Avoiding bank runs in transition economies:
The role of risk neutral capital

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Abstract

In a general equilibrium model with risk neutral and risk averse agents, we show that if banks issue both demand deposits and equity, then free banking is run-proof and efficient. In particular, we obtain the first best insurance solution if there is adequate risk neutral capital. If sufficient risk neutral capital is unavailable, then a partial suspension of convertibility is optimal. In general, therefore, policies like capital adequacy norms and deposit insurance are neither necessary nor desirable. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Transition economies have a legacy of government control over the banking sector. Indeed, most, if not all, banks continue to be in the public sector in many
of these economies. And, even when they are outside the direct ownership of the government, they are assured of government help in times of distress. Historically, the role of banks in these economies has often been more political, than economic. Not surprisingly, banks in transition economies have large amounts of bad loans accumulated over the years. Loss making banks, however, were able to survive as government budgetary support wiped out their losses.

Propping up loss making banks essentially imply that depositors are assured of their returns. Thus, there is implicit (if not explicit) deposit insurance in these countries. Consequently, depositors no longer discriminate against badly performing banks, and bank managers do not worry about the loans they give out. The net result is that, good projects subsidize the bad ones, increasing the cost of capital in the economy. This inefficiency restricts investment, and hence, growth.

Most transition economies are going through fundamental structural changes in both the real and financial sectors. For an efficient financial sector, it is important that the banking sector is built on a solid foundation. For this, it is imperative that banks face hard budget constraints. To make this constraint credible, governments must commit to allow banks to fail, if they perform badly. This calls into question the policy of government funded deposit insurance.

Diamond and Dybvig (1983) forms the theoretical foundation used by policy makers to argue for government funded deposit insurance schemes. According to the Diamond–Dybvig paper, a lack of deposit insurance can lead to a panic run. In this paper, we show that such panic runs can be avoided even in the absence of deposit insurance. We allow banks to sell equity as well as deposits. The presence of equity effectively solves the bank-run problem. In particular, we show that there exists an amount of risk neutral capital that is sufficient to ensure a run-free optimal outcome, and allow risk averse depositors to be completely insured (i.e., get full insurance coverage). In case the amount of risk neutral equity is smaller, then also runs can be avoided, but with less than full insurance for depositors.

Our model highlights the importance of adequate capital, as a market outcome, in preventing bank runs. Many authors have analyzed capital adequacy as a means to reduce the costs of subsidized deposit insurance schemes (Buser et al., 1981; Sharpe, 1978), without arguing that with adequate capital, deposit insurance is unnecessary. We go on to show the irrelevance of deposit insurance even with aggregate uncertainty.

In Section 2 we briefly summarize the literature on bank runs. Section 3 describes the basic parameters of our model. Section 4 develops the operation of the bank. The next section calculates the minimum amount of risk neutral capital that will give us the full insurance model. Section 6 deals with a situation where there is not enough risk neutral capital to guarantee complete insurance to risk averse agents. Section 7 concludes the paper.
2. Bank runs

Bank runs were formally modelled in a paper by Diamond and Dybvig (1983), henceforth, referred to as DD. Consider a project, which has a certain long term (two period) return. If the project is liquidated in the short run (one period), the return rate is lower. Depositors do not know whether they will need funds after one period, and this introduces some uncertainty regarding whether, or not, the project will have to be liquidated in the short run. Depositors are risk averse, preferring a smaller spread in return. However, insurance is not available because information on liquidity needs is private and non-verifiable. Typically, a bank will allow deposit rates such that the short term rate is lower than the long term rate. The insurance aspect is captured by the fact that the short term return is lower than the short term rate on deposits, lower than the long term return on deposits, which, in turn, is lower than the return from the long term technology. However, there are multiple Nash equilibria, with one of them that of a panic run. This leads to pre-mature liquidation and economic loss. By providing insurance to depositors, a bank makes itself vulnerable to a run.

The bank is unable to prevent bank market failures because of its “sequential service constraint”, by which a depositor, when withdrawing money from the bank, can only be paid (or, serviced) according to his, or her, position in the line. Most importantly, the amount paid to an agent cannot depend on information about the length of the line behind this agent. DD then go on to show that government deposit insurance can solve the problem. In their scheme, the bank pays out depositors in the short term, without worrying about the long term claims. This is because, if there are too many who withdraw in the short run, the government taxes every one of them to ensure that there are enough resources to pay back the long term claims. Those who can wait to get paid, are thereby assured of being paid, and hence, there is no panic run. DD goes on to argue that, no one but the government can enforce tax collections. Hence, deposit insurance, which in this case is an assurance that long term claims will be paid, can only be undertaken by the government.

