Term structure views of monetary policy under alternative models of agent expectations

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Abstract

Term structure models and many descriptions of the transmission of monetary policy rest on the empirical relevance of the expectations hypothesis. Small differences in the perceived policy reaction function in VAR models of agent expectations strongly influence the relevance in the transmission mechanism of the expected short rate component of bond yields. Mean-reverting or difference-stationary characterizations of interest rates require large and volatile term premiums to match the observable term structure. However, short rate descriptions that capture shifting perceptions of long-horizon inflation evident in survey data support a more substantial term structure role for short rate expectations. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The term structure of interest rates is often characterized as a crucial transmission channel of monetary policy where accurate market perceptions of policy are required for effective policy execution. This description rests on three propositions: First, a short-term interest rate, such as the overnight federal funds rate, provides an adequate summary of monetary policy actions. Second, long-term bond yields are important determinants of the opportunity cost of investments. Third, under the expectations hypothesis, the anticipated path of the policy short-term rate is the main determinant of the term structure of bond rates. Although the empirical validity of each of these three propositions has been criticized, the accumulation of empirical evidence against the third proposition — the expectations hypothesis — is impressively large.\(^1\) If variations in current bond rates are not well-connected to current and expected movements in the policy-controlled short-term rate, then the conventional description of monetary policy transmission is vacuous.

In no-arbitrage formulations of the term structure, variation in bond rates due to current and expected movements in short rates is determined by the description of short rate dynamics. Under the standard finance assumption that short rates are mean-reverting, the average of expected future short rates is considerably smoother than historical bond rates. Thus, the expectations hypothesis is empirically rejected by tests which assume constant term premiums, and postwar shifts in the term structure are often attributed to sizable movements in ‘liquidity’ and ‘term’ premiums. However, postwar data are consistent with descriptions of short rate movements other than mean reversion. This paper shows that small differences in the specification of the stochastic process for the short rate strongly influence the relative importance of short rate expectations and residual term premiums in bond rate movements.

As an alternative to conventional descriptions of short rate dynamics, a simple class of time-varying descriptions of short rate behavior is examined. Given well-documented shifts in postwar monetary policy, it seems highly probable that the short rate expectations of bond traders are heavily influenced by shifting perceptions of monetary policy. Short rate descriptions that capture shifting perceptions of the long-run objective of monetary policy, formulated as

\(^1\) For instance, Campbell and Shiller (1991) compare estimated ‘theoretical’ spreads to actual spreads and conclude that the spread is too variable to accord with the expectations hypothesis. In addition, in regressions of long-rate changes on spreads, Shiller (1979), Shiller et al. (1983), and Campbell and Shiller (1991) find that coefficient estimates on the spread tend to be significantly different from the hypothesized value of one and frequently negative.
a long-run inflation target, are supportive of a substantial term structure role for short rate expectations.²

In addition to rejecting the implication that high bond rates in the 1980s reflect large term premiums, the preferred description of short rate behavior also does not support interpretations that perceived inflation targets were rapidly reversed in the 1980s. More likely, the long-run inflation target perceived by the market behaved similarly to survey data on long-run expected inflation. Survey data suggest agents were cautious in adjusting their perceptions of long-run inflation.

This paper illustrates the term structure implications of alternative representations of monetary policy perceptions of agents. Section 2 develops a discrete-time, no-arbitrage model of the term structure where the stochastic discount factor is related to a vector of macroeconomic variables. Forecasts of this state vector are generated by a VAR with flexible specifications of long-run behavior. Section 3 discusses three VARs with different characterizations of agent perceptions of long-run behavior. Owing to the different characterizations of perceived long-run behavior, the VARs embed perceived policy reaction functions with different implied inflation targets. Section 4 indicates how estimates of expected short-rate components of bond yields and conventional ‘residual’ estimates of term premiums depend critically on assumptions regarding the long-run behavior of policy targets in VAR proxies of agent expectations. Section 5 concludes.

2. A model of the term structure of interest rates

A discrete-time term structure model is outlined in this section to illustrate the pivotal role of long-run forecasts by VAR models in representing agent expectations. Bond yields are linked to expectations of monetary policy through a VAR proxy for expectations formation which contains an implicit policy reaction function. The implicit policy reaction function relates short term interest rates to deviations of economic variables from their long-run equilibrium values, including deviations of inflation from the long-run inflation goal of monetary policy. Thus, bond yields depend on monetary policy primarily through the influence of market perceptions of policy on short rate expectations.

² As developed later, shifting perceptions are represented in this paper as sequential learning, where agents test for changepoints in parameters of linear time series models. Other approaches to the apparent nonstationarity of interest rates include the fractional difference processes in Backus and Zin (1993), the multiple regimes models in Evans and Lewis (1995) and Bekaert et al. (1997), and the ‘central tendency’ factor of Balduzzi et al. (1998).
The price of an \( n \)-period nominal bond, \( P_{n,t} \), satisfies

\[
P_{n,t} = E_t[P_{n-1,t+1}M_{t+1}],
\]

where \( M_{t+1} \) is the nominal stochastic discount factor. Assuming \( M_{t+1} \) and the bond price are conditionally joint lognormally distributed, (1) can be represented as

\[
p_{n,t} = E_t\{m_{t+1} + p_{n-1,t+1}\} + \frac{1}{2}\text{Var}_t[m_{t+1} + p_{n-1,t+1}],
\]

with lower-case letters denoting the natural logarithms of the corresponding upper-case letters. The log yield-to-maturity on an \( n \)-period bond is equal to

\[
r_{n,t} = -\frac{1}{n}p_{n,t}. \tag{3}
\]

