Default risk and optimal debt management

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Abstract

The role of movements in real rates in explaining the relationship between long- and short-term interest rates is explored within a model of optimal government debt management. The government’s incentives to resort in the future to inflation and ex post debt taxation in order to reduce the real value of its nominal liabilities have an impact on term premia and hence on the short–long spread. In particular, default risk and, consequently, long-term interest rates increase with the size of outstanding debt and the level of real rates; in the presence of short maturities, indexed debt and anti-inflationary governments. Optimal maturity either lengthens or shortens with inflation risk, depending on the time profile of government expenditure, while it always lengthens with default risk. However, when the stock of debt is extremely large the compensation for default risk required by the agents on long-term bonds may be so high that only short-term debt can be issued. The implications of this model are consistent with the observed behavior of risk premia in some highly indebted countries. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

In the past decade economists have devoted considerable attention to the apparent existence of inflation premia embedded in rates of return on government debt. The presence of government’s inflationary incentives, related to different potential sources, has been at the heart of the analysis of the problem of time inconsistency in monetary and fiscal policies. Calvo and Guidotti (1990a,b) and Missale and Blanchard (1994), among others, have investigated the government’s incentives to use opportunistic inflation to reduce the ex post real value of its nominal obligations, and explored the role of debt maturity in managing inflation. Fewer works have been addressed to the analysis of default on government debt (either in the form of simple repudiation of debt obligations or some ex post tax on debt repayments) and, in particular, to its implications for debt management. When investors perceive the possibility of partial (or complete) default, they require compensation for such risk, which is incorporated in nominal interest rates on government debt. A pioneering work in this area is the paper by Calvo (1988), Giavazzi and Pagano (1990) and Alesina et al. (1990) consider the possibility of financial crises in a context of multiplicity of equilibria. Drudi and Prati (1999) analyze the problem with incomplete information. Alesina et al. (1992) document some evidence of risk premia on government bonds in the highly indebted OECD countries. More recently, Cole and Kehoe (1996) develop a model which explains the Mexican government’s inability to roll over its debt during December 1994 and January 1995 as a self-fulfilling debt crisis.

The risk of default on government debt is not immediately intuitive. It is a widespread opinion that the government would always prefer to inflate away its debt instead of defaulting on it explicitly, by simply not repaying or restructuring it. After all, even though default is a non-distortionary lump-sum tax, it also has very high costs in terms of loss of reputation for defaulting governments, income redistribution and risk of bankruptcies in the financial sector. However, in a number of countries several indicators seem to suggest that default premia sometimes arose on bonds issued by the Treasury. Alesina et al. (1992) find statistical evidence that high stocks or rapid accelerations of government debt are associated with an increase of the return on government debt relative to the private return. Moreover, they find that, for the subset of highly indebted countries only, default premia are negatively correlated with debt maturity; for a larger set of countries there is no definite correlation between risk premia and debt maturity. In the case of Italy, default risk premia can be detected in the spread between swap and bond rates or in the differential between bonds issued by the Italian Treasury and those issued by other governments or supernational agencies. The high and time-varying spread between
the yields on floaters issued by the Treasury and short-term rates might be a further indication of the presence of default premia.  

In this paper we investigate the emergence of default risk premia under the hypothesis that the government has the option to reduce its liabilities also by means of inflation. In our model the government weighs an exogenous cost of default, which is proportional to the amount of repudiated debt, against the costs arising from inflation and income taxation. The microeconomic foundations of the cost associated with debt repudiation have been discussed in a number of papers on this topic. Defaulting governments lose reputation and may find it difficult to borrow in the future when necessary (Grossman and van Huyck, 1988; Chari and Kehoe, 1990; English, 1995, 1996). Default redistributes income away from debt-holders (Alesina, 1988; Drudi and Prati, 1999; Eichengreen, 1990; Tabellini, 1991). Default may lead to financial disruption in the banking sector, if financial institutions hold significant amounts of government debt in their portfolios (Alesina, 1988; Spaventa, 1988). Defaulting governments bear transaction costs associated with legal actions which may be undertaken when repudiation is open (Calvo, 1988).

Unlike in the existing literature on this topic, we study how the interaction between inflation incentives and risk of default affects debt management policies, in particular the choice of the maturity structure. One obvious policy implication of neglecting the emergence of default risk is that, in the presence of time-inconsistency problems, bonds indexed to domestic inflation or denominated in foreign currency may be used to avoid paying inflation premia on nominal debt. In fact, in Ireland, Belgium and Denmark a positive correlation between the share of foreign currency debt and the inflation differential is clearly observed (first panels in Figs. 1–3). At the same time, the increase in the share of foreign currency debt seems to have caused, at least in the case of Belgium and Ireland, a drop in the country rating, suggesting the concern that issuing bonds whose value cannot be reduced via inflation may increase default risk (second panels in Figs. 1–3). 2 On the other hand, a positive correlation

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1 The yield differential between the dollar-denominated bond issued by the Republic of Italy and the corresponding dollar-denominated bond issued by the World Bank increased from roughly 0.4 basis points in September 1994 to 0.7 in April 1995 to start falling soon after. Equivalently, the excess of the yield of the 10-year Treasury bond on the swap rate of the same maturity has been steadily rising from 0.2 basis points in September 1994 to 1.4 in April 1995 and decreased back to 0.5 in September 1995, showing perhaps that the markets feared a fiscal crisis during the political turmoil started at the end of 1994.

2 Figs. 1–3 are from Drudi and Prati (1997). While only few countries issued indexed bonds as a large share of their total debt, several issued foreign currency bonds on a large scale. Given the larger availability of data, they focus on three of the latter countries (Ireland, Belgium and Denmark). Inflation differentials are averages of differentials, weighted with the share of the different currencies which enter in the foreign currency debt of each country. Country ratings are from Institutional Investor.
between debt maturity and risk premium can be observed in Italy since the early 1990s, when the stock of debt reached unusually high levels and its rate of accumulation started accelerating (Figs. 4 and 5). This paper provides a theoretical explanation of these facts.

We distinguish situations of explicit default on debt from those of financial crisis. The former occur when the government does not fully repay its debt obligations. The latter originate when investors refuse to buy government debt. Default is triggered only when the burden of debt reaches a critical level. Before that point, the government prefers to refrain from defaulting and resort to

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3 The risk premium in Figs. 4 and 5 is computed as: 100 – the country rating.
inflation or income taxation. This feature allows us to rationalize the casual observation that default occurs very rarely (among the major economies, no government has ever defaulted since the Second World War). In our model, default is precipitated by an increase in interest rates. With stochastic and autocorrelated interest rates, risk premia appear once the level of rates is so high that a further increase in them would induce the government to default. In the extreme case when rates jump so much to force a future default in any state of the economy, a financial crisis emerges.\footnote{Similar results might apply as a consequence of shocks to government expenditure.}

In our setup, the introduction of indexed debt, as well as debt denominated in foreign currency, reduces the inflationary incentive and increases the risk of default. Similarly, high costs of inflation make equilibria with risk of default
more likely to occur. Therefore, the combination of large stocks of public debt, high real interest rates and anti-inflationary governments, or central banks, generally makes debt indexation less desirable.

Finally, we investigate the choice of optimal debt maturity. We assume that the government is able to issue short-term and long-term bonds, and that the release of information about real rates is after the repayment of short-term debt. Therefore, default risk affects long-term bonds only.

Unlike in the existing literature, in our context uncertainty about future rates assigns a role to debt maturity even in the presence of precommitment on government policies. Lack of precommitment on inflation either lengthens or shortens optimal maturity with respect to the full precommitment case. In fact,
under both regimes optimal maturity lengthens in the presence of decreasing patterns of government expenditure and shortens with increasing ones, as a more balanced distribution of government liabilities across periods improves tax smoothing. The absence of precommitment on inflation generally dampens these effects, since in this case better tax smoothing is achieved at the expense of larger inflationary biases. The possibility of emergence of default risk attaches an additional role to debt maturity. To this extent, a large share of long-term
paper in total debt represents a hedge against a steep rise in interest rates. However, when the stock of debt is extremely large, the compensation for default risk required by the public and embedded in long-term rates becomes unsustainable, so that only short-term debt can be issued. These predictions are consistent with the empirical findings of Alesina et al. (1992). They observe in fact a negative correlation between risk premia and debt maturity only for the subset of the most indebted countries (for which a default risk may be relevant) and no definite correlation for the entire sample. Moreover, our result provides a possible interpretation of the selection of debt maturity in Italy. During the 1980s, when the debt/GDP ratio was still relatively low (and the country rating was good), the choice of the maturity might have been dictated mainly by inflation considerations (and therefore did not exhibit any definite correlation with the risk). At the beginning of 1990s, the debt/GDP ratio passed the threshold of 100% and the debt maturity started exhibiting a clear positive correlation with the default risk.

The paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the situation where full precommitment on both inflation and debt repudiation policies on the part of the government is allowed. The equilibrium which results is efficient and involves, in the absence of uncertainty about future realizations of the real rate, perfect tax smoothing. In the presence of stochastic real rates, the maturity structure may act as a hedge against such a risk. Moreover, since neither inflation risk nor default risk is perceived by the public, long-term rates do not exhibit any term premium.

In Section 4 we consider the case of no risk of bankruptcy: the government is assumed to be able to precommit only its defaulting strategy but not its inflation policy. In the time-consistent equilibrium the government finds it optimal to reduce the amount of nominal obligations outstanding in period 2 by increasing tax revenues in period 1 relative to what is optimal under full precommitment. Optimal maturity can be either shorter or longer than in the case of full precommitment, since here better tax smoothing is achieved at the expense of higher inflation biases.

Section 5 addresses the case of no precommitment, in which the term premium may also account for a positive risk of default in the second period. Such a risk is shown to be increasing with the size of the debt, the level of real rates and the cost of inflation for the government. It decreases as the maturity lengthens. For this reason, optimal maturity is, in general, longer than in the case of partial precommitment.

Finally, Section 6 considers the case of indexed debt. Here the absence of inflationary means to reduce the real value of nominal debt obligations makes the emergence of default risk more likely.

Section 7 contains the results of some numerical simulations and Section 8 concludes the paper.
2. The basic model

2.1. The economy

Agents set nominal rates of return on government debt according to the following no arbitrage conditions:

\[ R_{01} = E_0 \left[ \frac{r_1 \Pi_1}{1 - \Theta_1} \right], \]

\[ R_{02} = E_0 \left[ \frac{r_1 r_2 \Pi_1 \Pi_2}{1 - \Theta_2} \right], \]

\[ R_{12} = E_1 \left[ \frac{r_2 \Pi_2}{1 - \Theta_2} \right], \]

where \( R_{ij} \) denotes the nominal interest factor (i.e., 1 plus the corresponding interest rate) between periods \( i \) and \( j \), \( \Pi_t \) denotes the inflation factor in period \( t \) (i.e., \( P_t = P_0 \Pi_1 \Pi_2 \cdots \Pi_t \), where \( P \) is the price level), and \( \Theta_t \) denotes the tax rate on debt maturing in period \( t \). \( E_t \) is the expectation operator, given the information set available to agents in period \( t \).

Real interest factors, \( r_t \), are exogenous with respect to monetary policy and follow a random walk

\[ r_t = r_{t-1} + \epsilon_t, \]

where \( \epsilon_t \) can take values \( \epsilon \) and \( -\epsilon \) with probability 1/2. Agents observe the current-period real interest rate, and formulate expectations about the future-period real interest rate when setting nominal returns on long-term government debt.

2.2. The government

The government is assumed to have a three-period horizon. Government expenditure occurs at three dates, period 0, period 1 and period 2. In period 0 the government does not levy any tax and the debt issued is equal to the expenditure, \( g_0 \). Debt issued in period 0 can be both short-term debt, maturing in period 1, and long-term debt, maturing in period 2. In period 1 the government finances a constant (exogenous) flow of expenditure, \( g_1 \), and repays the

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\(^5\) Eqs. (1)–(3) are first-order conditions of a simple intertemporal optimization problem, in which risk neutral agents choose a path for consumption and saving, given that the nominal rate of return on saving between periods \( i \) and \( j \) is \( R_{ij} \) and their time preference discount rate equals the real interest rate.
maturing debt by levying distortionary taxes on labor income, by using the revenue from inflation, or by issuing new short-term nominal debt (maturing in period 2). In addition, it may tax government debt ex post. Finally, in period 2 the government finances current expenditure, $g_2$, and repays the debt issued in periods 0 and 1 by resorting to conventional taxation, to the inflation tax, or by taxing both short-term and long-term outstanding debt. Therefore, the government’s budget constraints in the three periods are:

$$g_0 = D_{01} + D_{02}, \quad (5)$$

$$g_1 + \frac{D_{01}R_{01}}{\Pi_1} (1 - \Theta_1) = \tau_1 + D_{12}, \quad (6)$$

$$g_2 + \left( \frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2} \right) (1 - \Theta_2) = \tau_2, \quad (7)$$

where $D_{ij}$ denotes the value, at time $i$, of nominal public debt issued in period $i$ with maturity in period $j$, and $\tau_i$ and $\Theta_i$ are respectively the tax rates on labor income (normalized to one) and government debt in period $t$.

In period 0 the only decision faced by the government is choosing the maturity structure of the initial stock of debt necessary to finance the given public expenditure $g_0$. In periods 1 and 2 the government chooses a sequence of taxes on income and debt, and inflation taxes ($\tau_1, \tau_2, \Theta_1, \Theta_2, \Pi_1, \Pi_2$). In addition, in period 1 the government decides the amount of maturing debt to be rolled over to period 2.

The government’s optimal choice of instruments responds to the objective of minimizing the value of the following intertemporal cost function:

$$L^g = \beta^{-1} \left[ \frac{1}{2} \tau_1^2 + \frac{\gamma}{2} (\Pi_1 - 1)^2 \right] + \beta^{-2} \left[ \frac{1}{2} \tau_2^2 + \frac{\gamma}{2} (\Pi_2 - 1)^2 \right]$$

$$+ \alpha \left\{ \beta^{-1} \Theta_1 \frac{D_{01}R_{01}}{\Pi_1} + \beta^{-2} \left[ \Theta_2 \left( \frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2} \right) \right] \right\}, \quad (8)$$

where the time preference discount factor is assumed to be constant and equal to the real interest rate factor in period 1, i.e. $\beta = r_1$. $\alpha$ stands for the cost per unit of repudiated debt.

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$^6$ This assumption, implying that the government time preference discount rate does not coincide with the realized interest rate in period 2, allows us to introduce a new hedging motive into the analysis. Such a motive is particularly important in the case of full precommitment, since it assigns a role to debt maturity which it would not have otherwise.
The government’s policy affects welfare in three ways: non-linearly via the distortionary costs of taxation and inflation, and linearly via the cost of defaulting on government debt.\(^7\)

We characterize default as any action that reduces the value of government debt obligations and that was not contractually specified. The cost associated with such an action may have different interpretations. It could be thought of as transaction costs connected with legal actions, or political costs that have to be incurred because of the contract-breaking behavior of the government. Alternatively, default costs may be brought out by considerations of redistribution or risk of bankruptcies in the financial sector. Such costs arise if and only if default is unanticipated. They should therefore be modeled as a function of the amount of debt unexpectedly repudiated. However, results qualitatively analogous to those contained in this paper are obtained by specifying cost functions associated with surprise default only.

The timing of the game is the following. In period 0 all agents in the economy observe the real interest factor \(r_1\). Given the amount of public expenditure to be financed, \(g_0\), the government issues short-term and long-term debt, \(D_{01}\) and \(D_{02}\). Private agents set nominal returns on government debt according to Eqs. (1) and (2). In period 1 the government chooses inflation \(\Pi_1\), taxes on income and debt \(\tau_1\) and \(\Theta_1\). Then, \(r_2\) realizes and the amount of new short-term debt, \(D_{12}\), is issued. Private agents set \(R_{12}\) according to Eq. (3). Finally, in period 2 the government sets \(\Pi_2\), \(\tau_2\) and \(\Theta_2\).

