Whither nonlinear?

William A. Brock

Department of Economics, The University of Wisconsin, Madison, Wisconsin, MA, USA

I have been asked by the Editors to give some thoughts about the future of ‘complex systems approaches’ and nonlinearist methodology in Economic Science. Since this will be an essay that will attempt to stimulate the readers into making up their own thoughts, projections, and guesstimates, this essay shall be written in ‘English’. This tack is taken here because the writing down of a particular model, especially in mathematics, tends to ‘freeze’ the readers’ thoughts onto that particular model.

Before beginning, it is good to keep mental focus separated into two categories: (i) microphenomena and (ii) macrophenomena. The macro category is separated from the micro category by the degree of survival of the phenomena to aggregation. To put it another way, the phenomenon is classified as ‘macro’ if it survives a type of law of large numbers, i.e. there must be strong enough dependence across individual micro units in some kind of statistical sense so that the ‘averaging’ effect of aggregation does not ‘wash out’ the phenomenon of interest. In addition, in order to have macrodynamics, these limiting aggregates must have dynamical dependence.

It is useful to have some precise goals of the research in mind. In the spirit of complexity theory which emphasizes robust explanations for ‘scaling laws’ and ‘patterns’ as well as conventional economics which emphasizes policy relevant prediction and prediction out of sample, we shall discipline our peering into the future by concentrating on innovations that have promise for doing a better job of matching (‘explaining’) observed patterns and scaling laws as well as prediction.

This essay proceeds as follows. First, a rough attempt is made to illustrate what is meant by ‘complex systems’ for the purpose of this essay.

Second, although much complex systems based research is boundedly rational, a debate continues in economics on the costs and benefits of rational expectations modeling versus boundedly rational/evolutionary modeling (Kreps, 1997). The argument here will be to use computational advances and computer assisted methods to allow both approaches to compete in either a ‘nested’ testing setting or, perhaps, in a ‘Bayesian’ manner to give a decomposition of the variability and patterns seen in the data.

Third, the ‘Resilience Network’ view, which is a general ‘systems theoretic’ framework is sketched. This view is rather like that of control systems engineering but with a temporal hierarchy of time scales and lots of ‘nonlinearity’, ‘heterogeneity’, and ‘adaptability’, added.
It will be argued here that recent advances in computational techniques, computer assisted deductive techniques, and computer assisted inferential techniques as well as lower costs of access to more extensive data, will propel the further development of nonlinearist and complexity-based types of research, simply because the cost is dropping.

References to some literature that is moving in this direction will be given throughout this article. In order to economize on references we shall reference surveys and urge the reader to take a look at specific works mentioned in those surveys.

1. Complexity and complex systems

This essay reviews and projects for the future, ‘complex systems’-based research, in the sense defined by the Resilience Network (to be discussed below) and the two Santa Fe Institute volumes edited by Anderson et al. (1988), and Arthur et al. (1997), hereafter SFI(I) and SFI(II). First, the SFI(I), SFI(II) approach is discussed.

Since Sargent (1993) and Marimon in Kreps and Wallis (1997), cover somewhat related material, we shall be brief and attempt to focus on features not covered by these surveys that are helpful in peering into the future. We shall also skip over topics and tools covered in Dechert (1996) even though many of those tools may be useful for uncovering structure that might get swept into the residuals when standard estimation methods and structures are inappropriately imposed on data generated by some other mechanism such as complex dynamics.

Simply put, the complexity approach views the economy as made up of a large number of heterogeneous, interconnected individuals experimenting with different belief systems with a lot of bounded rationality. Much of the exploration is done with computer simulations rather than analytics.

In biology (e.g. scaling in Kauffman (1993)) and the natural sciences (e.g. ‘1/f noise’ in Bak (1996), which is closely related to ‘Granger’s typical spectral shape’ in Sargent (1987)) the emphasis is on how complicated interconnected systems with lots of interacting entities can generate ‘simple’ patterns, e.g., ‘scaling laws’, that possess a degree of ‘universality’, i.e. a certain measure of independence of the generated patterns from particular details of the system. Here ‘1/f’ noise will be loosely used to denote any process that generates a frequency spectrum that scales roughly like 1/f for small frequencies f — hence the name ‘1/f noise’. Much of the theorizing is about forces that lead a system to be in some kind of ‘poised’ state, which ‘relaxes’ with a power-law distribution of ‘relaxation events’.