McCulloch and Yu (1998), MY from now, convincingly argue that even in the DD context, government deposit insurance is unnecessary. The way deposit insurance works is precisely by getting rid of the sequential service constraint faced by the bank. In other words, the government, after learning about the entire length of the line, can, in some way, track down the agents who have made withdrawals, and collect back a part of the money paid out to them! The basic problem with this mechanism is that withdrawing agents will have to wait to consume, not knowing whether, or not, the government will ask them to return a part of their withdrawals – a highly unrealistic assumption. MY shows that the same outcome can be obtained by having a system of contingent bonus schemes, by which each withdrawing agent is paid a low amount as they come
up in the line, and a subsequent bonus whose amount is determined by the total
length of the line. In DD’s insurance scheme, the bank pays out more and the
government collects back the excess; in MY’s scheme, the bank pays out a
lower amount in the first step and then an additional amount in the second
step. The net amount with the depositors in both cases is exactly the same.
Wallace (1988) has a similar criticism to the deposit insurance scheme envis-
gaged by DD. Jacklin (1993) argues that, the outcome of the deposit insurance
scheme can be mimicked by a firm selling shares, when shares are tradeable in
the short term.

The essential disagreement between DD and the critics ¹ relates to the types
of contracts that investors and institutions can get into. In DD, the bank, the
depositors and the government sign deposit contracts. Here, an essential part
of the contract stipulates that the government will, if necessary, take away a
part of the deposit returns of early withdrawers. In MY, only the bank and the
depositors seemingly enter into a contract, which stipulates that early with-
drawers will be paid in two stages. In Jacklin also, the firm and the investors
enter into a contract which pays all investors a dividend in the short run, but
allows them to trade these dividends against long term claims (shares).

As it now stands, even when there is no explicit deposit insurance, most
transition country depositors (as in India, or Thailand) are implicitly assured
that their banks are too important to the national economies and will not be
allowed to go under. The obvious moral hazard problems of such systems are
evident from the recent East Asian financial crisis. MY and Jacklin argue that
the only government involvement one needs is that of enforcement of volun-
tary contracts. This is a non-trivial issue as enforcement is not costless. In DD,
this is glossed over by assuming that, on the one hand, taxing short term de-
positors needs no resources to implement, and on the other hand, all other
contractual arrangements are infinitely costly.

The theoretical literature has a number of papers analyzing the role of
government regulation in the banking sector. In particular, they argue that
allowing for contracting among the depositors and the bank can solve most of
the problems without any explicit need for the government to step in. Alonso
(1996) and Cooper and Ross (1998) have banks with depositors who accept a
positive probability of bank runs, as an equilibrium outcome. Nicolo (1996),
much like MY (and Jacklin), has state contingent deposit claims, where the
banks infer about the state from the type of withdrawals made by the depos-
itors in the short run. Villamil (1991) allows for suspension of convertibility to
prevent bank runs, but has no aggregate uncertainty. In our model, depositors
are assured of withdrawal amounts independent of the actual states. The

¹ See Bhattacharya and Thakor (1994) for a survey of the banking literature.
credibility of the assurance comes from the presence of bank equity, and does not need any government involvement.

The DD model tries to ensure for the depositors an outcome as close as possible to the insurance market equilibrium. In insurance schemes, as distinct from risk sharing, agents who are more risk averse transfer a part, or all, of their risk to those who are less risk averse. In the DD model, all agents have identical risk aversion, and it is not clear who insures whom. In our paper, by explicitly introducing agents with two different attitudes to risk, we reformulate the analysis as a true insurance problem.

There are two important features of bank deposits that allow us to treat them as an insurance mechanism. A bank offers deposit holders a promised return on their deposits and a guarantee that they will be able to withdraw on demand. In both MY and DD, the bank, however, offers a contingent return to the depositors. Moreover, in all these exercises, the late withdrawers are residual earners, the actual amount that they get depending on what happens in the short run. In DD, as well as those of its critics, the method of preventing runs makes the return to depositors uncertain, in both the short and the long terms. However, being risk averse, it is in the interest of depositors to reduce the risk of future uncertain returns. In our bank this is avoided by the owners of the bank, its equityholders, being the residual earners. Bank equity plays two crucial roles. First, it makes the promise to late withdrawers credible, preventing them from running the bank. Second, it allows a part of the depositors’ risk to be transferred to the risk neutral equity owners.

Both Jacklin (1987) and MY refer to a disintermediation possibility. This is a problem shared by the deposit insurance scheme as well as the contingent bonus scheme. Their solution is to assume that agents are forced to deposit their entire fund with the bank. We continue with this assumption in our paper. However, we show that, given the credible promises on deposits, risk averse agents investing in deposits, and risk neutral in equity, is a Nash equilibrium outcome. Going back to our observation about the enforcement of voluntary contracts, we will assume that contracts that are ex ante efficient, and voluntarily signed, will not be allowed by the government (or the legal institutions) to be breached.

3. The model

Ours is a three period model, 0, 1, 2. There is a continuum of agents in the interval [0, 1]. Agents are either risk averse, or risk neutral. \( \theta \) proportion of people are risk averse, and \( 1 - \theta \) are risk neutral. Also, they are either of type 1, or type 2. Type 1 agents derive utility from consumption in period 1 only, and type 2 agents from consumption in period 2 only. In period 0, each agent faces a probability \( t \) of being type 1. An alternative interpretation is that \( t \) proportion
of agents will be of type 1 and $1 - t$ proportion will be of type 2. We make the following assumption.

**Assumption 1.** $F(t)$ is uniform, on $[0, 1]$.

The distribution of $t$ is common knowledge in period 0. However, in period 1, the type of an agent is known privately to the agent only. Table 1 describes the agent classification in period 1.