\footnotesize

\begin{center}
\begin{tabular}{l l l}
0 & 1.1 & 1.2 & 2 \\
\hline
\(r_1\) realizes & government & \(r_2\) realizes & government sets \\
government issues \(D_{01}, D_{02}\) & sets \(\tau_1, \Pi_1, \Theta_1\) & government sets & \(\tau_2, \Pi_2, \Theta_2\) \\
public sets \(R_{01}, R_{02}\) & & issues \(D_{12}\) & public sets \(R_{12}\) \\
\hline
\end{tabular}
\end{center}

\footnotesize

\(^7\) Assuming quadratic costs of taxation and inflation is quite standard: they could be thought of as originated by an underlying production technology, increasing and concave in labor and money. Debt repudiation is observed very rarely. The choice of linear default costs allows us to easily generate emergence of default only after high inflation costs have been suffered. Notice that this feature can be reproduced alternatively by specifying a lump-sum cost of default (which occurs whenever repudiation is open, independently of the amount of defaulted debt). Such specification is more appropriate if political costs or costs linked to loss of reputation are assumed to be the main concern of defaulting governments.
In period 1 the government decides over taxation and inflation; this choice determines the amount of short-term debt to be rolled over to period 2. Then, the real rate $r_2$ realizes and the new stock of debt is issued. This timing should be interpreted as if the shock to the real rate is observed only at a time when the budgetary process can no longer be revised. Therefore, although the government issues $D_{12}$ after $r_2$ is known, the amount of the new stock of debt is actually predetermined. This assumption allows us to generate different default risk profiles for short- and long-term debt and to differentiate among cases of financial crisis. In particular, in our setup short-term debt is immune from bankruptcy risk and default risk premia arise on long-term debt only. In addition, in cases of financial crisis, we are able to distinguish between situations in which there is a crisis at time 0 (no debt can ever be issued) and cases in which the government is able to issue only risk-free short-term debt. In such a circumstance, after the realization of the shock, a financial crisis may either arise or be avoided.⁸

In the sections which follow we consider four cases. First, we investigate the trivial case in which the government can fully precommit its action both in terms of inflation and in terms of default. In the second case the government is assumed to be able to precommit its defaulting policy: a regime in which debt repudiation is an option not available to the government. The third case addresses the situation where no precommitment is allowed: the government is free to rely on both inflation and ex post debt taxation. Finally, we analyze the case in which government debt is totally indexed, that is, a regime where inflation can be precommitted.

3. The case of full precommitment

In this section we briefly analyze the first-best case where complete precommitment on the part of the government is possible and, therefore, policies chosen in period 0 are credible. This case serves as a benchmark against which to evaluate the outcomes that arise under more realistic setups. The intertemporal government budget constraint is given by

$$g_2 + D_{02}r_2^2 + (D_{01}r_1 + g_1 - \tau_1)r_2 = \tau_2.$$  

It is interesting to notice that our assumption about how information is released allows us to generate results similar to those obtained by Diamond (1991) for corporate debt. However, unlike in his paper, we do not have to rely on imperfect information and exogenous liquidity shocks. In particular, our prediction is that low risk countries issue long-term debt in order to eliminate the risk of default; medium risk countries are unable to issue long-term debt and cannot do better than roll over short-term debt, which is perceived as less risky by bond-holders; high risk countries are unable to issue any type of debt.

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Eq. (9) is obtained by combining Eqs. (6) and (7), taking into account that under full precommitment inflation and default are perfectly anticipated. The problem of the government in period 1 is to minimize loss function (8), subject to budget constraint (9).

The first best choice of taxes on income and debt and inflation rates implies

\[
\tau_1 = \frac{(D_{01}r_1 + g_1)(r_1 + \epsilon^2/r_1) + D_{02}r_1^2 + g_2}{1 + r_1 + \epsilon^2/r_1},
\]

\[
E_1[\tau_2] = \frac{(D_{01}r_1 + g_1)r_1 + (D_{02}r_1^2 + g_2)(1 + \epsilon^2/r_1)}{1 + r_1 + \epsilon^2/r_1},
\]

\[\Theta_1 = \Theta_2 = 0,\]

\[\Pi_1 = \Pi_2 = 1.\]

Under a full precommitment regime, optimal debt taxation and inflation are zero. In the absence of uncertainty about future realizations of the interest rate (i.e. $\epsilon = 0$), it would be optimal to achieve perfect smoothing of taxes over time. With stochastic real rates, instead,

\[\tau_1 \geq E_1[\tau_2]\]

depending on whether

\[D_{01}r_1 + g_1 \geq D_{02}r_1^2 + g_2,\]

(or, equivalently, $D_{12} \geq 0$) since the uncertainty about $r_2$ makes the optimal amount of short-term debt maturing in period 1 rolled over to period 2 too small compared with what would be necessary to achieve perfect tax smoothing.

In period 0 the decision faced by the government is choosing the maturity structure of the given initial stock of public debt $g_0$. Since the presence of uncertainty introduces a distortion by providing an incentive in period 1 not to smooth taxes completely over time, debt maturity plays a role in such a context. As a matter of fact, maturity will be optimal if the government in period 1 is able to achieve perfect tax smoothing by selecting $D_{12} = 0$, so that the risk associated with the uncertain realization of $r_2$ is eliminated. This implies

\[D_{01}r_1 + g_1 = D_{02}r_1^2 + g_2\]

if $|g_1 - g_2| < g_0 r_1^2$.  

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9 Since period 2 real interest rate is not observed at time 1, the government optimizes formulating expectations about the outcomes associated with the two possible realizations of $r_2$.

10 If $|g_1 - g_2| > g_0 r_1^2$, the optimal share of long-term paper in total debt is at a corner with $D_{02}/g_0 = 1$, if $g_1 - g_2 > 0$, and $D_{02}/g_0 = 0$, if $g_1 - g_2 < 0.$
Since inflation risk, as well as default risk, is perceived to be null, term premia are systematically equal to zero under this setup (i.e. \( R_{01} = r_1 \) and \( R_{02} = r_1^2 \)).

4. Partial precommitment: The choice between income taxation and inflation

We consider now a case of partial precommitment where the government is able to make commitments regarding ex post debt taxation only.

With partial precommitment time inconsistency of government behavior may arise because of the presence of nominal debt, which provides an incentive to the government to resort in the future to inflation in order to reduce the real value of nominal debt obligations.

The problem of the government consists in choosing a sequence of inflation and income taxes \( (\Pi_1, \Pi_2, \tau_1, \tau_2) \) to minimize loss function (8) subject to budget constraints (5)–(7), where now nominal returns on government debt reflect the total absence of uncertainty about default.

In order to characterize time consistent policies we solve the government’s problem starting from period 2.

4.1. The last period problem

The government in period 2 faces budget constraint (7), where the only non-predetermined variables are \( \Pi_2 \) and \( \tau_2 \). Given the objective of minimizing the value of the cost function

\[
\frac{1}{2} \tau_2^2 + \frac{\gamma}{2} (\Pi_2 - 1)^2
\]

subject to Eq. (7), the optimal choice of period 2 inflation is given by

\[
\gamma (\Pi_2 - 1) \Pi_2 = \tau_2 \left( \frac{D_0 R_{02}}{\Pi_1 \Pi_2} + \frac{D_{12} R_{12}}{\Pi_2} \right).
\]

Unlike the previous case, here the absence of precommitment of period 2 variables leaves the government free to resort to inflation in order to reduce the real value of its obligations (recall that in period 2 the nominal value of outstanding debt, as well as the nominal interest factors, \( R_{02} \) and \( R_{12} \), is a predetermined variable). Eq. (10) implies that the marginal cost of inflation equals

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11 The implication of zero slope of the yield curve crucially hinges on the hypotheses of total absence of uncertainty and agents’ risk neutrality.
the cost reduction from the tax cut that is induced by the associated larger inflation tax (including the fall in the real value of debt obligations maturing in period 2). The gains resulting from reducing the real value of government obligations are, however, at least on average, an illusion. The market perceives the future incentive to inflate on the part of the government and, at the time the nominal debt is being issued, nominal interest rates reflect future inflation. The only possibility of error comes from the uncertainty about the realization of the period 2 real interest rate, which the public faces when setting nominal returns on long-term government debt. This implies that for government bonds issued in period 0, we have in equilibrium

\[ R_{01} = r_1 \Pi_1, \]

\[ R_{02} = r_1 \Pi_1 \hat{R}_2, \]

where \( \hat{R}_2 \) is the average equilibrium nominal interest factor in period 2. Furthermore, the equilibrium interest factor for debt issued in period 1 satisfies

\[ R_{12} = r_2 \Pi_2. \]

