Valuation of adjustable rate mortgages with automatic stretching maturity

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Abstract

In Hong Kong, 35% of residential mortgage loans are adjustable rate mortgages with variable tenor (VRT). That is, with a change in interest rates, the loan adjusts its maturity and principal payment such that the monthly installment remains the same. In other words, instead of bearing a volatility on monthly payments as in a fixed-tenor variable payment (VRP) mortgage, VRT mortgagors bear interest rate risk by bearing a tenor risk. In this paper, we analyze the valuation of this type of mortgages and the results are compared with the conventional VRP mortgages. We find VRT loans are less expensive from a borrower’s perspective than VRP loans, but the difference between the loans becomes less significant if a tenor cap is added to the VRT loan. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The mortgage market is now the largest private debt market in many countries around the world. In terms of the value of the outstanding debt, the US mortgage market is approximately equal in size to the value of the Treasury debt or the stock market, which totaled more than US$4.7 trillion in 1997. At the other end of the world, it is estimated that outstanding mortgage loans in Hong Kong amount to about HK$420 billion (approximately US$54 billion), and make up nearly 50% of the loan portfolio of local banks. As the Hong Kong government intends to help developing the mortgage-backed securities market, it is of great importance for participants in the market to understand the pricing and riskiness from the underlying assets. In this regard, this paper contributes to the literature by analyzing an interesting variant of conventional adjustable rate mortgages (ARMs) which is common in Hong Kong, namely an ARM with variable tenor. ¹ While the analysis in this paper may seem to be more relevant to Hong Kong in the first instance, it is also relevant in the US and in many other countries with a similar product in the mortgage and securitized asset markets.

There are primarily two types of ARMs in Hong Kong: variable payment (VRP) type and variable tenor (VRT) type. ² The amounts of interest on both VRP loans and VRT loans are computed as for a floating rate instrument. That is, the interest amount will be charged according to the current interest rate plus a spread. The differences between these two types of loans are significant though subtle. The monthly installment of a VRP loan is recalculated based on the current interest rate as in the case of an ordinary ARM in the US. On the other hand, the tenor of a VRT loan varies with the current interest rate while keeping the monthly installment fixed. Therefore, VRT borrowers pay a fixed installment amount during the lifetime of mortgage loan, where the installment amount is determined by the initial tenor and the interest rate (plus spread) when the loan is originated.

The VRT type of loans has been among the menu of choices in the US since late seventies when ARMs became popular. Yet this type of loan has never taken a major role in the US, and is usually treated as another variant in the family of ARMs. A reason for this is that there are regulatory problems for major lenders of mortgage funds when the maturity of this type of loans gets extended due to a significant increase in interest rates. Under the

¹ The analysis of an ARM is not a straightforward one since the payoffs depend upon the actual path of the underlying state variables. Among others, previous studies which have investigated the valuation of ARM include Sa-Aadu and Sirmans (1989), Schwartz and Torous (1989, 1991), and Kau et al. (1990, 1993).
² See Huang et al. (1998) for a detailed discussion on the Hong Kong mortgage markets.
Financial Institutions Reform, Recovery, and Enforcement Act of 1989 (Public Law 101–73), thrifts are given more freedom in the type of residential property loans they can make, but these mortgages cannot extend beyond 40 years nor exceed 125% of the initial appraisal unless amortization is adjusted periodically. As for commercial banks, they usually avoid such (potentially) long-term real estate lending because of their dependence on short-term funds. In addition, by law prior to deregulation, long-term mortgage loans made by commercial banks were limited to a certain percentage of their time deposits or capital.3

The situation, however, is quite different in Hong Kong. Probably due to the oligopolistic industry structure, financial institutions offer limited choices to borrowers.4 In fact, prior to March 1998, VRP and VRT loans were the only two choices in Hong Kong, and fixed rate mortgages (FRMs) were not available.5 In contrast with ARMs in the US, ARMs offered in Hong Kong have no cap, floor, or teaser. The absence of FRMs and cap/floor/teaser feature might foster a stronger demand for the VRT type of mortgages. In fact, of the total outstanding balance to date, around 35% of the ARMs in Hong Kong are of the VRT type.

