A note on nonstationarity, structural breaks, and the Fisher effect

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Abstract

We find empirical evidence that inflation, nominal, and real interest rates in the US are trend-stationary with a structural break in both the unconditional mean and the drift rate of a deterministic trend, which occurs shortly after the change in operating procedures of the Fed in September 1979. This finding casts some doubts on cointegration tests of the long-run Fisher effect conducted in recent studies, since the results of these tests can be affected by the existence of common structural breaks in the series. We propose an alternative test of the Fisher effect, based on a VAR representation in appropriately detrended variables. We find strong support for the Fisher effect both in the medium term and in the long term. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The Fisher equation is one of the oldest and simplest equilibrium asset pricing models. It relates the one-period nominal interest rate to the sum of the
ex ante real interest rate and expected inflation. One of the most popular interpretations of the Fisher equation is that if the ex ante real interest rate is determined in the real sector by ‘technology and tastes’, then the nominal interest rate is related one-for-one to expected inflation.

Despite the general acceptance of the Fisher effect in theory, a stable one-for-one relationship between nominal interest rates and inflation has proven extremely difficult to establish empirically. Typically, estimated slope coefficients in regressions of nominal interest rates on various measures of expected inflation are substantially less than the hypothesised value of one, or, alternatively, real rates are negatively correlated with expected inflation.¹

Recently, empirical studies have focused on nonstationarities of the data and tested the Fisher effect as a long-run equilibrium relationship using cointegration techniques.² If inflation and nominal interest rates are unit root processes that contain permanent disturbances and the Fisher effect holds in the long run, then nominal interest rates and inflation ought to be cointegrated with a unit cointegration vector. In this case, the series move one-for-one in the long run such that their permanent disturbances cancel out leaving the real rate stationary. Although a number of these studies finds empirical support for the long-run Fisher effect, the result of a stationary real interest rate has been obtained by assuming that inflation and the nominal interest rate are nonstationary processes.

A potential difficulty in assessing the time series properties of inflation and interest rates is the existence of structural breaks in the form of infrequent changes in the mean or the drift rate of the series due to distinct exogenous events (oil price shocks, shifts in monetary or fiscal policy regimes etc.). As Perron (1989) showed, standard stationarity tests are biased towards nonstationarity since they misinterpret structural breaks as permanent stochastic disturbances. The issue of stationarity is important for choosing testing strategies. For example, assume that nominal interest rates and inflation are trend-stationary with a common structural break in their deterministic components, i.e. the unconditional mean or the time-trend of the series. A test of the Fisher effect based on a cointegration regression would suggest in this case that the series are cointegrated, i.e. they share a common stochastic trend. However, the finding of cointegration is due to the fact that both series are subject to a common deterministic shock and

¹ See Fama and Schwert (1977), Mishkin (1981), Fama and Gibbons (1982), Huizinga and Mishkin (1984, 1986) and Kandel et al. (1996), among others. Accounting for tax-effects leads to even stronger rejections of the Fisher effect, since in that case the coefficient of inflation should be higher than unity.

does not necessarily indicate a long-run relationship between the stochastic components of the series.

The purpose of this paper is twofold. First, to investigate the univariate time series properties of inflation and interest rates, allowing for structural breaks of unknown timing in the series. Second, to propose a new test of the Fisher effect in a stationary framework that allows for both short-term and long-term adjustments of the variables. Using quarterly data for the US from 1960:Q1 to 1995:Q3, we find that inflation, 3-month nominal and real interest rates are trend-stationary with a structural break in both the unconditional mean and the drift rate in the early 1980s. This result contrasts with Garcia and Perron (1996) study which identifies two structural breaks in the mean of inflation and the real rate, and Evans and Lewis (1995) study which reports evidence of stationarity for the real rate with one structural break in the mean but no evidence of stationarity for inflation and nominal interest rates. The difference in results can be explained by the difference in the alternative hypothesis. Whereas these authors test nonstationarity against the alternative of structural shifts in the mean, we test nonstationarity against the joint hypothesis of structural shifts in both the mean and the drift rate of the series. Thus, our test includes Evans and Lewis’ test as a special case under the alternative hypothesis.

