Seigniorage and conventional taxation with multiple exogenous shocks

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Abstract

The correlation between seigniorage and conventional taxation is investigated in a dynamic optimizing model which contains three shocks (to government expenditure and to the deadweight losses associated with conventional taxation and seigniorage), asymmetric costs of adjusting revenue, and potential absence of the availability of debt. Solutions and impulse response functions from the theoretical model are used to construct a three-variable structural vector autoregression (VAR) and identify the empirical reduced-form VAR. Judged by the parameter estimates, empirical impulse response functions, forecast error variance decompositions, and historical decompositions of the time series, identification is successful and the econometric results are supportive of the theoretical model. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Macroeconomic applications of the theory of optimal taxation imply that marginal deadweight losses should be equated across all available tax
instruments and across time periods. Recently, these propositions have been developed in models of optimum seigniorage and conventional taxation, in which monetary and fiscal policies are coordinated to minimize the total excess burden of financing an exogenous path of government spending. The original static model of seigniorage and conventional taxation developed by Phelps (1973) has thus effectively enjoyed renewed interest in the aftermath of temporal investigations of conventional taxation pioneered by Barro (1979). The recent ‘traditional’ model of seigniorage and conventional taxation – as developed by Mankiw (1987), Grilli (1989), Poterba and Rotemberg (1990), and Trehan and Walsh (1990) – considers the effects of changes in government spending under the assumption that seigniorage and conventional tax revenues are freely determined in each period. In response to changes in government spending, seigniorage and conventional tax revenues optimally move in the same direction in order to equate the marginal deadweight losses of the two revenue sources. Empirical analysis has focused attention on this theoretically positive correlation. Unfortunately, the traditional model does not perform well since empirical results suggest that seigniorage and conventional taxation are as likely to be negatively correlated as they are to be positively correlated.

When the traditional optimum taxation model fails empirical tests, the conclusion is usually that certain governments are acting sub-optimally. Another possibility, however, is that the optimization model under investigation is incomplete. This paper examines this alternative by developing and testing a ‘general’ model which introduces three new features: (1) multiple sources of shocks, (2) costs associated with adjusting revenues, and (3) potential absence of the availability of debt.

First and foremost, the model in this paper explicitly considers three stochastic shocks – one to government expenditure, one to the deadweight losses associated with conventional taxation, and one to the deadweight losses associated with seigniorage – to determine whether governments are optimizing in the presence of multiple disturbances. The traditional model of optimum taxation considers only the first of these and implicitly assumes that there are no shocks to the deadweight losses of taxation. Trehan and Walsh (1990) introduce shocks to deadweight losses as an explanation for the residual in equations relating seigniorage and conventional taxation, and point out that this residual may therefore be correlated (positively or negatively) with seigniorage and/or conventional taxation. However, Trehan and Walsh (1990) do not develop an explanation for the shocks or consider the effects of the shocks on seigniorage and conventional taxation in any detail. The general model presented here therefore contains implications on how governments respond to various types of shocks, not just shocks to expenditure.

Second, the model introduces costs of adjusting the amount of seigniorage and conventional tax revenue collected, which essentially allows for short run deviations from long run equilibrium. The adjustment costs are introduced to
capture the idea that instantaneous changes in conventional tax revenues are more difficult to achieve than changes in seigniorage. As noted in Poterba and Rotemberg (1990, p. 4): ‘Income tax schedules are often legislated several years in advance. This commitment is in part the result of time lags in the legislative process’. On the other hand, ‘the Federal Reserve can react quickly to changed circumstances: time lags are shorter’ (p. 4). When Poterba and Rotemberg (1990) conclude that the traditional optimum seigniorage tax model generally fails empirical tests, they advance this notion as an explanation: ‘governments are unable to adjust the structure of taxes frequently enough to enforce the first-order conditions implied by optimization models’ (p. 15). By introducing asymmetric adjustment costs, the model allows for gradual changes in conventional taxation over several periods compared to seigniorage.

Third, the availability of debt – which is not an issue in the traditional model – influences the adjustment process. When debt is freely available, borrowing smooths revenue over time, as advanced by Barro (1979). One criticism of the traditional model, however, is that debt may be constrained in some countries. Even in some countries in which debt is available, there may be a political preoccupation focusing on the budget balance period-by-period rather than intertemporally. When debt is not available, the budget must be balanced in each period, and seigniorage optimally smooths conventional tax revenue in the presence of asymmetric adjustment costs.

The theoretical model is carefully developed with an eye toward econometric estimation. In this sense, the theory is advanced as a positive description of reality and is then used to determine whether it describes what is empirically observed. If so, the model is used to quantify features of the data. Section 2 develops the general model of optimum seigniorage and conventional taxation using a linear–quadratic approach for which solution methods are known. Some simple examples of the model are also presented. This section illustrates, through the use of theoretical impulse response functions, that the model of multiple shocks and adjustment costs captures a richer array of possibilities than has been entertained in the past. Simulations of the model produce theoretical correlations between seigniorage and conventional taxation depending on the various shocks. Section 3 then presents solutions of the model in the form of a three-variable structural vector autoregression (VAR) and identifies the estimable reduced-form VAR. Long-run identifying restrictions based on the theoretical model are used to extract estimates of the structural VAR from the reduced-form VAR, with special attention to an overidentifying restriction suggested by the traditional model with one shock and to qualitative (yet under-identifying) restrictions from the general model with three shocks. Section 4 introduces data for US, UK, Brazil, and Argentina, and conducts preliminary statistical analysis. Section 5 applies the econometric specification developed in Section 3 to the data introduced in Section 4. The empirical results provide actual impulse response functions which can be compared to the theoretical
impulse response functions of Section 2. Historical decompositions of the time series are used to examine correlations between seigniorage and conventional taxation depending on the various shocks. Hence, two important expository features are impulse response functions and correlation coefficients in both theoretical and empirical dimensions. Section 6 concludes.

The existing literature examines the correlation between seigniorage and conventional taxation by estimating an intratemporal Euler equation derived from a theoretical model. This approach is fairly restrictive, however, because it does not describe the responses of different taxes to various exogenous shocks. In contrast, this paper uses the structural VAR methodology precisely because it demonstrates how seigniorage and conventional taxation respond to various shocks, both contemporaneously and over time, and supports an examination of the correlation. Since the VAR is more general than the Euler equation approach, it can be more illuminating. For example, even if empirical results do not confirm theoretical implications, there is potentially interesting descriptive information on how revenues actually respond to various shocks. The main empirical finding in this paper is that the model which includes multiple shocks outperforms the traditional model that examines only a shock to government expenditure.

2. A general model of optimum government behavior

This section develops a quadratic dynamic optimization problem for a government acting as a welfare maximizer. The model is set up such that the Euler equations are stochastic linear difference equations. Using the methods of Hansen and Sargent (1998), the system of linear difference equations is solved for paths of the government’s decision variables.

To ensure that the Euler equations are linear, all variables are expressed as ratios to national output. The government is assumed to face an exogenously determined stochastic sequence of expenditure requirements, \( \{ g_t \}_{t=0}^{\infty} \), where \( g_t \) is the ratio of government spending to output in period \( t \). Its task is to choose plans for conventional tax revenues as a fraction of output, \( \{ \tau_t \}_{t=0}^{\infty} \), and seigniorage as a fraction of output, \( \{ s_t \}_{t=0}^{\infty} \), that satisfy an intertemporal budget constraint.\(^1\) Government debt is generally permitted, denoted as a fraction of output with \( \{ d_t \}_{t=0}^{\infty} \).

\(^1\) Note that the sequence \( \{ g_t \}_{t=0}^{\infty} \) is not a choice variable in the government’s optimization problem. In the tradition of Barro (1979), all of the recent papers on optimum seigniorage and conventional taxation—e.g., Mankiw (1987), Grilli (1989), Poterba and Rotemberg (1990), and Trehan and Walsh (1990)—assume that \( \{ g_t \}_{t=0}^{\infty} \) is exogenous and take \( \{ \tau_t \}_{t=0}^{\infty} \) and \( \{ s_t \}_{t=0}^{\infty} \) as the choice variables. This is generally done to keep the problem tractable, which will become particularly important in this paper when identifying restrictions are imposed on the reduced-form VAR in Section 3.
Thus, there are three financing sources at the government’s disposal in each period: conventional tax revenue, seigniorage, and borrowing. The first two of these are associated with deadweight losses of welfare from the usual public finance considerations, as well as costs of adjusting revenues from period to period. The third instrument is limited by the solvency condition, an intertemporal budget constraint which precludes perpetual debt finance, and some borrowing costs. The government is modeled as a social optimizer in the sense that excess burdens, adjustment costs, the solvency condition, and borrowing costs are taken into account when taxes are levied.

Conventional taxation and seigniorage are associated with quadratic deadweight losses:

$$(1/2)[a\tau_t^2 + 2T_t\tau_t + b\sigma_t^2 + 2\sigma_t\sigma_t],$$

where $a$ and $b$ are positive coefficients that determine the marginal excess burdens of taxation (the usual distortionary effects of taxes on decisions concerning the supply of labor or output and the demand for cash balances) and $\{T_t\}_{t=0}^\infty$ and $\{S_t\}_{t=0}^\infty$ are stochastic disturbances to deadweight losses which reflect the fact that raising a given amount of revenue as a fraction of output is costlier in some periods than in others. The disturbance $T_t$ is a shock to the deadweight losses of conventional tax revenues, which could represent shocks to administrative and collection costs or macroeconomic shocks that indirectly affect conventional tax revenues. Kenny and Toma (1997), for example, demonstrate that variation in collection costs associated with conventional taxation helps explain the temporal patterns of both conventional taxation and seigniorage. As another example, $T_t$ may be higher during recessions in order to capture the idea that the burden of taxation is higher. The disturbance $S_t$ is a shock to the deadweight losses of seigniorage, which could represent shocks to the costs of operating the central bank or shocks to the demand for money or to the banking and payments system. In equilibrium, the marginal deadweight loss from each source of revenue will be equal, and more revenues will be raised by the less costly instrument. In order for more revenue to be raised through

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2 The seminal paper on optimum seigniorage and conventional taxation by Phelps (1973) models the government as a welfare maximizer given households that are utility maximizers and derives the deadweight losses from the effects of taxation on household utility. As with the recent literature on seigniorage and conventional taxation – e.g. Mankiw (1987), Grilli (1989), Poterba and Rotemberg (1990), and Trehan and Walsh (1990) – this paper assumes functional forms of the deadweight losses confronting government in order to simplify the problem in preparation for empirical work. This is clearly a short-cut approximating the alternative general equilibrium analysis, but, as pointed out in the literature, is also somewhat more general in that losses confronting the government may include collection and enforcement costs, distributional consequences of taxes, and political costs of imposing taxes. For a general equilibrium model with distorting taxes and a discussion of its complexities, see Chapter 14 of Hansen and Sargent (1998).
conventional taxation than through seigniorage, $b$ must usually be greater than $a$, although the magnitude of the shocks $\{T_t\}$ and $\{S_t\}$ affects the analysis as well.