As there were no FRMs prior to 1998 in Hong Kong, VRT mortgages were perceived to provide a positive social service to the general public in terms of risk sharing between borrowers and lenders, since a VRT loan can be seen as a hybrid of ARM and FRM. Although VRT loan borrowers are paying a fixed amount over time, they are not fully shielded from interest rate volatility. Instead, they bear interest rate risk in a slightly different way. Volatility of cash flows is simply transformed into volatility of paying tenor or maturity. And mortgage originators remain exposed to interest rate risk: negative amortization will occur when interest rates reach a relatively high level and the fixed installment cannot cover the interest payment. In other words, the outstanding mortgage balance will increase (instead of decrease) when the loan interest rate rises to a level where the fixed monthly payment is insufficient to cover the accrued interest.

Given a possibility of negative amortization, it is clear that a potential problem with VRT loans is that there is a chance that the mortgage maturity will be extended without bound into the future when interest rates rise significantly from the initial levels. This scenario will pose a problem for the

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3 Since deregulation, however, legal restrictions play only a minor role in most banks’ decisions as to mortgage lending.
4 A further investigation of the reasons why this situation arises in the first place is interesting, but we will not pursue the issue here.
5 In March 1998, upon the promotion of the Hong Kong Mortgage Corporation, local financial institutions began to offer the so-called fixed-rate ARM, or ‘FARM’, with a fixed feature for the first three years.
lending institution’s interest rate risk position as its duration gap widens significantly. As a means of interest rate risk management, a tenor cap would usually be imposed on VRT mortgages. Whenever the interest payment exceeds the fixed payment or the current tenor exceeds the tenor cap, a VRT loan would automatically switch into a VRP loan. When interest rates decline, and the payment falls back to the level of the original payment, the loan would then switch back to a VRT loan again.

Despite the implication of interest rate risk position on financial institutions is different, VRT loans with the tenor cap feature are treated no differently from VRP loans. While borrowers are offered a choice between the two types of loan products, banks charge the same spread on both VRT loans and VRP loans. Likewise, in the secondary mortgage market, agencies do not differentiate between these two types of instruments. In so doing, these institutions essentially assume that the variable tenor feature is not of much value for ordinary borrowers. However, in this paper we show that such an assumption is valid only when a tenor cap is imposed.

It may be argued that VRT loans imply a lower default risk as the loans provide a smoother cash flow tilt for borrowers. Therefore, banks tend to charge a lower premium on this type of loans to compensate for the lower default risk. Motivated by practical considerations, however, we do not consider this effect in this paper. Specifically, as shown in Huang et al. (1998), defaults are commonly perceived to be negligible in Hong Kong, so the effect of credit risk on the pricing of VRT loans would be minimal. This is possibly due to the fact that there is a 70% limit on the loan-to-value ratio in Hong Kong. The observed low default rate in Hong Kong may also be a result of the high socio-economic cost associated with local views on defaults.

There is, to our knowledge, no previous research concerning a variable tenor mortgage loan, with the exception of Brennan and Schwartz (1977) who examined the value of an extendible clause for loans. In general, they treated this clause as an option and found it to be worthy of some value. Compared with this clause, VRT loans are more interesting since they internalize the extension and contraction of the tenor according to varying economic conditions. Conceptually, this self-adjustable extension or contraction of the tenor leads to a change in the duration of mortgage pass-throughs and therefore a very different interest rate risk exposure relative to typical VRP loans.

The organization of this paper is as follows. In Section 2, we lay out the valuation problems to be solved. The VRT mortgages will be evaluated and compared with the VRP mortgages. The main results and the implications are discussed in Section 3. We will illustrate how the tenor cap feature changes the value of VRT mortgages. The value of the prepayment option will also be investigated and compared. Concluding remarks are set out in Section 4.
2. Adjustable rate mortgages: VRP vs VRT

The interest payment on an ARM varies with the prevailing market interest rate conditions. In this section, we will lay down the way VRP loans, VRT loans, and the tenor capped alternative pay back their principal amounts. In order to simplify the presentation, we will follow Schwartz and Torous (1991) and assume the mortgagor pays back the loan continuously based on the interest rate. The following notations are used in this paper: $M(t)$ denotes the outstanding balance, $r(t)$ is the risk-free interest rate, $C(t)$ is the continuous payment rate, and $m$ is the margin (markup) of a particular loan.