For example, Bak’s favorite metaphor is a sandpile sitting on a table which is fed by an external source of sand dropped upon it from above. The pile evolves from a flat state on the table to self-organized criticality where each grain tends to be near to a critical angle relative to nearby grains as sand continues to rain down upon the pile while the ‘excess’ sand rolls off the edge of the table. Sandslides appear at criticality. Size measures of sandslides satisfy a power-law-type scaling relationship that looks like a ‘1/f’ phenomenon.

Sandpile-type mathematical models are like threshold cellular automata with external forcing. See Bak’s work with Scheinkman and Woodford, reported in his book and references (1996) as well as Scheinkman and Woodford (1994) for applications to explain thick tail phenomena in economics, for example, inventory dynamics along production chains. A contrasting approach to inventory dynamics along production chains using dynamical systems theory is in Sosnovtseva and Mosekilde (1997) and their references, especially to work of Sterman, Mosekilde, and others on the Sloan School’s ‘beer game’.

Notice the contrast in emphasis between complexity theorizing, for example, with sandpile-type metaphors which stresses how very complicated interconnected systems can generate robust simple scaling relationships for observables and chaos theory which stresses how simple rules can generate very complicated dynamics. Besides the sandpile metaphor, there are many models of scaling phenomena, with associated notions of ‘poisedness’ and ‘criticality’ in the sciences (Anderson, 1996; Kauffman, 1993; Durlauf in SFI(II)).

Bak (1996) argues, however, that the sandpile approach does not require any ‘outside tuning’ to criticality unlike theories of scaling based on, for example, Ising-type interacting particle systems models, which require setting of some parameter, for example, a ‘temperature’, to push the system to criticality. Consider, for example, the metaphor of boiling of water where temperature must be set at the boiling point and phase transition scaling in this metaphor is revealed by a power-law distribution of bubble sizes.

However, the necessity for outside tuning is not quite clear cut. For discrete choice settings in economics, Brock’s paper in SFI(II) contains a brief discussion of a theory to endogenize the intensity of choice where choice intensity is determined by a supply side interacting with a demand side much as in a market for any other resource which is demanded and supplied.

On the supply side, higher choice intensity, i.e. more precise choice amongst alternatives requires more resources. On the demand side more heterogeneity and wider variation of values of the choices increases demand for more precise choosing amongst the alternatives. Forces similar to those equilibrating supply and demand in markets determine the equilibrium quantity and implicit price of choice intensity. This kind of theorizing guides the analyst towards uncovering economic forces that determine the intensity of choice in bounded rationality analysis.

In any event, prediction analysis and impulse-response analysis, for example, local-perturbation analysis of a piece of the system becomes very different when
it is at criticality in contrast to when it is far from criticality. Bak argues that for systems with gigantic state spaces which have evolved to criticality as in the sandpile metaphor that prediction of the dynamics of a shock to a localized chunk of the pile will depend on what is happening to parts of the pile far away from that particular chunk. This is so because, at criticality, there are events, called ‘avalanches’ (sand slides) distributed in size according to a power law.

Thus, the near decomposability (a block dominant diagonal-type property) or the small number of operative modes (as in dynamical systems theoretic analysis) is not available to simplify impulse-response analysis and perturbation analysis. Hence it would seem worthwhile to generalize the work of Gallant, Rossi, and Tauchen; and Potter surveyed in Brock SFI(II) for systems poised at criticality. One could also examine the temporal ‘spatial’ shape of k-step ahead predictions in response to a pulse today at one part of the system for the Rnet view (to be discussed below) and the ‘poised state’ view.

Some code words and code phrases that might indicate a complex systems approach include: (i) complex adaptive systems, (ii) genetic programming; (iii) neural nets, (iv) artificial life, (v) genetic algorithms, (vi) self-organized criticality, (vii) scaling laws, (viii) edge of chaos, (ix) poised states, (x) phase transition, (xi) universality, and (xii) renormalization group.

In economics applications, the complexity approach is more simulational, makes heavy use of computer technology, and is less analytic in contrast with ‘conventional’ economics which imposes more rationality and is more analytic. This approach is illustrated in SFI(I), and SFI(II). Besides the two SFI books the reader should take a look at Holland (1995) for the basic idea of Complex Adaptive System which plays an important role in one complexity-based approach to economics (e.g. Arthur et al. in SFI(II)).