Each risk averse agent has an endowment of 1 in period 0 and nothing in other periods. Each risk neutral agent has an endowment of $K$ in period 0 and nothing in any other period. The total endowment in the economy in period 0 is $\theta + (1 - \theta)K$.

Let

$$\theta' = \frac{1 - \theta}{\theta}.$$  

(1)

For each unit of resource invested in period 0, the return is $R (> 1)$ in period 2. Alternatively, the investment may be liquidated in period 1 in which case only 1 can be recovered. Thus, the technology is constant returns to scale and, the long term return is greater than the short term one. Also, pure storage is costless and does not yield any extra return. This technology is available to everyone. Also observe that, there is no uncertainty in the technology.

Let $c_{it}$ denote the consumption of a type $i$ agent in period $i$, $i = 1, 2$. Given our assumption on the consumption requirements of agents, $c_{12}$ or $c_{21}$ are irrelevant. The superscript $a$ will denote the risk averse agents, while $n$ will denote the risk neutral ones. The expected utility of a risk averse agent in period 0 is

$$EU^a = \int_0^1 [tu(c^a_{11}) + (1 - t)\rho u(c^a_{22})]dF(t).$$

(2)

This follows from Assumption 1. Given our assumption on the preferences of agents, it follows that we only need to consider $c^a_{11}$ and $c^a_{22}$. $\rho$ is the discount rate, $0 < \rho < 1$.

Similarly, for the risk neutral agent,

$$EU^n = \int_0^1 [tc^n_{11} + (1 - t)\rho c^n_{22}]dF(t).$$

(3)

<table>
<thead>
<tr>
<th></th>
<th>Risk averse</th>
<th>Risk neutral</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>$\theta t$</td>
<td>$(1 - \theta) t$</td>
<td>$t$</td>
</tr>
<tr>
<td>Type 2</td>
<td>$\theta(1 - t)$</td>
<td>$(1 - \theta)(1 - t)$</td>
<td>$1 - t$</td>
</tr>
<tr>
<td>Total</td>
<td>$\theta$</td>
<td>$1 - \theta$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1
Assumption 2. \( \rho R > 1 \).

Assumption 2 guarantees that agents prefer to be type 2 or, the long term returns are sufficiently high. For risk averse agents, the issue is similar to the problem of insurance. Being type 2 is a “win” situation, while being type 1 is a “loss”. However, since the information regarding types is private, an insurance market with risk averse agents only, will fail (Diamond and Dybvig, 1983). In our model, we will investigate how far this insurance market can be mimicked by the presence of risk neutral agents. The amount of risk neutral capital, measured by \( (1 - \theta)K \), will play a crucial role. It will also help us in defining what one means by the notion of capital adequacy in banks.

For getting explicit solutions we will use another assumption in the paper.

Assumption 3. \( u^a = (1 - s)x^{(1-s)}, s > 1 \).

If each agent invests on her own, then

\[
EU^a = t^e u(1) + (1 - t^e)\rho u(R) \equiv U^a,
\]

\[
EU^n = [t^e.1 + (1 - t^e)\rho R]K \equiv U^n,
\]

where \( t^e \) is the expectation of \( t \). With Assumption 1, \( t^e = (1/2) \).

4. The bank

A bank in our model is an institution that can sell shares and (demand) deposits. These are issued in period 0. Deposit claims in any period are senior to claims by the shareholders in that period. For each unit invested in deposits, an agent receives either \( r_1 \) in period 1 and zero in period 2, or zero in period 1 and \( r_2 \) in period 2. Shares are long term assets (irredeemable in period 1), while deposits can be liquidated in period 1 if the depositor so wishes. Banks, however, can offer dividends \( v_1 \geq 0 \) and \( v_2 \geq 0 \) to shareholders, in periods 1 and 2, respectively. We will carry out the analysis with one (aggregate, or representative) bank, which will make zero profits because of competitive pressures.

Suppose every agent buys the issue of the bank. Then the bank’s proceeds from selling shares and deposits is the total endowment of the economy, \( \theta + (1 - \theta)K \). This the bank invests in the available technology. Since the technology offers a positive net return in period 2 only, in period 1, the liquid value of its investment is the same as what it received in period 0. This is also the value it can disburse in period 1. The bank’s liabilities in period 1 are the demands made by the depositors in period 1 and, the amount of dividends committed to by the bank, for period 1 (\( v_1 \) per share).
While depositors can liquidate their holdings in period 1 if they so want (depending on their type), shareholders cannot. Thus type 1 shareholders will be left with an asset which will be redeemable only in period 2. The utility value to them from the ensuing consumption in period 2 is zero. Ideally, they would like to trade this asset with type 2 agents.

For the moment, assume that all risk averse agents buy deposits and all risk neutral agents buy equity. A bank run will occur in period 1 if type 2 depositors withdraw their deposits in period 1. They will do so if they are better off withdrawing in period 1, rather than wait for period 2. To prevent this from happening, first \( r_1 \) must be less than \( r_2 \). Second, type 2 depositors must be convinced that they will be paid their \( r_2 \) in period 2.