We shall restrict our analysis to default-free but prepayable ARMs. That is, we consider a mortgage as a combination of its equity value, $S(t)$, and a prepayment option, $F(t)$. At the end of this section, we will discuss how to take a prepayment option into account in the valuation. We will confine our discussions of these mortgages to a rational prepayment framework as in Kau et al. (1993), i.e., without developing a realistic prepayment model. Also, we eliminate the possibility of default because a satisfactory treatment of default would require the value of the property as a state variable. This restriction allows us to illustrate the main issues without the ideas being clouded by the enormous complexity of a model that includes both the interest rate and the property value as true state variables.

2.1. Variable payment loan

We consider a fully amortizing mortgage with an initial principal of $M(0)$ and an initial term to maturity of $\tau(0)$ years, with a continuously compounded interest rate of $r(0)$. In this case, the ARM’s payout rate over the initial adjustment period is

$$C(0) = \frac{(r(0) + m)M(0)}{1 - \exp\left(- (r(0) + m)\tau(0)\right)}$$

since

$$M(0) = \int_0^{\tau(0)} C(0) \exp\left(- (r(0) + m)(\tau(0) - s)\right) ds.$$
The outstanding balance declines over time as follows:

\[
\frac{dM(t)}{dt} = (r(t) + m)M(t) - C(t)
\]

subject to the terminal condition of \(M(\tau(0)) = 0\), i.e., the loan being fully amortized in \(\tau(0)\) years. In general, given a fixed \(\tau(0)\), the loan’s outstanding balance at any time \(t, 0 < t < \tau(0)\), is given by

\[
M(t) = \int_{t}^{\tau(0)} C(s) \exp \left( - (r(t) + m)(\tau(0) - s) \right) ds.
\]

Eq. (1) implies that the payment rate \(C(t)\) will vary according to

\[
C(t) = \frac{(r(t) + m)M(t)}{1 - \exp \left( - (r(t) + m)(\tau(0) - t) \right)}.
\]

It is worth noting here that a ‘rational pricing’ or ‘perfect foresight pricing’ of this loan without the prepayment option is

\[
M(0) = \int_{0}^{\tau(0)} C(s) \exp \left( - \int_{0}^{s} (r(l) + m) dl \right) ds,
\]

which will hold if there is no interest rate risk. In general, as we take the interest rate risk into account, the valuation of the VRP loan, given the risk-neutral probability, would be

\[
S(t) = \hat{E} \left( \int_{t}^{\tau(0)} C(s) \exp \left( - \int_{t}^{s} (r(l) + m) dl \right) ds \right).
\]

where \(\hat{E}(\cdot)\) denotes the expectation operator over possible paths of the risk-adjusted interest rate.

2.2. Variable tenor loan

Assume the initial contracted payment stream is \(C\). If we denote the loan amount at time 0 after the closing date as \(M(0)\) and the initial tenor as \(\tau(0)\), we have \(C\) determined by the following equation:

\[
C = \frac{(r(0) + m)M(0)}{1 - \exp \left( - (r(0) + m)\tau(0) \right)}
\]

as

\[
M(0) = \int_{0}^{\tau(0)} C \exp \left( - (r(0) + m)(\tau(0) - s) \right) ds.
\]

This is the equation which also holds in the VRP case, except that \(C\) will be fixed throughout the mortgage life.
Here, the outstanding balance declines over time as follows:

\[
\frac{dM(t)}{dt} = (r(t) + m)M(t) - C. \tag{3}
\]

There is, however, another ‘constraint’ on the outstanding balance of the loan. As the interest rate changes, the maturity date \(\tau(t)\) is adjusted according to

\[
M(t) = \int_t^{\tau(t)} C \exp\left(-\left(r(t) + m\right)(\tau(t) - s)\right) ds,
\]

or \(\tau(t) = \infty\) when

\[
M(t) > \int_t^{\infty} C \exp\left(-\left(r(t) + m\right)(\tau(t) - s)\right) ds,
\]

i.e., when the transversality condition does not hold. Note that in the latter case, the fixed payment \(C\) cannot meet the interest payment, and negative amortization will necessarily occur. As for the previous case, it is also possible that the fixed payment \(C\) cannot cover the interest payment, i.e., negative amortization can occur under some interest rate paths.