While it appears that complexity-based research will thrive in the future, it is difficult to isolate just what it is about ‘complexity’ that separates it from other closely related subjects of statistical mechanics, dynamical systems theory, and pattern formation. We gave an attempt to separate one type of complexity theorizing (self-organized criticality) from the above dynamical systems theory.

In order to get some feel for the similarities and differences, the reader is urged to take a look at the Santa Fe Institute’s WWW site, www.santafe.edu and look at the work of Arthur, Crutchfield, Kauffman, Mitchell et al., as well as looking at SFI(I) and (II).

Economist readers of this Journal who have a background in dynamical systems theory should examine James Crutchfield’s WWW site http://www.santafe.edu/~jpc and take a look at his papers and references. Here, one will find formalizations of different notions of ‘complexity’. One will also find attempts to pinpoint what is really ‘new’ about recent complexity theorizing from subjects such as dynamical systems theory, statistical mechanics (phase transitions and renormalization group), and pattern recognition theory. See, especially, the paper by Nimwegen et al. (1997), ‘Statistical Dynamics of the
Royal Road Genetic Algorithm. This paper identifies a mechanism for metastability (alternation of the dynamics between periods of stasis and brief periods of rapid change) that is different than the usual one of a population cloud moving across local maxima on a rugged fitness landscape.

2. Rational expectations versus bounded rationality/evolutionary modelling

While much of complexity-based work is boundedly rational and evolutionary, it is important to realize, however, that there is nothing in the idea of a large number of interacting agents in intensive feedback connectivity possibly generating robust patterns that implies restrictions to a bounded rationality framework.

One could just as well entertain the same research style in a rationality-based intertemporal dynamic heterogeneous agent general equilibrium framework (e.g. Marimon (1989) and references) with an emphasis on interconnective structure, and, perhaps, even allowing the model agents to self-sort themselves into different ‘locations’ in order to endogenize the connection structure. One could seek conditions on such structures so that observables reproduce the patterns in data that we seek to understand. The emphasis on bounded rationality may be due to computational and analytical limitations which are being removed as evidenced by recent articles appearing in journals like Macroeconomic Dynamics. See especially the inaugural first two issues in 1997. See also the General Equilibrium 40th Anniversary Conference (1994) to see the span of general equilibrium theory and its extensions.

While any older economist who has followed the work of Herbert Simon, Richard Nelson, Roy Radner, Sidney Winter, Richard Day, and others knows that bounded rationality is an old and respected tradition, it is fair to say that rationality-based traditions have been dominant (and still are dominant) in economics.

However, we are now at the point where computer technology (fast and cheap computers) and statistical and econometric technology (the bootstrap, method of simulated moments, nonparametrics) has made serious bounded rationality/evolutionary economic research with a strong empirical, computer-assisted analytic, and experimental component possible.

One might even say that an evolutionary/bounded rationality bandwagon has started. Before leaping onto the bandwagon, it is wise to step back and to ask some harsh questions. This is especially so because the same advances in computational technology should allow more realistic heterogeneity to be introduced into rationality-based intertemporal equilibrium theorizing.

It is argued here that the future lies with a general theory that nests both approaches where elements of both approaches compete on the basis of economic-based performance measures that are determined within the general theory. As computational costs drop one can imagine using data to attach
something like ‘Bayes factors’ for each approach for each dataset application (Coop and Potter, 1998; Hilborn and Mangel, 1997). Common sense suggests that both are needed to explain reality but computational costs have limited our ability to attach relative weights in a data relevant manner.

The general theory must do enough better at tasks of interest such as explaining patterns in dataset and out-of-sample prediction in order to pay the extra costs of the extra ‘free parameters’ carried by the general theory (Sargent, 1993).

A natural economic way to do this nesting is to assume that agents in the general theory must pay costs in order to obtain fully rational expectations. They must pay costs in order to understand the world they live in well enough to even form ‘fundamental’-type expectations, much less fully rational expectations which require an even higher level of cognitive understanding.

Fitness measures for each type of expectation are introduced which are based on the net profits from acting on each. This generates an evolutionary dynamical system. Expectational strategies that do better at earning net profits for their users may drive out those who do worse.