In addition to the deadweight losses, the government encounters quadratic costs of adjusting conventional and seigniorage revenues represented by

$$\frac{1}{2}[c(\tau_t - \tau_{t-1})^2 + k(s_t - s_{t-1})^2],$$

(2)

where $c$ and $k$ are positive coefficients. These adjustment costs are introduced to capture the idea that rapid changes in conventional tax revenues are more difficult to achieve than changes in seigniorage ($k < c$). The government is able to raise or lower conventional revenues, but only at a cost that increases with the magnitude of the change. This is broadly interpreted as macroeconomic costs associated with temporary employment and output effects of tax changes, the costs of changing revenue within the existing tax infrastructure (such as the collection and enforcement bureaucracies), perhaps expanding or contracting the infrastructure, and political costs of legislating tax changes. The government is also able to raise or lower seigniorage, but once again only at a cost that increases with the magnitude of the change. This is broadly interpreted as macroeconomic costs associated with temporary employment and output effects of seigniorage changes, the costs of changing seigniorage within the central bank, and the political costs of altering inflation.

Finally, borrowing is associated with quadratic costs above normal risk-free interest costs:

$$\frac{1}{2}[q(d_t - d_{t-1})^2],$$

(3)

where $q$ is a positive coefficient representing borrowing costs. This might represent administrative and brokerage costs. In this paper, however, it will be used as a device to allow debt to be either freely determined or completely unavailable, although any intermediate borrowing constraint is possible. For most industrial countries with well developed financial markets, these borrowing costs are likely to be trivial ($q \approx 0$). In some developing countries, however, borrowing is less likely to be available, and one way to capture this is to make it prohibitively costly ($q \approx \infty$).

The government’s objective is to choose stochastic processes $\{\tau_t, s_t, d_t\}^\infty_{t=0}$ to minimize the present discounted value of the welfare, adjustment, and borrowing costs in Eqs. (1)–(3):

$$E_0 \sum_{t=0}^\infty \beta^{t/2} [a\tau_t^2 + 2T_t\tau_t + b s_t^2 + 2S_t s_t + c(\tau_t - \tau_{t-1})^2$$

$$+ k(s_t - s_{t-1})^2 + q(d_t - d_{t-1})^2]$$

(4)
subject to the periodic budget constraint:

$$g_t = \tau_t + s_t + d_t - (1 + \delta)d_{t-1}$$  (5)

and given $d_{-1}$, an initial condition. The parameter $\beta$ is the discount factor. The parameter $\delta$ is the growth rate of the outstanding debt/output ratio due to interest accrual in excess of output growth, and is assumed to be constant for simplicity. This $\delta$ must be positive, indicating that the real interest rate in the economy exceeds the real growth rate of the economy, in order for current debt as a portion of output to be bounded. Furthermore, note that the definition of $\delta$ in real terms does not allow for government revenue from the erosion of the nominal outstanding debt due to inflation. In this framework, erosion of the nominal debt is fully compensated by higher interest rates, or the debt is either indexed or denominated in foreign currencies.

The government’s optimization problem can be solved using Lagrange multipliers. The constraints in Eq. (5) are associated with a vector multiplier process $\{\lambda_{it}\}_{i=0}^\infty$. Because the constraints are required to hold in all states of the world, these multipliers are stochastic processes. The government solves the problem by choosing stochastic processes for $\{\tau_t, s_t, d_t\}_{i=0}^\infty$ and the multipliers $\{\lambda_{it}\}_{i=0}^\infty$. The Euler equations and the intratemporal condition from the optimization problem comprise a system of three stochastic linear difference equations with three unknowns ($d_t, \tau_t,$ and $s_t$). A solution to the system exists provided that $\{\tau_t\}, \{s_t\}$, and $\{d_t\}$ are of exponential order less than $1/\sqrt{\beta}$, and therefore converge to stable values.

Rather than algebraically solving the system described, this paper makes use of the dynamic programming methods developed in Hansen and Sargent (1998). In particular, the social planning problem here takes the form of a linear-quadratic control problem known as the ‘discounted optimal linear regulator problem’. The MATLAB program solvea.m provided with Hansen and Sargent (1998) solves the discounted optimal linear regulator problem by iterating on the Bellman equation until converging to a solution. Programming is done by specifying the parameters of the model, so several examples are presented next: the traditional model, the general model in which debt is freely determined, and the general model in which debt is not available. Each example considers both temporary and permanent shocks as relevant extremes for persistence of the shocks, and autoregressive parameters between 0 and 1 can be handled by the model as well.

The traditional model of optimum seigniorage and conventional taxation has been concerned only with government spending shocks, so the focus is on $\{g_t\}$. A temporary change in government spending is modeled as a shock to $\{g_t\}$ that does not persist, so $\{g_t\}$ follows a ‘white noise’ process with positive mean. A government acting as a welfare maximizer will optimally finance temporary spending almost entirely with debt, although conventional taxes and seigniorage will rise slightly in accordance with the increase in the discounted present value.
of expenditures. The impulse response functions are therefore flat lines at the magnitude of the increase in conventional taxation and seigniorage. A permanent change in government spending is modeled as a shock to \( \{g_t\} \) that does persist, so \( \{g_t\} \) follows a ‘random walk’ process with positive mean. The government will optimally not use any debt to finance the shock, but will raise conventional taxes and seigniorage to completely finance the increase in government spending. The impulse response functions are once again flat lines at the magnitude of the increase in conventional taxation and seigniorage. The traditional model of optimum seigniorage and conventional taxation therefore results in conventional tax constancy (or smoothing), as advanced by Barro (1979), along with seigniorage tax constancy (or smoothing), as advanced by Barro (1989). Furthermore, since conventional tax revenue and seigniorage are always moving in the same direction and by the same proportion, seigniorage and conventional tax revenue are optimally positively correlated.\(^3\)

The general model allowing for multiple shocks and adjustment costs presents a richer array of possibilities than the traditional model. Fig. 1 presents impulse response functions for the general model with freely available debt, showing the effects of temporary shocks in the left column and the effects of permanent shocks in the right column. Row 1 presents the responses to government spending shocks. Note that seigniorage rises to its new equilibrium immediately because there are no adjustment costs, but conventional taxes rise gradually over several periods due to the high costs of changing conventional taxes. The difference between the long run equilibrium level of conventional tax revenue and the actual level of conventional tax revenue is financed with debt. Note also that the paths of adjustment are identical across the two shocks, but that the magnitudes (shown along the vertical axes) are different because the change in the discounted present value of government spending is much smaller with a temporary shock than with a permanent shock. Row 2 shows the responses to shocks to the deadweight losses associated with conventional taxation, \( \{T_t\} \), and row 3 shows responses to shocks to the deadweight losses associated with seigniorage, \( \{S_t\} \). Note that temporary shocks cause temporary adjustments of the revenue sources as the government lowers revenues from the source experiencing the shock in order to reduce deadweight losses, but that the long run relationship is virtually unchanged. On the other hand, permanent shocks require revenues to be permanently reconfigured such that a shock to the deadweight losses of conventional taxation (seigniorage) will optimally reduce

\(^3\)In programming the traditional model in the Hansen and Sargent (1998) software, \( \{T_t\} \) and \( \{S_t\} \) are nonstochastic and equal to zero, and \( c, k, \) and \( q \) are set arbitrarily small. In addition, \( \beta(1 + \delta) = 1 \), so there is no drift in the ratio of debt to output. The steady state equilibrium is \( \tau = (b/(a + b))g \) and \( s = (a/(a + b))g \). For example, when \( \{g_t\} \) is assumed to have a mean of 0.30 and \( b = 2a \), the steady state equilibrium is \( \tau = 0.20 \) and \( s = 0.10 \).
reliance on conventional taxation (seigniorage) and increase reliance on seigniorage (conventional taxation) by an equal magnitude, allowing for some short-run adjustments. \(^4\)

To allow an even richer array of possibilities, the general model is extended to consider the complete absence of the availability of debt. \(^5\) Fig. 2 presents impulse response functions for the general model without debt. Again, row 1

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\(^4\)The examples set \(b = 2a\) and \(\beta(1 + \delta) = 1\). As a benchmark, \(c = a\) and \(k\) is arbitrarily small. (In models of this type, the adjustment costs are usually ignored so the only costs of conventional taxation are the deadweight losses; \(c\) is therefore not likely to exceed \(a\).) To allow debt to be freely determined, \(q\) is set arbitrarily small. Without loss of generality, the mean of \(\{g_t\}\) is set to 0.30, and the means of \(\{T_t\}\) and \(\{S_t\}\) are set to zero.