As can be seen, the loan bears a great similarity with the ARM in that it adjusts its principal according to Eq. (3). On the other hand, the loan bears similarity with the FRM in a way that the payment is fixed according to \(r(0) + m\) and the initial tenor \(\tau(0)\). In a sense, the loan can be taken as a FRM, the only difference being that it has a path-dependent random maturity. When the interest rate goes up, instead of having a higher payment level, the tenor of the loan is lengthened. Similarly, when the interest rate goes down, the tenor of the loan is reduced instead of a lower payment level. As we will see below, this partially immunizes the loan from interest rate risk.

Let \(\tau^*\) denote the ex post realized tenor. Given \(\tau^*\) and without the prepayment option, we have the ‘rational pricing’ of the VRT loan as

\[
M(0) = \int_0^{\tau^*} C \exp\left(-\int_0^s \left(r(l) + m\right) dl\right) ds \tag{4}
\]

when we do not consider the interest rate risk. It should be obvious that the uncertainties in this case are the potentially varying mortgage tenor and the interest rate risk. Therefore, the risk neutral valuation of the VRT mortgage without the prepayment option would be

\[
S(t) = \hat{E}\left(\int_t^{\tau^*} C \exp\left(-\int_t^s \left(r(l) + m\right) dl\right) ds\right),
\]

where the expectation is now taken over the possible paths of the risk-adjusted interest rate and the associated mortgage tenor.
2.3. Variable tenor loan with tenor cap

As mentioned above, theoretically, there is a possibility that the outstanding balance of a VRT loan always remains positive, i.e., the tenor may be extended without bound. In practice, a cap of 25 or 30 years would usually be applied when the interest rate, and thus the variable tenor, become too large. When this tenor cap is applied, the loan automatically switches to a VRP loan until the interest rate decreases to an appropriate level, after which the loan again becomes a VRT loan. The interest rate at which this switch happens is determined by the following condition:

\[(r(t) + m)M(t) = C,\]

where \(M(t)\) is now the principal amount. That is, the loan remains a VRT loan when \(r(t) < r(t)\) and the mortgagor continues to pay at the rate \(C(t) = C\) as determined in Eq. (2), while the loan becomes a VRP loan when \(r(t) > r(t)\) and the mortgagor will pay at a rate

\[C(t) = \frac{(r(t) + m)M(t)}{1 - \exp\left(- (r(t) + m)(\tau - t)\right)},\]

where \(\tau\) is the predetermined cap on the tenor.

The tenor cap feature limits the interest rate risk exposure of lenders, as it constrains the extent to which the principal payment is delayed when the interest rate becomes high. Borrowers of such a loan would enjoy part of the flexibility provided by the variable tenor feature, as the loan has the possibility of tenor contraction while retaining some of the possibility of extension. Thus the value of the mortgage loan can be evaluated in the risk-neutral probability space as

\[S(t) = \hat{E}\left(\int_t^{\tau^*} C(s) \exp\left(- \int_t^s (r(l) + m) \, dl\right) ds\right),\]

where \(C(s)\) is the payment rate determined as in the previous paragraph and \(\tau^*\) is the realized tenor.

2.4. Prepayment option

The analysis to this point has not considered the fact that residential mortgages are typically prepayable. In general, if we omit the default option, a mortgage loan can be regarded as a combination of the loan contract and a prepayment option. That is,

\[S(M(t), r(t), m) = P(M(t), r(t), m) + F(M(t), r(t), m),\]

where \(S(\cdot)\) is the equity value of the loan, \(P(\cdot)\) is the loan’s value to the lender, and \(F(\cdot)\) is the value of the prepayment option. Note that this
equation will also enable us to determine the spread $m$ in equilibrium. Since $P(M(0), r(0), m)$ should be equal to $M(0)$ at the time of the mortgage origination, so the equilibrium value of $m$ can be found from the following equation:

$$S(M(0), r(0), m) = M(0) + F(M(0), r(0), m).$$

The prepayment rate of mortgages in the Hong Kong market has been at a very high level. The conditional prepayment rate has been above 20% for the past few years. It is therefore very important for us to evaluate the value of the prepayment option when we evaluate a particular mortgage. In this paper, we will not try to evaluate the mortgages with a realistic modeling of prepayment behavior. Instead, we will assume that at any time, a borrower can terminate the loan by paying the lender the current unpaid balance plus any interest that may have accrued. That is, following Kau et al. (1990, 1993), LeRoy (1996) and many others, we will only consider the so-called optimal or rational prepayment, as determined by the financial contract itself, and not the ‘suboptimal’ but more realistic prepayment assumptions. Stanton (1995) has shown how suboptimal prepayment assumptions could potentially be explained by further exogenous considerations, while Huang and Xia (1996) showed how suboptimal prepayment of the ARM instruments could be modeled in the US setting. The rational prepayment assumption adopted here may seem to be less than realistic in the first instance, however, it does provide a useful benchmark for the main issues we analyze here.

As acknowledged in the literature, valuation of the prepayment option is not an easy task since it is an American style option. Different methods have been used to evaluate the value of an American option, such as the finite difference method (Hull and White, 1990), the lattice method (Broadie and Detemple, 1996), and the Monte Carlo methods (Broadie and Glasserman, 1997). The contract feature of variable tenor in a VRT loan, however, poses a new dimension of difficulty in the evaluation of the American option as most of the existing numerical methods are based on a fixed maturity date. In the next section, we will describe how the problem can be solved.

3. Valuation and results

In our analysis, we will only analyze the loan value at the origination. The loan value after origination can be obtained by resetting the mortgage principal and the initial tenor properly. Also, throughout our analysis, we have set the initial principal amount as $1$ million. Any loan with a different initial principal amount could be figured out proportionally, as the loan value is homogenous of degree one in the principal amount.
3.1. *A trinomial tree for the interest rate*

The set of state variables specifying the term structure of interest rates is the relevant information set for valuing the different contracts discussed above. We assume the term structure is generated by a single factor, the risk-free spot interest rate \( r_t \). Its risk-adjusted dynamics are given by

\[
dr = a(b - r)dt + c\sqrt{r}dz.
\]

This is a mean reverting process, with \( b \) signifying the long-term trend for the interest rate, \( a \) representing the speed of adjustment, and \( c \) controlling for the variance of the standard Wiener process \( dz \). This specification of the dynamics of \( r_t \) is treated thoroughly in Cox et al. (1985).

Any instrument based on the above interest rate dynamics can be evaluated with the trinomial tree method based on Hull and White (1990). The American feature of the prepayment option can be easily incorporated into this trinomial tree by comparing the remaining outstanding balance with the discounted value of future payments. The major difficulty in an ARM type instrument is the path dependence arising from a varying interest payment. That is, the concurrent payment amount (or the concurrent tenor) would depend on the current interest rate as well as on the remaining outstanding balance, which in turn depends on the particular interest rate path taken.

In order to solve the path dependence problem, Hull and White (1993) suggest that an additional dimension (variable) could be added in the lattice approach. In each lattice node, we keep a record of the set of asset values corresponding to different potential values of the additional variable. The asset value corresponding to an unrecorded point would be derived using interpolation. The path dependence in this particular case would be dealt with by the addition of the principal amount variable. In each node, we first calculate the potential range of the remaining balance and then the number of nodes in the additional dimension would evenly spread out in this range. The calculation of the minimum and maximum is based on the principle that a higher interest rate will consume the principal more slowly, which is true for both VRT loans and VRP loans. Specifically, the minimal of the current node will come from the minimal of the lower branch (lower interest rate), while the maximal of the current node will come from the maximal of the upper branch (higher interest rate).\(^8\)

It should be noted that since a VRT loan without a tenor cap could potentially involve a very long or even infinite maturity, this will then pose a

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\(^8\) Two exceptions are for the two higher branches next to the lower bound and the two lower branches next to the upper bound, as these nodes get their minimal/maximal from points two branches away.
technical problem in the computation of loan values. Therefore, we set a maximum maturity on such a loan to 75 years in the following computations. As 75 years is a relatively lengthy period of time and thus, we can reasonably assume that a VRT mortgage can be paid off within the specified horizon. We have also tried different maximum maturities in order to check the robustness of the model, e.g., 80 and 90 years, but we found that the inputs do not affect the results significantly.