A recursive expression for log yields can be obtained by substituting the yield-price relationship into Eq. (2):

\[
r_{n,t} = \frac{1}{n}E_t\{-m_{t+1} + (n - 1)r_{n-1,t+1}\} - \frac{1}{2n}\text{Var}_t(m_{t+1} - (n - 1)r_{n-1,t+1}). \tag{4}
\]

Noting that for \( n = 1 \), \( p_{n-1,t+1} = p_{0,t+1} = 0 \), the yield on a one-period bond is

\[
r_{1,t} = E_t(-m_{t+1}) - \frac{1}{2}\text{Var}_t(m_{t+1}). \tag{5}
\]

Using the initial condition (5), the difference equation (4) can be solved to express the yield of an \( n \)-period bond as

\[
r_{n,t} = \frac{1}{n}E_t\left[\sum_{i=0}^{n-1} r_{t+i}\right] + \theta_{n,t}, \tag{6}
\]

where the last component

\[
\theta_{n,t} = -\frac{1}{2n}\sum_{i=1}^{n-1}\text{Var}_t((n - i)r_{n-i,t+i}) + \frac{1}{n}\sum_{i=1}^{n-1}\text{Cov}_t(m_{t+i},(n - i)r_{n-i,t+i}). \tag{7}
\]

is the term premium.\(^3\) Thus, Eq. (6) explicitly identifies the roles of expected future short rates and the term premium in longer maturity bond yields.

To enable use of VAR forecast systems, we use a discrete-time variant of the ‘affine’ class of bond pricing models, such as discussed by Backus et al. (1996)

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\(^3\)The first term in \( \theta_{n,t} \) is a Jensen’s inequality term while the second contains the more conventional liquidity, risk, or term premium. Generally, we will use the phrase ‘term premium’ to refer to \( \theta_{n,t} \).
and Campbell et al. (1997), where bond log yields are linear functions of state variables. Consequently, the stochastic discount factor, \( m_{t+1} \), is specified to be a linear function of a vector of state variables, \( x_t \).

\[
-m_{t+1} = \phi_0 + \phi'x_t + w_{t+1},
\]

(8)

where \( w_{t+1} \) is a serially uncorrelated innovation. The discount factor shock is a weighted average of shocks to the state vector, \( \omega_{t+1} \), and an additional innovation term, \( v_{t+1} \),

\[
w_{t+1} = \beta'\omega_{t+1} + v_{t+1},
\]

(9)

where \( \omega_{t+1} \) and \( v_{t+1} \) are uncorrelated at all leads and lags. The shocks have zero means, and the variance of \( v_{t+1} \) is constant, \( E_t(v_{t+1}^2) \equiv \sigma_v^2 \). The conditional covariance of the state innovations is denoted as \( E_t(\omega_{t+1}\omega_{t+1}') \equiv \Sigma_{\omega,t+1} \), where the covariances may be time varying,

\[
Var_t(m_{t+1}) = \beta'\Sigma_{\omega,t+1}\beta + \sigma_v^2,
\]

\[
= b_0 + b'x_t + \sigma_v^2,
\]

(10)

and the second line in (10) imposes the affine restriction that any time variations in the variances of state innovations are due to square-root processes, as in Cox et al. (1985) and Pearson and Sun (1994).

Because all forecast models of the state vector in this paper contain interest rate feedback rules to represent predictable policy responses, we assume one of the state variables, \( x_{1,t} \), is the one-period interest rate, \( r_{1,t} \). This imposes restrictions on elements of \( \phi_0 \) and \( \phi \). To illustrate, substitute Eqs. (8) and (10) into Eq. (5), giving

\[
r_t = E_t( - m_{t+1} ) - \frac{1}{2} Var_t(m_{t+1}),
\]

\[
= \phi_0 + \phi'x_t - \frac{1}{2}[b_0 + b'x_t + \sigma_v^2].
\]

(11)

If the covariance matrix of the state innovations is constant, \( b \) in Eq. (10) is a zero vector. In this instance, the one-period interest rate is equal to the first element of the state vector, \( x_{1,t} = r_t \), if \( \phi_0 = (1/2)[b_0 + \sigma_v^2] \) and \( \phi \) is a selector vector with one in the first element, \( \phi_1 = 1 \) (and zeros elsewhere). As an alternative example, suppose the first element of \( b \) is a positive constant, \( b_1 > 0 \), so that the variance of the stochastic discount factor is a linear function of the level of \( x_{1,t} \) (and the remaining elements of \( b \) are zero). To again equate the one-period interest rate to the first state variable, the only alteration required of the restrictions for the constant variance case is to redefine the first element of the selector vector: \( \phi_1 = 1 + (1/2)b_1 \).

Returning to the expected short-rate terms in the bond rate equation (6), the expectations of bond traders are represented by a VAR forecasting system. The
vector of macroeconomic state variables, \( x_t \), is a nonexplosive VAR process
about the endpoint vector, \( \mu^{(t)}_{x_t} \),
\[
x_t = Hx_{t-1} + (I - H)\mu^{(t-1)}_{x_t} + \omega_t,
\]
where the coefficient matrix, \( H \), has all eigenvalues less than or equal to one in magnitude. Following Kozicki and Tinsley (1998), the endpoint, \( \mu^{(t)}_{x_t} \), is the limiting conditional expectation of the vector \( x_t \),
\[
\mu^{(t)}_{x_t} \equiv \lim_{k \to \infty} E_t x_{t+k},
\]
with \( I \) indexing the time subscript of the information set on which expectations are conditioned.