\(^5\)The models with and without debt represent extremes along a continuum of possibilities; for most countries, reality is probably somewhere in between.
Fig. 2. Theoretical impulse response functions, general model without debt.

presents the responses to government spending shocks, row 2 shows the responses to shocks to deadweight losses of conventional taxation, and row 3 shows responses to shocks to deadweight losses of seigniorage. Note that the flexibility of seigniorage vis-à-vis conventional tax revenue enables seigniorage to smooth conventional tax revenue in the absence of debt. For the case of a temporary shock to government spending, the government is required to temporarily raise taxes, and the increases in the two taxes together must completely finance the increase in government spending. Since the conventional tax is costly to adjust, seigniorage actually rises more. The subsequent adjustment involves a gradual reduction of the conventional tax rate back to its equilibrium, but seigniorage overshoots its equilibrium and approaches its equilibrium from below. For a permanent shock to government spending, the government permanently raises both taxes. Since the conventional tax is costly to adjust, there will be a longer adjustment period for it to reach its new equilibrium, so in order to equate marginal costs of taxation across the two taxes, seigniorage overshoots its equilibrium and subsequently declines as
Table 1
Correlations between seigniorage and conventional taxation based on theoretical simulations

<table>
<thead>
<tr>
<th>Model</th>
<th>Type of shock</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional model</td>
<td>Temporary shock to $g$</td>
<td>+ 1.00</td>
</tr>
<tr>
<td></td>
<td>Permanent shock to $g$</td>
<td>+ 1.00</td>
</tr>
<tr>
<td>General model with debt</td>
<td>Temporary shock to $g$</td>
<td>+ 0.99</td>
</tr>
<tr>
<td></td>
<td>Permanent shock to $g$</td>
<td>+ 0.99</td>
</tr>
<tr>
<td></td>
<td>Temporary shock to $T$</td>
<td>- 0.08</td>
</tr>
<tr>
<td></td>
<td>Permanent shock to $T$</td>
<td>- 0.99</td>
</tr>
<tr>
<td></td>
<td>Temporary shock to $S$</td>
<td>- 0.04</td>
</tr>
<tr>
<td></td>
<td>Permanent shock to $S$</td>
<td>- 0.99</td>
</tr>
<tr>
<td>General model without debt</td>
<td>Temporary shock to $g$</td>
<td>+ 0.94</td>
</tr>
<tr>
<td></td>
<td>Permanent shock to $g$</td>
<td>+ 0.98</td>
</tr>
<tr>
<td></td>
<td>Temporary shock to $T$</td>
<td>- 1.00</td>
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<tr>
<td></td>
<td>Permanent shock to $T$</td>
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<td>Temporary shock to $S$</td>
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<td></td>
<td>Permanent shock to $S$</td>
<td>- 1.00</td>
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Note: Simulations are run using the program asimul.m provided with Hansen and Sargent (1998). The correlation is the average correlation coefficient for 50 simulations of 100 periods each.

conventional taxation increases. The importance of seigniorage in smoothing conventional tax revenue reemerges in the examination of shocks to the deadweight losses of conventional taxation. Conventional taxes decrease and seigniorage increases to make up the revenue shortfall when there is a temporary shock, and seigniorage gradually approaches its equilibrium as conventional taxation gradually approaches its equilibrium when there is a permanent shock. By the nature of the balanced-budget constraint, the responses due to shocks to deadweight losses of seigniorage are exactly the opposite of the responses to shocks to deadweight losses of conventional taxation in every respect.  

In order to summarize the empirical implications of the various models and shocks, Table 1 presents correlation coefficients between seigniorage and conventional tax revenue from simulations of the models. The three models are listed in the first column, the relevant shocks are listed in the second column, and the correlation is presented in the third column. Three major conclusions can be drawn from this table. First, there is a strong positive correlation in response to government spending shocks, regardless of the model under consideration. Second, permanent shocks to $\{T_t\}$ and $\{S_t\}$ result in a strong negative

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6 To preclude debt, $q$ is set arbitrarily high. Otherwise, the settings are the same as the ones for the model in which debt is freely available.
correlation in both the model with debt and the model without debt, because taxes are permanently reconfigured. Third, the correlation is close to zero for temporary shocks to the deadweight losses when debt is available to smooth the total revenue because only one of the two taxes is changing very much, but is strongly negative when debt is not available to smooth the total revenue because the two taxes are forced to balance the budget and are therefore forced to move in opposite directions in equal magnitudes. The main point of Table 1 is that there are theoretical reasons why conventional tax revenues and seigniorage are sometimes positively correlated, sometimes negatively correlated, and sometimes uncorrelated. The models in this paper confirm the positive correlation for cases in which the shocks come in the form of shocks to government spending. However, the models also demonstrate that conventional tax revenue and seigniorage will be uncorrelated to strongly negatively correlated if the shocks come in the form of shocks to the deadweight losses of conventional taxation and seigniorage, and that the correlation is affected in the case of temporary shocks by whether or not debt is available.

3. Econometric specification of the model

This section develops the econometric specification of the general model of seigniorage and conventional taxation. It uses solutions for paths of conventional tax revenues and seigniorage, along with the path of government spending, to set up a three-variable structural vector autoregression (VAR). It then identifies the structural VAR from an estimable reduced-form VAR using restrictions suggested by the model.

There are in fact several ways to approach empirical estimation of the model. In the literature on optimum seigniorage, the typical approach is to examine and estimate the Euler equations of the model in order to recover the correlation between $\tau_t$ and $s_t$. However, initial empirical estimation of the Euler equations from the constrained minimization problem in Eqs. (4) and (5), using the generalized method of moments (GMM), does not produce very precise estimates of the parameters and occasionally produces point estimates with implausible values. Since the model predominantly extends the optimum seigniorage literature by introducing multiple shocks, this paper instead proceeds to examine the innovations representation of the model, estimate the reduced-form VAR, and identify the structural VAR using restrictions indicated by the model. This technique places relatively few weak restrictions on the data, thereby allowing the model to be representative of a broader class of models with multiple shocks. For example, the quadratic form in Eq. (1) might be

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7 For details, see Chapter 3 of Click (1994).
considered misspecified if deadweight losses are in reality higher-order or non-linear functions of \( \tau_t \) and \( s_t \). (Poterba and Rotemberg (1990) and Trehan and Walsh (1990) both assume more general constant elasticity functions.) Or, there may be short-run deviations from long-run equilibrium not captured by adjustment costs in Eq. (2). Under such circumstances, however, the structural VAR methodology will still be able to answer such questions as: ‘How important are the shocks?’ and ‘How do conventional tax revenues and seigniorage respond to the shocks?’ In addition, the correlation between \( q_t \) and \( s_t \) can still be assessed with regard to the various shocks. Maximum likelihood estimation (MLE) of the general equilibrium provides an alternative technique using the innovations representation, but would put stronger restrictions on the VAR in order to estimate the model’s parameters. This may again be too restrictive if the hypothesized model is an approximation of the correct model, so this paper does not recover the deep parameters but instead estimates the dynamic multipliers (for conventional taxation and seigniorage with respect to the three exogenous shocks) which represent nonlinear functions of the deep parameters.\(^8\)

Solutions to the minimization problem of Eqs. (4) and (5) express contingency plans for \( \{\tau_t, s_t, d_t\}_{t=0}^{\infty} \) as functions of expectations of the exogenous variables \( \{g_t, T_t, S_t\}_{t=0}^{\infty} \) and the initial conditions for \( \tau_{-1}, s_{-1}, \) and \( d_{-1} \). Formulations of the contingency plans as functions of known current and past variables can be derived by specifying expectations of the exogenous variables \( \{g_t, T_t, S_t\}_{t=0}^{\infty} \) as functions of known current and past variables. (See the discussion of projections of geometric distributed lags in Chapter XI of Sargent (1987).)

Assume that \( \{g_t\}, \{T_t\}, \) and \( \{S_t\} \) are orthogonal covariance stationary processes and that expectations of future variables are linear least squares projections of future variables onto information available in period \( t \). Consider the case in which the variables have autoregressive representations: \( \rho_g(L)g_t = u_{gt} \), \( \rho_T(L)T_t = u_{Tt} \), and \( \rho_S(L)S_t = u_{St} \) where \( u_{gt}, u_{Tt}, \) and \( u_{St} \) are white noise, and \( \rho_g(L), \rho_T(L), \) and \( \rho_S(L) \) are polynomials in the lag operator. Given these processes, there exist ARMA representations for conventional tax revenues, \( \tau_t \), and seigniorage, \( s_t \), such that

\[
\begin{align*}
\alpha_g(L)\tau_t &= [\xi_{\tau g}(L) \quad \xi_{\tau T}(L) \quad \xi_{\tau S}(L)]R(L)u_t, \\
\alpha_s(L)s_t &= [\xi_{s g}(L) \quad \xi_{s T}(L) \quad \xi_{s S}(L)]R(L)u_t,
\end{align*}
\]

where \( R(L) = \text{diag} \left[ \rho_g(L)^{-1} \rho_T(L)^{-1} \rho_S(L)^{-1} \right] \), \( R_0 = I \), and \( u_t = [u_{gt} \quad u_{Tt} \quad u_{St}]' \). The programs provided with Hansen and Sargent (1998) implement the autoregressive specifications of the exogenous processes in finding the solution to the optimization problem.

\(^8\) For further information on the relationship between VAR and MLE, see Chapter 8 of Hansen and Sargent (1998) or Anderson et al. (1996).
The ‘law of motion’ of the vector \( x_t = [g_t \tau_t s_t] \)' is based on the autoregressive representation for \( g_t \) and the ARMA representations for \( \tau_t \) and \( s_t \). Let \( A(L)x_t = B(L)R(L)u_t, \) where \( A(L) = \text{diag} [1 \ x_t(L) \ x_s(L)], \) \( A_0 = I, \) and

\[
B(L) = \begin{bmatrix}
1 & 0 & 0 \\
\xi_{tg}(L) & \xi_{sT}(L) & \xi_{ss}(L) \\
\xi_{sg}(L) & \xi_{sT}(L) & \xi_{ss}(L)
\end{bmatrix} \tag{8}
\]

The VAR representation of the structural model is therefore \( [B(L)R(L)]^{-1}A(L)x_t = u_t \), which is an estimable form, where \( E[u_t u_t'] = \Omega = \text{diag} [\omega_o \ \omega_T \ \omega_s] \). The MA representation of the structural model is therefore:

\[
x_t = A(L)^{-1}B(L)R(L)u_t = D(L)u_t, \tag{9}
\]

where \( D_0 = B_0 \). The estimated reduced form of the VAR is represented by \( A(L)x_t = \eta_t \), where \( \eta_t \) is a \( 3 \times 1 \) vector of white noise, \( E[\eta_t \eta_t'] = \Sigma, \) and \( A_0 = I \). The MA representation of the reduced form is, where \( \Gamma_0 = I \):

\[
x_t = A(L)^{-1}\eta_t = \Gamma(L)\eta_t. \tag{10}
\]

Identification of the model is achieved by matching the reduced form representation with the structural representation. From Eqs. (9) and (10), \( \Gamma(L)\eta_t = D(L)u_t \) and \( \eta_t = B_0u_t \), which implies:

\[
\Sigma = E[B_0u_t u_t'B_0'] = B_0\Omega B_0', \tag{11}
\]

\[
\Gamma(L)B_0 = D(L), \tag{12}
\]

\[
\Gamma(L)\Sigma \Gamma(L)' = \Gamma(L)B_0\Omega B_0'\Gamma(L)' = D(L)\Omega D(L)'. \tag{13}
\]

Restrictions on \( D(L) \) based on the theoretical model can thus be used to extract estimates of \( D(L) \) and \( \Omega \) from the unrestricted \( \Sigma \) matrix and the MA representation, \( \Gamma(L) \).