3.2. Comparisons among VRP, VRT, and VRT with cap

Using monthly data of Hong Kong prime rate from 1981–1998, the maximum likelihood estimates of the parameters for the risk-adjusted CIR interest rate process are as follows: \(a = 0.4205\), \(b = 0.0870\), and \(c = 0.0665\). These are the values used in this paper. The following discussions present our results on the values of the loans and equilibrium spreads, as well as the values of the prepayment options and the tenor cap. Arbitrage principle requires that the claims exchanged be of equal value for both parties to accept the transaction. The equilibrium spreads (or markups) are generated exactly according to this principle at the date when both parties close the loan.9

3.2.1. Value of the loans

What are the prices for these different products (i.e., VRP, VRT, and VRT with tenor cap)? These can be found from Figs. 1–3 when there are no prepayment options, and Figs. 4–6 when we allow for rational prepayments. These figures are generated with an initial outstanding balance \(M(0)\) of $1 million, and the spread has been set as negative 50 basis points.10 The tenor caps are kept at 30 years unless otherwise noted.

In general, VRP loans are much more expensive for borrowers than VRT loans. With the chosen parameters, the difference could amount to $100,000 for a typical 15-year loan when we do not consider the prepayment option. This difference becomes dramatically less and the amount decreases to around $10,000 for a 15-year loan when a tenor cap of 30 years is added to the VRT mortgage. The values of VRP loans have a flatter slope as seen from the figures, and therefore are less sensitive to interest rates. With a lower initial interest rate, we typically find a lower loan value. This is true as a lower interest rate

9 See Kau et al. (1990, 1993) for how equilibrium conditions can be used to generate the equilibrium spread.

10 In real life, there will be a positive spread added on top of this spread due to transaction costs, so we rarely see a negative spread in mortgage markets. See Stanton (1995) for the importance of such transaction costs.
means a lower immediate payment on the outstanding principal, and a higher interest rate means the opposite. In addition, we observe that loans with longer maturity tend to have lower loan values.

An interesting feature of VRT loans without tenor caps is that they have a larger interest rate sensitivity when initial interest rates are lower. In cases
where the initial tenor is long enough, VRT loans exhibit the property of negative convexity. The intuition being that the lender is offering a much larger compensation to the borrower when interest rates are lower. That is, the lender is subject to a much higher interest rate risk when initial interest rates are low,
and therefore, the initial payment level is lower. This negative convexity feature does not hold when the initial tenor is sufficiently short. Furthermore, when the initial tenor is short, the loan value at a lower initial interest rate is

\[ \text{Value} = 10^{-5} \times 9.5, 9.0, 8.5, 8, 7.5, 7, 6.5, 6, 5.5, 5 \]

\[ \text{Interest Rate} = 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16 \]

\[ \text{Value} = 9.9 \times 10^{-5} \times 9.5, 9.85, 9.8, 9.75, 9.7, 9.65, 9.6, 9.55, 9.5 \]

\[ \text{Interest Rate} = 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16 \]

Fig. 5. Value of VRT loans with prepayment option. (Note: The values of a VRT loan with a prepayment option for various initial maturities with an initial interest rate ranging from 1% to 15% are plotted. From top to bottom, the lines indicate the values of 5-, 10-, 15-, 20-, 25- and 30-year loans.)

Fig. 6. Value of VRT loans (with tenor cap) with prepayment option. (Note: The values of a VRT loan with a 30-year tenor cap and a prepayment option for various initial maturities with an initial interest rate ranging from 1% to 15% are plotted. From top to bottom, the lines indicate the values of 5-, 10-, 15-, 20-, 25- and 30-year loans.)

\[ \text{Value} = 10^{-5} \times 9.5, 9.0, 8.5, 8, 7.5, 7, 6.5, 6, 5.5, 5 \]

\[ \text{Interest Rate} = 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16 \]

\[ \text{Value} = 9.9 \times 10^{-5} \times 9.5, 9.85, 9.8, 9.75, 9.7, 9.65, 9.6, 9.55, 9.5 \]

\[ \text{Interest Rate} = 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14, 0.16 \]

\[ 11 \text{ When interest rates begin to rise, the lender is effectively losing a lot by giving a protection to the borrower.} \]
higher than the loan value at a higher initial interest rate. The explanation for this is that for a very short initial tenor, when the initial interest rate is high enough, the mortgagors would have a higher chance of repaying the mortgage in a shorter period as the interest rate lowers. This retraction of tenor is of value to borrowers. For the lower initial interest rate case, since the installment is smaller, the retraction is less likely to happen. Therefore, borrowers are paying a premium for the VRT loan in the case of a lower interest rate than in the case of a higher interest rate.