The endpoint representation is sufficiently general to encompass more traditional VAR forecasting systems in which elements of \( x_{t+1} \) are classified as either I(0) or I(1). In particular, if an element of \( x_{t+1} \) is I(0), then the corresponding element of \( \mu^{(t)}_{x_t} \) will be a constant; if an element of \( x_{t+1} \) is I(1), then the corresponding element of \( \mu^{(t)}_{x_t} \) will be a linear combination of elements of \( x_t \). The endpoint representation also encompasses cases in which elements of \( x_{t+1} \) may be subject to sporadic mean shifts with the corresponding elements of \( \mu^{(t)}_{x_t} \) deterministic but subject to occasional shifts. Such specifications are convenient for explicitly accounting for changes in the perceived long-run inflation goals of monetary policy within a VAR framework. Three competing VAR specifications, differing primarily according to endpoint assumptions, are examined in detail in the next subsection.

As earlier, we continue the convention that the short-term interest rate is the first element of the state vector, \( r_t = r_1 x_t \), where \( r_1 \) is a selector vector with one in the first element. Multistep forecasts of short rates can be constructed using the state-variable VAR. Expression (12) implies
\[
E_t r_{t+i} = r_1 [H^i x_t + (I - H^i)\mu^{(t)}_{x_t}].
\]
Substituting into (6), the \( n \)-period yield can be rewritten in terms of the vector of state variables as
\[
r_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} r_1 [H^i x_t + (I - H^i)\mu^{(t)}_{x_t}] + \theta_{n,t},
\]
Eq. (15) relates bond yields in \( t \) to contemporaneous observations on state variables, including the one-period rate in \( t \). Predictions of yields, conditional on data through \( t - 1 \), can be constructed as
\[
E_{t-1} r_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} r_1 [H^{i+1} x_{t-1} + (I - H^{i+1})\mu^{(t-1)}_{x_t}] + E_{t-1} \theta_{n,t}.
\]
This term structure model has two noteworthy features. First, because the short rate equation in the VAR approximates agent perceptions of the monetary policy reaction function, monetary policy influences bond yields through a dependence of short rate expectations on expected policy. Second, if the conditional variance of the stochastic discount factor is a linear function of the level of the state vector, as illustrated earlier, then the model admits also the potential for time-varying term premiums.

3. VARs with fixed and time-varying policy goals

This section discusses the use of VARs to approximate agent expectations, especially longer-horizon expectations. Because expectations models of the term structure of interest rates embed long-horizon forecasts, the limiting conditional forecasts, or endpoints, of the VAR play a crucial role in term structure models. In particular, standard VAR specifications frequently used in empirical macroeconomic studies embed an assumption that the market believes the long-run policy goal for inflation has not and will not change over time. This assumption is contradicted by survey data on long-horizon inflation expectations. This section shows how to generalize VAR specifications to permit expectations to be influenced by shifting perceptions of monetary policy goals. Operationally, such shifts are mapped into time-varying intercepts of the VAR equations.

Typically, forward-looking models, including those of the term structure of interest rates, contain implicit assumptions on long-horizon behavior. These long-run conditions are usually determined by the decision to include variables in level or differenced formats. This choice of dynamic formats has minimal impact on short-horizon VAR forecasts for variables that are highly persistent, such as interest rates or inflation. However, the dynamic format can have a substantial effect on long-horizon forecasts, as shown in Kozicki and Tinsley (1998) for univariate autoregressive models. Dynamic specifications are similarly important when VAR forecasts are used to proxy for long-horizon market expectations.

Kozicki and Tinsley (1998) indicate that a convenient way to examine long-run assumptions in a time series model is to identify the endpoints of the series.

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4 In general, the standard approaches include variables such as interest rates and inflation in levels in VARs. When, as is usually the case, point estimates of the VAR coefficients are consistent with mean-reversion of VAR variables, the VARs implicitly embed an assumption that the VAR endpoints for interest rates and inflation are constant. Since the endpoint of the inflation process reasonably can be assumed to correspond to the long-run policy goal for inflation, these VARs implicitly assume that the long-run policy goal for inflation is a constant. Clarida et al. (1997, 1998) estimate policy rules assuming a constant inflation target since 1979.
being modeled. As introduced in the previous section, the endpoint of the vector of time series, \( x_t \), is denoted \( \mu_{I(t)} = \lim_{k \to \infty} E_{I} X_{t+k} \) where the expectation is conditioned on information set \( I \).\(^5\) If a series is mean reverting or \( I(0) \), then the endpoint of the series will be a constant. This feature is just a restatement of the property that long-run forecasts of a mean-reverting series converge to a constant, which is equal to the mean of the series in large samples. By contrast, if a series is difference stationary, \( I(1) \), then, in a univariate autoregressive model, the endpoint of the series will be a moving average of recent observations of the series. In a multivariate VAR, the endpoint of an \( I(1) \) series will be either a moving average of the series or a linear combination of moving averages of the series and other series with which it is cointegrated.

VARs can be readily rewritten to explicitly reveal dynamic adjustments of deviations from endpoints. An advantage of this reformulation is that alternative characterizations of long-run behavior can be easily implemented and interpreted. For example, the VARs considered in this section contain an implicit policy reaction function which relates deviations of the short rate from its endpoint to deviations of inflation and capacity utilization from their respective endpoints. The inflation endpoint thus plays the role of the perceived policy target for inflation. The implicit endpoint restrictions imposed by \( I(0) \) assumptions correspond to an assumption that the perceived policy target for inflation is fixed and equal to the sample mean of inflation. An alternative assumption that inflation is \( I(1) \) corresponds to a characterization that the perceived policy target for inflation is a moving average of inflation.\(^6\) Relaxing the implicit endpoint restrictions imposed by \( I(0) \) or \( I(1) \) assumptions allows additional characterizations of time variation in the perceived policy goal for inflation.