Suppose that, \( r_1 < r_2 \), and the type 2 depositors wait till period 2. The bank’s resources at the end of period 1 is the amount left over after paying type 1 depositors and the committed dividend payments of period 1. If \( E \) is the total endowment of the economy, then this is equal to

\[
E - r_1 \theta t - v_1 (1 - \theta)K = \theta + (1 - \theta)K - r_1 \theta t - v_1 (1 - \theta)K.
\]

In period 2, given the technology, this will become

\[
[E - r_1 \theta t - v_1 (1 - \theta)K]R.
\]

For type 2 depositors to wait, it must be the case that

\[
r_2 \theta (1 - t) \leq [E - r_1 \theta t - v_1 (1 - \theta)K]R.
\]

Thus, for all realisations of \( t \), for which the following holds, type 2 depositors will not run the bank in period 1:

\[
t \leq \frac{R(1 + \theta'K) - v_1 \theta'KR - r_2}{Rr_1 - r_2} \quad \text{if} \quad Rr_1 - r_2 > 0. \tag{6}
\]

In this model, the parameters are, \( K, R, \theta' \), while the endogenous variables are \( r_1, r_2, v_1 \). Define \( t \) to be the value of \( t \) such that (6) holds with equality. Then, if we can ensure that the parametric configurations, coupled with the solution to the endogenous variables, are such that \( t \geq 1 \), then for all realisations of \( t \), it will pay the type 2 depositors not to run the bank in period 1.

5. Capital adequacy

Suppose that type 2 depositors are confident that they will be paid in period 2 and, hence, do not run the bank in period 1. In other words, they believe that banks have full (unlimited) liability to the depositors. We continue to assume that risk neutral agents buy equity only, while the risk averse agents are depositors. Then, the shareholders of the bank become residual income earners of period 2. Define \( \pi(t) \) to be the amount of residual income earned per unit of
capital. In period 2, a shareholder earns \(v_2 + \pi(t)\) per unit of capital. Being residual income earners, \(\pi\) could be positive or negative.

Total return to shareholders in period 2 is

\[
(1 - \theta)K(v_2 + \pi(t)) = [\theta + (1 - \theta)K - r_1 \theta t - v_1 (1 - \theta)K]R - r_2 \theta(1 - t).
\]

Then, for each \(t\), given \(r_1, r_2, v_1\) and \(v_2\),

\[
v_2 + \pi(t) = \frac{R(1 - r_1 t) - r_2 (1 - t)}{\theta K} + (1 - v_1)R. \tag{7}
\]

This expression allows us to make a couple of interesting observations. First, observe that, in period 1, type 1 risk neutral agents will have \(v_1 K\) in cash, and will own period 2 redeemable assets of book value \(v_2 K + E \pi(t) K\). They do not have any use for period 2 value and, hence, will want to sell their assets to those who want to consume in period 2. This will allow a market for ex-dividend shares to function in period 1 where, type 1 risk neutral agents will sell their period 2 claims to type 2 risk neutral agents. Suppose, type 2 depositors protected by unlimited liability, do not withdraw cash in period 1. Type 1 depositors, on the other hand, will not buy assets redeemable in period 2. Thus, only the type 2 risk neutral agents, each holding \(v_1 K\) of cash, will demand these shares. The supply of such shares, \(x^s\), is given by \(^2\)

\[
x^s = t(1 - \theta)K
\]

and the demand, \(x^d\), by

\[
x^d = \frac{(1 - t)(1 - \theta)K v_1}{p},
\]

where \(p\) is the price of ex-dividend shares in period 1. The money value of the total demand for shares from type 2 risk neutral agents is worth \((1 - t)(1 - \theta)K v_1\). Solving for \(p\), by equating demand and supply, one gets

\[
p = \frac{1 - t}{t} v_1.
\]

Thus, for each \(t\),

\[
c_{11}(t) = (v_1 + p)K
\]

and

\[
c_{22}(t) = \left(1 + \frac{v_1}{p}\right)K(v_2 + \pi(t)).
\]

\(^2\) Jacklin (1993).
Substituting the value of \( p \) and for \( v_2 + \pi(t) \) from (7), we get
\[
c_{n1}^n = \frac{v_1}{t}K
\]
and
\[
c_{n2}^n = \frac{1}{\theta'(1-t)}[R(1-r_1t) - r_2(1-t)] + \frac{(1-v_1)R}{(1-t)}.
\]

With unlimited liability, the expected utility of risk neutral shareholders, from (3) and the values of \( c_{n1}^n \) and \( c_{n2}^n \), is
\[
E^u = \rho R K + v_1(1 - \rho R)K + \frac{\rho}{\theta'}[R(1-r_1t^e) - r_2(1 - t^e)],
\]
where \( t^e \) is the expected value of \( t \) (equal to 1/2 with our assumption on the distribution of \( t \)). There are two important things about (8). First, observe that \( E^u \) is decreasing in \( v_1 \) as \( \rho R > 1 \), by assumption. Second, if the bank were to issue no deposits, and behave like the firm in Jacklin (1993), then the third expression on the right-hand side of (8) will vanish. In other words, risk neutral shareholders, by choosing \( v_1 = 0 \) and not issuing deposits, can achieve an expected utility of \( \rho R K \). Thus any solution, where the bank has unlimited liability, must ensure that (risk neutral) shareholders get at least \( \rho R K \). \( \rho R K \) is then the reservation utility level of the shareholders.