As for VRT loans with tenor caps, by comparing Fig. 6 with Fig. 4 and Fig. 5, we note that the loans’ values are bounded between those of VRP and VRT without tenor caps. When the initial tenor is long and the initial interest rate is low, this would imply a lower installment and therefore increases the chance of the tenor being capped. When the tenor is capped, the VRT loan becomes a VRP loan, we therefore expect the value of the VRT loans with tenor caps to be similar to value of the VRP loan with a lower interest rate and a longer initial tenor. When the initial tenor is very short, the chance of being tenor-capped is small. Therefore we expect the properties of VRT loans without a tenor cap to be preserved when the initial tenor is sufficiently short. Also, as the initial tenor increases to more than 10 years, the convexity takes a form similar to that of a VRP loan when initial interest rates are low. As the initial interest rate increases, these loans behave more like VRT loans without tenor caps by having a negative convexity.

In summary, we find the values of these loans to differ with different initial interest rates, whether the loan has a prepayment option or not. According to economic conditions, the variable tenor feature gives borrowers valuable flexibility to either extend or retract their mortgage payments. The tenor cap feature brings a VRT loan closer to a VRP loan both in terms of value and the sensitivity to varying interest rates and initial tenor. Therefore, the results here show that it is justifiable to treat VRP loans and VRT loans with tenor caps as the same type of loans.

3.2.2. Spread in equilibrium

Next, we ask what is the equilibrium spread that lenders can charge such that borrowers are indifferent to the choice of taking or not taking a mortgage. This spread would give us a sense of the expensiveness of the mortgage from a borrower’s perspective. The fact that borrowers are willing to pay a positive spread suggests that the loans are less expensive under similar terms. An equilibrium spread with different loan instruments would also give us a sense of whether the market spreads of different instruments are reasonable or not.

We found that the equilibrium spread for a typical VRP loan without a prepayment option is zero, regardless of the maturity and the initial loan rate. The equilibrium spread for a typical VRP loan with an optimal prepayment
option is slightly less than, but insignificantly different from, zero. For a VRT loan without a prepayment option, the equilibrium spread is between 16 to 42 basis points as shown in Table 1. For a typical VRT loan with optimal prepayment, however, the equilibrium spread is between 28 to 43 basis points as shown in Table 2. This suggests in another way that a VRT loan is much less expensive than its VRP counterpart. The value of the VRT feature, however, is curtailed to a great extent if the tenor cap feature is added. The equilibrium spreads for VRT loans with a 30-year tenor cap, with or without a prepayment option, are a little higher than for the VRP loan, although again not significantly different from zero. This suggests that it is rational to treat VRP loans and VRT loans with tenor caps as the same.

### Table 1

<table>
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<tr>
<th>Interest rate</th>
<th>Initial tenor (year)</th>
<th>5.0</th>
<th>10.0</th>
<th>15.0</th>
<th>20.0</th>
<th>25.0</th>
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3.2.3. **Value of the prepayment options**

We find that the prepayment options are indeed valued differently for different products. In particular, we find that the options for VRT loans (both with and without tenor caps) are positive, compared to almost zero value options for VRP loans. Figs. 7–9 plot the prepayment option values for the VRP, VRT, and VRT with tenor cap loans for various initial interest rates. It can be seen that the values for the VRP loans’ prepayment option are about zero. As for the VRT type of loans, it can be seen that the option values for VRT with a tenor cap are higher than those without when the initial tenor is five years. However, as the initial tenor increases, the prepayment options are generally worth more for VRT loans without tenor caps. That is, a VRT with a
tenor cap should have a higher value than a VRT without a tenor cap when the initial tenor is short.