Since market interest rates reflect actual equilibrium inflation, we have, combining Eq. (10) with Eqs. (7), (12) and (13),

\[ \gamma (\Pi_2 - 1) \Pi_2 = \left( \frac{D_{02} r_1 \hat{R}_2}{\Pi_2} + D_{12} r_2 \right)^2 + g_2 \left( \frac{D_{02} r_1 \hat{R}_2}{\Pi_2} + D_{12} r_2 \right). \]  

Eq. (14) shows that \( \Pi_2 \) and \( \tau_2 \) are positively linked at optimum. Furthermore, \( \Pi_2 \) increases with \( r_1 \) and decreases with \( \gamma \). Higher realizations of period 2 real interest rate (i.e. \( \epsilon > 0 \)) are associated with higher values of \( \Pi_2 \) if and only if \( D_{12} > 0 \). \(^{12}\)

4.2. The intermediate period problem

In period 1 the government formulates its time consistent policy taking into account that in equilibrium \( \Pi_2 \) will be chosen according to Eq. (14), which shows that \( \Pi_2 \) is a function of the stock of nominal debt maturing in period 2. However, while setting policy variables in period 1, the government is not able to observe the realization of the real interest rate in period 2. It therefore has to form expectations over two possible outcomes.

Let \( \tau_{2,s} \) and \( \Pi_{2,s} \) denote income tax and inflation in the last period under the two possible realizations of the real rate (\( s = l, h \) respectively, when \( r_2 = r_1 - \epsilon \))

\(^{12}\) In fact, a higher \( r_2 \) makes the government budget constraint tighter or looser depending on whether the government borrows or lends in period 1.
and \( r_2 = r_1 + \epsilon \). Let us also define \( r_{2,l} \equiv r_1 - \epsilon \) and \( r_{2,h} \equiv r_1 + \epsilon \). Then, the problem of the government in period 1 is to minimize expected social loss

\[
\frac{1}{2} \tau_1^2 + \frac{\gamma}{2} (\Pi_1 - 1)^2 + \frac{1}{2r_1} \left[ \frac{1}{2} \tau_{2,l}^2 + \frac{\gamma}{2} (\Pi_{2,l} - 1)^2 + \frac{1}{2} \tau_{2,h}^2 + \frac{\gamma}{2} (\Pi_{2,h} - 1)^2 \right]
\]

subject to budget constraints (6) and (7) and the incentive compatibility constraint (10), where the perfect-foresight condition (13) holds (of course, (6), (7) and (10) apply for both realizations \( r_{2,l} \) and \( r_{2,h} \)). This problem involves the choice of \( s_1, P_1, D_{12}, s_2, s \) and \( P_2; s \). (s = l, h).

The first-order conditions for optimization with respect to \( P_1, D_{12}, P_2; s \) imply respectively

\[
\gamma(\Pi_1 - 1)\Pi_1 = \tau_1 \frac{D_{01}R_{01}}{\Pi_1} + r_1^{-1}E_1 \left[ \tau_2 + 2\lambda r_1 (2\tau_2 - g_2) \right] \frac{D_{02}R_{02}}{\Pi_1\Pi_2}, \tag{16}
\]

\[
\tau_1 = r_1^{-1}E_1 [\tau_2 r_2 + 2\lambda r_1 (2\tau_2 - g_2)r_2], \tag{17}
\]

\[
\begin{align*}
\tau_{2,s} & = \frac{D_{02}R_{02}}{\Pi_1\Pi_{2,s}} + 2\lambda_s \left( 2\tau_{2,s} - g_2 \right) \frac{D_{02}R_{02}}{\Pi_1\Pi_{2,s}} + \gamma(\Pi_{2,s} - 1)\Pi_{2,s} \\
& \quad + \frac{\gamma}{2} (\Pi_{2,h} - 1)^2 \\
& = \tau_{2,s} \frac{D_{02}R_{02}}{\Pi_1\Pi_{2,s}} + 2\lambda_s \left( 2\tau_{2,s} - g_2 \right) \frac{D_{02}R_{02}}{\Pi_1\Pi_{2,s}} + \gamma(2\Pi_{2,s} - 1)\Pi_{2,s} \tag{18}
\end{align*}
\]

where \( \lambda \) is the Lagrange multiplier associated with the incentive compatibility constraint (10). Eqs. (16) and (18) say that the marginal cost of inflation, at optimum, equals the marginal benefit from the tax reduction associated with the larger inflation tax and the fall in the real value of debt obligations. Eq. (17) implies that, compared with the first-best optimum, even in the absence of uncertainty it is no longer optimal to smooth out taxes completely over time. In particular, the time consistent equilibrium exhibits higher (lower) taxes in period 1 relative to period 2 whenever \( \lambda > 0 \) (\( \lambda < 0 \)). Further analysis of the first-order conditions shows that the intertemporal distribution of taxes is directly related to whether the government borrows or lends in period 1. In fact, solving (18) for \( \lambda_s \) and using (14), we obtain \( \tau_1 \geq E_1 [\tau_2] \) (i.e. \( \lambda_s \geq 0 \), \( s = l, h \)) if and only if \( D_{12} \geq 0 \). This inequality indicates that the only time inconsistency problem that matters to the government in period 1 for altering the intertemporal distribution of taxes is the one concerning \( D_{12} \). In particular, if \( D_{12} > 0 \), the government in period 2 is provided with an additional (to \( D_{02} \)) incentive to increase \( \Pi_2 \). As a result, the government finds it optimal to lower the amount of nominal obligations left in period 2 by reducing its borrowing in period 1. This can be done by raising tax revenues in period 1. This phenomenon is what has been recognized in the literature as debt aversion (Calvo and Guidotti,
The opposite reasoning applies if $D_{12} < 0$, since a negative $D_{12}$ provides an incentive to deflate.

4.3. The first period problem: the optimal maturity structure

Eqs. (14) and 16, 17, 18, together with budget constraints (6) and (7), characterize the time consistent policy for the government in period 1 as a function of the maturity structure of initial debt and the exogenous variables of the model.

The optimal maturity structure for the initial stock of government debt is the one which minimizes the expected discounted sum of periods 1 and 2 social losses, subject to budget constraints (6) and (7) and incentive-compatibility constraints (14) and (16)–(18).

To investigate how optimal debt maturity and term-spread depend on exogenous variables, we numerically simulate the model. The results are summarized in Table 1.

In column 1 we report our benchmark case, which is characterized by the following parameter values: the initial stock of debt, $g_0$, is assumed to be equal to 60% of gross national product (GNP), government expenditure in both periods, $g_1$ and $g_2$, equals 40% of GNP, and the inflation cost, $\gamma$, equals 4. The real interest factor, $r_1$, and the shock, $\epsilon$, are assumed to be equal to 1 and 0.05, respectively. We also report the results obtained for a case of low debt ($g_0 = 20\%$) and a case of high debt ($g_0 = 80\%$). For each size of debt we analyze situations where government expenditures (or deficits) exhibit increasing and decreasing patterns (i.e. $g_1 = 0\%$, $g_2 = 80\%$, and $g_1 = 80\%$, $g_2 = 0\%$).

The simulations show that optimal debt maturity in the absence of precommitment on inflation can be either shorter or longer than optimal maturity under full precommitment. Under both regimes, optimal maturity lengthens in the presence of decreasing patterns of government expenditure (or deficit) and shortens with increasing deficits, as a more balanced distribution of government liabilities across periods improves tax smoothing. The absence of precommitment on inflation in general dampens these effects, since better tax smoothing is achieved at the expense of larger inflationary biases. Changes in equal proportion in $g_1$ and $g_2$ are shown to affect optimal maturity only slightly; the corresponding results are not reported in the table. The effects of changes in $g_0$ on optimal maturity are negligible in the presence of a balanced distribution of government expenditures over time (i.e. $g_1 = g_2$). In contrast, with downward-sloping (upward-sloping) time profiles of deficits, optimal maturity tends to shorten (lengthen) as $g_0$ increases.

