sample of 24 continuing and 11 terminating Australian life insurers during the period 1988–1996, a period of intense product competition and industry rationalisation leading to consolidation and recovery. The put option values appear to be robust to various sensitivity tests, which demonstrate that put option values distinguish between the samples of terminating and non-terminating firms and are associated with other components of the fair value of life insurance businesses.

The put options values, when incorporated into a broader valuation model of financial institutions to calculate the intrinsic value of equity, can be used as a benchmark to assess the effectiveness of alternative static and dynamic solvency test models. However the effectiveness of these models are sensitive to the use of accounting or actuarial-based capital measures of the fair value of life insurance businesses. The International Accounting Standards Committee, in developing a new accounting standard for life insurance contracts, is currently
considering both measures. The empirical results suggest that the IASC should require life insurance firms to report both measures under the new accounting standards.

The empirical results also have broader implications for management. The option of firms to voluntarily sell funds is potentially important, and appears to provide management with a valuable decision tool to evaluate their option to terminate or acquire life insurance business. Since the models are relatively easy to calculate on the basis of publicly available information and as the accounting academic and professional community gradually comes to be more familiar with them, we expect that these estimates will gradually come into greater use.

The analysis presented in this paper can be extended in a number of dimensions. First, the put option values can be used to provide reliable estimates of the valuable option to terminate investment in life insurance companies and other kinds of financial intermediary which have fixed redeemable contingent liabilities. Second, when used in conjunction with a model of risk capital developed by Merton and Perold (1993), this analysis can be applied to analyse the financing, capital budgeting and risk-management decisions of other types financial services firms. More research is needed to validate these results for other samples of life insurance firms, other kinds of financial service firms, and in other institutional settings.

Acknowledgements

We acknowledge preliminary discussions with Mahmoud Ezzamel, T.S. Ho and Bob Scapens.

Appendix A. Methodology for calculating put option termination value

Assume that a life insurance fund comprises a portfolio of simple endowment policies.\(^7\) \(A\) denotes the total, actuarially-determined present value of their accrued liabilities. \(F\) denotes the market value of assets in the insurance fund and \(0.3E\) denotes the proportion of shareholder’s capital funds that are tied up in the fund. At a voluntary termination by take-over, and assuming that the fund is sufficiently funded \((F + 0.3E > A)\), the firm gains \(F\), transfers assets of value \(A\) to the acquiring firm. Net proceeds are

\[
F - \min(A, F + 0.3E)
\]  

\(^7\) Negative values for \(A\) or \(0\) indicate non-finite values for the put option value.
or equivalently,
\[ F - A + \max[A - (F + 0.3E), 0]. \tag{A.2} \]

Expression (A.2) highlights the nature of the firm’s put option. Its net insurance policy liability is \( F - A \); however, at the termination date it can transfer its liability of \( A \) to the acquiring firm in return for only \( F + 0.3E \). To solve for the value of a life insurance fund it first is necessary to specify the dynamics for accrued liabilities and the assets backing the fund. Let \( S \) denote the sum \( F \) a constant fraction 0.3\( E \). \( S \) follows the diffusion process

\[ dS = (C_S + \alpha_S)S \, dt + \sigma_S S \, dz_S, \tag{A.3} \]

where \( \alpha_S \) is a standard drift term attributable to the normal rate of return on the insurance fund’s invested assets, and where \( C_S \) is the rate (as a fraction of \( S \)) of regulatory required firm funding of the insurance fund liabilities. Denote by \( \alpha_A \) the expected rate of return on a bond with a payoff stream identical to the expected growth rate in the present value of accrued fund liabilities. If interest rates were non-stochastic, then \( \alpha_A = r \). Denoting the net growth rate in termination values attributable to demographic factors as \( C_A \), the total growth rate in \( A \) would be \( C_A + r \). In the non-stochastic steady state, \( C_A \) equals \( r \), and \( A \) would remain constant. The evolution of \( A \) is summarised by the process

\[ dA = (C_A + \alpha_A)A \, dt + \sigma_A A \, dz_A. \tag{A.4} \]

The stochastic component of (A.4) is due to uncertainty regarding long-term interest rates and future liabilities. Denote the correlation coefficient between \( dz_A \) and \( dz_S \) as \( \rho \).