It is important to understand what is happening here. Risk neutral agents are residual income earners in period 2. There are \((1-\theta)(1-t)\) of them (type 2) around in period 2. Being residual income earners, they not only get their own \( v_2 K + \pi(t) K \), they also get what was due to the type 1 risk neutral agents – who are not interested in consumption in period 2. This is possible because the latter sell off their shares to type 2 risk neutral agents. This is not difficult to see when \( v_1 > 0 \). But, we have just argued that \( v_1 = 0 \). This essentially means that shareholders are signing a contract that says the following: in period 2 they will get \( v_2 K + \pi \), and in period 1 they get nothing. Ex ante, such a contract gives them the maximum utility. Ex post, however, if they turn out to be type 1 in period 1, they will want to break the contract by selling their shares and getting a price in period 1.

Since \( v_1 = 0 \), no risk neutral equity owner will be able to buy their shares. However, depositors can withdraw money from the bank in period 1 and buy these shares. Only type 2 depositors will have this incentive. This too will lead to a bank run, as type 2 depositors will withdraw in period 1. Alternatively, it will lead to disintermediation in the short run. To prevent this, we need a restriction on the possible trades in period 1 (Jacklin, 1993). Simply put, shares should be non-transferrable in period 1 (or, more like long term bonds with no market for them in period 1). With \( v_1 = 0 \), type 2 shareholders are already precluded from trade. The trading restriction prevents type 2 depositors from buying shares in period 1 and, hence, they have no incentive to withdraw de-
posits in period 1. Such an assumption is implicit in our algebra; we now make it explicit.

**Assumption 4.** Bank shares are non-transferable in period 1.

One question still remains. Even though ex ante, all shareholders will prefer to sign a contract which allows Assumption 4, ex post, they will want to breach it if they are type 1. This is where contract enforcement becomes important, something that requires institutional (legal) backing. The risk neutral shareholders play a role similar to Selgin’s (1996, Chapter 11) corn merchants. In Selgin (1993), the bank issues IOUs to period 1 depositors, who trade them against corn from the merchants, who later (period 2) redeem them at the bank. These merchants are outside the banking system, and the rationality of their actions is not modelled. In our general equilibrium setup, on the other hand, they are an integral part of the banking mechanism.

Competition among banks will ensure that $E^n = \rho RK$. This can be achieved with $v_1 = 0$, and most importantly, $R(1 - r_1) - r_2(1 - r^2) = Z = 0$. To see this, write the bank’s problem as follows:

$$
\begin{align*}
\text{(P) Maximise } & \quad EU^n = \int_0^1 [tu(c_{11}) + (1 - t)\rho u(c_{22})] dF(t), \\
\text{subject to } & \quad EU^n \geq \rho RK \text{ and unlimited liability, i.e., } t \geq 1.
\end{align*}
$$

Recall that $c_{11} = r_1$, and $c_{22} = r_2$.

Suppose a solution to (P) exists. Competitive banking with unlimited liability will guarantee that $EU^n$ is no greater than $\rho RK$. This will be achieved with $Z = R(1 - r_1) - r_2(1 - r^2) = 0$, and $v_1 = 0$. Using the relationship between $r_1$ and $r_2$ thus obtained, maximisation of $EU^n$ gives the following first order condition:

$$u'(r_1) = \rho Ru'(r_2). \quad (9)$$

As noted in Diamond and Dybvig (1983), this is a necessary condition for the first best solution to the depositors’ problem. We now need to ensure that unlimited liability is credible, and type 2 depositors do not run the bank. For this, two things need to be satisfied $-1 < r_1 < r_2 < R$ (it pays type 2 depositors to wait for period 2) and $t \geq 1$ (type 2 depositors will wait). From (9), $\rho R > 1$, and $Z = 0$, the first requirement follows. For the second requirement, observe that plugging in the value of $r_2$ in terms of $r_1$ from $Z = 0$, and putting it into the value of $t$ in (6), one gets

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3 See Eq. (6).
In Eq. (10) everything is a parameter, excepting $t$. To see how the mechanism works, and to get explicit solutions, we use Assumption 3.

**Proposition 1.** Suppose Assumptions 1–4 hold. Let $Q \equiv (\rho R)^{1/s}$. If $K \geq K_0 \equiv [R - Q]/[(R + Q)\theta']$, then a Nash equilibrium exists where risk averse agents buy deposits, risk neutral agents buy equity and, there is no bank run. The equilibrium deposit rate pair $(r_1^*, r_2^*)$ is given by $r_1^* = [2R]/[Q + R]$ and $r_2^* = Qr_1^*$, and period 1 dividends are given by $v_1^* = 0$.