If the structural shocks can be modeled as unit root processes, implying that they have permanent effects, then the autoregressive representations of the variables are: \( \rho_D(L)(1 - L)g_t = u_{gt}, \rho_T(L)(1 - L)\tau_t = u_{Tt}, \) and \( \rho_S(L)(1 - L)s_t = u_{st}. \) In this case, the ‘law of motion’ of vector \( x_t = [g_t \tau_t s_t]' \) is \( A(L)x_t = B(L)R(L)(1 - L)^{-1}u_t \) and the VAR representation is

\[
[B(L)R(L)]^{-1}A(L)\Delta x_t = u_t, \tag{14}
\]

where \( \Delta = (1 - L) \). The reduced form of the VAR is therefore estimated in first differences: \( A(L)\Delta x_t = \eta_t \). Identification then proceeds as in Eqs. (11)–(13).

The distinction between reduced-form VARs \( A(L)x_t = \eta_t \) and \( A(L)\Delta x_t = \eta_t \) involves the structural specification of the econometric model. If the data are
stationary, shocks have temporary effects and \( A(L)x_t = \eta_t \) should be estimated; if the data are difference-stationary, shocks have permanent effects and \( A(L)\Delta x_t = \eta_t \) should be estimated.\(^9\) For the most part, the theoretical models in Section 2 investigating temporary shocks produce temporary effects on conventional taxes and seigniorage. Although this is strictly true only for the models without debt (see Fig. 2), the permanent effects in the models with debt are so small that they may be approximated in empirical work by restricting them to be zero (see Fig. 1). Hence, econometric models estimated in levels can be matched against these. Since all of the theoretical models in Section 2 investigating permanent shocks produce permanent effects on conventional taxes and seigniorage (see both Figs. 1 and 2), econometric models estimated in differences can be matched against them.

Identifying restrictions are required in order to extract estimates of \( D(L) \) and \( \Omega \) from \( \Gamma(L) \) and \( \Sigma \). In the remainder of this section, restrictions are applied to the long run parameters, \( D(1) \). From Eq. (13), it is evident that \( \Gamma(1)\Sigma\Gamma(1)' = D(1)\Omega D(1)' \) and the twelve parameters in \( D(1)\Omega D(1)' \) must be reduced to no more than the six unique entries in \( \Gamma(1)\Sigma\Gamma(1)' \). A just-identified system thus requires six identifying restrictions. For models estimated in levels, the long-run parameters represent cumulative responses; for models estimated in differences, they represent permanent responses. In addition, however, considerable attention is paid to the contemporaneous parameters, \( B_0 \), implied by the long run parameters and the VAR, given by Eq. (12), \( B_0 = \Gamma(1)^{-1}D(1) = A(1)D(1) \).\(^10\)

Normalizations and the exogeneity of government spending are imposed first. Three elements can be set through normalizations, so normalization along the diagonal in \( D(1) \) imposes \( D_{gg}(1) = D_{tt}(1) = D_{ss}(1) = 1 \). Since the response of conventional tax revenues to a shock in \( T_t \) and the response of seigniorage to a shock in \( S_t \) are positive under this normalization, the shocks must be interpreted as favorable shocks, in contrast to the model in Section 2 where shocks are adverse. Since the theoretical model has been developed under the assumption that shocks to government spending are exogenous, every element in the polynomial distributed lags \( D_{gT}(L) \) and \( D_{gS}(L) \) is required to be zero. In empirical work, this is easily imposed when the VAR is estimated. In identification, this

\(^9\)In addition, if the data are difference-stationary but cointegrated, \( A(L)x_t = \eta_t \) should be estimated with a cointegrating vector included, producing a vector error correction model (VECM). If some data series are stationary and other series are difference-stationary, mixed models should also be considered.

\(^10\)For more on long-run identifying restrictions, see Shapiro and Watson (1988) and Blanchard and Quah (1989). In addition, Faust and Leeper (1997) describe some problems of long-run restrictions imposed on finite-horizon data and suggest that short-run restrictions be taken seriously. For more on mixing contemporaneous and long-run restrictions, see Gali (1992).
implies that \( b_{qT} = b_{qS} = D_{qT}(1) = D_{qS}(1) = 0 \), where \( b_{qT} \) and \( b_{qS} \) are elements of \( B_0 \). Hence, there are three normalization restrictions and two exclusion restrictions applied to the long-run parameters at this point. These restrictions are sufficient to exactly identify one element of \( \Omega \), two additional long-run parameters, and three more contemporaneous parameters: \( \omega_q = \gamma_{11} \), \( D_{qg}(1) = \gamma_{21}/\gamma_{11} \), \( D_{qg}(1) = \gamma_{31}/\gamma_{11} \), \( b_{qg} = A_{11}(1) + A_{12}(1)\gamma_{21}/\gamma_{11} + A_{13}(1)\gamma_{31}/\gamma_{11} \), \( b_{qg} = A_{21}(1) + A_{22}(1)\gamma_{21}/\gamma_{11} + A_{23}(1)\gamma_{31}/\gamma_{11} \), and \( b_{qg} = A_{31}(1) + A_{32}(1)\gamma_{21}/\gamma_{11} + A_{33}(1)\gamma_{31}/\gamma_{11} \), where the \( \gamma \)'s denote the elements of \( I(1)\Sigma I(1)' \). These parameters are theoretically positive, but empirical implementation does not impose this as a restriction. With the five restrictions applied to the long-run parameters and two additional parameters thereby exactly identified, there are two unidentified long-run parameters remaining (\( D_{qg}(1) \) and \( D_{qT}(1) \)). Similarly, with one element of \( \Omega \) identified, there are two unidentified elements remaining (\( \omega_T \) and \( \omega_S \)). Identification must therefore proceed, and additional identifying restrictions must be considered.

Additional restrictions on the elements of \( D(1) \) are suggested by the traditional model, which does not include much of a role for shocks to \( T_q \) or \( S_T \). This could imply either that the shocks do not exist or that the shocks do not affect tax-setting behavior if they do exist. The former interpretation might imply that \( \omega_T = \omega_S = 0 \). However, this cannot be implemented econometrically because, although the order conditions are satisfied, the rank conditions are not satisfied.\(^{11} \) The latter interpretation might imply that every element of \( D_{Tq}(L), D_{qS}(L), D_{qg}(L), \) and \( D_{qT}(L) \) is zero, overriding the normalizations along the diagonal. This cannot be implemented econometrically because \( D(L) \) would not be invertible. However, either one of these interpretations implies that \( D(1)\Omega \bar{D}(1)' \) is singular, and this can be empirically examined by looking at the determinant of \( I(1)\Sigma I(1)' \). Furthermore, feasible restrictions on the \( D(L) \) matrix include zero restrictions on both \( D_{qg}(1) \) and \( D_{qT}(1) \). This allows conventional taxation to be affected by government spending shocks and conventional tax shocks, and allows seigniorage to be affected by government spending shocks and seigniorage shocks. This is more than the traditional model allows, but is less than the full model under consideration in this paper allows, so thus corresponds to a ‘generous version’ of the traditional model. (Note that this in fact is not as severe as requiring every element of \( D_{qg}(L) \) and \( D_{qT}(L) \) to be zero, although this would be feasible as well.) Furthermore, the restriction that \( D_{qS}(1) = D_{qT}(1) = 0 \) is used for its empirical feasibility, and does not correspond to a special case of the theoretical model in Section 2. Along with the three normalization restrictions and two exclusion restrictions already imposed, these

\(^{11} \) Such restrictions identify \( \omega_q \), and \( D_{qg}(1) \) and \( D_{qT}(1) \) are overidentified, but \( D_{qS}(1) \) and \( D_{qT}(1) \) are unidentified.
two restrictions based on a generous version of the traditional model bring the total number of restrictions to seven. Thus, there is one overidentifying restriction on \( D(1) \) that can be statistically tested, and is specifically the test of the hypothesis that \( D_{cs}(1) = D_{st}(1) = 0 \). If estimation does not fail the test of overidentification, the parameters can be examined to determine whether they appear reasonable vis-à-vis the traditional model. If estimation fails the test of overidentification, the generous version of the traditional model can be rejected based on the data.

