In the winter of 1996 I was asked to undertake one of my least favourite tasks. I was asked to check the validity of undocumented computer code for the pricing and hedging of exotic options. During the course of my investigations I discovered that the ‘lookback’ algorithm implemented was in fact that due to Simon Babbs. However, I knew that the paper containing the algorithm had not been published. I knew something of the history of this paper and that I was not alone in thinking that it should be published. Further, I believed that there was a need to get into the public domain papers, such as Simon’s, that were clearly making a significant contribution to the practice, as well as the theory of Financial Economics; indeed, the field of Computational Finance was developing in its own right. As a consequence, I agreed to produce a broadly interpreted JEDC Special Issue on Computational Aspects of Complex Securities.

Classifying by technique, the papers may be thought of as being divided into three inter-related classes. The first class utilises discretization, through the use of finite difference methods, which of course includes lattices. The second is optimization based, and is driven by the use of shadow prices, both directly and indirectly. The final group can be thought of as ‘surrogate approximation’ methods, which have been designed explicitly for complex instruments. These papers, not surprisingly, have much in common with those published in Zenios (1997), and it is well worthwhile reading his introduction to gain additional insight to the contents of this Special issue.

The first paper of the discretization group is by Simon Babbs, who uses a binomial approximation scheme to value lookback options with a particular emphasis on finite date sampling. This paper was one of the earliest to demonstrate just how different finite date sampling prices could be from continuously sampled ones. Also using a binomial model for underlying returns, Stylianos Perrakis and Jean Lefoll investigate optimal perfect hedging portfolios and optimal exercise policy for American call options, in the presence of transaction costs and dividends. In contrast with the binomial approach of Babbs, and Perrakis and Lefoll, Robert Zvan, Kenneth Vetzal and Peter Forsyth, present a very general implicit finite difference method for solving the partial differential equation formulation of contingent claims prices with general algebraic
constraints on the solutions. In particular, they focus on a variety of barrier options.

A central problem for micro-economists, and in particular financial economists, is that of the determination and application of optimum consumption and portfolio rules under uncertainty. In the theory of linear inequalities, the existence of a relationship between the primal system and its dual has been well known since at least the publication of Gordan (1873). In the field of optimization Von Neumann (1947) and Gale et al. (1951) derived the duality theorem of linear programming. In this context Dantzig (1963) explicitly discussed the relationship between Lagrange multipliers and duality. He stated that ‘If the linear programming problem is attacked by the method of Lagrange multipliers, we find that the multipliers, if they exist, must satisfy a ‘dual’ system...’. It is therefore natural to expect that duality will play an important role in Computational Finance, and this has, indeed, proved to be the case. Given that even the simplest non-trivial problem usually has at least a budget constraint, we must expect to see generalised Lagrange multipliers appearing, which the economist immediately recognises as shadow prices. In this Special issue we have four papers that are explicitly optimization based, and their analyses are driven by the use of shadow prices, both directly and indirectly.

Following Merton (1969,1971) optimum consumption and portfolio rules under uncertainty are frequently obtained by maximizing expected utility from terminal wealth and/or consumption, by working in continuous time, and modelling the evolution of uncertainty through the use of Brownian motion. In general, the rules are derived from a stochastic dynamic programming problem that requires the solution of a Hamilton–Jacobi–Bellman equation that is rarely easy to solve. The difficulties are further increased when there are constraints on the level of wealth, borrowing and short-selling, etc.