**Proof.** Choose $r_1 = r_1^*$, $r_2 = r_2^*$ and $v_1 = 0$. We first show that it does not pay a risk neutral agent, $j$, to buy deposits if all other risk neutral agents are buying equity and, all risk averse agents are buying deposits. If $j$ buys deposits, her period 1 consumption will be $r_1^*K$ if she turns out to be type 1, and $r_2^*K$ in period 2 if she is of type 2. This gives her an expected utility in period 0 of

$$\frac{1}{2}[r_1^* + \rho r_2^*]K = \frac{RK(1 + \rho Q)}{Q + R} < \rho RK$$

given Assumption 2. If $j$ buys equity instead, she gets $\rho RK$. This follows from (8) where $v_1 = 0$ and $Z = 0$. The fact that $Z = 0$ follows from using $\tau = (1/2)$ and using the values of $r_1^*$ and $r_2^*$. It also follows from (8) that $j$ gets at least her reservation utility.

We now show that no risk averse agent will deviate. Given $v_1 = 0$ and Assumption 3, a risk averse agent will not buy equity. The other alternative is to invest directly in technology, and getting a consumption of 1 if type 1, and $R$ if type 2. This will give a lower utility than buying deposits, as is evident from the fact that $1 < r_1^* < r_2^* < R$, since $1 < Q < R$. Intuitively, the risk averse agent by buying deposits, gets a narrower spread than by investing in the technology.

To show there is no run, we have to show that $t \geq 1$. With $v_1 = 0$,

$$t = \frac{R(1 + \theta'K) - r_2^*}{Rr_1^* - r_2^*}.$$  

Plugging in the values of $r_1^*$ and $r_2^*$, and using $K \geq [R - Q]/[(R + Q)\theta']$, one gets $t \geq 1$.

Thus, if banks have unlimited liability, depositors can be fully insured. For unlimited liability to be credible, there must be enough risk neutral capital. The latter depends on two things – the proportion of risk neutral to risk averse agents, $\theta'$, and the proportion of risk neutral to risk averse capital in the economy, $K$, per representative agent. This suggests that, if the risk neutral capital is not sufficient, then the depositors cannot be fully insured.
However, note that, if $K < K_0$, it does not follow that there has to be a positive probability of a run on the bank. Recall that there is a positive probability of a run if $t < 1$. For this, it must be the case that $(1 + \theta' K) < r_1$. This follows from (6), after putting $v_1 = 0$. Thus, a run can always be avoided if one chooses $r_1 = (1 + \theta' K)$. In Diamond and Dybvig, $\theta' = 0$, and hence, zero probability of a run implied that $r_1 = 1$. However, this meant that the outcome was inefficient (no insurance). Thus, the question that remains is the following: Suppose that $K < K_0$. Does it mean that $r_1 = (1 + \theta' K)$, which allows for partial insurance, is the only solution when $K < K_0$? We study this question in the next section.

6. Less than adequate capital

For this section we will continue to assume that Assumptions 1–4 hold; in addition, $K < K_0 \equiv [R - Q]/[(R + Q)\theta']$. This ensures that capital is not adequate, i.e., $t < 1$, if we choose $r_1 = r_1^*$ and $r_2 = r_2^*$. We will, instead, allow the banks to suspend convertibility (of deposits) if they want to. The period 0 contract with depositors will include an “option clause” which will allow banks to suspend convertibility if the realisation of $t$ is greater than $t_s$. Observe that in the previous section, this $t_s$ was implicitly fixed at values greater than 1 and, hence, irrelevant.

We will pose the problem generally, and allow any $t_s$. For all realisations $t \leq t_s$, the depositors will be able to withdraw $r_1$. If $t > t_s$, proportion $t - t_s$ will receive $b \leq r_1$ and the proportion $t_s$ will receive $r_1$. Given Assumption 3, $b > 0$. Since the bank does not know the type of each depositor, it will implement this policy by giving the first $t_s$ depositors $r_1$ and the remaining depositors who want to withdraw in period 1 will receive $b$. Again, to prevent type 2 depositors from withdrawing in period 1, the following must be true: $r_1 < r_2$, and for $t > t_s$,

$$\left[1 + \theta' K - r_1 \min(t, t_s) - b \max(t - t_s, 0) - v_1 \theta' K\right]R \geq r_2(1 - t),$$

which can be written as

$$\left[1 + \theta' K - r_1 t + (r_1 - b) \max(t - t_s, 0) - v_1 \theta' K\right]R - r_2(1 - t) \geq 0. \quad (11)$$

---

4 With Assumption 3, $u(0)$ is negative infinity. With any $t_s < 1$, this implies a positive probability of $t > t_s$, and depositors will get a negative infinity utility with positive probability. In other utility functions, where $u(0)$ is bounded below, $b$ could be equal to zero – a complete suspension of convertibility rather than a partial one as is necessary here. As will become clear, our utility function actually makes the point we are trying to state in this paper more difficult. The final result is, therefore, stronger!
Observe that this expression is similar to the one leading up to Eq. (6), with a modification for the option clause. Also, if the expression above is satisfied for some \( t > t_s \), then, given \( r_1 \geq b \), it is satisfied for all \( t \).

The condition \( r_1 \geq b \) is important. If \( r_1 > b \), we have a non-trivial solution to the (voluntarily contracted) suspension of convertibility, in the sense that some type 1 depositors get paid less than others. If, on the other hand, \( r_1 = b \), then every type 1 depositor (or any body who wants to withdraw in period 1) gets the same amount, implying that the option clause solution is a trivial one, i.e., \( t_s = 1 \).