The remainder of this section discusses three different VAR specifications to illustrate the long-horizon implications of alternative endpoint specifications. The VARs differ in their assumptions about long-run behavior. In particular, each incorporates a different characterization of the inflation endpoint, which represents the market perception of the long-run policy target for inflation. The VARs are similar in that each contains the same three variables: a one-month nominal interest rate, \( r \), inflation, \( \pi \), and the rate of capacity utilization, \( y \). All interest rates referenced in this paper are end-of-month zero-coupon Treasury bond yields.\(^7\) Monthly inflation is measured by the BEA chain-weighted

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\(^5\) Strictly speaking, the infinite horizon subscript is applicable only for stochastic processes that do not contain unbounded deterministic trends.

\(^6\) This assumes no cointegration between capacity utilization, interest rates, and inflation. See the discussion on this point later in the section.

\(^7\) Interest rates for 1960m\(_1\)-1991m\(_1\) are from McCulloch and Kwon (1993). For comparability, these data were extended to 1997m\(_9\) by applying the cubic spline estimator described in McCulloch (1975) to end-of-month yields. Our thanks to Mark Fisher for his generous assistance.
deflator for personal consumption expenditures, and monthly capacity utilization by the FRB index for manufacturing.

3.1. The fixed endpoints VAR

The general structure of this VAR resembles the representative small-scale VARs used by macroeconomists such as Bernanke and Blinder (1992) and Fuhrer and Moore (1995) to represent the responses and effects of monetary policy. Such VAR specifications, with all variables included in levels, are typically fixed endpoint specifications as the estimated coefficients are usually consistent with mean reversion.

The top rows of Table 1 summarize estimates of univariate twelve-order autoregressions for the one-month bond rate, \( r \), the inflation rate, \( \pi \), and the manufacturing utilization rate, \( y \). The point estimates of the coefficients of the lagged levels of each variable, \( b \), are consistent with mean-reverting behavior.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autoregressions under alternative endpoint specifications$^a$</td>
</tr>
<tr>
<td>( \Delta x_t = c + bx_{t-1} + b^*(L)\Delta x_{t-1} + a_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x_t )</th>
<th>( c )</th>
<th>( b )</th>
<th>( b^*(1) )</th>
<th>( R^2 )</th>
<th>( \text{SEE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t )</td>
<td>0.246</td>
<td>-0.038</td>
<td>0.065</td>
<td>0.07</td>
<td>0.684</td>
</tr>
<tr>
<td>(2.4)</td>
<td>( -2.5)</td>
<td>(0.3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_t )</td>
<td>0.310</td>
<td>-0.076</td>
<td>-1.85</td>
<td>0.29</td>
<td>1.64</td>
</tr>
<tr>
<td>(2.2)</td>
<td>( -2.5)</td>
<td>( -4.1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y_t )</td>
<td>2.65</td>
<td>-0.032</td>
<td>0.544</td>
<td>0.18</td>
<td>0.783</td>
</tr>
<tr>
<td>(3.6)</td>
<td>( -3.6)**</td>
<td>(5.1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fixed endpoints**

| \( r_t' \) | -0.019 | -0.087 | 0.116 | 0.09 | 0.689 |
| ( -0.5) | ( -3.5)** | (0.4) |
| \( \pi_t' \), $^c$ | -0.002 | -0.228 | -1.12 | 0.32 | 1.60 |
| ( -0.0) | ( -4.2)** | ( -2.1) |
| \( \pi_t - \pi_t' \), $^d$ | 0.016 | -0.105 | -1.40 | 0.28 | 1.65 |
| (0.2) | ( -3.0)** | ( -3.1) |

**Shifting endpoints**

$^a$The one-month nominal bond rate is denoted by \( r \), the PCE inflation rate by \( \pi \), and the manufacturing capacity utilization rate by \( y \). Monthly inflation and utilization rates are seasonally adjusted, and the sample spans are 1955m2–1997m9 for \( \pi \) and \( y \), and 1966m1–1997m9 for \( r \). Entries in parentheses are \( t \)-ratios. Endpoint constructions are discussed in the text, where \( x_t' \) denotes the endpoint or long-horizon expectation of \( x \) as conditioned on information available in \( t \).

$^b$A unit root in endpoint deviations is rejected at 90% (**), 95% (**), or 99% (***), significance levels.

$^c$Inflation endpoints (calendar time).

$^d$Inflation endpoints as perceived in ‘real time’.

Under this specification, a long-horizon forecast will converge to the fixed endpoint, \( -c/b \). For the sample spans underlying Table 1, the estimated endpoints are \( r_\infty = 6.5, \pi_\infty = 4.1, \) and \( y_\infty = 82.8. \)

Selected coefficients of a fixed endpoints VAR for \( r, \pi, \) and \( y, \) using a common sample span of 1966\text{m}1−1997\text{m}9, \(^9\) are shown in the top panel of Table 2. \(^{10}\) The short rate equation plays the role of a policy reaction function where a short-term interest rate, controlled by the policy authority, responds to deviations of inflation from a fixed long-run policy target for inflation and to deviations of output from trend.