One can locate sufficient conditions for fully rational expectations to drive out all other expectational schemes. If agents have finite memory, are risk averse, and are not infinitely sensitive to small changes in net profits across expectational strategies, then one can show that while fully rational expectations may loom large in the population, there are phases when the system is near a steady state where rational expectations do not cover their cost of acquisition.

Forces, such as outside shocks, may knock the system away from this steady state or instabilities may slowly build up so the system starts moving enough dynamically to make it worthwhile to pay the cost of acquisition of rational expectations. When enough agents switch to ‘forward’ looking expectations, i.e. rational expectations, then the system is ‘stabilized’ and moves back to a ‘steady-state’ phase where the net gain to the acquisition of rational expectations drops. The process repeats.

A key difference in this type of approach from existing literature on bounded rationality is that it builds on the extensive work in REE theory by expressing beliefs in terms of deviations from a structural REE model.

For example, in finance, these deviations can be viewed as each REE trader’s beliefs about how the deviations from REE by the rest of the trading community might show up in equilibrium prices. In this sense a general theory is fully rational in the sense that truly rational traders must take into account the behavior of other traders in the trading community in order to calculate whether it is worthwhile to pay the costs of attaining fully rational expectations.

This behavior leads to a decomposition of excess returns on the risky asset. This decomposition for excess returns consists of an ‘REE’ part which is a Martingale Difference Sequence (MDS) plus an ‘endogenous dynamical’ part which is contributed by the general theory. The MDS part of the decomposition corresponds to received Efficient Markets Theory in more specialized contexts.
Ongoing work in finance (e.g. SFI(II)) is uncovering classes of adaptive learning dynamical models which are consistent with the main stylized facts of returns, volatility of returns, and volume of returns. This consistency not only imposes empirical discipline on the model building process, but also moves the models closer to actual econometric implementation.

As computational technology advances, it should be easier to impose more empirical discipline and move towards serious econometric implementation (rather than mere calibration) of general theories which nest both rational expectations and evolutionary/bounded rationality. See, for example, Baak (1998) who nests conventional rational expectations models of cattle cycles in a general model with some ranchers holding a type of boundedly rational expectations and finds evidence consistent with the presence of boundedly rational ranchers.

The nesting strategy enables the bounded rationality researcher to borrow from the large econometric literature developed in the rational expectations literature to estimate specific models and test for the ‘significance’ of the ‘extra’ parameters added by bounded rationality.

One can conduct econometric investigations by setting up rational expectations as the null and testing the ‘significance’ of the extra parameters of bounded rationality or, one can set the bounded rationality theory up as the null and test for the ‘significance’ of parameters associated with rational expectations. Indeed, one could even take a ‘Bayesian’ view and let the two theories compete on posterior odds.

At the risk of repeating, most of the existing literature on bounded rationality is rather ad hoc and evidence is adduced mostly in an anecdotal manner rather than rigorous econometric testing. In contrast, the work suggested here naturally suggests econometric methodology and rigorous testing by nesting the rational expectations econometrics within the general theory. It is now easy to set up a nested testing situation where the null hypothesis is that 100% of the agents are conventional rational expectations and the alternative is that a ‘significant’ fraction of agents are using a form of expectational strategy whose structure is not conventional rational expectations but whose structure the data can speak to. See Brock and Hommes (1997) as well as my review of related work in SFI(II) (see especially my references to work of Baak and Chavas) for an initial foray into this type of work.

Advances in computer-based inferential techniques such as the bootstrap and computational Bayes methods should stimulate this kind of research which has been constrained by high costs of computation. Look at Horowitz’s chapter on bootstrap and Geweke’s chapter on computational Bayes methods in Kreps and Wallis (1997). In view of this it seems safe that we can predict more work in the nonlinearist domain like that of Coop and Potter (1998) as well as that in SFI(I) and SFI(II). We now turn to a related, but different emerging strand of research.
3. The resilience network

The Resilience Network is a group of ecologists and economists associated with the Beijer Institute of Sweden. They are unified by exploration of concepts of ‘resilience’ where ‘resilience’ refers to the ability of a system to restore itself when buffeted by shocks. See Holling (1997) for a most recent entryway into this literature. Holling et al. (1997) compare ecological and social systems from this point of view.