We proceed as before. Observe that (11) guarantees that shareholders in the bank obtain non-zero returns in period 2. Once again, for the moment assume that all risk neutral agents buy equity and trade in ex-dividend shares in period 1. For any \( v_1 \) and \( t \), like before, the consumption of type 1 risk neutral agents will be \( v_1/t \). Now, from (3) and (11),

\[
EU^n = v_1K(1 - \rho R) + \rho RK + \rho \frac{2R - Rr_1 - r_2}{2\theta^r} + \frac{\rho R(r_1 - b)}{2\theta^r}(1 - 2t_s + t_s^2).
\]

Given Assumption 2, like in the previous section, \( v_1 = 0 \), and the risk neutral agents, by themselves (i.e., without issuing deposits) can obtain \( \rho RK \). Hence, we must have \( EU^n \geq \rho RK \). From (12), therefore,

\[
2R - Rr_1 - r_2 + R(r_1 - b)(1 - 2t_s + t_s^2) \geq 0.
\]

If \( t_s = 1 \), the left-hand side of (13) collapses to what we had described as \( Z \) in the previous section.

Again, suppose that all risk averse agents buy deposits. If \( t \leq t_s \), then with probability \( t \) they will be type 1 and obtain a utility equal to \( u(r_1) \) and with probability \( 1 - t \) they will be type 2 and obtain a utility \( \rho u(r_2) \). If \( t > t_s \), and they are of type 1, they will get \( u(r_1) \) if they come to the bank soon enough. If reaching the bank is a random event, this probability will be \( t_s/t \). With probability \( (t - t_s)/t \), they will be late and get \( u(b) \). If they are of type 2, they are assured of \( r_2 \). Thus,

\[
EU^a = \int_0^{t_s} [tu(r_1)]dF(t) + \int_{t_s}^1 \left\{ t \left[ \frac{t_s}{t}u(r_1) + \frac{t - t_s}{t}u(b) \right] \right\} dF(t) + \int_0^1 [(1 - t)\rho u(r_2)]dF(t).
\]

Using Assumption 1, (14) can be written as
\[ \text{EU}^a = \left( t_s - \frac{t_s^2}{2} \right) u(r_1) + (1/2)(1 - t_s)^2 u(b) + (1/2)\rho u(r_2). \] (15)

Also, for type 2 depositors not to indulge in a panic run to the bank, they must be convinced that (11) holds for all \( t \). In particular, if it holds for \( t = 1 \), it holds for all \( t \). Putting \( t = 1 \) in (11), we get
\[ (1 + \theta K)R - Rr_1 + R(r_1 - b)(1 - t_s) \geq 0. \] (16)

The competitive bank’s problem is then a simple one – maximise \( \text{EU}^a \) given by (15), subject to (13) and (16). However, now we need an additional constraint, regarding the maximum value of \( t_s \): The definition of \( t_s \) implies that it be restricted to values that are no greater than 1, i.e., \( 1 - t_s \geq 0 \). If \( \lambda \) is the Lagrange multiplier for (13), \( \eta \) for (16), and \( \alpha \) for the restriction that \( 1 - t_s \geq 0 \), the following must hold:
\[
\left( t_s - \frac{t_s^2}{2} \right) u'(r_1) - \lambda R(2t_s - t_s^2) - \eta Rt_s = 0, \] (17)
\[
\frac{1}{2} \rho u'(r_2) - \lambda = 0, \] (18)
\[
\frac{1}{2} (1 - t_s)^2[u'(b) - 2\lambda R] - \eta R(1 - t_s) = 0, \] (19)
\[
(1 - t_s)[u(r_1) - u(b) - 2\lambda R(r_1 - b)] - \eta R(r_1 - b) - \alpha = 0. \] (20)

As in Proposition 1, we can easily show that if \( v_1 = 0 \), and we maximise (15) subject to (13) and (16), then we can support a Nash equilibrium where all risk neutral agents buy equity and risk averse agents buy deposits. The proof is exactly as in Proposition 1. The only thing to check is that \( r_2 > r_1 \). This follows directly from combining (17) and (18), since \( \rho R > 1, \eta \geq 0 \), and \( t_s > 0 \).

However, we are here more interested in the value of \( t_s \). Specifically, is the option clause an equilibrium outcome?

**Proposition 2.** Let \( K < K_0 = [R - Q]/[(R + Q)\theta^0] \), and Assumptions 1–3 hold. Then, the optimal \( t_s \) equal to 1 is always a solution. However, the depositors do not obtain full insurance, i.e., \( u'(r_1) \neq \rho R u'(r_2) \).

**Proof.** First, observe that Eqs. (17)–(20) are always satisfied at \( t_s = 1 \), provided \( \alpha = 0 \). Indeed, we will now argue that \( \alpha \) is always equal to zero. Suppose instead that \( \alpha > 0 \). Then \( t_s = 1 \). But then, either one of these two must hold for

\[ 6 \text{ If } t_s = 0, \text{ the option clause is irrelevant since then } b \text{ becomes like } r_1! \]
(20) to be satisfied: \( \{ \eta < 0 \text{ and } r_1 > b \} \) or, \( \{ \eta > 0 \text{ and } r_1 < b \} \). Both are invalid since \( \eta \geq 0 \), by virtue of being the Lagrange multiplier, and \( r_1 \geq b \), by definition. The fact that \( z \) is never positive, does not rule out \( t_s = 1 \). It does, however, suggest that there could be instances where \( t_s < 1 \) is a solution.