In a mean-reverting VAR, the implied policy target for inflation is equivalent to the fixed endpoint for inflation, assuming information on policy targets is symmetrically available to all public and private sector agents. As shown in the top panel of Fig. 1, the fixed endpoint for inflation over the 1966–1997 sample is estimated to be a bit less than 5% at an annual rate. \(^{11}\) A feature of fixed endpoints is that forecasts formulated in any period of the sample will eventually converge to the same endpoint. This property is illustrated in the top panel of Fig. 2, where two multiperiod forecasts of monthly inflation rates are shown. The first forecast originates in 1972\text{m}1 and the second in 1979\text{m}10, when the dramatic change in the policy operating procedures of the Fed was announced by Paul Volcker. Despite the very different history of inflation preceding each forecast, the assumption of the fixed endpoints VAR is that the underlying policy target for inflation is believed to be constant.

One check on whether the VAR does a reasonable job at replicating long-run agent expectations is to compare VAR-based forecasts of 10 yr inflation rates to survey data on long-horizon inflation expectations. This is done in Fig. 3. The top panel of Fig. 3 shows VAR-based predictions of 10 yr inflation rates.

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\(^8\) In remaining empirical work, the mean of the utilization rate is removed, implying a fixed endpoint of zero.

\(^9\) Models using a short-term interest rate to approximate the operating policy of the Federal Reserve often omit observations before 1966 due to changes in the behavior of the interbank market for overnight funds prior to the mid-1960s; see discussion of the relationship between the federal funds rate and the Federal Reserve discount rate in Tinsley et al. (1982). The VARs in Table 2 contains six lags of each variable. In the case of the fixed endpoints model, the \( p \)-values of likelihood ratios indicate that a three-lag specification is rejected in favor of six lags, but a six-lag specification is not rejected against the alternatives of 9 or 12 lags.

\(^10\) Sims (1992) and Christiano et al. (1994) include commodity price indexes as an additional determinant of interest rate policy responses. Although we explored relative commodity price indexes in preliminary work, inclusion raised several specification issues, such as the well-known problem (Prebisch-Singer) of long-run negative trends in relative commodity prices, without contributing essential insights into the issue of time-varying endpoints.

\(^11\) Using a similar calculation, Clarida et al. (1998) estimated a constant inflation target slightly larger than 4% at an annual rate over 1979:10–1994:12.
Table 2
VARs under alternative endpoint specifications*

\[ \Delta x_t = c + A \Delta x_{t-1} + A^*(L) \Delta x_{t-1} + a_t \]
\[ \bar{x}_t = x_t - x_{t0} \]

<table>
<thead>
<tr>
<th>Regressor coefficients</th>
<th>( \pi )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>( A )</td>
<td>( A^*(1) )</td>
</tr>
<tr>
<td>( \Delta r )</td>
<td>( -0.007 )</td>
<td>( -0.033 )</td>
</tr>
<tr>
<td></td>
<td>( -0.1 )</td>
<td>( -1.8 )</td>
</tr>
<tr>
<td>( \Delta \pi )</td>
<td>( 0.313 )</td>
<td>( 0.037 )</td>
</tr>
<tr>
<td></td>
<td>( 1.1 )</td>
<td>( 0.7 )</td>
</tr>
<tr>
<td>( \Delta y )</td>
<td>( 0.379 )</td>
<td>( -0.056 )</td>
</tr>
<tr>
<td></td>
<td>( 3.73 )</td>
<td>( -3.2 )</td>
</tr>
</tbody>
</table>

| Moving average endpoints | \( \Delta r \) | \( -0.018 \) | \( -0.462 \) | \( -0.176 \) | \( 0.030 \) | \( 0.254 \) | \( 0.668 \) |
|                         | \( 0.5 \)   | \( -3.3 \)  | \( -2.4 \)  | \( 3.4 \)   | \( 3.1 \)   |
| \( \Delta \pi \)       | \( 0.013 \) | \( 0.111 \) | \( -1.94 \) | \( 0.042 \) | \( 0.298 \) | \( 1.78 \) |
|                         | \( 0.1 \)   | \( 0.3 \)   | \( -9.8 \)  | \( 1.8 \)   | \( 1.4 \)   |
| \( \Delta y \)         | \( -0.020 \) | \( -0.000 \) | \( 0.059 \) | \( -0.028 \) | \( 0.562 \) | \( 0.639 \) |
|                         | \( -0.6 \)  | \( -0.0 \)  | \( 0.8 \)   | \( -3.4 \)  | \( 7.2 \)   |

| Shifting endpoints      | \( \Delta r \) | \( -0.002 \) | \( -0.069 \) | \( -0.276 \) | \( 0.029 \) | \( -0.261 \) | \( 0.032 \) | \( 0.196 \) | \( 0.664 \) |
|                         | \( -0.7 \)  | \( -2.2 \)  | \( -1.6 \)  | \( 1.7 \)   | \( 3.0 \)   | \( 3.3 \)   | \( 2.0 \)   |
| \( \Delta \pi \)       | \( 0.044 \) | \( 0.023 \) | \( 0.184 \) | \( -0.112 \) | \( -1.64 \) | \( 0.066 \) | \( 0.098 \) | \( 1.77 \) |
|                         | \( 0.5 \)   | \( 0.3 \)   | \( 0.4 \)   | \( -2.4 \)  | \( -7.1 \)  | \( 2.5 \)   | \( 0.4 \)   |
| \( \Delta y \)         | \( -0.036 \) | \( -0.091 \) | \( 0.302 \) | \( -0.006 \) | \( 0.064 \) | \( -0.014 \) | \( 0.390 \) | \( 0.631 \) |
|                         | \( -1.1 \)  | \( -3.1 \)  | \( 1.9 \)   | \( -0.4 \)  | \( 0.8 \)   | \( -1.5 \)  | \( 4.2 \)   |

*The 3 x 1 vector, \( x \), contains the one-month nominal bond rate, \( r \), the PCE inflation rate, \( \pi \), and the manufacturing capacity utilization rate, \( y \). Monthly inflation and utilization rates are seasonally adjusted, and the sample span is 1966m1–1997m9. Entries in parentheses are t-ratios. Endpoint constructions are discussed in the text, where \( x_{t0} \) denotes the endpoint or long-horizon expectation of \( x \) as conditioned on information available in \( t \).

historical inflation, and a series of spliced survey measures of long-horizon inflation expectations.\(^{12}\) Whereas the survey data have been gradually trending down, the VAR-based predictions are relatively flat and barely deviate from the fixed endpoint. This feature suggests that predictions from the fixed endpoint VAR may not be very good proxies for agent expectations.