However, there are many attractions to such a methodology. One is ‘...that if the ‘geometric Brownian motion’ hypothesis is accepted, then a general ‘separation’ or ‘mutual-fund’ theorem can be proved such that, in this model, the classical Markowitz–Tobin mean–variance rules hold without the objectionable assumptions of quadratic utility or of normality of distribution of prices’ (Merton, 1971, p. 374). This enables one to undertake much portfolio analysis with confidence. However, it is well known that the implementation of Markowitzian concepts is difficult, particularly because of the problems in obtaining reliable asset return forecasts and estimates of the covariance matrix. As a means of overcoming these weaknesses, Berç Rustem, Robin Becker and Wolfgang Marty provide a means for considering rival representations of the future. They derive and present an algorithm for the determination of robust portfolio policies based on a min–max strategy, which does not rely on any specific properties of the assets and, although not implemented here, can also incorporate transaction costs.
A second important feature is the ability to introduce the ‘embedding’ approach, due to Cvitanić and Karatzas (1992), that uses the mathematical tools of continuous-time martingales, convex analysis and duality theory. Lucie Teplá, building on their duality results, characterizes optimal intertemporal portfolio policies for investors with CRRA utility facing borrowing and/or shortsale restrictions. She then uses her results to identify the relationships between the constrained and unconstrained portfolios, which is of considerable computational importance.

Merton’s approach is particularly attractive to individuals with a mathematics or mathematical sciences background. However, in a traditional economics degree ‘mathematical methods of physics’ would not normally be taught. In my view, with some admittedly deep thinking, the economist searching for a mathematical model to describe the dynamic evolution of an asset price over time, is necessarily and naturally led to the concept of semi-martingales, and martingales in particular. Complementing Merton’s methodology there is the approach initiated and developed by Foldes (1978a,b, 1990), which is based on the martingale properties of shadow (or marginal utility) prices.

I asked Lucien Foldes to derive the Black and Scholes formula using his methodology, as it is rather different from that usually found, and because the shadow price approach gives greater visibility and transparency to the underlying economics. To achieve this objective a number of important foundational issues of the economics of Contingent Claims Analysis are necessarily addressed in the Introduction to the paper. The paper goes far further than simple option pricing. Capital budgeting has always been a key part of finance. Today we talk and write about real options, which are an increasingly important aspect of project evaluation. As a further example of the utility of the martingale property of shadow prices a formula for ‘indivisible’ project evaluation is developed.

In contrast with the previous papers in this Special issue, motivated by the demands for portfolio insurance in incomplete markets, Roko Aliprantis, Donald Brown and Jan Werner, establish a necessary and sufficient condition for the existence of a price-independent, minimum cost portfolio insurance policy and show how to construct it. Further, they also derive a characterization of incomplete derivative markets in which the minimum-cost portfolio insurance strategy is independent of arbitrage-free security prices. The characterization developed by them relies on the theory of lattice-subspaces. Their work can be viewed as a generalization of the ‘vector calculus’ of Kruizenga (1964), Kruizenga and Samuelson (see Samuelson, 1965; Garman, 1976). The results are obtained through an explicit linear programming formulation, and once again show the importance of shadow prices, both with regard to computation, as well as economic interpretation.

The papers described so far have a particularly important feature in common. Where approximations are used they are ‘natural approximations’. That is to say, they are essentially generated through the use of a Taylor’s series expansion,
which relies on the function being both analytic and mathematically tractable. Mathematical tractability is clearly a non-trivial requirement that is not often fulfilled. An elegant approach to this difficulty is to find a surrogate problem that is closely connected, but is less demanding. The third group of papers fall into this category, and represent a broad spectrum of approaches, which I think of as ‘surrogate approximation’ methods.

It is well known that Taylor’s series based approximations are by no means optimal, for example with respect to error behaviour. One of the most important alternatives is the derivation of orthogonal polynomial expansions, particularly Chebyshev polynomials, which have the equal-error (or ripple) property. However, in much of finance it is assumed that uncertainty evolves as a function of Brownian motion. It is therefore far more natural to consider polynomial approximations based on Hermite polynomials. In order to obtain prices and Greeks speedily, one must often decide whether it is better to approximate the payoff function and use the true probability density function, or act vice versa. Serge Darolles and Jean-Paul Laurent explore this question, and investigate the optimality of polynomial approximations generated by the underlying stochastic process, illustrating the strengths and weaknesses of the approach.