13 The results about debt maturity reported in Table 1, as well as those in Table 2, are rounded to 5%, due to the use of a coarse grid in the numerical approximation.
<table>
<thead>
<tr>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>40</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>60</td>
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<td>30</td>
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<tr>
<td>20</td>
<td>40</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>80</td>
<td>40</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>80</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 1: Optimal debt maturity and term-spread under partial precommitment (in parentheses, optimal debt maturity under full precommitment)

<table>
<thead>
<tr>
<th>$D_{0; \theta_0}$</th>
<th>$D_{1; \theta_1}$</th>
<th>$D_{2; \theta_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>60%</td>
<td>50%</td>
</tr>
<tr>
<td>45%</td>
<td>40%</td>
<td>45%</td>
</tr>
<tr>
<td>55%</td>
<td>55%</td>
<td>55%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spread</th>
<th>$H_1$</th>
<th>$H_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>1.093</td>
<td>1.050</td>
</tr>
<tr>
<td>0.012</td>
<td>1.093</td>
<td>1.050</td>
</tr>
<tr>
<td>0.012</td>
<td>1.093</td>
<td>1.050</td>
</tr>
</tbody>
</table>
The results of changes in $g_1$ and $g_2$ on the long-short spread, measured by the difference $R_{02}^{1/2} - R_{01}$, are strictly associated with optimal maturity: the spread systematically decreases as the maturity shortens. In particular, the term-spread is positive whenever the base for $\Pi_2$ at optimum (i.e. $D_{02}R_{02} + D_{12}R_{12}$) exceeds the base for $\Pi_1$ (i.e. $g_0r_1$). This is in general the case for low-enough levels of initial debt $g_0$ and high-enough values of $g_1$ relative to $g_2$, that is, for steep-enough downward-sloping time profile of government expenditures (or deficits). For the same reason, the term-spread decreases as $g_0$ increases.

5. No precommitment: The case of bankruptcy risk

When no precommitment is allowed, investors require compensation for both inflation and default risk, which is embedded in nominal returns on government debt. In order to find the subgame perfect equilibrium of the game, we solve the government’s problem by backward induction.

5.1. The last period problem

In the last period, the government chooses $\Pi_2$, $\tau_2$ and $\Theta_2$ to minimize the value of the cost function

$$\frac{1}{2} \tau_2^2 + \frac{\gamma}{2} (\Pi_2 - 1)^2 + \alpha \Theta_2 \left( \frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2} \right)$$

subject to its period 2 budget constraint, Eq. (7).

The government’s optimal policy in the last period is characterized by the following first-order conditions:

$$\tau_2 - \lambda \geq 0 \ (= 0 \text{ if } \tau_2 > 0),$$

$$\gamma(\Pi_2 - 1)\Pi_2 - (\tau_2 - g_2) \left( \lambda + \frac{\alpha \Theta_2}{1 - \Theta_2} \right) \geq 0 \ (= 0 \text{ if } \Pi_2 > 0),$$

$$\alpha - \lambda \geq 0 \ (= 0 \text{ if } \Theta_2 > 0),$$

where $\lambda$ is the Lagrange multiplier associated with the budget constraint.

---

14 The term-spread is negative whenever $\Pi_1$ exceeds $\Pi_2^\ast$. This result could be modified by assuming risk averse investors, who require extra-compensation for the uncertainty associated with future realizations of inflation and interest rates. Usually, it is such extra-premium that generates positive term-spreads. Since our focus is on default risk, the hypothesis of risk neutral agents allows us to simplify the analysis without affecting the qualitative features of the results.
Two possible situations may arise from the solution to this system of equations. Solving Eqs. (20)–(22), all with equality sign, together with budget constraint (7), gives

\[ \tau_2 = x, \]  
\[ \gamma(\Pi_2 - 1)\Pi_2(1 - \Theta_2) = x(\tau_2 - g_2), \]  
\[ \Theta_2 = 1 - \frac{(\tau_2 - g_2)\Pi_2}{D_{02}R_{02}\Pi_1^{-1} + D_{12}R_{12}}. \]  

\( \tau_2 \) and \( \Pi_2 \) can be interpreted as the maximum amount of income and inflation taxes that the government is willing to levy in period 2. Therefore, the government’s problem has an interior solution and \( \Theta_2 > 0 \) whenever the total stock of debt maturing in period 2, \( D_{02}R_{02}\Pi_1^{-1} + D_{12}R_{12} \), is larger than the maximum amount of income taxes (net of current expenditure) and inflation that the government is ready to levy, namely \( (\tau_2 - g_2)\Pi_2 \).

If instead \( D_{02}R_{02}\Pi_1^{-1} + D_{12}R_{12} < (\tau_2 - g_2)\Pi_2 \), then the government’s problem has a corner solution and \( \Theta_2 = 0 \), since the maximum amount of income and inflation taxes exceeds the maturing stock of debt. Under such circumstance, optimal inflation and income tax rates are given by the solution to the following system of equations:

\[ \gamma(\Pi_2 - 1)\Pi_2 = \tau_2(\tau_2 - g_2), \]  
\[ \frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2} + g_2 = \tau_2, \]  

which solved for \( \Pi_2 \) gives back the incentive compatibility constraint under partial precommitment, Eq. (10).

5.2. The intermediate period problem

In period 1 the government chooses inflation \( \Pi_1 \), taxes on income and debt, \( \tau_1 \) and \( \Theta_1 \), and the amount of maturing debt to be rolled over to period 2, \( D_{12} \), to minimize (8), subject to its period 1 and period 2 budget constraints, Eqs. (6) and (7), and the incentive compatibility constraints in period 2 (either (26) and (27) or (23)–(25), depending on whether the problem in period 2 admits a corner or an interior solution). Recalling that in period 1 the government does not observe \( r_2 \), its objective is given by the following expected social loss:
where $\theta_{2,s}$ is the ex post tax on debt maturing in period 2 when $r_{2,s}$ realizes $(s = l, h)$. Obviously, expected loss (28) is minimized taking into account that budget and incentive compatibility constraints must hold for both $r_{2,l}$ and $r_{2,h}$.

The first-order condition with respect to $s_1$ gives

$$
\tau_1 \leq a,
$$

which implies that $a$ is the maximum amount of income taxes that the government is ready to levy in period 1. As in period 2, the government’s problem can have either an interior solution with $\theta_1 > 0$ or a corner solution with $\theta_1 = 0$. In period 1 the government also chooses inflation tax $\Pi_1$ and how much debt $D_{12}$ to roll over to the last period. Since in equilibrium $R_{01} = r_1 \Pi_1$, from (29) it follows that, in the absence of default, $D_{12}$ must be at least equal to $D_{01}r_1 + g_1 - a$.

In the following sections, we consider only equilibria in which the government finds it optimal to borrow in period 1 (i.e. $D_{12} > 0$). The analysis of the case, empirically less important, with optimal lending in period 1 can be provided by the authors upon request.

In order to investigate all situations which in principle may arise, we need to introduce some notation.

Let $\Pi_{1s}^{p}$, $\tau_{1s}^{p}$, $D_{12s}$, $\Pi_{2s}^{p}$, $(s = l, h)$ represent the solution to the problem under partial precommitment (i.e., the solution to the system of Eqs. (10) and (16)–(18), where perfect foresight conditions (11)–(13) hold). By investigating first-order conditions (14) and (16)–(18) it immediately appears that if $D_{12s}^{p} > 0$, then $\tau_{1s}^{p} > \tau_{2s}^{p} > \tau_{2s}^{l}$.

Three possibilities may thus occur.

5.2.1. Equilibrium with no risk of default

In this section, we investigate the conditions under which it is never optimal for the government to repudiate its debt.