\(^{12}\) In the 1980s, Richard Hoey, an economist at Drexel Burnham Lambert, conducted a survey of market participants which asked for forecasts of inflation in the second five years of a 10 yr forecast horizon. This series was used prior to 1990m7. The remainder of the spliced series is the long-run expectations series from the Survey of Professional Forecasters (SPF), currently published by the Federal Reserve Bank of Philadelphia.
3.2. The moving average endpoints VAR

The fundamental difference between the fixed endpoints VAR and the moving average endpoints VAR is that the latter incorporates unit root restrictions.
Fig. 2. VAR multiperiod predictions of 1-month inflation rate.

on the 1-month interest rate and inflation rate, $r$ and $\pi$. Returning to the top of Table 1, the format of the autoregressions in $r$, $\pi$, and $y$, is the same as that required for an ADF test of unit root behavior. Indeed, the $t$-ratios of the coefficients of the lagged dependent variable indicate that a unit root hypothesis cannot be rejected for both the short-term interest rate and
Cointegration between $r$ and $n$ has not been imposed. A reasonable assumption might be that after-tax real rates are stationary. This would imply that $r$ and $\pi$ are cointegrated with a cointegrating vector that reflects tax rates. Since tax rates have changed over time, $r$ and $\pi$ would not be cointegrated with a constant cointegrating vector. Time-varying cointegrating restrictions are beyond the scope of this paper.

\footnote{Cointegration between $r$ and $\pi$ has not been imposed. A reasonable assumption might be that after-tax real rates are stationary. This would imply that $r$ and $\pi$ are cointegrated with a cointegrating vector that reflects tax rates. Since tax rates have changed over time, $r$ and $\pi$ would not be cointegrated with a constant cointegrating vector. Time-varying cointegrating restrictions are beyond the scope of this paper.}

Fig. 3. Concatenated VAR predictions of 10 yr inflation rates.
In the present discussion, the interesting property of this specification is that the endpoints of $r$ and $\pi$ are now also $I(1)$. In each period, the conditional endpoint of a multiperiod inflation forecast is a moving average of inflation in the months just prior to the forecast period.\footnote{As shown in Kozicki and Tinsley (1998), the moving average endpoint is precisely the Beveridge-Nelson ‘permanent’ component of unit-root stochastic processes. By construction, deviations from moving average endpoints of $I(1)$ series are stationary.} This moving average property is displayed in the middle panel of Fig. 1, where it is seen that the inflation endpoint — which functions as the market view of the long-run policy target — now moves very closely with recent inflation history. The two multiperiod forecasts of inflation generated by the moving endpoints VAR are presented in the middle panel of Fig. 2. Whereas the long-run inflation forecast in 1972$m_1$ is now about 1% lower than the fixed endpoint in the top panel, the moving average endpoint for inflation forecasts originating in 1979$m_{10}$ is an alarming 10%, owing to the large run-up in historical inflation in preceding months.

An implication of the moving-average property of the endpoint is that VAR-based 10 yr inflation forecasts will follow recent inflation history. The middle panel of Fig. 3 illustrates this property. VAR-based predictions of 10 yr inflation rates follow actual inflation quite closely. Unlike predictions based on the fixed endpoints VAR, 10 yr inflation predictions based on the moving average endpoints VAR are quite variable. In fact, the long-horizon predictions are more variable than survey data. Whereas survey data suggest that long-horizon inflation expectations adjust slowly, with a significant lag compared to actual inflation, predictions based on the moving average VAR adjust quite rapidly.

3.3. A shifting endpoints VAR

The third VAR specification admits endpoints that can differ from either the fixed endpoints implied by the mean-reverting specification or the moving average endpoints provided by the difference-stationary specification. The format of an autoregression or VAR can be written to explicitly account for deviations from endpoints, according to
\[ \Delta x_t = c + A(x_{t-1} - x^{(t-1)_x}) + A^*(L)\Delta x_{t-1} + a_t, \]
where $A^*(L)$ denotes a polynomial in the lag operator, $L^i x_t = x_{t-i}$. When expressed in this format, it is clear that endpoints need not be constrained to be fixed or moving averages.

In some instances, direct measurements of the implied endpoints or long-horizon forecasts of agents are available. For example, under the expectations hypothesis, bond rates contain ex ante long-horizon forecasts of short rates.
Thus, as suggested in Kozicki and Tinsley (1998), one measure of the short-term interest rate endpoint is the average of expected short rates from $t + n$ to $t + n'$, for $n' > n$,

$$\hat{r}_{t} = \frac{n'(r_{n',t} - \theta_{n'}) - n(r_{n,t} - \theta_{n})}{n' - n},$$

(18)

where $\theta_{n}$ is an estimate of a constant term premium for an $n$-period bond. For the current study, estimates of the shifting endpoints of the short-term interest rate are based on the average of expected short rates between the 5 yr and 10 yr maturities. One check of the reasonableness of this interest rate endpoint is that deviations of the one-month bond rate from this endpoint should be stationary if agents’ bond rate forecasts in a given period are internally consistent. The fourth equation listed in Table 1 indicates that the interest rate deviations from the constructed series of nominal rate endpoints reject the hypothesis of unit root behavior at a 99% significance level.