According to Hammersley and Handscomb (1964): ‘The real use of Monte Carlo methods as a research tool stems from work on the atomic bomb during the second world war’. Since then Monte Carlo methods, especially in the field of numerical integration, have become increasingly important, due in a major part to the ever-increasing power of computers. With the advent of compound-option-based instruments and securitization, the need to compute very high-order integrals is immense. Currently the only technique that we have to solve this type of problem is Monte Carlo. Recently the use of quasi-random numbers has received considerable attention in computational finance. However, there is no obvious method for computing the standard error of the estimate. Ken Seng Tan and Phelim Boyle address the termination criterion for the number of points to use, and compare and contrast their proposals against a known benchmark derivative contract. The authors’ work gives promising results even for high dimensions.

For almost a decade, investment in equities and equity related financial products has substantially increased. Further, increasingly complex instruments, ‘exotics’, have been issued. Many of these instruments have American style exercise features. It is rare for analytic valuation formulae to exist. The last two papers here consider American style options, one being theoretical in its orientation and the other empirical, both emphasizing the need to implement surrogate approximation.

Barrier options have their origins in the work of Merton (1973) and Black and Cox (1976). Today they are usually thought of within the context of exotic derivatives. However, they are also fundamentally important in the area of credit derivatives (see, for example, Selby, 1983). Even in their simplest form
their computational requirements are intense compared with those of the Black and Scholes and similar formulae. When coupled with early exercise privilege the derivation of a fast and accurate algorithm is a formidable task. To obtain such an algorithm Bin Gao, Jing-zhi Huang, and Marti Subrahmanyam investigate the pricing and hedging of American barrier options by obtaining an analytic representation for the value and hedge parameters of barrier options, using the decomposition technique of separating the European option value from the early exercise premium. The results are exciting. Their methodology dominates lattice methods, in terms of both accuracy and efficiency, and should be compared and contrasted with that of Zvan, Vetzal and Forsyth.

The important place ascribed to the normal distribution in statistical theory is well justified on the basis of the central-limit theorem. However, often it is not known whether the basic distribution is such that the central-limit theorem applies or whether the approximation to the normal distribution is good enough that the resulting confidence intervals, and tests of hypotheses based on normal theory, are as accurate as desired. Often an experimenter does not know the form of the basic distribution and needs statistical techniques, which are applicable regardless of the form of the density. These techniques are called non-parametric or distribution-free methods. (Mood and Graybill, 1963). Further, classical statistical methods are often relatively insensitive to the assumption of normality. Nonparametric methods have high efficiency relative to normality-based classical techniques and frequently have higher efficiency in other situations. Therefore it is very surprising, to me, that financial econometricians have only recently used nonparametric methods.

Complementing the paper of Gao, Huang, and Subrahmanyam, Mark Broadie, Jérôme Detemple, Eric Ghysels and Olivier Torrès suggest a new and different strategy for dealing with the pricing of American options and the characterization of the exercise boundary. In order to estimate option prices and exercise boundaries their strategy is to apply nonparametric statistical methods to market data. This approach has the flavour of the ‘arbitrage-free’ methodology so popular in the interest rate derivatives area. It has obvious attractions over conventional American option pricing models. The authors also compare and contrast their results with parametric analyses.

My prime criterion for the inclusion of a paper, after significant contribution, is its readability by a research student, rather than a specialist in the field. This has brought dual pressures on the authors. They have had to make both significant contributions to the literature, as well as to make them generally accessible. As a means of helping the authors, with one deliberate exception, I have reviewed each paper as carefully as possible, after there was a unanimous positive recommendation for publication by the referees.

The one exception is the well-cited paper of Hayne Leland and John Cox, written in 1982. Although presented in the privately circulated Proceedings of the Seminar on the Analysis of Security Prices, Center for Research in Security Prices,
University of Chicago, it has remained unpublished for a variety of reasons. For almost 20 years many financial economists have frequently used and referred to the paper, in areas as diverse as portfolio insurance and risk premium analysis. In contrast with the other papers in this Special issue, it complements them by considering “... the inverse problem: given any specific dynamic strategy, can we characterize the results of following it through time?” I am certain that its publication will ensure not only its availability to a wide audience, but will also continue to further research into fundamental issues, especially those within the optimum consumption and portfolio rules under uncertainty paradigm.

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