We now show that \( \eta > 0 \). Suppose, instead, that \( \eta = 0 \). Then, from combining (17) and (18), and (18) and (19) given \( \hat{a} = 0 \), \( u'(r_1) = \rho Ru'(r_2) \) and \( u'(b) = \rho Ru'(r_2) \), implying \( b = r_1 \). Using Assumption 3,

\[
r_2 = Qr_1,
\]

where \( Q \) was defined as \( (\rho R)^{(1/\theta)} \). Also, from (13), using \( b = r_1 \), and the fact that \( \lambda > 0 \) (from Assumption 3), we get

\[
r_2 = 2R - Rr_1.
\]

This implies that

\[
r_1 = \frac{2R}{R + Q}.
\]

From (16), and \( r_1 = b \), we get

\[
1 + \theta'K \geq r_1 = \frac{2R}{R + Q},
\]

which, in turn, implies that \( K \geq K_0 \). This leads to a contradiction.

So, \( \eta > 0 \). Given Assumption 3, we know that \( r_1, r_2 \) (and \( b \)) will all be positive.\(^7\) Recall that, given Assumption 3, \( \lambda > 0 \). This completes the proof.

With \( t_s = 1 \), and Eq. (16) holding with equality (\( \eta > 0 \)), it follows that \( r_1 = 1 + \theta'K \). In the last section, where \( K \geq K_0 \), we also had the same result. However, there, \( K \), and hence \( \theta'K \), was large enough to allow \( r_1 \) and \( r_2 \) to satisfy the (full) insurance condition of \( u'(r_1) = \rho Ru'(r_2) \). In Eq. (17), the full insurance condition can never be satisfied if \( \eta > 0 \). Our two propositions argue that \( \eta \) will be equal to zero if and only if \( K \geq K_0 \), i.e., there is enough equity in the bank. The problem with the DD model was that they were trying to achieve a full insurance outcome, without there being any risk neutral capital. Not surprisingly, such a setup was prone to (market) failure.

Why is \( t_s = 1 \)? Suppose that \( t_s < 1 \). This necessarily means that \( r_1 > b \). But this implies that in period 1, the agents, if they are type 1, get either a high return of \( r_1 \) or a low return of \( b \). Consider another bank that reduces \( r_1 \) slightly (to \( r_1 - \epsilon \)) and increases \( b \) (to \( b + \delta \)) such that conditions (13) and (16) continue to hold.\(^8\) Depositors being risk averse, one can show that this will improve

\(^7\) Of course, with \( t_s = 1 \), value of \( b \) is irrelevant.

\(^8\) This can be done by choosing \( \epsilon t_s = \delta (1 - t_s) \).
their period 0 expected utility of period 1. Moreover, the second bank will be able to further increase $t_s$ in this situation. Hence $t_s$ moves towards 1.

7. Conclusion

In this paper, we have reformulated the bank run problem of DD, and allowed banks to issue both debt and equity. We go on to show that the presence of equity prevents a bank run, even without government deposit insurance. In addition to this, equity plays another major role. It allows risk averse depositors to be insured. If equity is large enough, depositors can be fully insured, i.e., they obtain full insurance coverage against liquidity shocks and have perfect consumption smoothing. If the amount of equity is less, then also runs are prevented but depositors are only partially insured. The solution we obtain is also efficient, as we maximize the depositors’ utility by keeping the (bank’s) shareholders at their opportunity cost.

The findings in our paper have important implications for transition economies trying to develop a sound financial market, insulated against financial crises as recently experienced in the East Asian markets. A popular approach is to devise insurance guarantee schemes that will prevent bank runs. The pitfalls of such schemes in the presence of moral hazard have been highlighted in Kane (1989). Indeed, Demirg-Kunt and Detragiache (1997) provide evidence that countries with an explicit deposit insurance scheme have been more prone to financial market failures.

Our model suggests that, in the present world system of global banks, there is enough risk neutral global capital to ensure that risk averse depositors in any one country can be offered full insurance against liquidity shocks, provided there is competition among banks. Many transition economies, unwilling to open up their financial markets to global competition, are trying to provide full insurance to depositors by adopting the left over regulations of currently developed countries. However, these regulations are a legacy of a world order, when banks had to be regulated for political, rather than pure economic reasons. More importantly, such regulations were initially made when competition among banks, especially with foreign ones, was not the order of the day. It is more important for transition economies to generate competition, rather than devising policies to obtain the market outcome without free competition among banks.

The sufficient amount of (risk neutral) capital, to provide full insurance to depositors, is dependent on the risk aversion of the depositors, the amount of

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9 As in the case of pure insurance, if banks are not competitive, depositors cannot be fully insured.
resources with them, the distribution governing liquidity shocks, etc. This makes the job of defining a uniform capital adequacy norm across all countries dangerous. On the other hand, banks left to themselves, but facing the threat of competition, will develop the types of contract necessary to tackle the problem of (inadequate) capital, if any.

References