It is more difficult to obtain direct measurements of long-horizon endpoints for agents’ forecasts of inflation. Although several surveys of expected 5–10 yr inflation emerged in the 1980s, contiguous survey estimates of expected long-horizon inflation are not available for earlier decades. To generate backcasts of agent expectations of long-run inflation, bond traders are assumed to be ‘boundedly-rational’ statistical agents, who sequentially test for intercept shifts in an autoregressive model of inflation. The statistical learning model is based on a univariate autoregressive model of inflation. The intuition behind the assumed learning behavior is that agents infer an error in their current estimate of the long-run policy goal of inflation when they observe a string of prediction errors for inflation of the same sign, and adjust their outlook accordingly. These shifts in the long-run perceptions induce nonstationarity in the mean of the inflation process.

Central bankers of developed economies are generally cautious and slow to change either operational policies or strategic objectives. Thus, changes in long-run policy objectives are likely episodic with frequencies that are more appropriately measured in half-decades rather than months. This inference seems to be consistent with the small number of policy regimes typically identified in postwar analyses of policy, such as Huizinga and Mishkin (1986). A simple representation of episodic policy shifts in the inflation process as considered by agents is

$$\Delta\pi_{t} = b_{0} + \sum_{k} \delta(t - \tau_{k})\Delta b_{0,\tau_{k}} + b_{1}\pi_{t-1} + b(L)\Delta\pi_{t-1} + a_{t}.$$  

(19)

To represent possible shifts in the intercept, the dummy variable notation, $\delta(.)$, denotes the Heaviside switching function where $\delta(t - \tau_{k}) = 0$ if $t < \tau_{k}$. If a change in the intercept is detected for period $\tau_{k}$, the function is switched ‘on’ in all subsequent periods, $\delta(t - \tau_{k}) = 1$ if $t \geq \tau_{k}$. 

In the absence of intercept shifts, the long-run inflation rate defined by (19) is \( \pi_\infty = -b_0/b_1 \). However, if intercept shifts are sequentially detected, the evolution of expected long-run inflation is determined by the current estimate of the intercept, \( \pi_\infty^{(t)} = -[b_0 + \sum_{t_i < t_i} \Delta b_0, t_i]/b_1 \). By not presuming ex ante knowledge of the number of distinct policy regimes, the changepoint learning model allows agents to be less informed about possible policy regimes than would be the case for a shifting-regimes analysis.\(^{15}\)

In the learning simulations referenced in this paper, agents test for intercept shifts in a twelve-order autoregression in inflation using critical values extrapolated from Andrews (1993). The search by simulated agents for significant changepoints is in ‘real time’ as new observations are added in expanding monthly samples over the period 1954m1−1997m9. To account for heterogeneous expectations among bond traders, agents are viewed as subscribing to one of \( m \) market forecast ‘newsletters’. Before testing for a new inflation endpoint, each newsletter accumulates a different minimum number of monthly observations since the date of the last estimated changepoint. The minimum number of observations used by the \( i \)th newsletter is denoted \( s_i \). Results discussed here are based on fifteen newsletters, \( m = 15 \). The minimum sample collected since the last changepoint ranges from 12 months for newsletter 1 to 96 months for newsletter 15: \( s_i = 6(1 + i) \).

An important by-product of this simulated model of sequential, ‘real-time’ learning by agents is that it generates both the date of an estimated changepoint and the date of the subsequent recognition of that changepoint, when sufficient observations have been collected to satisfy the relevant critical value of the changepoint test. The concatenation of shifted endpoints in the estimated inflation model is termed the ‘calendar time’ endpoint series, and the concatenation of estimated endpoints when perceived by the learning agents is termed the ‘real time’ endpoint series.

A sampling of shifting inflation endpoints generated by competing forecast newsletters is shown in Fig. 4. The top panel indicates the inflation endpoints generated by the newsletter that collects a minimum of 30 months of observations after detecting a changepoint before testing for a subsequent shift. The middle and bottom panels correspond to newsletters with minimum observation collections of 60 and 90 months respectively. As indicated, newsletters with a smaller number of minimum observations have a shorter lag in detecting changepoints, as indicated by the shorter horizontal displacements between the calendar time endpoints series (bold line) and the ‘real time’ endpoints series (dashed line). Newsletters using smaller minimum samples also can generate

\(^{15}\) Regime-shifting specifications, such as proposed by Hamilton (1989), typically presume ex ante knowledge of the number of policy regimes, a constraint that is difficult to verify empirically and to implement in real time policy analysis.
more volatile long-run forecasts with frequent adjustments of estimated endpoints. This difference depends on the underlying behavior of historical inflation, as can be seen by comparing the frequency and size of estimated changepoints in the 1970s and early 1980s with the infrequent endpoint adjustments by all newsletters after the mid-1980s.

Fig. 4. Shifting inflation endpoints used by competing forecast newsletters.
An aggregate inflation endpoint series is constructed by weighted averages of the different newsletter estimates of long-run inflation,

\[ \hat{\pi}_{\infty}^{(t)} = \sum_{i=1}^{m} n_{i,t} \hat{\pi}_{i,\infty}^{(t)}, \quad \sum_{i=1}^{m} n_{i,t} = 1, \]  

(20)

where \( n_{i,t} \geq 0 \) is a nonnegative weight indicating the proportion of bond traders subscribing to the \( i \)th newsletter in month \( t \).