One might expect linearist methods to work quite well when restoration ability is large (e.g. that part of macroeconomics which is based upon generalizations of the classical stochastic growth model). One might be cautious with linearist methods when resilience is small (e.g. when one suspects an impending ‘flip’ due to an impending bifurcation).

The Rnet approach to complexity looks more like dynamical systems theory and nonlinear control systems theory with a lot of emphasis on locating patterns of spatial and temporal ‘lumpiness’, where ‘space’ is interpreted in a wide sense. See, for example, Holling’s (1992) paper reprinted in Samson and Knopf (1996), hereafter denoted as H(1996).

Many other papers in Samson and Knopf (1996) will give the reader an idea of ecological modeling which focuses on hierarchies of positive feedback loops (e.g. DeAngelis et al., 1980) and the structure of the dynamical loop diagram for each application (as in, e.g., Puccia and Levins, 1985).

While highly interconnected interactive pathways play a major role in the Rnet research style, there is a lot of stress upon simplifying complexity by looking for evidence of ‘lumpiness’ in spatial and temporal data and studying structuring processes that produce such patterns (e.g. Holling (1987), Holling’s paper in Samson and Knopf (1996)). Look especially at the log/log space/time scaling plot of H(1996, p. 356), called a ‘Stommel’ plot, and at the adaptive cycle diagram of H(1996, p. 393).

It is beyond the scope of this paper to explain adaptive cycle diagrams as used by Holling. All we can say here is that they are used as schematics to label the dynamical phases that ecosystems tend to follow that Holling believes are quite common and which are documented via case studies in Gunderson et al. (1995).

Stommel-type plots are constructed by plotting measures of temporal (spatial) activity (for example, the spectrum in time and in space) on the horizontal axis (the vertical axis). H(1996) shows that for ecosystem time/space data one tends to get an upward trending of ‘lumps’ on the plot which suggests that most activity takes place in time/space clumps with lower frequency temporal activity being associated with larger domain spatial activity. See Clark (1985) for a very nice discussion of Stommel-like plots, their uses, and techniques for their construction.

As an aside Stommel-type diagrams may be a useful way to adduce evidence for or against the ‘sandpile’ view versus the Rnet view. For a sandpile at a given
time/space scale, data on time length and size of avalanches generated by this pile at self-organized criticality may generate a less lumpy pattern on a Stommel-type plot than Rnet-type models. It is beyond the scope of this short paper to say anything more about Stommel diagrams and how they might be used to adduce evidence for or against a particular class of generating mechanisms such as a hierarchy of sandpile-type dynamics in contrast to a hierarchy of differential equation-type dynamics each having a small number of basins of attraction at each time/space scale.

This attempt to simplify complexity by application of spectral analysis in time and ‘space’ in order to identify ‘clumps of high spectral power’ is analogous to application of spectral analysis in macroeconomics to locate regions of high spectral power. See, for example, Sargent (1987, pp. 279–283). If one added a notion of ‘space’, did a spatial spectral analysis to locate regions of high spatial spectral power, and then attempted to find a scaling relationship between the temporal and spatial regions of high spectral power, then one would be close to the Rnet research style of simplification of complexity.

To put it another way, the structuring of temporal variables looks much like a directed tree in graph theory where spectral analysis is used to identify ‘gaps’ in the tree for purposes of simplification and where variables lower down on the tree are ‘slaved’ to those above. I.e. the time scale of variables above a fixed level is slow enough that a good approximation is to treat them as constants relative to the time scale at that fixed level. This construction introduces a hierarchy of potential bifurcation parameters at higher time scales on the directed tree which structure the processes at the lower time scales on that directed tree.

Yet, another way of viewing the Rnet approach to complex systems research is by way of Herbert Simon as discussed by Gunderson et al. (1995). Simon stresses that many complex systems have an hierarchical structure which can be translated into a near-decomposable structure where the interaction matrix of the system can be rearranged into almost block diagonal form with tightly interacting blocks on the diagonal with loose interactions on the off-diagonals. The ‘big element’ matrices on the diagonal correspond to high-frequency dynamical clusters, whereas the ‘small element’ terms in the ‘off-diagonal matrix blocks’ correspond to low-frequency dynamics across that block group.

