On the investment–uncertainty relationship in a real options model

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Abstract

It appears to be widely accepted in the real options literature that an increase in uncertainty should have an inhibiting effect on investment. Our article demonstrates that the notion of a negative uncertainty–investment relationship is not always correct. We show that in certain situations, an increase in uncertainty can actually increase the probability of investing, and thereby have a positive impact on investment. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The relationship between uncertainty and investment has been of interest to economists for a long time. The issue has been addressed in various ways in the literature, as discussed by Caballero (1991). This article is limited to one such approach: the “real options” approach to investment decisions, pioneered by McDonald and Siegel (1986), Dixit (1989), Pindyck (1988), Dixit and Pindyck (1994), etc.

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In this “real options” approach, the investment opportunity is viewed as an (American) option to invest, which must be exercised optimally. The exercise policy determines the investment in new projects. This policy is frequently (for an infinite-horizon setting) of the form: invest if the level of earnings (or NPV of project), say $x$, exceeds some critical value $x^*$. This critical value $x^*$, of course, depends on the parameters of the economy, particularly important being the level of uncertainty or the volatility of the project being considered. In terms of option theory, the investment rule can be equivalently stated as follows: invest when the value of the project exceeds its cost by an amount equal to the option value of waiting to invest.

In the financial options literature, it has been shown that a higher level of uncertainty increases option value, and this leads to a more distant critical value for option exercise (for American options). Consistent with this intuition, the real options literature also predicts a negative relationship between uncertainty and investment, since greater uncertainty increases the value of the option to wait. One finds repeated references to the negative investment–uncertainty relationship in the literature. Some examples are: “… this leads to the important implication that an increase in uncertainty raises the option value and thereby discourages new investment” (Mauer and Ott, 1995, p. 582); “… the recent literature on irreversible investment has shown that increase in uncertainty lowers investment” (Caballero, 1991, p. 279); and “… based on previous findings by researchers of an inverse relationship between uncertainty and investment” (Metcalfe and Hassett, 1995, p. 1472). Thus the general prediction of the real options literature is that a higher level of uncertainty will have a negative effect on investment.

In order to gauge the overall effect of uncertainty on investment, one can look at the probability that investment will take place (i.e., the critical trigger value will be reached) within a specified time period. An increase (decrease) in this probability implies a positive (negative) effect on investment. In this article, therefore, we study the uncertainty–investment relationship by examining the effect of higher volatility on this probability measure.

2. The model

We use a canonical real options model of investment, along the lines of McDonald and Siegel (1986) or Dixit and Pindyck (1994), (Chapter 5), with two differences: (i) the state variable is earnings rather than firm value, and (ii) systematic risk is explicitly taken into account. The firm is considering an infinite-horizon investment project which generates a random net cash flow (or earnings) stream of $x_t$ per unit time. The earnings stream follows the stochastic lognormal process below:

$$dx_t = \mu x_t dt + \sigma x_t dz_t, \quad (1)$$
where \( \mu \) is the expected growth rate of the cash flow stream, \( \sigma \) the standard deviation of the growth rate, and \( dz \) the increment of a standard Weiner process. The level of uncertainty of the project (or of the earnings process) is measured by the volatility term \( \sigma \).

The project can be accepted at any time; when it is accepted, the firm can implement the project instantaneously at a cost of $1 (this is just a normalization; there is no loss of generality in assuming a unit investment cost). The risk-free interest rate is a constant \( r \). The correlation of the project with the market portfolio is \( \rho \) (i.e., \( dz \ dz_m = \rho \ dt \)), and the market price of risk is \( \lambda \) (defined in Merton, 1973).

In the above setting, the postponable project can be viewed as an (American) option to invest, which should be exercised optimally, i.e., when exercising the option generates a higher payoff than holding it. The firm’s investment decision is therefore equivalent to an optimal stopping problem: at what point is it optimal to implement the project? Alternatively, what is the optimal exercise policy for the American option to invest? In an infinite-horizon setting, this translates into some critical value of earnings (say, \( x^* \)) such that the firm should implement the project as soon as \( x \) reaches or exceeds this critical trigger level.

### 3. The critical investment trigger

With the above specifications, it can be shown (proof available from the author on request) that the project value (in capital budgeting terms, the NPV of the project when accepted) is given by

\[
\text{Project value} = \frac{x}{r + \lambda \rho \sigma - \mu} - 1. \tag{2}
\]

The value of the option to invest (i.e., value of project prior to acceptance), \( F(x) \), follows an ordinary differential equation of the form specified in McDonald and Siegel (1986) or Dixit and Pindyck (1994), (Chapter 5). Along with the appropriate boundary conditions (value matching and smooth pasting), the solution is given by (proof available from the author on request):

\[
F(x) = A x^\alpha, \tag{3}
\]

where

\[
\alpha = \frac{1}{2} - \frac{\mu - \lambda \rho \sigma}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu - \lambda \rho \sigma}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}
\]
and
\[ A = \frac{(x^*)^{1-\alpha}}{(r + \lambda \rho \sigma - \mu) \alpha}. \quad (5) \]

In Eq. (5), \( x^* \) is the optimal stopping boundary; that is, the optimal investment rule is to invest when \( x \) rises to \( x^* \). The boundary \( x^* \) is given by:
\[
x^* = \frac{\alpha}{\alpha - 1}(r + \lambda \rho \sigma - \mu). \quad (6)
\]

For the investment decision, the important result is Eq. (6), which gives us a closed-form expression for the optimal or critical investment trigger \( x^* \).