Let $\hat{\tau}_{2,s}$ and $\hat{\Pi}_{2,s}$ denote the solutions to the incentive compatibility constraints, Eqs. (26) and (27), when $D_{12} = D_{01}r_1 + g_1 - a$ and $r_2 = r_1 + \epsilon$. That is, $\hat{\tau}_{2,s}$ and $\hat{\Pi}_{2,s}$ are income and inflation taxes that the government would optimally implement in period 2 if it observes a high realization of the real interest rate and the amount of debt rolled over from the previous period is the minimum compatible with optimal income taxation in period 1 (i.e. $\tau_1 = a$).
Similarly, let \( \hat{s}_{2,l} \) and \( \hat{P}_{2,l} \) be the equilibrium value of \( s_2 \) and \( P_2 \) when \( D_{12} = D_{01}r_1 + g_1 - \alpha \) and \( r_2 = r_1 - \epsilon \).

**Proposition 1.** Assume \( D^p_{12} > 0 \). Then, the unique perfect equilibrium of the game never involves default if and only if

\[ \hat{s}_{2,h} \leq \alpha. \]

In this equilibrium the government chooses \( \tau^0_1, \Pi^0_1, \tau^0_{2,s} \) and \( \Pi^0_{2,s} \) (s = l, h), if \( \tau^0_1 \leq \alpha \). That is, the equilibrium is at a corner and coincides with the solution to the unconstrained system of first-order conditions under partial precommitment. 15 If instead \( \tau^0_1 > \alpha \), then optimization implies \( \tau_1 = \alpha, \tau_{2,s} \) and \( \Pi_{2,s} \) (s = l, h) will be determined according to Eqs. (26) and (27), where \( D_{12} = D_{01}r_1 + g_1 - \alpha \) is imposed (i.e. \( \tau_{2,s} = \hat{s}_{2,s} \) and \( \Pi_{2,s} = \hat{P}_{2,s} \)). The public sets nominal returns on government debt according to Eqs. (1)–(3), assuming zero risk of default (i.e. \( \Theta_1 = \Theta_2 = 0 \)).

**Proof.** See Appendix A. \( \square \)

Proposition 1 provides the necessary and sufficient condition for the game to admit an equilibrium with no risk of default in the presence of borrowing by the government in period 1. A more compact but equivalent condition is that for both realizations of \( r_2 \)

\[
\frac{D_{02}R_{02}}{\Pi_1 \Pi_2} + \left( \frac{D_{01}R_{01}}{\Pi_1} + g_1 - \alpha \right) r_2 + g_2 \leq \alpha,
\]

that is, the intertemporal budget constraint can be always satisfied by relying on inflation and income taxation only.

5.2.2. Equilibrium with default risk

Suppose now that under a high realization of \( r_2 \), even by choosing \( D_{12} = D_{01}r_1 + g_1 - \alpha \) (i.e. as small as possible) the real value of the total stock of outstanding debt in period 2 is larger than the maximum amount of income taxes that the government is willing to levy. That is,

\[
\frac{D_{02}R_{02}}{\Pi_1 \Pi_2} + \left( \frac{D_{01}R_{01}}{\Pi_1} + g_1 - \alpha \right) r_2 + g_2 > \alpha.
\]

---

15 Notice that \( \tau^0_1 \leq \alpha \) implies \( \tau^0_{2,s} \leq \alpha \) (s = l, h). Furthermore, from incentive compatibility constraints (26) and (27) it follows that \( \tau^0_{2,s} \leq \alpha \) implies \( \Pi^0_{2,s} \leq \Pi^0_{2,h} \), where \( \Pi^0_{2,s} \) is given by the solution to (24) and (25) with \( r_2 = r_1 - \epsilon \) for \( s = l \) and \( r_2 = r_1 + \epsilon \) for \( s = h \).
if and only if \( r_2 = r_{2,h} \) (i.e., \( \hat{r}_{2,h} > \alpha \) and \( \hat{r}_{2,l} < \alpha \)). \(^{16}\) Then, the government has to resort to ex post debt taxation if \( r_2 = r_{2,h} \) occurs.

The problem of the government thus consists in choosing the optimal defaulting policy, that is, how much to tax short-term debt maturing in period 1 as opposed (or in addition) to taxing debt maturing in period 2. In making this choice the government faces the following trade-off. If it taxes away, fully or partially, its short-term debt maturing in period 1, the associated cost will be proportional to the amount of defaulted debt, \( \Theta_1D_{01}R_{01}\Pi_1^{-1} \). It may alternatively decide to roll over that amount to period 2. Then, depending on the realization of the real interest rate, total outstanding debt may prove unsustainable. If, and only if, this is the case, the government has to resort to default and bear the cost of taxing the stock of debt rolled over from period 1. This reasoning provides the intuition for the following.

**Lemma 1.** Assume that condition (31) holds. Then the optimal strategy for the government is to tax away long-term debt maturing in period 2 as much as necessary and never repudiate short-term debt maturing in period 1.

**Proof.** See Appendix A. □

The result stated in the lemma relies on the fact that repudiation in period 2 occurs with probability 1/2. Therefore, before having observed the realization of the real interest rate in period 2, it is less costly to roll over short-term debt which cannot be repaid by current taxation rather than to default on it.

The following proposition provides the conditions under which an equilibrium with partial default on long-term debt exists indeed.

**Proposition 2.** Let condition (31) hold. Then the game admits a unique perfect equilibrium which implies risk of partial default if and only if \( \exists \) a pair \((\Theta^*_2, \Pi^*_2)\), \( \Theta^*_2 \in [0, 1] \), which solves incentive compatibility constraints in period 2, Eqs. (23)–(25) with \( D_{12} = D_{01}r_1 + g_1 - \alpha \), such that

\[
(1 - \Theta^*_2)\frac{D_{02}R_{02}}{\Pi_1\Pi_2^2} + \left(\frac{D_{01}R_{01}}{\Pi_1} + g_1 - \alpha\right) r_2 + g_2 \leq \alpha 
\]

(32)
is satisfied both when \( r_2 = r_{2,l} \) and \( r_2 = r_{2,h} \), and holds with equality for \( r_2 = r_{2,h} \).

In this equilibrium, the government never repudiates short-term debt maturing in period 1 (i.e. \( \Theta_1 = 0 \)).

\(^{16}\) If both \( \hat{r}_{2,h} \) and \( \hat{r}_{2,l} \) were greater than \( \alpha \) no sustainable equilibrium with long-term debt would ever be attained.
Optimal ex post tax on long-term debt is zero when \( r_2 = r_{2,l} \) (i.e. \( \Theta_{2,l}^* = 0 \)). Furthermore, the government chooses \( \tau_1 = \alpha \), \( \tau_{2,h} = \alpha \), \( \Pi_{2,h} = \Pi_{2,h}^* \), \( \Theta_{2,h} = \Theta_{2,h}^* \), and \( \tau_{2,l} \) and \( \Pi_{2,l} \) according to Eqs. (26) and (27), where \( D_{12} = D_{01} r_1 + g_1 - \alpha \) is imposed.

The public sets nominal returns on government debt according to Eqs. (1)–(3), where \( \Theta_1 = 0 \), \( \Theta_{2,l} = 0 \) and \( \Theta_{2,h} = \Theta_{2,h}^* \).

**Proof.** See Appendix A. \( \square \)

### 5.2.3. Financial crisis

The cases addressed in Propositions 1 and 2 do not exhaust all the possible situations which in principle may arise.

In particular, two cases of financial crisis may be conceived. If the maximum amount of income taxes that the government is willing to levy is not large enough to satisfy the government’s intertemporal budget constraint in any state of the economy (i.e., for any realization of period 2 real interest rate), then a case of financial crisis emerges. Under such a circumstance, the government is not able to issue any debt in period 0, since default is expected to be total with probability one (i.e. \( \Theta_{2,h} = \Theta_{2,h}^* = 1 \)).

Suppose instead that the fiscal regime becomes unsustainable when a high realization of the real rate in period 2 occurs (i.e., condition (31) is met for \( r_2 = r_{2,h} \)), but condition (32) in Proposition 2, necessary and sufficient for having an equilibrium with risk of default, is not satisfied (i.e. \( \Theta_{2,h}^* \geq 1 \)). Then, a different case arises, characterized by the emergence of a risk of financial crisis. In this situation, the government is able to issue debt in period 0, since \( \Theta_1 \) and \( \Theta_{2,l} \) are expected to be 0. However, it might not be able to borrow in period 1, since, if \( r_2 = r_{2,h} \) realizes, optimal default on outstanding debt will be total (i.e. \( \Theta_{2,h} = 1 \)), and the return required by the public for holding government debt will become infinity. \( \Delta \)

### 5.3. The first period problem

The existence of the option for the government to rely on default has important implications for the choice of the maturity structure.