We can now contrast the Rnet view with the ‘sandpile’ type of ‘complexity’ view. The Rnet view stresses a ‘Marshallian’ type of partial equilibrium analysis, where something like an implicit block version of the dominant diagonal property of dynamic intertemporal general equilibrium analysis holds. This contrasts with a dynamical setting ‘poised’ at a ‘critical’ state such as a sandpile or an interacting particle system with ‘tuning’ parameter set at the edge of a phase transition.

A related way of thinking about the clumpiness (hierarchy) structure of complex systems is discussed by Roughgarden (1996). Roughgarden combines sequential colonization with higher degree of competition between more similar
morphs to generate body size clumping in computational simulative ecological experiments.


A third way of developing and using complexity-based approaches is discussed in Brown’s (1995) book, Macroecology. Brown shows how complexity-based approaches using patterns such as species/area curves (a scaling law that relates number of species to area in biogeography) can be used to produce predictions of extinctions caused by climate change. Looking at traditional economic issues as the division of labor, the extent of the market, relationships between the size of the variety spectrum of tradable and non-tradeable goods, the minimum efficient scale of production of a particular good, production hierarchies, and the ‘new’ economic geography (cf. Krugman’s article in SFI(II)) though the lens of Brown’s book may shed new light on these old problems.

Nonlinear methodologies seem to be key to all of these views of the world. Recall that in the hierarchical view activity is organized by dynamical systems operating at a hierarchical variety of temporal and spatial scales (even though the number of these scales is relatively small) with the temporal variables at one level acting as bifurcation parameters for the next faster level below it. Thresholds, alternative stable states, bifurcations, and other nonlinear phenomena are endemic.

In hierarchical types of dynamical systems, linearist methodology at one spatial/temporal scale may work well provided one is not near the edge of a bifurcation. However, linearist methodology is dangerous if one is near the edge of a bifurcation, especially where the state is near a ‘hard’ loss of stability where it moves a relatively great distance to a new stable state. Modification of conventional benefit/cost analysis in public economics to take into account bifurcations and hysteresis effects caused by alternative stable states is just beginning.

A start on this task is given by the Rnet team of Carpenter et al. (1998) where the impact of parameter uncertainty on parameters associated with possible alternative stable states, and response lag length upon cost/benefit analysis of policies to control nonpoint pollution is studied. Ludwig (1995) and Pizer (1996) show that irreversibility itself, much less possible alternative stable states, looms large as a precautionary force, especially when there’s parameter uncertainty about the ‘true’ model. Decreasing costs of computational-based research should lead to more of this kind of policy-based work in the future.

Here is an attempt to peer into the future to guess what this kind of research development may look like in expectational analysis and how it may change policy analysis where expectational effects may loom large. Work of Chiarella and co authors, especially Flaschel and Khomin, makes a nice launch point for the discussion. Chiarella and Khomin (1997) review work that criticizes the
standard forward looking ‘jump variable’ technique of rational expectations analysis. They argue that one should replace the term ‘rational expectations’ by ‘full knowledge of the model expectations’. See Judd (1997) for many examples of jump variable techniques in rational expectations analysis as well as computational methods in general.

Now, suppose we follow de Fontnouvelle (1996) and introduce a hierarchy of costs for different levels of knowledge and different levels of approximation to the ‘true model’ with full knowledge of the true model costing a huge amount. We can introduce heterogeneity in this hierarchy of costs across different agents in the economy. After all, some agents, by virtue of their profession or location in the economic web of interconnected relationships and information flows may have much lower costs for some levels of the information hierarchy than others.

Now, introduce payoff measures for each level of knowledge which consist of action gross revenue minus costs of knowledge acquisition and let these modes compete in an evolutionary dynamic similar to that in SFI(II).

We now have an outline of a view of economic dynamics where some agents may have costs low enough to purchase ‘full knowledge of a good approximation to the true model’ expectations, especially if temporal variability is large enough in their sector so that the extra gain in prediction is large enough to cover the extra costs of a more precise level of knowledge about the true dynamical system which is most probably stochastic.

Note that we are deliberately vague about how agents gain access to different levels of knowledge about the true dynamics by paying different levels of costs or by being located in advantageous parts of the economic network. This knowledge could be purchased by some agents in the manner of Walters (1986) by paying short run costs to conduct ‘probing experiments’ to learn more about the part of the dynamical system of interest to those agents. It could be purchased by a set of agents hiring an ‘expert’ to unearth information of relevance to those agents. Whatever the case, a key issue and key quantity is information about the system.