In order to investigate the dynamic relation between inflation and the nominal interest rate, we employ a vector-autoregressive (VAR) model using appropriately detrended stationary variables. Based on the impulse–response function of the VAR, we derive correlations between the nominal interest rate and inflation at various horizons and test the Fisher effect as a dynamic relationship. Our results suggest that the Fisher effect holds in the medium to the long term. The speed of adjustment of interest rates to inflation shocks is found to be considerably higher than suggested in previous studies.

The remainder of the paper is as follows. Section 2 presents the data and their univariate time series properties. In Section 3 we discuss implications of unit root tests for modelling the relationship between interest rates and inflation. Section 4 presents the results of the VAR analysis and introduces a new dynamic test of the Fisher effect. Section 5 investigates the robustness of the VAR model to various specifications. Section 6 summarizes the principal findings and concludes.

2. Time series properties of the data

The empirical analysis is based on quarterly data for the United States from 1960:Q1 to 1995:Q3. Nominal interest rates are three-month Treasury-Bill rates. Inflation is the one quarter ahead change in the log of the Consumer
Price Index (CPI). All data are from the OECD database and were downloaded from Datastream International. ³

Augmented Dickey–Fuller (ADF) tests for unit roots are reported in columns headed \( t \) (with constant) and \( t_\tau \) (with trend). Column \( t(\tau) \) reports Zivot and Andrew tests, allowing for a break in both the mean and the drift rate of the trend. Column \( T\tau \) reports the estimated date of a structural break. Under column \( a \) we show the estimated AR-1 coefficient of the series. Numbers in the parentheses are the number of lags, \( k \), in the ADF regression, chosen according to Campbell and Perron (1991). Numbers in brackets are asymptotic standard errors. Data are quarterly from OECD. Sample: 1960:Q1–1995:Q3.

Next, we apply the sequential ADF test of Zivot and Andrews (1992) which accounts for structural breaks in the data with endogenous timing. ⁵ This test can be conducted using three variants of the alternative hypothesis:

Model A: The series is trend-stationary with a break in the mean.
Model B: The series is trend-stationary with a break in the drift rate of the trend.
Model C: The series is trend-stationary with a break in both the mean and the drift rate of the trend.

Formally, the sequential ADF test equation of model C reads:

³ The Datastream codes are USOCPCONF for CPI, and USOCTBL for T-Bill rates. Quarterly inflation was multiplied by 400 to obtain annualized figures.

⁴ Both the Campbell–Perron and the AIC criterion suggest inclusion of five lags. The more conservative BIC criterion, however, indicates three lags. The ADF \( t \)-statistic is in this case \(-2.50\) and cannot reject a unit root in the nominal interest rate. Test results for inflation and the real interest rate do not suffer from this problem.

⁵ A similar test has been developed by Peron and Vogelsang (1992).
$y_t = b_0 + b_1 t + b_2 D_{t, \tau} + b_3 D_{t, \tau} t + a y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + u_t,$

where $D_{t, \tau}$ is a dummy variable that takes the value 0 when $t \leq T\tau$ and 1 else, $T$ is the number of observations and $\tau \in (0, 1)$ is the relative timing of the structural break. Model A (the case considered in Evans and Lewis) assumes $b_3 = 0$ and model B assumes $b_2 = 0$, i.e. both models are nested within model C. The null hypothesis in all three models is that the series has a unit root, i.e. $a = 1$.