As a matter of fact, under a no precommitment regime high realizations of the real rate may generate a risk of default which is larger the shorter the maturity (since a larger stock of debt has to be rolled over at the new interest

\[ \text{Notice that this equilibrium relies on the assumption that the cost of financial crisis is not excessively high. In particular, such cost must be low enough to make the government prefer to risk financial crisis with probability 1/2 rather than to default on short-term debt.} \]
rate). In particular, default risk may emerge only in the presence of short maturities. If this is the case, the government will find it optimal to increase the fraction of long-term paper in total debt up to the level which is compatible with an equilibrium with no risk of default. Therefore, optimal maturity under no precommitment is in general longer than under partial precommitment for all parameter values such that equilibria with risk of default are attained. Optimal maturity tends to lengthen with increases in the real interest rate. Nevertheless, there may be cases in which real rates are so high that no equilibrium with long-term debt is achieved. Under such a circumstance, the only sustainable equilibrium may be one involving risk of financial crisis. In this equilibrium optimal maturity drops to zero since government is able to issue short-term debt only.

Table 2 illustrates this intuition. The parameter values characterize a case of large government indebtedness and downward-sloping time profile of current expenditures (i.e. $g_0 = 80, g_1 = 80, g_2 = 0; \alpha = 0.845, \gamma = 4, \epsilon = 0.05$).

<table>
<thead>
<tr>
<th>Real interest rate</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{02}/g_0$</td>
<td>55%</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
<td>90%</td>
<td>0%</td>
</tr>
<tr>
<td>Spread</td>
<td>-0.0073</td>
<td>-0.0066</td>
<td>-0.0049</td>
<td>-0.0030</td>
<td>-0.0010</td>
<td>-</td>
</tr>
</tbody>
</table>

In the presence of low levels of the real interest rate, the model admits equilibria with no risk of default. Optimal maturity is constant and coincides with optimal maturity under partial precommitment. As real rates increase, a default risk emerges at the maturity which was optimal in the presence of lower rates, and the government optimally increases the share of long-term paper in total debt. At rates equal to 6%, no equilibrium with long-term debt exists.

---

18 Since the aim of our numerical simulations is to provide a feeling about the sensitivity of our results to qualitative changes in the relevant parameters of the model, their values were simply picked in order to make our intuition to emerge as clear as possible. Furthermore, we chose $g_1 = 80, g_2 = 0$ for two reasons. First, a decreasing pattern of public expenditure suits well with the need of highly indebted governments (for which default risks may emerge) to reduce their stocks of debt. Second, it allows us to obtain, in equilibrium, a positive value for $D_{12}$; such situation is not only more appealing but also, in the no precommitment case, analytically more tractable.

19 Given the parameter values assumed in the numerical exercise, the model admits equilibria with default risk for levels of the real rate at least equal to 4.8%. Nevertheless, optimal maturity starts lengthening when real rates are greater than 1%. As a matter of fact, at those levels of the real rate, in period 1 the marginal cost of taxation (under partial precommitment) would exceed the cost of default (i.e., $\tau \phi > \alpha$). Therefore, government in period 1 would find it optimal to set $\tau_1 = \alpha$. Under such a circumstance, a longer maturity, by improving tax smoothing, may reduce government loss.
(i.e., condition (32) cannot be satisfied for any \(D_{02} > 0\)), and only short-term debt can be issued under the risk of financial crisis.

Notice that optimal maturity will exhibit a similar evolution if government debt (instead of real rates) increases: when the size of debt is small, no default premia are required and optimal maturity is determined only by hedging and inflation considerations. With larger stocks of debt, the government lengthens the maturity in order to avoid the emergence of default premia. If government indebtedness were to reach extremely high levels, compensation for default risk embedded in long-term rates would be unbearable, and only short-term debt could be issued.

Nominal interest rates on long-term bonds increase, and therefore the term-spread reduces in absolute value, as real rates increase. 20

6. The case of indexed debt

In the presence of real indexation of government debt, the incentive of the government to reduce the real value of its nominal obligations through inflationary means disappears. Hence, equilibria with zero inflation are always achieved. The problem of the government consists in choosing a path of taxes on income and debt \(\tau_1, \tau_2, \Omega_1, \Omega_2\) to minimize loss function (8), subject to its budget constraints, Eqs. (6) and (7), where nominal rates now reflect the total absence of inflation risk.

The government’s optimal policy in the last period is characterized by one of the following two situations. If \(D_{02}R_{02} + D_{12}R_{12} + g_2 \leq \alpha\), then the government’s problem has a corner solution and \(\Omega_2 = 0\), since the maximum amount of income tax (net of current expenditure) that the government is ready to levy in period 2 exceeds the stock of maturing debt. If instead \(D_{02}R_{02} + D_{12}R_{12} + g_2 > \alpha\), then the solution is at an interior with \(\tau_2 = \alpha\) and \(\Omega_2 = 1 - (\alpha - g_2)(D_{02}R_{02} + D_{12}R_{12})^{-1}\).

In period 1 the government chooses taxes on income and debt, \(\tau_1\) and \(\Omega_1\), and the amount of maturing debt to roll over to period 2, \(D_{12}\), to minimize (8), subject to its period 1 and period 2 budget constraints and the incentive compatibility constraints in period 2.

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20 The presence of default risk makes the long-term nominal rate to increase; furthermore, the higher the default risk the higher such rate. This result is consistent with the observed behavior of risk premia. In our context, since nominal rates on short-term debt are, because of the inflation profile, in general higher than those on long-term debt (on a yearly basis), the result is that the spread is negative and therefore it reduces in absolute value as the default risk increases. However, in the absence of inflation (i.e., if the government could precommit on inflation) the term premium would be positive and increasing in the default risk.
The first-order conditions with respect to $s_1$ and $D_{12}$ give, respectively

$$\tau_1 \leq z, \quad \tau_1 = r_1^{-1} E[\tau_2 r_2],$$

which imply the same tax smoothing behavior as under full precommitment, subject to the constraint that the marginal cost of income taxation cannot exceed the marginal cost of default.

In the following section, some numerical simulations illustrate how the presence of indexed debt affects the results.

7. Numerical simulations

To investigate how the risk of default depends on exogenous variables like $g_0$, $\gamma$, $\epsilon$, and on the presence of indexed debt, we numerically simulate the model.

Table 3 reports the minimum levels of the real interest rate at which equilibria with default risk emerge, for different parameter values, with and without real indexation of government debt. In order to isolate the effect on the risk of default due to a change in the parameters, we assume the same maturity structure (55% of long-term debt) throughout. In parentheses we report the corresponding default premia, measured approximatively by the difference $R_{02}(\Pi_1 \Pi_2^{-1})^{1/2} - R_{01} \Pi_1^{-1}$, where $\Pi_2 = (\Pi_2, l + \Pi_2, h)/2$. The benchmark case is characterized by the same parameter values as in Table 2.

The table shows that the risk of default increases with the size of debt and the cost of inflation. This finding would suggest that the combination of very high levels of public debt and anti-inflationary governments, or central banks, makes default premia more likely to arise. More volatile real interest rate processes increase the risk of default.

If government debt is totally indexed, the real value of nominal obligations cannot be lowered by means of inflation taxes and repudiation is more likely to occur at lower interest rate levels. The higher the variance of the real rate shock, the greater the impact of debt indexation on the risk of default. Furthermore, the simulations show that government losses tend to decrease as the fraction of indexed debt increases, since the presence of real indexation of government debt eliminates the distortion associated with the inflationary incentive. Nonetheless, whenever fiscal sustainability is doubtful and the inflationary bias is not excessively large, having real indexation of government debt

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21 The effect of the cost of inflation on the default risk is small, however. This is due to the fact that in equilibrium inflation is anticipated (at least on average). Therefore, the conditions under which the government is not able to satisfy its budget constraint with conventional taxes and has to rely on default are independent of the size of inflation that it optimally implements.
may be suboptimal (i.e. in the presence of equilibria with default risk and very high values of \( \gamma \), government losses turn out to be greater with real indexation than without it).