In ecological management, one has in mind a large highly interconnected stochastic dynamical system operating on a mosaic-like structure. Suppose one represents this stochastic dynamical system with an interconnected collection of Puccia/Levins dynamical loop diagrams operating at a ‘clumpy’ hierarchy of temporal and ‘spatial’ scales where ‘space’ is metaphorically interpreted in a wide sense which may even include a ‘spatial’ structure in varieties of the same ‘good’. In ecology many articles in Samson and Knopf (1996) fit this dynamical ‘mosaic-like’ view where ‘variety’ is replaced by ‘species’.

Return now to the general view of a dynamical heterogeneous agent-based economy with different levels of information about the ‘true’ dynamical system evolving evolutionarily through fitness measures based upon actual profits net of acquisition cost along a sequence of ‘temporary equilibria’ ground out by the forces of equilibration of supply and demand at each point in time and at each point in ‘space’.
In order to think in a more focused manner assume a form of ‘transferable utility’ welfare measure which is the weighted sum at a point in time of net benefits generated by each agent where the cost of acquisition of the information level for each agent is subtracted off of the welfare measure for that agent. This total welfare measure varies along the sequence of temporary equilibria.

This larger view nests conventional rational expectations general equilibrium theory in the sense that if the costs of acquiring ‘full knowledge of the true model expectations’ was zero for each agent, then we would have full rational expectations equilibrium and we could carry on with analysis and policy in the established framework. Policy analysis would focus on the management of externalities and the design of incentivization schemes subject to inescapable informational constraints so that all cost causers bear their own costs as much as possible within a mechanism design framework.

In the larger view policy attention gets turned toward use of government not only to enforce contracts, define property rights, and manage externalities, but also to lubricate information flows. Policy would be focused on information channels where government could assist in reducing duplication costs of information acquisition. This is standard and could come out of a noisy rational expectations framework where the level of information possessed by an agent corresponds to the precision of that agent’s signal and that precision, in turn is a function of that agents expenditure on information acquisition.

What are the ‘new’ ingredients in this ‘general view’, if any? As in de Fontnouvelle (1996), an extra ‘layer’ of dynamics in the proportion of agents using each type of information is ‘added onto’ the ‘underlying’ dynamics given a fixed information structure. Thus, we get a two-level hierarchy of dynamics where the first level is the underlying economic dynamics given the information structure, and the second level is the dynamics of the information structure itself.

It is plausible in some applications such as finance, that the dynamics of the information structure (the information of different types of traders) is slow relative to the dynamics of the economy itself (the dynamics of the ticker tape). Once the idea of evolutionary dynamics of the information structure is added to conventional economic dynamics, then it is natural to think about modeling, in an evolutionary view, perhaps, dynamics as ‘higher’ levels such as institutional formation (e.g. North in SFI(II)). This level of dynamics would be slower yet.

In the evolutionary computation literature which uses versions of the genetic algorithm (cf. Holland, 1995) there are three main operations: (i) replication, (ii) mutation, and (iii) crossover (‘sex’). If one imagines replication taking place on a faster time scale than mutation and crossover, one has a 2-level hierarchy of time scales. In order to get this kind of adaptive system to mimic ingredients of ‘jump variable’ phenomena, which seems necessary for events such as ‘common knowledge’ movements in highly organized asset markets such as stock and futures markets, we can go in two main directions.
One direction and perhaps, the most speculative, is to introduce ‘introspection’ with cognitive costs increasing with higher levels of iteration towards full common knowledge. See Volume I of Kreps and Wallis (1997) for discussions of modeling common knowledge. Another direction is to introduce a fast time scale relative to the time scale of observable data so that ‘selection pressure’ can operate fast enough within that time scale to mimic rapid movements in economic variable such as stock market prices to commonly understood news about impacts on fundamentals.

Let us now make some wild guesses, not only about future research directions, but also the impact of the research lines discussed here upon policy analysis.

The Rnet works cited above and the detailed case studies in Gunderson et al. (1995) have a common theme that agents must form, at best approximate models and then act on them. As agents act on approximate, but wrong models, there will be a small instability in some cases where there is an unstable feedback caused by a large number of agents acting on approximate models. This instability continues to grow until a ‘surprise’ (a bifurcation, perhaps) occurs.