If the exclusion restrictions based on the traditional model fail the test of overidentification, further investigation based on the model with multiple shocks and adjustment costs is warranted. In particular, the zero restrictions on \( D_{cs}(1) \) and \( D_{st}(1) \) must be dropped. Dropping one restriction or the other would leave the VAR exactly identified. However, the relationship between the two parameters can be examined following the method of Davis and Haltiwanger (1996), and this is less arbitrary and less restrictive than setting either \( D_{cs}(1) \) or \( D_{st}(1) \) to zero. The system provides a nonlinear mapping between \( D_{cs}(1) \) and \( D_{st}(1) \) such that

\[
D_{st}(1) = \frac{(\gamma_{23} - \gamma_{21}/\gamma_{11}) - (\gamma_{33} - \gamma_{31}/\gamma_{11})D_{cs}(1)}{(\gamma_{22} - \gamma_{21}/\gamma_{11}) - (\gamma_{23} - \gamma_{21}/\gamma_{11})D_{cs}(1)},
\]

where the \( \gamma \)'s again denote the elements of \( \Gamma(1)\Sigma\Gamma(1)' \). Mappings from \( D_{cs}(1) \) and \( D_{st}(1) \) into \( \omega_T \) and \( \omega_S \) can also be made:

\[
\omega_T = \frac{(\gamma_{22} - \gamma_{21}/\gamma_{11}) - (\gamma_{33} - \gamma_{31}/\gamma_{11})D_{cs}(1)^2}{(1 - D_{cs}(1)^2D_{st}(1)^2)},
\]

\[
\omega_S = \frac{(\gamma_{33} - \gamma_{31}/\gamma_{11}) - (\gamma_{22} - \gamma_{21}/\gamma_{11})D_{st}(1)^2}{(1 - D_{cs}(1)^2D_{st}(1)^2)}.
\]

Hence, the four unidentified parameters – \( D_{cs}(1), D_{st}(1), \omega_T, \) and \( \omega_S \) – are jointly determined. The method of Davis and Haltiwanger (1996) examines the sets of \( D_{cs}(1), D_{st}(1), \omega_T, \) and \( \omega_S \) which represent solutions to the nonlinear equations (15)–(17). Since the system is underidentified, the method restricts the sets considered by simultaneously imposing qualitative restrictions on the four parameters.

Since \( D_{cs}(1), D_{st}(1), \omega_T, \) and \( \omega_S \) are jointly determined, the general model with multiple shocks can be examined by imposing qualitative restrictions on the estimated parameters that are suggested by the structural model. If there are no values of \( D_{cs}(1), D_{st}(1), \omega_T, \) and \( \omega_S \) which simultaneously satisfy the qualitative restrictions, the general model can be rejected, and alternative investigations might be considered. However, if there are values of \( D_{cs}(1), D_{st}(1), \omega_T, \) and \( \omega_S \) which simultaneously satisfy the qualitative restrictions, the parameter
values can be examined to determine whether they appear reasonable for the general model. Although the structural VAR is underidentified when there are many values of $D_{sS}(1)$, $D_{sT}(1)$, $\omega_T$, and $\omega_S$ which satisfy the qualitative restrictions, the four parameters can thus be identified within certain intervals. The qualitative restrictions used here are sign and boundary restrictions. In particular, the theory in Section 2 and the examples shown in Figs. 1 and 2 clearly indicate that $D_{sS}(1)$ and $D_{sT}(1)$ are the opposite signs of $D_{sS}(1)$ and $D_{sT}(1)$, respectively, thus providing sign restrictions. Although this is unambiguously correct for the models when debt is not available and for permanent shocks when debt is available, it is likely to be true for the models with temporary shocks when debt is available as well. Furthermore, the model implies that the magnitudes of $D_{sS}(1)$ and $D_{sT}(1)$ will at most match the magnitudes of the responses of $D_{sS}(1)$ and $D_{sT}(1)$, respectively, thus providing boundary restrictions. This is again evident for the models without debt and for the permanent shocks when debt is available. This lower boundary is less convincing for the models with temporary shocks when debt is available, but is useful in empirical work given that these models are estimated in levels, which forces the permanent response to be zero. Hence, two qualitative restrictions are: $-1 \leq D_{sS}(1) \leq 0$ and $-1 \leq D_{sT}(1) \leq 0$. Since standard deviations cannot be negative, two additional qualitative restrictions are $\omega_T \geq 0$ and $\omega_S \geq 0$. The qualitative examination of the general model with multiple shocks thus asks whether there are any solutions for $D_{sS}(1)$, $D_{sT}(1)$, $\omega_T$, and $\omega_S$ in the nonlinear equations (15)–(17) which simultaneously satisfy these four restrictions.

If the investigation of the long-run parameters reveals that there are values of $D_{sS}(1)$, $D_{sT}(1)$, $\omega_T$, and $\omega_S$ which simultaneously satisfy the long-run qualitative restrictions suggested by the structural model, an examination of the contemporaneous parameters is warranted. Since the long run parameters and the VAR imply what the contemporaneous parameters are, the appropriateness of the ranges for $D_{sS}(1)$, $D_{sT}(1)$, $\omega_T$, and $\omega_S$ can be further evaluated using qualitative restrictions on $b_{sT}$, $b_{sS}$, $b_{sT}$, and $b_{sS}$. Based on the theoretical models (again see Figs. 1 and 2), four additional qualitative restrictions are available. Since $D_{sT}(1)$ and $D_{sS}(1)$ are both normalized to unity, shocks to the deadweight losses of conventional taxation and seigniorage would have positive contemporaneous effects as well, though not exceeding the magnitude of unity. Hence, two qualitative restrictions are: $0 \leq b_{sT} \leq 1$ and $0 \leq b_{sS} \leq 1$. The theory also implies that $b_{sS}$ and $b_{sT}$ are the opposite sign of $b_{sS}$ and $b_{sT}$. Furthermore, the magnitude of $b_{sS}$ is bounded by the magnitude of $b_{sS}$ in the model of asymmetric adjustment costs; unfortunately, no similar statement can be made for $b_{sT}$. Hence, two final qualitative restrictions are: $-1 \leq -b_{sS} \leq b_{sS} \leq 0$, and $b_{sT} \leq 0$. If the contemporaneous qualitative restrictions narrow down the ranges for $D_{sS}(1)$, $D_{sT}(1)$, $\omega_T$, and $\omega_S$, identification of the model is more exact.
4. Data and preliminary analysis

Four countries have been chosen to examine the models of seigniorage and conventional taxation: the US, the UK, Brazil, and Argentina. For the US and UK, two countries which have typically been studied in the traditional literature, two measures of seigniorage are used: the change in currency and the change in the monetary base. For Brazil and Argentina, two countries which have not typically been studied in the traditional literature but for which seigniorage is generally considered more important, data are more limited. For Brazil, seigniorage is only the change in currency. For Argentina, seigniorage is the change in M1. This section characterizes the data. The first part describes the data and sources. The second part examines the stationarity of the series in an effort to determine how the VARs should be run. Section 5 then proceeds to econometrically estimate the reduced-form VARs.

The US has the highest quality data available annually for the period 1929–1994. The government spending series is total government expenditure less net interest paid, and comes from National Income and Product Accounts (NIPA) (US Bureau of Economic Analysis). Conventional tax revenue is total tax revenue of the federal government less the transfer from the Federal Reserve to the Treasury (seigniorage), and also comes from NIPA. Seigniorage is calculated both as the change in currency and as the change in the monetary base. Data for currency come from Historical Statistics of the United States (US Bureau of the Census, 1975) and are updated using the Economic Report of the President (US Council of Economic Advisors, various issues). Data for the monetary base come from Friedman and Schwartz (1963), and are updated using International Financial Statistics (International Monetary Fund, various issues). The output measure is GNP, from NIPA.

Data for the UK are generally poorer quality than data for the US, but do exist annually for the long time horizon 1872–1990. All data series come from Mitchell (1988) and are updated using Annual Abstract of Statistics (UK Central Statistical Office). Government expenditure is reported as total expenditure and conventional tax revenue is reported as total revenue in the consolidated accounts. Two measures of seigniorage are again used, one as the change in currency and one as the change in the monetary base. Output is GNP at factor cost.

Data for Brazil are available for 1913–1985 on an annual basis. Total government receipts and total government expenditure come from Ludwig (1985) and are updated using Anuario Estatistico do Brasil (Ministerio da Economia). Seigniorage is the change in currency, reported as ‘money issued’ in Ludwig and Anuario Estatistico do Brasil. The output series is GDP and comes from Mitchell (1983), and is updated using International Financial Statistics.

Data for Argentina come from Domenech (1986) and are available annually for the period 1914–1984. Government spending is national government expenditure and excludes social security payments. Conventional tax revenues are
Table 2
Augmented Dickey–Fuller (ADF) unit root tests

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Series</th>
<th>Mean</th>
<th>ADF $\tau_1$</th>
<th>$\rho$</th>
<th>ADF $\tau_0$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (1929–1994)</td>
<td>Expenditure/Output</td>
<td>0.171</td>
<td>-4.76**</td>
<td>0.71</td>
<td>-4.59**</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>Revenue/Output</td>
<td>0.158</td>
<td>-2.22</td>
<td>0.91</td>
<td>-2.59</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ Currency/Output</td>
<td>0.004</td>
<td>-4.20**</td>
<td>0.66</td>
<td>-4.07**</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ Base/Output</td>
<td>0.007</td>
<td>-4.24**</td>
<td>0.48</td>
<td>-3.56*</td>
<td>0.60</td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>Expenditure/Output</td>
<td>0.222</td>
<td>-4.26**</td>
<td>0.85</td>
<td>-3.39*</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Revenue/Output</td>
<td>0.194</td>
<td>-3.31</td>
<td>0.91</td>
<td>-1.75</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ Currency/Output</td>
<td>0.003</td>
<td>-4.93**</td>
<td>0.49</td>
<td>-4.28**</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ Base/Output</td>
<td>0.004</td>
<td>-4.80**</td>
<td>0.63</td>
<td>-4.72**</td>
<td>0.65</td>
</tr>
<tr>
<td>Brazil (1913–1985)</td>
<td>Expenditure/Output</td>
<td>0.103</td>
<td>-3.58*</td>
<td>0.70</td>
<td>-3.36*</td>
<td>0.74</td>
</tr>
<tr>
<td>(0 lags)</td>
<td>Revenue/Output</td>
<td>0.090</td>
<td>-5.21**</td>
<td>0.50</td>
<td>-5.19*</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ Currency/Output</td>
<td>0.014</td>
<td>-4.09*</td>
<td>0.63</td>
<td>-3.94**</td>
<td>0.65</td>
</tr>
<tr>
<td>Argentina (1914–1984)</td>
<td>Expenditure/Output</td>
<td>0.133</td>
<td>-4.29**</td>
<td>0.57</td>
<td>-2.60</td>
<td>0.81</td>
</tr>
<tr>
<td>(0 lags)</td>
<td>Revenue/Output</td>
<td>0.095</td>
<td>-4.41**</td>
<td>0.56</td>
<td>-3.12*</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ M1/Output</td>
<td>0.037</td>
<td>-4.41**</td>
<td>0.55</td>
<td>-3.53*</td>
<td>0.70</td>
</tr>
<tr>
<td>Critical values</td>
<td>95% confidence level</td>
<td>-3.50</td>
<td>-2.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 observations</td>
<td>99% confidence level</td>
<td>-4.15</td>
<td>-3.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical values</td>
<td>95% confidence level</td>
<td>-3.45</td>
<td>-2.89</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>100 observations</td>
<td>99% confidence level</td>
<td>-4.04</td>
<td>-3.51</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * denotes significant at 5% level, ** denotes significant at 1% level.