Column $t(\tau)$ of Table 1 presents the results of the sequential ADF test with a structural break of unknown timing for inflation, nominal interest rates, and the ex post real rate (model C). The unit root hypothesis can be rejected at the 5% level against the alternative of stationarity with a shift in the mean and the drift rate of the deterministic trend for all three series. Based on the lowest value of the $t$-statistic, the break point is estimated at 1980:Q3 for the nominal and real interest rate, and at 1981:Q3 for inflation, two years after the introduction of money supply targeting by the Fed in September 1979. Estimates of the AR-1 coefficients, $a$, suggest that the series are stationary processes after adjusting for structural breaks. The minimum values of $a$, which are the relevant ones if the series exhibit a structural break, are: 0.40 for inflation, 0.63 for the nominal rate and 0.28 for the real rate. However, contrary to Garcia and Perron (1996), who find that the real rate is stationary with two shifts in the mean in 1973 and 1981, our results suggest that the real rate is stationary around a negative linear trend with one upward shift in 1980:Q3. This is in line with evidence presented in Evans et al. (1994), based on US and UK data covering more than one century, that the long-term mean of the real interest rate exhibits a downward trend. To the extent that the real rate reflects the productivity of investment, this finding can be explained along the lines of the neoclassical growth model as the result of an increase in capital intensity. The finding of two shifts in the mean of the real rate in the Garcia and Perron study may be the result of the sample choice by the authors (1961–1986). Truncating the sample at 1986, the real rate can be plausibly approximated by a random process around a constant mean with one downward shift in 1973 and one upward shift in 1980–81. When we extend the sample to 1995, however, the real rate appears to exhibit a negative trend after 1981 – see Fig. 1.

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6 Stationarity of the real interest rate could not be rejected against the alternative of model A, confirming the result of Evans and Lewis (1995). Also, note that, under the alternative of model A, inflation and nominal interest rates seem to contain a unit root, as reported by Evans and Lewis. Similar results were obtained for model B. These results are available from the author upon request.
3. Structural breaks and the long-run Fisher effect

Evidence of structural breaks in inflation and interest rates which make the series appear as unit root processes has important implications for tests of the Fisher effect. In particular, if a broken linear trend is common in both series, cointegration analysis may lead to the wrong inference of a common stochastic trend, although the series are trend-stationary. Assume, for example, that one-quarter ahead inflation, \( \pi_t \), and the three-month nominal interest rate, \( R_t \), are given by

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\pi_t = \pi(t) + \pi_t^a,
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This casts some doubts on the results of a number of studies which find cointegration between interest rates and inflation, e.g. Mishkin (1992), Wallace and Warner (1993), Engsted (1995), Crowder and Hoffman (1996), Daniels et al. (1996).

We define \( \pi_t \) as the proportional rate of change in the CPI between \( t \) and \( t+1 \), and \( R_t \) as the yield on a one-period zero-coupon bond purchased at time \( t \) which matures at time \( t+1 \).
\[
R_t = \beta(t) + R^a_t,
\]

where \( x(t) \) and \( \beta(t) \) are deterministic trend polynomials allowing for a structural break at time \( T^* \), as specified in model C of the sequential ADF test, and \( \pi^a_t, R^a_t \) are stationary, zero-mean, stochastic processes. Assuming rational expectations, a test of the Fisher effect as a long-run relationship is equivalent to a unit root test of the ex post real interest rate, \( r_t \), given by

\[
r_t \equiv \beta(t) - \beta(t) + R^a_t - \pi^a_t.
\]

Note that \( r_t \) is trend-stationary by definition, since \( \pi^a_t, R^a_t \) are stationary. Moreover, \( r_t \) is free of structural breaks if the deterministic trend-polynomials of \( R_t \) and \( \pi_t \) cancel out exactly, i.e. if \( \beta(t) - \beta(t) \) is constant (or zero). Provided that \( \beta(t) - \beta(t) \) is constant, any linear combination \( \beta(t) - \beta(t) + R^a_t - \gamma \pi^a_t \), for arbitrary values of \( \gamma \), will also be trend-stationary. Hence, due to the existence of a common structural break in \( R_t \) and \( \pi_t \), a test for cointegration between \( R_t \) and \( \pi_t \) cannot reject the existence of a long-run Fisher effect. This result, however, is misleading, since the Fisher effect ought to hold only if \( \gamma = 1 \), i.e. if there is a one-for-one relationship between the stochastic components \( R^a_t \) and \( \pi^a_t \) of the series. On the other hand, the finding of no cointegration does not necessarily indicate that \( r_t \) is nonstationary, since nonstationarity of \( r_t \) may result from the fact that \( \beta(t) - \beta(t) \) is not constant. If inflation and interest rates are trend-stationary with common structural breaks, a more appropriate test of the Fisher effect should be carried out in a stationary framework, allowing for dynamic adjustments of the variables.