One could imagine developing an approach where agents are connected by an additional feedback from the market as in Brock and Hommes (1997) and Brock’s survey in SFI(II) or an additional feedback from building adaptive models of neighbors’ behavior as in Darley and Kauffman and Durlauf in SFI(II). These extra dynamical pathways cause the system’s collection of models to be modified by the data generated by the evolution of the system itself, which, in turn, feeds into the dynamics of the system. It is possible to imagine research into pathways which may lead to poised states as in Darley and Kauffman. As in Nimwegen et al. (1997) one might uncover interesting mechanisms for metastability.

4. Possible policy implications

Impacts upon policy that seem likely from a computationally based complexity approach include (i) an increased focus on the dynamics of the whole distribution of welfare rather than a specific moment such as mean welfare as well, (ii) increased attention to searching for potential pathways for occurrence of abrupt changes as in mechanisms for metastability (cf. Durlauf (1997)). I.e. there will be more concern with use of computer assisted analytics and computer assisted inferential techniques to locate boundaries of parameter space consistent with existing data which indicates stasis but near ‘hidden’ emergent nonlinearities where conventional approximation techniques are dangerous.

Consider, for example, a macro policy debate like Symposium (1997) between those who fear growing the economy more will ignite an inflationary spiral and those who feel the opposite. The stakes are high because a bit more growth might remove a lot of harmful unemployment and increase production.
Stiglitz, in Symposium (1997), reported that the CEA conducted econometric studies that suggested that one could grow the economy a little bit more without fear of igniting an inflationary spiral. He states, ‘… the view, more common in nonacademic circles, that the NAIRU is like a precipice: take one step over it, and you fall into a spiral of rapidly accelerating inflation … The evidence simply does not support this view … the world is not only continuous but approximately linear …. Thus small mistakes have only small consequences’.

Walters (1986), as well as Easley and Kiefer (1988), and the research reviewed above might suggest a ‘probing’ strategy to check this out with especial attention to probing for possible indicators that might indicate a ‘poised state’ of the economy. Indicators might be sought that would indicate nearness to a bifurcation point that is not revealed by the current data analysis.

The work on valuation of irreversibility under potential alternative stable states might cause a shift in the burden of proof upon those acts which have the potential to generate irreversibilities. Indicators of pathways towards social interactions in expectations formation which might cause alternative stable states could be sought after. Perhaps high-frequency survey work on expectations would be in order while growing the economy a bit more. If such high-frequency survey work indicated a start towards a shift in inflationary expectations then policy would quickly back-off and there would be publically credible mechanisms set in place to convince the markets that policy would ‘turn on a dime’.

Expectational survey work could also include questions tuned to measure the size of social interactions in expectation formation, if any. This may give some kind of indicator how tightly interconnected relevant actors’ expectations are. This could be important in practice, because the more tightly interconnected expectations are, the more likely is a ‘surprise’ burst of expectation revision that might be hard to reverse as well as effects that would be hard to forecast with conventional tools (see Durlauf in SFI(II)).

Computational work could be done to approximate something like Coop and Potter’s ‘Bayes factors’ for the different views by drawing on histories and datasets of inflationary processes. Welfare distributions under the different views could then be calculated as in Pizer (1996) perhaps, in order to present policy makers with informative information on the distribution of potential costs and benefits. This might lead to further scrutiny of any sources of potential irreversibilities such as a possible ‘flip’ into an alternate ‘stable state’ as in the Carpenter et al. (1998) lake, or sluggishness in changing policy if leading indicators such as some of these discussed above show caution.

This highly speculative discussion of a policy debate is intended to suggest to the reader, in a summary way, how complexity-based research might be developed and how it might impact policy discussions in the future. Even if the speculations given in this article all turn out to be off the mark, it still seems safe to project that as the costs of doing this kind of work continue to fall, the supply
curve most likely will drop. Hence, the only way that the quantity of this type of work can fall is for the demand curve to fall further and it is hard to see any forces on the horizon that will push the demand curve down at all, much less further.

Acknowledgements

W.A. Brock would like to thank the NSF under Grant # SES-9122344 and the Vilas Trust for financial support.

References

Arthur, W., Durlauf, S., Lane, D., 1997. The Economy as an Evolving Complex System: II. Addison-Wesley, Redwood City, CA.