The data for the US reject the null hypothesis of a unit root in all series except the ratio of conventional tax revenue to output. Visual inspection reveals that there is most likely a structural shift upward in conventional taxation associated with the end
Table 3
Perron–Vogelsang unit root tests allowing structural break

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Series</th>
<th>With trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t_a$</td>
</tr>
<tr>
<td>US (1929–1994)</td>
<td>Revenue/Output with break in 1941</td>
<td>$-6.16^{**}$</td>
</tr>
<tr>
<td>Critical values</td>
<td>95% confidence level</td>
<td>$-4.26$</td>
</tr>
<tr>
<td>50 observations</td>
<td>99% confidence level</td>
<td>$-5.10$</td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>Revenue/Output with break in 1919</td>
<td>$-4.56^*$</td>
</tr>
<tr>
<td>Critical values</td>
<td>95% confidence level</td>
<td>$-4.19$</td>
</tr>
<tr>
<td>100 observations</td>
<td>99% confidence level</td>
<td>$-4.91$</td>
</tr>
</tbody>
</table>

Note: * denotes significant at 5% level, ** denotes significant at 1% level.

of World War II, and a structural change in 1941 produces the highest value of a $t$-statistic on the shift variable. A Perron–Vogelsang unit root test allowing for an endogenously-determined structural break in 1941 is therefore presented in Table 3. Based on this test, the hypothesis of a unit root can be rejected in favor of the alternative hypothesis of stationarity around a changing mean. Hence, the conclusion is that all series for the US are stationary, once allowing for a structural break in the ratio of conventional tax revenues to output.

The data for the UK also reject the null hypothesis of a unit root in all series except the ratio of conventional tax revenue to output. Visual inspection reveals that there is most likely a structural shift upward in conventional taxation associated with the end of World War I. The Perron–Vogelsang unit-root test for a shift in 1919, which produces the highest $t$-statistic on the shift variable, is presented in Table 3. Based on this test, the hypothesis of a unit root can be rejected in favor of the alternative hypothesis of stationarity around a changing mean.

13 The unit-root tests allowing for an endogenously-determined structural break are discussed in Perron and Vogelsang (1992). The test regression implemented here is the innovational outlier model in Eq. (7) of their paper. As recommended by Perron and Vogelsang after considering different ways of selecting the order of the appropriate autoregression, the number of lags has been chosen by a test of significance of the last included lag, denoted $k = k(t)$ in their paper. The critical values at the 5% and 1% levels, based on Table 4 of Perron and Vogelsang (1992), are shown at the bottom of the table.

14 I am grateful to an anonymous referee for suggesting this line of inquiry.
mean. Hence, the conclusion is that all series for the UK are stationary, once allowing for a structural break in the ratio of conventional tax revenue to output.

Unit root tests for Brazil and Argentina more clearly reject the hypothesis of a unit root. The data for Brazil reject the null hypothesis of a unit root for all series, and are therefore considered stationary. The data for Argentina reject the null hypothesis of a unit root in all series when a trend is included. Since the trends are statistically significant, all series are therefore considered stationary.

On the whole, the unit root tests suggest that the data series are stationary (allowing for structural breaks in the US and UK ratios of conventional tax revenue to output). This conclusion in turn suggests that the shocks are temporary (except for the single permanent changes in the US and UK ratios of conventional tax revenue to output) and that the VARs should be estimated in levels. As a result, no further consideration of models estimated in differences or of cointegration among variables is necessary.15

5. Empirical results

The econometric specification developed in Section 3 is here applied to the annual data for the US, UK, Brazil, and Argentina described in Section 4. The strategy is to first examine the parameters that are exactly identified by the normalization and exogeneity restrictions and to test the overidentifying restrictions suggested by the generous version of the traditional model (Table 4), then to subsequently estimate parameter intervals based on the general model for the VARs which fail the test of overidentification (Tables 5 and 6). Finally, we examine the impulse response functions for the most successful VARs (Fig. 3), and correlations between seigniorage and conventional taxation for historical decompositions of the time series (Table 7).

The VARs have been estimated in levels, and the equations include constants and logarithmic time trends. Models for the US and UK also include dummy variables to allow for the structural shifts uncovered for 1941 and 1919, respectively. All of the VARs impose exogeneity of government expenditure, as indicated by matrices \( A(L) \) and \( B(L) \).

Table 4 presents the results for the overidentified VARs. For all of these models, \( \Gamma(1)\Sigma\Gamma(1)' = D(1)\Omega D(1)' \) has a nonzero determinant so is empirically

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15 For comparison, mixed models for the US and UK in which \( g_t \) and \( s_t \) are temporary shocks but \( T_t \) is always a permanent shock have also been estimated by putting the first difference of \( \tau_t \) in the VAR along with the levels of \( g_t \) and \( s_t \). However, these mixed models do not improve the estimation of the VAR as much as allowing for one permanent change in the \( \tau_t \) series and interpreting all other changes as temporary, so the results are not reported.
Table 4
Parameter estimates based on overidentified model

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Seigniorage measure</th>
<th>LR test (p-value)</th>
<th>$D_{sp}(1)$ (s.e.)</th>
<th>$D_{sg}(1)$ (s.e.)</th>
<th>$b_{sg}$</th>
<th>$b_{sp}$</th>
<th>$b_{sg}$</th>
<th>$\sqrt{\omega_g}$</th>
<th>$\sqrt{\omega_r}$</th>
<th>$\sqrt{\omega_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (1929–1994)</td>
<td>Δ Currency/Output(*)</td>
<td>57.576</td>
<td>0.2464</td>
<td>-0.0135</td>
<td>0.53</td>
<td>0.00</td>
<td>0.01</td>
<td>0.0512</td>
<td>0.0149</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.037)</td>
<td>(0.012)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>45.653</td>
<td>-0.1556</td>
<td>-0.1651</td>
<td>0.53</td>
<td>-0.01</td>
<td>0.02</td>
<td>0.0512</td>
<td>0.0234</td>
<td>0.0141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.058)</td>
<td>(0.035)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>Δ Currency/Output(*)</td>
<td>2.196</td>
<td>0.4792</td>
<td>0.0090</td>
<td>0.18</td>
<td>0.01</td>
<td>0.00</td>
<td>0.2464</td>
<td>0.0527</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.138)</td>
<td>(0.020)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>0.511</td>
<td>0.4910</td>
<td>0.0103</td>
<td>0.18</td>
<td>0.01</td>
<td>0.00</td>
<td>0.2464</td>
<td>0.0511</td>
<td>0.0070</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.474)</td>
<td>(0.020)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil (1913–1985)</td>
<td>Δ Currency/Output(*)</td>
<td>55.704</td>
<td>0.6141</td>
<td>0.1598</td>
<td>0.33</td>
<td>0.13</td>
<td>0.03</td>
<td>0.0371</td>
<td>0.0230</td>
<td>0.0188</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.074)</td>
<td>(0.060)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina (1914–1984)</td>
<td>Δ M1/Output (*)</td>
<td>69.237</td>
<td>0.5524</td>
<td>0.2380</td>
<td>0.53</td>
<td>0.13</td>
<td>0.15</td>
<td>0.0337</td>
<td>0.0184</td>
<td>0.0457</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.066)</td>
<td>(0.164)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Asterisk denotes VAR for which impulse response functions are presented in Fig. 3. Standard errors are determined using the likelihood-based function described in Section 8.5 of Doan (1992).
Table 5
Interval estimates based on general model

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Seigniorage measure</th>
<th>Range of $D_{s,t}(1)$</th>
<th>Range of $D_{t,t}(1)$</th>
<th>Range of $\sqrt{\omega_T}$</th>
<th>Range of $\sqrt{\omega_S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (1929–1994)</td>
<td>$\Delta$ Currency/Output(*)</td>
<td>− 1, − 0.53</td>
<td>− 0.24, − 0.22</td>
<td>0.0144, 0.0148</td>
<td>0, 0.0021</td>
</tr>
<tr>
<td>(4 lags)</td>
<td>$\Delta$ Base/Output</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>$\Delta$ Currency/Output(*)</td>
<td>0</td>
<td>0</td>
<td>0.0527</td>
<td>0.0047</td>
</tr>
<tr>
<td>(4 lags)</td>
<td>$\Delta$ Base/Output</td>
<td>0</td>
<td>0</td>
<td>0.0511</td>
<td>0.0070</td>
</tr>
<tr>
<td>Brazil (1913–1985)</td>
<td>$\Delta$ Currency/Output(*)</td>
<td>− 0.91, − 0.06</td>
<td>− 0.58, 0</td>
<td>0.0148, 0.0230</td>
<td>0, 0.0187</td>
</tr>
<tr>
<td>(2 lags)</td>
<td>$\Delta$ M1/Output(*)</td>
<td>− 0.32, − 0.23</td>
<td>− 1, 0</td>
<td>0.0108, 0.0154</td>
<td>0.0378, 0.0457</td>
</tr>
<tr>
<td>Argentina (1914–1984)</td>
<td>$\Delta$ M1/Output(*)</td>
<td>− 0.32, − 0.23</td>
<td>− 1, 0</td>
<td>0.0108, 0.0154</td>
<td>0.0378, 0.0457</td>
</tr>
<tr>
<td>(2 lags)</td>
<td>$\Delta$ Base/Output</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Note: Asterisk denotes VAR for which impulse response functions are presented in Fig. 3. ‘none’ signifies that long-run restrictions cannot be satisfied.

invertible, immediately suggesting that the traditional model is inadequate. The formal likelihood ratio test (LR Test) of overidentification, which is distributed $\chi^2(1)$, also enables us to reject the generous version of the traditional model in most cases.\(^{16}\) For the US, Brazil, and Argentina, there are strong rejections of the overidentifying restriction. For the UK, however, the overidentifying restriction cannot be rejected.