The test proposed in this paper is based on a VAR model representation in appropriately detrended variables, and thus, provides a framework from which incisive evidence regarding both the short-term and the long-term relationship between nominal interest rates and inflation can be obtained. In order to remove the structural break from the series before fitting the VAR model, we estimate the OLS regressions

\[
\pi_t = b_{n,0} + b_{n,1}t + b_{n,2}D_{t,c^*_n} + b_{n,3}D_{t,c^*_n}t + \pi^a_t,
\]

\[
R_t = b_{R,0} + b_{R,1}t + b_{R,2}D_{t,c^*_R} + b_{R,3}D_{t,c^*_R}t + R^a_t,
\]

where \( D_{t,c^*_n} \) takes on the value 0 before 1981:Q3 and unity afterwards; and \( D_{t,c^*_R} \) takes on the value 0 before 1980:Q3 and unity afterwards. The residuals of these regressions, \( \pi^a_t, R^a_t \), are zero-mean stationary by construction and can be used in the VAR analysis.

Fig. 1 plots quarterly inflation, the nominal interest rate and the ex post real rate, along with their fitted segmented trends.
4. VAR tests of the Fisher effect

Assume that $\pi_t^a$ and $R_t^a$ have a VAR($j$) representation:

$$B(L)\hat{y}_t = u_t,$$

where $\hat{y}_t = (\pi_t^a, R_t^a)'$, $B(L)$ is a $(2 \times j)$ matrix of $j$th order polynomials in the lag operator $L$ and the residual vector $u_t = (u_{1,t}, u_{2,t})'$ is i.i.d. normal with zero mean and covariance matrix $\Sigma$. The Vector Moving Average representation of (6) with orthogonal innovation processes reads

$$y_t = C(L)e_t,$$

where $C(L) \equiv B(L)^{-1}D$, $e_t = (e_{1,t}, e_{2,t})' = D^{-1}u_t$, $E(e_t) = 0$, $\text{Var}(e_t) = I$, and $D$ is the lower triangular Choleski matrix satisfying $\Sigma = DD'$. 

In order to test the Fisher effect, we compute dynamic correlations, $\rho_k$, between $R_{t+k}$ and $\pi_{t+k}$ generated by an innovation in $\pi$ at time $t$. These can be calculated as the ratios of the cumulated impulse–response functions of $R$ and $\pi$ to an inflation shock, $e_{1,t}$, $k$ periods after the shock:

$$\rho_k = \frac{d\Sigma_{i=0}^k R_{t+i} / de_{1,t}}{d\Sigma_{i=0}^k \pi_{t+i} / de_{1,t}} = \frac{\Sigma_{i=0}^k C_{21,i}}{\Sigma_{i=0}^k C_{11,i}}, \quad k = 1, 2, \ldots, \infty.$$

Dynamic correlations can be interpreted in a straightforward way: they express the response of $R$ to an orthogonal shock in $\pi$ relative to the response in $\pi$ to its own shock $k$ periods after the shock occurs. If the Fisher effect holds at each point in time, then $\rho_k$ should equal unity for all $k$. If the Fisher effect holds only in the long run, then $\rho_k$ should converge to unity for increasing $k$.