It is well known in the literature that VRP loans do not have much interest rate sensitivity (e.g., see Chiang et al., 1997). As a result, their prepayment option values should be close to zero. For VRT type of loans,

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Fig. 7. Prepayment option value for VRP loans. (Note: The values of a prepayment option on a VRP loan for various initial maturities with an interest rate ranging from 1% to 15% are plotted. The maturities considered are 5, 10, 15, 20, 25 and 30 years. As can be seen, the option has a zero value across interest rates and maturities.)
Fig. 8. Prepayment option value for VRT loans. (Note: The values of a prepayment option on a VRP loan for various initial maturities with an interest rate ranging from 1% to 15% are plotted. The maturities considered are 5, 10, 15, 20, 25 and 30 years. As can be seen, the relationship among the prepayment option values, initial interest rates and initial maturity is much less smooth in this case than for VRT loans with tenor caps as shown in Fig. 9. This is in fact due to: (1) the lattice structure of the trinomial tree estimator, (2) the interpolation of minimum/maximum values in each node, and (3) the endogenous termination of the valuation problem. This again illustrates intricacies of option valuations involving path dependence problems.)

Fig. 9. Prepayment option value for VRT loans with tenor cap. (Note: The values of a prepayment option on a VRT loan with a 30-year tenor cap for various initial interest rates and initial maturities are plotted. For example, the top line indicates the values of the prepayment option on a 5-year (initial maturity) loan with an initial interest rate ranging from 1% to 15%. As can be seen, the option value approaches zero for all initial interest rates when initial maturity increases.)
however, the prepayment option will have values which vary with the initial interest rate and initial maturity. This is because the outstanding balance of VRT loans will decrease more slowly than that of the VRP, as the borrowers can delay their payment due to future interest rate changes. In turn, such a feature implies that the option to prepay the larger outstanding balance has a larger value. When the initial tenor is short, however, a VRT loan would repay its balance very fast. This is especially relevant in the case of the lower initial interest rate since at a lower the interest rate, the principal amount is paid off at a higher speed. Therefore, this option would have a higher value.

3.2.4. Value of the tenor cap

As mentioned before, it is reasonable to treat VRP loans and VRT loans with a 30-year tenor cap as the same, that is, there is a value associated with the tenor cap. However, how sensitive is its value? To examine this issue, Fig. 10 plots the case for an initial tenor set at 15 years, and we then consider different tenor caps at 15, 20, 25, 30, 35 and 40 years. It is obvious from Fig. 10 that, as the initial interest rates rise, the value of the VRT loan is not greatly affected by the choice of tenor cap. On the other hand, when initial interest rates are lower, we see that the longer the tenor cap, the lower the VRT loan values. The explanation is that as the tenor cap increases, the lender is offering more protection to borrowers when initial interest rates are low.

Fig. 10. Value of the tenor cap for VRT loans with prepayment option. (Note: The values of a 15-year (initial maturity) VRT loan with a prepayment option for various tenor caps with an interest rate ranging from 1% to 15% are plotted. From top to bottom, the lines indicate the loan values with 15-, 20-, 25-, 30-, 35- and 40-year tenor cap.)
4. Concluding remarks

The popularity of VRT mortgages, coupled with nearly non-existent FRMs, make the Hong Kong mortgage market unique. In this paper, we have examined and evaluated different types of ARMs available in Hong Kong. Intuitively, without FRMs, VRT mortgages are normally associated with risk averse borrowers who presumably prefer a stable payment stream. The results of this paper show that taking a VRT loan is a less expensive way of financing from borrowers’ perspective, as it allows a constant payment and gives borrowers the flexibility of delaying or speeding up the principal payment, conditional on the current interest rate environment.

As VRT loans without a tenor cap can potentially involve a very long or even infinite maturity when negative amortization occurs, the industry practice is to introduce a cap on the tenor. We find the VRT loan loses most of its attractiveness once a tenor cap is added. Although the borrowing cost of a VRT loan with tenor cap is still less expensive than that of a VRP loan, the difference is minimal. Therefore, while there are significant differences between VRP and VRT loans, the industry practice of considering VRP loans and VRT loans with tenor caps as equivalent is supported. Furthermore, we find that both the values of variable tenor and tenor cap to be sensitive to interest rates, and are more valuable with lower initial interest rates.

References