Following the analysis in Merton (1973) and letting \( P(A, S) \) denote the value of the life insurance put, one can show that \( P \) must satisfy the partial differential equation

\[
\frac{1}{2} P_{AA} A^2 \sigma_A^2 + \frac{1}{2} P_{SS} S^2 \sigma_S^2 + P_{AS} A \sigma_A \sigma_S \rho - rP + (r + C_A)AP_A \\
+ (r + C_S)SP_S = 0, \tag{A.5}
\]

where subscripts on \( P \) denote partial derivatives and \( r \) denotes the rate of return on instantaneously riskless bonds. The terms \( C_A \) and \( C_S \) have effects analogous to those of (negative) proportional dividends in the standard option pricing model.

The boundary conditions for \( P \) are:

(a) At a point of exercise of the put (i.e. termination of the plan), \( P = A - S \).
(b) The limit of \( P \) as \( S \) approaches infinity is zero.
(c) The limit of \( P \) as \( A \) approaches zero is zero.
(d) The rule for voluntary termination is chosen to maximise the value of the life insurance put option.
For general specifications for \( C_A \) and \( C_S \), (5) must be solved numerically (see Section 3). In the special case that \( C_A \) and \( C_S \) are constant, (5) has an analytic solution that can be shown to have the general form (McDonald and Siegel, 1986):

\[
P(A, S) = (1 - K)A(S/A)^{1/2}K^{-\varepsilon},
\]

where \( K \) is the ratio of \( S/A \) at which the option is exercised. Eq. (A.6) will satisfy p.d.e. (A.5) for

\[
\varepsilon = -\left[\frac{C_S - C_A}{\sigma^2} - \frac{1}{2} - 2 \frac{C_A}{\sigma^2}\right]^{1/2} + \left(\frac{1}{2} - \frac{C_S - C_A}{\sigma^2}\right),
\]

\[
\sigma_2 = \sigma_A^2 + \sigma_S^2 - 2\rho\sigma_A\sigma_S.
\]

These conditions are derived by solving the quadratic equation that is generated by substituting (A.6) into (A.5). Choosing \( K \) to maximise the value of the option results in the condition (Marcus, 1985):

\[
K^* = \frac{\varepsilon}{\varepsilon - 1}.
\]

Eq. (A.6) gives the value of the life insurance fund under simplifying assumptions of constant \( C_S \) and \( C_A \). Given estimates of the parameters in (A.6) and (A.8) one can assess the value of the insurance to the shareholders of the firm.

Eq. (A.8) gives the condition for voluntary termination of the life insurance fund. Second-order conditions require that \( \varepsilon < 1 \). One must further restrict \( \varepsilon \) to be negative since a feasible \( K^* \) must be positive (because \( A \) and \( S \) are always positive). Thus, \( \varepsilon < 0 \), which implies \( 0 < K^* < 1 \) so that the put will be exercised only for \( S < A \), that is, if fund assets plus 30% of net worth fall below accrued benefits. Parameters that result in nonnegative values for \( \varepsilon \) would imply that the option would never be exercised.

Eqs. (A.6) and (A.8) generalise the formula for the perpetual American put option presented in Merton (1973). For the special case presented in Eq. (A.6), the value of the termination option increases with \( C_A \) and decreases with \( C_S \). Conversely, the ratio of \( S/A \) at which it is optimal to terminate falls with \( C_A \) and increases with \( C_S \).

Eq. (A.6) can be used to generate actuarially fair values of the put for any initial values of \( A \) and \( S \). If a fund is increasing in size (large positive \( C_A, C_S \)), then the dollar gain from a termination for any given ratio of \( S/A \) is larger. If the fund is growing, it pays to wait to terminate, and the ratio \( S/A \) must be smaller to induce early termination. Thus, one should expect voluntary termination decisions to be more frequent where funds are shrinking. These results can be verified analytically: Equal (algebraic) increases in \( C_A \) and \( C_S \) always increase the value of \( P(A, S) \) and lower the termination ratio, \( K^* \).
References