Note that identification of $\rho_k$ presupposes an orthogonal transformation of the innovation vector $u_t$. The method of orthogonalization employed here is the standard Choleski decomposition of the covariance matrix. This method imposes a specific causality structure into the VAR such that, using the ordering $(\pi_t, R_t)$, $\pi_t$ responds only to its own contemporaneous shocks, whereas $R_t$ responds to contemporaneous shocks to both $\pi_t$ and $R_t$. The Choleski restriction used here could be plausibly rationalized by arguing that the price level is sticky, i.e. predetermined for at least one quarter.9

A parsimoniously parameterized VAR(3) model was estimated in the detrended variables $(\pi_t^a, R_t^a)$. The order of the VAR was chosen based on the Schwarz criterion.10 The hypothesis that the nominal interest rate is not

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9 Results based on the inverse ordering do not differ significantly – see Section 5.
10 Inflation shows a weak seasonal pattern. However, inclusion of deterministic seasonal dummies does not affect the qualitative results of the following analysis.
Granger causal to inflation could not be rejected at the 10% significance level, whereas Granger noncausality from inflation to the nominal interest rate could easily be rejected at the 1% significance level.  

Fig. 2 plots dynamic correlations between $R_{t+k}$ and $\pi_{t+k}$ generated by an orthogonal shock to inflation at time $t$ for $k = 1, \ldots, 20$ quarters. Dotted lines in the graph represent 95% significance bands calculated by bootstrap simulations. The correlations are not significantly different from unity for horizons longer than five quarters after the shock occurs, suggesting that the Fisher

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11 See also Crowder and Hoffman (1996), who cannot reject strong exogeneity, i.e. long-run exogeneity of inflation and (short-run) Granger-noncausality of the interest rate. This result contrasts with Daniels et al. (1996), who find weak evidence of bi-directional causality using a VAR(3) model. One possible reason for this is that the authors estimate a VAR in first differences with a restriction on the cointegration vector, whereas we estimate a VAR in trend-adjusted, stationary levels.

12 In each run we draw a random sample $\tilde{y}_t$ with replacement from the empirical distribution of estimated residuals $\pi_t^d$ and $R_t^d$ of regressions (4) and (5), re-estimate the VAR using the vector $\tilde{y}_t$ and calculate impulse response functions and dynamic correlations based on the orthogonal decomposition $\tilde{e}_t = D^{-1}B(L)\tilde{y}_t$ where $D$ and $B(L)$ are VAR estimates of the artificial data. After 2000 runs, we calculate standard errors of dynamic correlations from their sampling distributions and construct 95% significance bands.
effect holds in the medium to the long run. The fact that dynamic correlations are significantly less than one at shorter horizons is consistent with evidence of previous studies that real rates and inflation are negatively correlated.\textsuperscript{13} However, contrary to those studies, the results of the present analysis suggest that this negative correlation is only a short term phenomenon.

Our results also differ significantly from results of previous studies regarding the speed of adjustment of nominal interest rates to inflation shocks. Crowder and Hoffman (1996), for example, report adjustment lags of six to eight years for the nominal interest rate to adjust fully to an inflation shock. To the contrary, this study finds much faster adjustments of interest rates to inflation shocks (by a factor of five to six), and, consequently, lower persistence of real interest rates. One reason for the contrasting results would seem to be the fact that we estimate a VAR in detrended, stationary variables, whereas Crowder and Hoffman (1996) estimate a VAR in first differences that contain per definition a much higher proportion of permanent shocks. Since there is evidence of structural breaks in the data which have the appearance of permanent shocks, failure to recognise this in modelling the relationship between the variables may lead to substantial, spurious, lags in adjustment.