Comparing the default premia illustrated in the table is not very informative, since they refer to different real interest rate levels. Notice, however, that the presence of debt indexation, together with a high cost of inflation and a high volatility of the interest rate process, has the effect of increasing the default premium. In particular, for a real rate of 4.8%, the default premium, which in the benchmark case equals 0.0013, goes up to 0.0017 when \( \gamma = 8 \), and to 0.0173 when \( \epsilon = 0.1 \).

**8. Concluding remarks**

In this paper we construct a model of optimal debt management where the inability of the government to precommit its policies generates time inconsistency of government behavior. The potential for time inconsistency exists because the presence of nominal debt provides an incentive for the government to resort in the future to inflation, in order to reduce the real value of nominal debt obligations. In addition, the government may resort to ex post debt taxation if the fiscal regime becomes unsustainable.

We assume that future realizations of the real interest rate are uncertain. The cost of government borrowing depends on such realizations and so, consequently, does the incentive of the government to inflate away the stock of outstanding debt. Moreover, the presence of large stocks of debt associated with high levels of the real rate makes the sustainability of the fiscal regime doubtful and leads the market to perceive a positive risk of default.

We show that different patterns of the term spread may be generated by different types of precommitment which the policymaker is able to enter into.

Whenever precommitment is imperfect (i.e. the government cannot commit either its inflationary or defaulting policy, or both), the model is able to generate overreactions of long-term rates to movements in short interest rates. This finding is consistent with the evidence of time-varying and sometime excessive term spreads.
In particular, if, on the one hand, the government is able to fully precommit its action in terms of both inflation and default, then the term structure reflects the total absence of risk (i.e. long-term interest rates move together with short-term interest rates). On the other hand, if the government can only precommit its defaulting strategy, then an inflation risk is perceived by the market and embedded in nominal rates. Long-term rates react more or less than short-term rates to changes in such risk depending on the time profile of government expenditures (or deficits). Finally, the paper examines the case in which there is no precommitment. This situation introduces a discontinuity in the long-short spread behavior, since when real rates reach the maximum level compatible with fiscal sustainability a positive default risk adds to the original term premium. Such risk increases with the size of the debt, the cost of inflation for the government and the volatility of the real interest rate process.

When government debt is totally indexed, inflation risk disappears but default risk becomes more likely. As a consequence, issuing indexed debt may not always result to be optimal, in general depending on the sustainability of the fiscal regime and the cost of inflation for the government.

We show that optimal debt maturity also is heavily conditioned by the precommitment regime we consider. Under full precommitment, since future realizations of the real interest rate are uncertain, an appropriate choice of debt maturity can serve as a hedge against such a risk. Optimal maturity helps improving tax smoothing by balancing government liabilities across periods. Lack of precommitment on inflation may have the effect of either shortening or lengthening the optimal maturity, depending on the distribution of government expenditures over periods. Since default risk increases as the maturity structure of the debt shortens, optimal maturity under bankruptcy risk is in general longer than in the case in which debt repudiation policies can be precommitted or are very much unlikely.

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Appendix A. Appendix

Proof of Proposition 1. Recall that first-order conditions (14) and (16)–(18) imply, when $D_{12} > 0$, $\tau_1^p > \tau_{2,h}^p > \tau_{2,l}^p$ (debt aversion).
If: Suppose \( \hat{\tau}_{2,h} \leq \alpha \). The solution to the government’s problem under no
precommitment gives \( \tau_t \leq \alpha, t = 1, 2 \). Two situations may thus occur. If \( \tau''_t \leq \alpha \),
then debt aversion implies \( \tau''_{2,s} \leq \alpha (s = l, h) \). Therefore, the equilibrium under
bankruptcy risk is at a corner and coincides with the solution to the partial
precommitment problem. From incentive compatibility constraints (23)–(25) it
follows that \( \Theta_{2,s} = 0 (s = l, h) \): no default occurs in any state of the economy.
If instead \( \tau''_1 > \alpha \), then optimality in the presence of the option of debt repu-
diation requires \( \tau_1 = \alpha \). Further, since \( \hat{\tau}_{2,h} \leq \alpha \) by assumption, the government
in period 1 will always find it optimal to set \( D_{12} = D_{01}r_1 + g_1 - \alpha \) and \( \Theta_1 = 0 \).
By doing so, it avoids bearing the cost of default in both periods. Given
\( D_{12} = D_{01}r_1 + g_1 - \alpha \), the solution to the government’s problem in period 2 is
again at a corner, with \( \tau_{2,s} = \hat{\tau}_{2,s}, \Pi_{2,s} = \Pi_{2,s} \) and \( \Theta_{2,s} = 0 (s = l, h) \).

Only if: Suppose \( \hat{\tau}_{2,h} > \alpha \). Equilibrium in period 1 implies \( \tau_1 \leq \alpha \) and hence
\( D_{12} \geq D_{01}r_1 + g_1 - \alpha \). Therefore, in the absence of default, the budget con-
straint in period 2 implies \( \tau_{2,s} \geq \hat{\tau}_{2,s}, s = l, h \). But, since \( \hat{\tau}_{2,h} > \alpha \) by assumption,
from incentive compatibility constraints (23)–(25) it follows that \( \tau_{2,h} = \tau''_{2,h} = \alpha, \Pi_2 = \Pi_{2,h} \),
and \( \Theta_{2,h} > 0 \): the solution in period 2 is at an interior and default
occurs, at least in the presence of a high realization of the real rate. □

**Proof of Lemma 1.** When condition (31) holds, the problem faced by
the government in period 1 is to minimize the cost function (28), given \( \tau_1 = \alpha \) and
\( \tau_{2,h} = \alpha \). By substituting budget constraints (6) and (7), where condition (13)
holds, for \( \Theta_1D_{01}R_{01}\Pi_1^{-1} \) and \( \Theta_{2,h}(D_{02}R_{02}(\Pi_1\Pi_2)^{-1} + D_{12}r_{2,h}) \) into (28) (\( \Theta_{2,l} = 0 \),
by assumption), and differentiating with respect to \( D_{12} \) gives \(-\alpha + \alpha/2 \). The first
term in this expression is the marginal benefit (negative cost) for the govern-
ment if it does not default today, whereas the second term is the expected cost
of defaulting tomorrow rather than today. Since the first term is greater than
the second term, it is always optimal to choose \( D_{12} \) as large as possible, which
implies \( \Theta_1 \) as small as possible. □

**Proof of Proposition 2.** If: Let \( (\Theta^*_{2,s}, \Pi^*_{2,s}) \), \( \Theta^*_{2,s} \in [0, 1) \), solve incentive com-
patibility constraints in period 2, Eqs. (23)–(25) when \( D_{12} = D_{01}r_1 + g_1 - \alpha \) and
\( r_2 = r_{2,s} \), \( s = l, h \). If condition (31) holds, then, because of debt aversion, it must
be the case \( \tau''_1 > \alpha \). Therefore, first-order condition in period 1, Eq. (29), implies
\( \tau_1 = \alpha; \Theta_1 = 0 \) and \( D_{12} = D_{01}r_1 + g_1 - \alpha \), because of the lemma. The government
in period 2 faces budget constraint (7), where \( D_{12} = D_{01}r_1 + g_1 - \alpha \) and
\( R_{12} = r_2\Pi_2(1 - \Theta_2) \), because of perfect foresight. Now, if \( r_2 = r_{2,s} \) realizes, then
condition (31) implies that the equilibrium is at a corner, with \( \Theta_{2,l} = 0 \), and \( \tau_{2,l} \)
and \( \Pi_{2,l} \) given by incentive compatibility constraints (26) and (27). Condition
(32) is satisfied with \( \Theta^*_{2,l} = 0 \) and the inequality sign. If instead \( r_2 = r_{2,s} \), then
the solution is interior: \( \tau_{2,s} = \alpha, \Theta_{2,h} = \Theta^*_{2,h}, \Pi_{2,h} = \Pi^*_{2,h} \), and condition (32)
holds with equality.
Only if: Suppose that condition (32) does not hold, for at least one realization of \( r_2 \). Then, the intertemporal budget constraint cannot be satisfied for any \( \Theta_2^* \in [0, 1) \). □

References