Intuitively, it is obvious that a higher level of uncertainty will increase the critical trigger level \( x^* \) (as can be verified by differentiating Eq. (6) with respect to \( \sigma \)), and thereby have a negative effect on investment. However, there is an additional effect of the higher volatility: because of higher volatility, the variable \( x \) is now more likely to reach the critical level \( x^* \). This is similar to the “realization effect” of Metcalf and Hassett (1995), and will have a positive effect on investment. Thus, there are two effects of higher volatility on investment, one negative and the other positive. To get an idea of the overall effect, we examine the probability of investment next.

4. The probability of investing

The probability of reaching the critical level \( x^* \) (i.e., probability of investing) within some time period \( T \) is given by (this can be derived from Harrison (1985), (pp. 11–14))
\[
\text{Prob}(\text{Inv}) = \Phi \left( \frac{\ln(x_0/x^*) + (\mu - 1/2\sigma^2)T}{\sigma \sqrt{T}} \right)
+ \left( \frac{x^*}{x_0} \right)^{2\mu/\sigma^2 - 1} \Phi \left( \frac{\ln(x_0/x^*) - (\mu - 1/2\sigma^2)T}{\sigma \sqrt{T}} \right), \quad (7)
\]
where \( x_0 \) is the starting (or time 0) value of \( x \), and \( \Phi(\cdot) \) the area under the standard normal distribution. Substituting for \( x^* \) from Eq. (6), we get
\[
\text{Prob}(\text{Inv}) = \Phi \left( \frac{\ln[x_0(1 - 1/\alpha)/(r + \lambda \rho \sigma - \mu)] + (\mu - 1/2\sigma^2)T}{\sigma \sqrt{T}} \right)
+ \left( \frac{r + \lambda \rho \sigma - \mu}{x_0(1 - 1/\alpha)} \right)^{2\mu/\sigma^2 - 1} \Phi \left( \frac{\ln[x_0(1 - 1/\alpha)/(r + \lambda \rho \sigma - \mu)] - (\mu - 1/2\sigma^2)T}{\sigma \sqrt{T}} \right). \quad (8)
\]
Eq. (8) gives the probability of investment occurring within time $T$, in terms of the parameters of the economy and the project. A higher (lower) probability implies a greater (smaller) chance of project acceptance, hence a positive (negative) effect on investment. Since the sign of the derivative $[d(\text{Prob})/d\sigma]$ cannot be determined unambiguously, it is not clear how a higher $\sigma$ will affect the probability of investing. We therefore have to use numerical results to illustrate the uncertainty–investment relationship.

5. Numerical analysis

We start with the following base case parameter values: $\mu = 0, r = 10\%, \rho = 0.7, \lambda = 0.4, x_0 = 0.1$, and $T = 5$ yr. We use $\mu = 0$ because we wish to focus on volatility effects and not growth effects; $\rho = 0.7$ reflects a project imperfectly (but positively) correlated with the market, which is a good description of the majority of projects; and $\lambda = 0.4$ is the approximate historical average (see Bodie et al., 1996, p. 185). Note that using a different value of $x_0$ will result in different $\text{Prob}(\text{Inv})$, but will make no difference to the relationship between $\sigma$ and $\text{Prob}(\text{Inv})$, which is what we are interested in. With the above parameter values, we computed $\text{Prob}(\text{Inv})$ for different values of $\sigma$. The results are displayed in Fig. 1.

From Fig. 1, we see that the probability of investing is initially an increasing function of volatility, but after a certain point (about $\sigma = 0.39$ for the base case) it becomes a decreasing function of volatility. Therefore, for low levels of

![Fig. 1. Probability of investing as a function of volatility. Parameters: $\mu = 0$, $r = 10\%$, $\rho = 0.7$, $\lambda = 0.4$, $x_0 = 0.1$, and $T = 5$ yr.](image-url)
uncertainty, an increase in uncertainty increases the probability of investing and thereby has a positive effect on the expected rate of investment. With the base case parameters, the direction of the overall effect of volatility on investment is thus ambiguous. This result is also robust to the exact choice of parameter values, as was confirmed by repeating the computations for wide range of parameter values around the base case. This illustrates our main result: An increase in uncertainty might actually speed up investment, contrary to what the literature generally predicts.

The effects of the various parameters can be summarized as follows: the uncertainty–investment relationship is more likely to be positive when (i) the current level of uncertainty \( \sigma \) is low, (ii) \( \lambda \) is high, (iii) \( \rho \) is high, (iv) \( r \) is high, (v) \( \mu \) is low, and (vi) \( T \) is short. We also find that the trigger \( x^* \) is always an increasing function of \( \sigma \), as predicted.

To summarize, this article has demonstrated that a higher level of uncertainty might have a positive effect on investment (in the sense of increasing the probability of investing), particularly for low-growth and low-risk projects. The economic implication is that more uncertainty is not always bad for investment, because under certain conditions, it can actually increase the probability of investing. For example, in the case of a low-risk low-growth firm (such as that found in a regulated industry like the utilities sector), a moderate increase in uncertainty might increase the probability of investing, and thus have a positive effect on investment.

We would like to add a caveat here: what the article has shown is that the probability of investing might be an increasing function of the uncertainty or volatility \( \sigma \) in certain scenarios. However, from this result, it is difficult to reach a firm conclusion regarding the effect on aggregate investment. Recall that our result was based on a single-project partial-equilibrium model. In order to make any inference regarding the effect on aggregate investment, we would have to extend the model to a multi-firm or industry general equilibrium setting. Not only would this complicate the model enormously (see Cecchetti (1993) for a discussion of these complications), we would also need to incorporate the impact of other factors such as the arrival rate for the projects or characteristics of the firms in the industry. Moreover, these factors would themselves have to be endogenous in a complete general equilibrium model. Since it is beyond the scope of this paper, we leave the extension of the model to a general equilibrium setting for future research on this topic.

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