Table 4 also presents values of $D_{sg}(1)$ and $D_{sg}(1)$ along with standard errors. Thus, statistical tests of the hypotheses that $D_{sg}(1) > 0$ and $D_{sg}(1) > 0$ can be performed. Values of $b_{sg}$, $b_{tg}$, and $b_{sg}$ implied by the long run parameters are presented as well, and the last three columns provide values of $\sqrt{\omega_{sp}}$, $\sqrt{\omega_T}$, and $\sqrt{\omega_S}$. For the US, some estimates of $D_{sg}(1)$ and $D_{sg}(1)$ have the wrong sign. Of the two models, the one using currency is the most sensible because the response of conventional tax revenues to a government spending shock is significantly

\(^{16}\) In overidentified VARs, $D(1)\sqrt{\Omega}$ does not precisely factor $\Gamma(1)\Sigma(1)'$. The test of the overidentifying restriction is $LR = n[\ln(\det(D(1)\sqrt{\Omega})) - \ln(\det(\Gamma(1)\Sigma(1)'))]$ where $n$ is the number of observations and $D(1)$ is restricted such that $D_{sg}(1) = D_{sg}(1) = 0$. Exogeneity of government spending is imposed in both the restricted model and the unrestricted model, so $D_{sg}(1)$ and $D_{sg}(1)$ are the same in both. Hence, the likelihood ratio tests the hypothesis that $D_{sg}(1) = D_{sg}(1) = 0$ against the alternative that there is a mapping between $D_{sg}(1)$ and $D_{sg}(1)$.
Table 6
Intervals of contemporaneous parameters based on intervals identified in Table 5

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Seigniorage measure</th>
<th>Range of $b_{sT}$</th>
<th>Range of $b_{sT}$</th>
<th>Range of $b_{sS}$</th>
<th>Range of $b_{sS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (1929–1994)</td>
<td>Δ Currency/Output(*)</td>
<td>0.47, 0.48</td>
<td>0.01, 0.02</td>
<td>−0.05, 0.24</td>
<td>0.44, 0.51</td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>None, None</td>
<td>None, None</td>
<td>None, None</td>
<td>None, None</td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>Δ Currency/Output(*)</td>
<td>0.22</td>
<td>0.02</td>
<td>−0.57, 0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>0.23</td>
<td>0.00</td>
<td>−0.09, 0.55</td>
<td></td>
</tr>
<tr>
<td>Brazil (1913–1985)</td>
<td>Δ Currency/Output(*)</td>
<td>0.34, 0.47</td>
<td>−0.14, 0.22</td>
<td>−0.20, 0.19</td>
<td>0.44, 0.62</td>
</tr>
<tr>
<td>Argentina (1914–1984)</td>
<td>Δ M1/Output(*)</td>
<td>0.64, 0.67</td>
<td>−0.05, 0.56</td>
<td>−0.19, 0.42</td>
<td></td>
</tr>
</tbody>
</table>

Note: Asterisk denotes VAR for which impulse response functions are presented in Fig. 3. ‘None’ signifies that long-run restrictions cannot be satisfied.

Table 7
Correlations between seigniorage and conventional taxation based on historical decompositions of series

<table>
<thead>
<tr>
<th>Country (Period)</th>
<th>Seigniorage measure</th>
<th>Total</th>
<th>Without shocks</th>
<th>Due to $g$</th>
<th>Due to $T$</th>
<th>Due to $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (1929–1994)</td>
<td>Δ Currency/Output(*)</td>
<td>0.104</td>
<td>−0.047</td>
<td>0.664</td>
<td>0.306</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>−0.514</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>UK (1872–1990)</td>
<td>Δ Currency/Output(*)</td>
<td>0.322</td>
<td>0.971</td>
<td>0.191</td>
<td>−0.152</td>
<td>−0.482</td>
</tr>
<tr>
<td></td>
<td>Δ Base/Output</td>
<td>0.183</td>
<td>0.802</td>
<td>0.041</td>
<td>−0.612</td>
<td>−0.201</td>
</tr>
<tr>
<td>Brazil (1913–1985)</td>
<td>Δ Currency/Output(*)</td>
<td>−0.056</td>
<td>−0.153</td>
<td>0.765</td>
<td>−0.574</td>
<td>−0.286</td>
</tr>
<tr>
<td>Argentina (1914–1984)</td>
<td>Δ M1/Output(*)</td>
<td>0.224</td>
<td>0.997</td>
<td>0.871</td>
<td>−0.312</td>
<td>−0.988</td>
</tr>
</tbody>
</table>

Note: Asterisk denotes VAR for which impulse response functions are presented in Fig. 3. ‘n.a.’ means ‘not applicable’.
positive and the response of seigniorage is, although negative, insignificant. Specifically, a one point temporary increase in the ratio of government expenditure to output originates as a 0.53 point increase and has a cumulative one point increase resulting from an autoregressive process. Although there is no contemporaneous response of conventional taxation (implying that the entire amount is financed with debt), the cumulative increase in government spending is financed 25% with an increase in taxes, leaving 75% to be financed with debt. For the UK, estimates of $D_{qg}(1)$ and $D_{sg}(1)$ are all positive and significant. A one point increase in government spending originates as a 0.18 point increase. Although there is a very small (0.01 point) increase in conventional taxes contemporaneously, the cumulative increase in government spending is eventually financed 48% with taxes (based on the VAR using currency) and 1% with seigniorage, and therefore 51% with debt.

For Brazil and Argentina, the results in Table 4 also indicate that $D_{qg}(1)$ and $D_{sg}(1)$ are positive and usually significant (the exception being $D_{sg}(1)$ in Argentina). In Brazil, a one point increase in government spending originates as a 0.33 point increase. Conventional taxes rise 0.13 points and seigniorage rises 0.03 points contemporaneously, leaving 0.17 points to be financed with debt. Cumulatively, conventional taxes rise 0.61 points and seigniorage rises 0.16 points, leaving 0.23 points to be financed with debt. In Argentina, a one point increase in government spending originates as a 0.53 point increase, and is contemporaneously financed with a 0.13 point increase in taxes and a 0.15 point increase in seigniorage, leaving 0.25 points to be financed with debt. Cumulatively, the one point increase in government spending is financed 55% with taxes, 24% with seigniorage, and 21% with debt. Based on these results and the time periods involved, Brazil and Argentina may therefore not have as much access to borrowing as the US and the UK in order to finance temporary shocks. (For Argentina, however, recall that the measure of seigniorage is the change in M1 and this may overstate the revenues.)

Table 4 also contains estimates of the standard deviations of shocks to the systems. The values of $\sqrt{\omega_q}$ suggest that government spending shocks are the most important in the UK, followed distantly by the US. The values of $\sqrt{\omega_T}$ suggest that shocks to the deadweight losses of conventional taxation are the most important in the UK, and the values of $\sqrt{\omega_S}$ suggest that shocks to the deadweight losses of seigniorage are the most important in Argentina.

For the US, Brazil, and Argentina, interval identification of $D_{qS}(1)$ and $D_{qT}(1)$ is particularly important given strong rejections of the overidentifying restriction. Table 5 presents ranges for the parameters which simultaneously satisfy the qualitative restrictions: $-1 \leq D_{qS}(1) \leq 0$, $-1 \leq D_{qT}(1) \leq 0$, $\omega_T \geq 0$, and $\omega_S \geq 0$. When such intervals are available, the general model of seigniorage

\[17 D_{qg}(1), D_{sg}(1), b_{qp}, b_{sp}, b_{sr} \text{ and } \sqrt{\omega_q} \text{ are exactly identified in the same manner as in Table 4.} \]
and conventional taxation is a potentially important description of government behavior. For the US, the VAR using currency does contain intervals which satisfy the qualitative restrictions. With respect to the monetary base, even taking the negative coefficients on $D_qg(1)$ and $D_qs(1)$ as given, there are no intervals which can simultaneously satisfy the qualitative restrictions. Hence, we effectively reject the general model when qualitatively tested using the monetary base, but cannot reject the general model when using currency. For Brazil and Argentina, each VAR contains parameter values which satisfy the qualitative restrictions. For the US, Brazil, and Argentina, then, the general model of seigniorage and conventional taxation is promising, and identification can proceed.

Table 6 presents intervals for the contemporaneous parameters which correspond to the intervals for cumulative parameters and standard deviations presented in Table 5. For most of the VARs, contemporaneous restrictions are typically satisfied. In particular, the qualitative restrictions on $b_{qT}$ and $b_{sS}$ are always satisfied. The qualitative restriction on $b_{sT}$ can be satisfied, but is automatically satisfied only for the UK and Argentina. Finally, the qualitative restriction on $b_{sT}$ can be satisfied for Brazil and Argentina but cannot be satisfied for the US and the UK. However, even when the restriction on $b_{sT}$ cannot be satisfied, it is nearly satisfied, which is therefore not too damaging considering that contemporaneous parameters are generally less precisely estimated in these VARs and probably represent short run deviations from equilibrium not accounted for by adjustment costs.\textsuperscript{18} Overall, the fact that qualitative restrictions are generally satisfied is evidence that the model with multiple shocks and adjustment costs may characterize the data.

Impulse response functions from the most successful model for each country are presented in Fig. 3. For the UK, the model imposing $D_qS(1) = D_sT(1) = 0$ is used. For the other countries, models which rejected the overidentifying restriction use a value of $D_qS(1)$ chosen to satisfy as many contemporaneous restrictions as possible. Shocks to $T_t$ and $S_t$ have been converted into adverse shocks so that the empirical impulse response functions can be directly compared to the theoretical impulse response functions in Figs. 1 and 2.