Table 2 reports variance decompositions of $\pi$ and $R$ at various horizons. The numbers in each row represent the fraction of the variance of the $k$ quarters ahead forecast error for each variable explained by each type of shock. Most of the variance of inflation is explained by its own shocks (around 95%). Nominal interest rates are driven mainly by their own innovations in the short run, which account for 62% of their variance. However, the variance proportion of nominal rates accounted for by inflation shocks increases significantly to 63% after four quarters and to 70% after eight quarters, thus supporting the earlier evidence of a medium-term to long-term Fisher effect. Fig. 3 plots the nominal

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Horizon (quarters) & Inflation & & & T-Bill Rate & \\
& Inflation & T-Bill & SE & Inflation & T-Bill \\
& shock & shock & SE & shock & shock \\
\hline
1 & 95.0 & 5.0 & 3.3 & 37.5 & 62.5 & 1.2 \\
4 & 93.5 & 6.5 & 3.7 & 63.3 & 36.7 & 2.6 \\
8 & 93.7 & 6.3 & 3.8 & 69.6 & 30.4 & 2.9 \\
12 & 93.7 & 6.3 & 3.8 & 70.2 & 29.8 & 3.0 \\
16 & 93.7 & 6.3 & 3.8 & 70.2 & 29.8 & 3.0 \\
20 & 93.7 & 6.3 & 3.8 & 70.2 & 29.8 & 3.0 \\
\hline
\end{tabular}
\caption{Variance decompositions\textsuperscript{a}}
\end{table}

\textsuperscript{a}Variance decompositions are based on a VAR(3) model in the detrended variables ($\pi_t^t, R_t^\tau$). SE: asymptotic standard error.

\textsuperscript{13} See references in footnote 1.
interest rate along with its inflation component, derived from the moving-average representation of $R_t^a$ by setting the interest rate shocks, $e_{2,t}$, equal to zero and adding the deterministic segmented trend component of the nominal interest rate. The apparent comovement of the series demonstrates that most historical episodes of interest rate changes are closely related to innovations in the rate of inflation.

5. Tests of robustness

To ensure robustness of the results, a series of sensitivity tests has been performed. In order to test robustness of the results to variations of the lag-length of the VAR, we estimated a VAR(4) and a VAR(2) model. In both models the hypothesis that the nominal interest rate adjusts to inflation shocks fully within five quarters cannot be rejected at the 5% level.

In order to test the robustness of the results to the ordering of the variables, the analysis was performed using the alternative ordering $(R_t^a, \pi_t^a)$. The impulse – responses and the variance decompositions of this experiment did not reveal significant differences, confirming our earlier results of Granger causality tests.

As a third exercise, a VAR(3) model in first-differenced variables was estimated, i.e. $\Delta\pi_t, \Delta R_t$. In order to account for the structural break in the series, a dummy variable was included which takes on the value one in 1980:
Q3–1981:Q3 and zero elsewhere. Although the variance decomposition of the nominal interest rate changed significantly, the hypothesis that the nominal interest rate responds one-for-one to shocks in inflation could not be rejected at the 5% significance level for horizons longer than seven quarters.

In a final experiment, we test the robustness of the results to changes in the data set. For this purpose, the analysis was repeated using returns on zero-coupon 3-month T-Bills from the McCulloch data set over the period 1960:Q1–1990:Q4. Although the speed of adjustment of the nominal rate to inflation shocks decreases somewhat, the Fisher effect could be rejected at the 5% significance level only for horizons of less than twelve quarters after the shock, thus supporting the qualitative results of our analysis.

6. Conclusion

We re-examined empirical evidence on the interest rate–inflation relationship in the United States. Our results suggest that inflation, real, and nominal interest rates are trend-stationary with a structural break in both the mean and the drift rate of a deterministic trend in the early 1980s, when the Federal Reserve introduced new operating procedures of money supply targeting. The dynamic effect of inflation on nominal interest rates is investigated using a VAR model in appropriately detrended, stationary variables. The empirical results lend strong support to the Fisher effect in the medium to the long term. A series of sensitivity tests confirmed the robustness of the results to changes in the specification of the VAR and to alternative data sets. Due to the stationarity of the series, estimates of the speed of adjustment of nominal interest rates to inflation shocks are considerably higher than previously reported in the literature. A general implication of our analysis is that failure to account for structural breaks in economic data may lead to wrong inference about long-run behavior and understatements of the speed of adjustment of variables to long-run economic relationships.

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14 This data set is also used by Evans and Lewis (1995).
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