For the US, impulse response functions for the VAR estimated using currency are presented. Specifically, $D_qS(1) = -0.95$, which sets $D_sT(1) = -0.22$, $b_{qT} = 0.48$, $b_{sT} = 0.02$, $b_{sS} = -0.01$, and $b_{qS} = 0.45$. Hence, only $b_{sT}$ violates its qualitative restriction (because it should be negative), and even then is not too far off. The impulse response functions show that conventional taxes gradually rise in response to a government spending shock, although subsequently appear

\textsuperscript{18} VARs imposing only contemporaneous restrictions have been investigated for all of these models. Results produce estimates of $b_{qg}$ and $b_{qs}$ with large standard errors and estimates of $b_{qs}$ and $b_{sT}$ which fall in large intervals.
Fig. 3. Empirical impulse response functions.
to dip below zero then rise slightly above zero before settling at zero. This pattern resembles the path of government spending after the shock, but revenues are also much smoother than the path of government spending after the shock, and is therefore supportive of the optimization model. The response of seigniorage also seems to follow this pattern, but is small (and may be statistically insignificant based on $D_{sg}(1)$). The responses to shocks to deadweight losses oscillate a bit much, but otherwise have appropriate patterns: in response to a shock to the deadweight loss of conventional taxation (seigniorage), conventional tax (seigniorage) revenues are generally lower and seigniorage (conventional taxation) is generally higher. Forecast error variance decompositions demonstrate that changes in conventional taxation are due 47% to $\{T_i\}$, 52% to $\{g_i\}$, and less than 1% to $\{S_i\}$ at the five year horizon. The decompositions demonstrate that changes in seigniorage are due 22% to $\{S_i\}$, 48% to $\{g_i\}$, and 30% to $\{T_i\}$ at the five year horizon. Hence, the variance decompositions demonstrate that all three shocks are important, so the traditional model considering only shocks to government spending is inadequate. Together, the impulse response functions and the variance decompositions support the model with multiple shocks and adjustment costs, particularly because of the introduction of shocks to deadweight losses of conventional taxation.\(^\text{19}\)

For the UK, either model could be used, so responses for the VAR using currency are presented for consistency. Recall that $D_{sS}(1) = D_{sT}(1) = 0$. This corresponds to $b_{sT} = 0.22$, $b_{sT} = 0.02$, $b_{rS} = -0.57$, and $b_{sS} = 0.76$. Of particular interest is the fact that the responses of seigniorage and conventional taxation to shocks affecting their own deadweight losses are perfectly appropriate. In addition, $b_{sT}$ violates the qualitative restriction (because it should be negative), but again is not too far off, and is not surprising given that $D_{sT}(1) = 0$. As with the US, there is gradual adjustment of conventional tax revenue after a shock to government spending, and this time the path of seigniorage is appropriate and supportive of the general model because $D_{sg}(1)$ is statistically significant. Once again, responses to shocks to deadweight losses also have the appropriate patterns overall. Variance decompositions again suggest that all three shocks are important. Forecast error variance decompositions demonstrate that changes in conventional taxation are due 41% to $\{T_i\}$, 57% to $\{g_i\}$, and 2% to $\{S_i\}$ at the five year horizon. Decompositions also demonstrate that changes in seigniorage are due 53% to $\{S_i\}$, 36% to $\{g_i\}$, and 11% to $\{T_i\}$ at the five year horizon. Hence, although the hypothesis that $D_{ss}(1) = D_{st}(1) = 0$ cannot be rejected, the impulse response functions and variance decompositions suggest that the model with multiple shocks and adjustment costs probably characterizes the data better than the traditional model, or at least that the generous version of the traditional model is better than the narrow version.

\(^{19}\) For a related model of taxes and government spending without seigniorage, see Braun (1994).
For Brazil, the VAR again uses currency. Specifically, $D_{sT}(1) = -0.50$, which sets $D_{sT}(1) = -0.38$, $b_{QT} = 0.38$, $b_{sT} = -0.004$, $b_{sS} = -0.0001$, and $b_{sS} = 0.53$. Hence, all qualitative restrictions are satisfied simultaneously. The impulse response functions in Fig. 3 also show generally appropriate paths. In response to a government spending shock, both sources of revenue increase sharply (rather than gradually as in the US and UK) and gradually return to their equilibria, a pattern resembling the model with debt constraints. Responses for shocks to $T_i$ and $S_i$ depict general movement in opposite directions followed by gradual adjustment to equilibrium, somewhat resembling the model with debt, yet somewhat resembling the model without debt. Forecast error variance decompositions demonstrate that changes in conventional taxation are due 57% to $\{T_i\}$, 38% to $\{g_i\}$, and 5% to $\{S_i\}$ at the five year horizon. The decompositions demonstrate that changes in seigniorage are due 82% to $\{S_i\}$, 8% to $\{g_i\}$, and 9% to $\{T_i\}$ at the five year horizon. Overall, the model with multiple shocks, adjustment costs, and some debt constraints probably characterizes the data well.

For Argentina, all contemporaneous and long run qualitative restrictions are again simultaneously satisfied, and the midpoint of the feasible range sets $D_{sT}(1) = -0.235$, $D_{sT}(1) = -0.98$, $b_{QT} = 0.64$, $b_{sT} = -0.03$, $b_{sS} = -0.13$, and $b_{sS} = 0.47$. The impulse response functions strongly resemble those from the theoretical model without debt. In response to a government spending shock, both sources of revenue increase, and conventional taxes return to their initial level gradually as seigniorage drops below its initial level and approaches equilibrium from the opposite side. Responses for shocks to $T_i$ and $S_i$ depict movement in opposite directions followed by adjustment to equilibrium. Forecast error variance decompositions demonstrate that changes in conventional taxation are due 54% to $\{T_i\}$, 30% to $\{g_i\}$, and 16% to $\{S_i\}$ at the five year horizon. The decompositions demonstrate that changes in seigniorage are due 83% to $\{S_i\}$, 9% to $\{g_i\}$, and 8% to $\{T_i\}$ at the five year horizon. Overall, once again, the model with multiple shocks, adjustment costs, and debt constraints characterizes the data well.

To further examine the empirical models under consideration, Table 7 presents correlations between seigniorage and conventional taxation for historical decompositions of the conventional tax and seigniorage series using the identification techniques implemented for the impulse response functions. The column ‘Total’ presents the correlations for the raw data. The column ‘Without Shocks’ presents correlations that would be observed as the estimated system achieves and maintains steady state equilibrium; this primarily represents trends. The remaining three columns show correlations due to the three identified shocks, which can be compared to the theoretical correlations in Table 1. Although there are some exceptions, most of the correlations have correct signs and appear reasonable. Empirical correlations due to shocks to government spending are all positive, and are especially high for the US, Brazil, and Argentina.
Weak correlations for the UK are surprising given the good results in the parameter estimates and the impulse response functions, but probably reflect generally lighter use of both taxes (and more reliance on debt) compared to Brazil and Argentina. Table 1 suggested that the correlations due to shocks to $T_t$ and $S_t$ should be near zero to highly negative depending on the availability of debt. Except in the US, where seigniorage has already been shown to be less important, the empirical correlations fit into this range. Taken as a whole, the results in Table 7 suggest that identification has been successful, particularly for the UK, Brazil, and Argentina, and that the model of optimum seigniorage and conventional taxation performs well once different sources of shocks are taken into account.

6. Conclusion

This paper studies how governments finance exogenous shocks in order to determine how seigniorage and conventional taxation covary. The theoretical part of the paper develops a model of public finance in which there are multiple shocks, asymmetric costs of adjusting revenue, and borrowing costs that capture the possibility that debt may be constrained. As with the traditional model of optimum taxation, seigniorage and conventional tax revenues are positively correlated when the important shocks are to government spending. However, the theoretical model also introduces shocks to deadweight losses associated with both forms of taxation, and in turn suggests that seigniorage and conventional tax revenue will be uncorrelated to highly negatively correlated when the important shocks take these forms. The addition of asymmetric costs of adjusting revenues suggests that there may be important short run deviations from long run equilibrium which alter the correlations. Furthermore, this makes the availability of debt a much more important issue than previously thought, because debt smooths conventional tax revenue when it is available but seigniorage smooths conventional tax revenue when debt is not available.

Solutions and impulse response functions from the theoretical model are used to construct a three-variable structural VAR (in government expenditure, conventional tax revenue, and seigniorage) and identify the structural VAR from an estimable reduced-form VAR. The theory asserts how governments optimally respond to shocks, and the empirical work examines the theory by determining whether the data can be characterized by any form of the model. Some aspects are statistically testable – such as whether $D_{eq}(1) > 0$ and $D_{eq}(1) > 0$, and whether $D_{eq}(1) = D_{eq}(1) = 0$ – and some aspects are qualitatively examined – such as whether $-1 \leq D_{eq}(1) \leq 0$, $-1 \leq D_{eq}(1) \leq 0$, $\omega_T \geq 0$, and $\omega_S \geq 0$ simultaneously. Judged by the parameter estimates, empirical impulse response functions, forecast error variance decompositions, and historical decompositions of the time series, identification of the reduced-form VAR by imposing long
run restrictions is successful and the econometric results support the theoretical model. The main empirical finding is that the model which includes multiple shocks outperforms the traditional model that examines only a shock to government expenditure. For the US, Brazil, and Argentina, a generous version of the traditional model is rejected, based on tests of overidentification, in favor of the more general model. Based on difficulty identifying a model for the US and the results of that model, however, seigniorage does not appear to be very important and might be appropriately left out of the public finance analysis. Despite being unable to reject the generous version of the traditional model, the VARs for the UK provide support for the general model because of the appropriate use of seigniorage and appropriate responses of seigniorage and conventional taxation to shocks affecting their own deadweight losses. Models for Brazil and Argentina are indicative of the general model with multiple shocks with some debt constraints. Taken together, the chief contribution of examining these varied cases is in demonstrating the importance of multiple exogenous shocks.

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