The relative contribution by each forecast newsletter, \( n_{i,t} \), to the aggregate inflation endpoint is estimated by relating movements in the after-tax endpoint of the nominal interest rate to the movements of the aggregate of newsletter inflation endpoints. That is, the long-run, after-tax nominal interest rate is assumed equal to the sum of the long-run real rate and the long-run inflation rate,

\[ (1 - t_x) r_{\infty}^{(t)} = \rho_{\infty} + \pi_{\infty}^{(t)}, \]  

(21)

where \( t_x \) is the tax rate on bond earnings and \( \rho_{\infty} \) is the long-horizon expectation of the after-tax real rate. Notice that this specification assumes after-tax real rates are stationary. Eq. (21) has two important properties. First, unlike the traditional Fisher equation, nominal rates will move more than one-for-one with expected inflation under positive tax rates.\(^{16}\) Second, shifting agent perceptions of the long-run policy goal for inflation, as captured by \( \pi_{\infty} \), are a likely source of shifts in the perceived endpoint of the nominal rate, \( r_{\infty} \).

The selection of forecast newsletters by agents is described by a stochastic discrete choice model. The utility of the \( i \)th newsletter to its subscriber subpopulation in period \( t \) is represented by

\[ u_i(t) = u_{i,t} + \epsilon_{i,t}, \quad i = 1, \ldots, 15, \]  

(22)

where \( u_{i,t} \) denotes that portion of utility that is related to observable variables. This component is assumed to consist of a constant taste preference for the \( i \)th newsletter, \( v_i \), and a quadratic penalty on deviations from the expected consensus forecast of long-run inflation.\(^{17}\)

\[ u_{i,t} = v_i + v_0 (\pi(i)_{\infty}^{(t)} - \pi_{\infty}^{(t)})^2, \quad i = 1, \ldots, 15, \]  

(23)

---

\(^{16}\) Crowder and Hoffman (1996) illustrate the historical relevance of the nonzero tax rate. Estimated tax rates are drawn from McCulloch and Kwon (1993). McCulloch (1975) found that the effective tax rate that best explains the prices of U.S. Treasury securities lies in the range, 0.22 to 0.30. As noted by Green and Odegaard (1997), tax rates fell substantially after 1986.

\(^{17}\) Traders are assumed to have disparate priors about the frequency of shifts in the long-run inflation objectives of policy but place some weight also on the long-run views of other agents. For more extensive discussion of the dynamic implications of discrete choice models with social interactions, see Brock and Durlauf (1995) and Brock and Hommes (1996).
To reduce computations in the estimation stage, the contraction estimate of the aggregate inflation endpoint in each month, \( \pi_{t}^{(n)} \), is proxied by the aggregate inflation endpoint generated in the preceding month, \( t - 1 \).

The residual portion of utility associated with the \( i \)th forecast newsletter, \( e_{i,t} \), is assumed to follow an extreme value distribution with a zero mean and variance proportional to \( \gamma^{-2} \). As shown in Anderson et al. (1992), the assumptions of discrete choice utility maximization and the extreme value distribution of residual utility imply that the relevant choice probabilities can be characterized by a multinomial logit model. Thus, the probability of selecting the \( i \)th forecast newsletter is

\[
n_{i} = \Pr \left( u(i) = \max_{j} u(j) \right),
\]

\[
= \Pr(\varepsilon_{1} - \varepsilon_{i} \leq u_{i} - u_{1}, \ldots, \varepsilon_{n} - \varepsilon_{i} \leq u_{i} - u_{n}),
\]

\[
= \frac{e^{\gamma u_{i}}}{\sum_{j=1}^{m} e^{\gamma u_{j}}}. \tag{24}
\]

Estimates of the parameters \((\rho_x, \gamma, v_i, i = 0, \ldots, m)\) defined by the system of equations (20)–(24) over the sample 1955m7–1997m9 are shown in Table 3. The estimate of the long-run after-tax real rate, \( \rho_x \), is 2.21. The frequency distribution of agents over newsletters is smoothed by constraining the constant taste preferences, \( v_i \) (\( i = 1, \ldots, m \)), to lie on a third-degree polynomial. Given that the choice probabilities are invariant to a constant addition to utility, the constant taste parameter associated with the newsletter using the largest minimum observation sample is normalized to zero, \( v_{15} = 0 \). Whereas the estimated constant taste parameters are all positive, the estimate of the coefficient of squared deviations of newsletter endpoints from the consensus endpoint, \( v_0 \), is significantly negative, indicating forecasts nearer the consensus estimate tend to attract more subscribers. Finally, the nonnegative precision parameter, \( \gamma \), is associated with the normalization, \((\sum_{j=0}^{m} v_{j}^{2})^{1/2} = 1.19\). The \( \gamma \) parameter indicates the concentration of agent preferences. That is, if \( \gamma \rightarrow + \infty \), the newsletter with the highest observable utility characteristics will attract all subscribers. Conversely, if \( \gamma \rightarrow 0 \), each newsletter will attract an equal market share of subscribers, on average.

As indicated in Table 4, the sample mean estimates of the newsletter subscriber shares, \( \bar{\eta}_{i,t} \), indicate that more than 40% of the simulated agents are
Table 3
Discrete choice selection of forecast newsletters

\[
(1 - t_x) r^{(i)}_x = \rho_x + \sum_{i=1}^{15} n_{i,t} \hat{\pi}^{(i)}_{t,x}
\]

\[
n_{i,t} = e^\beta_m \cdot \sum_{j=1}^{15} e^\beta_{mj}
\]

\[
u_{i,t} = v_i + v_0 (\hat{\pi}^{(i)}_{t,x} - \hat{\pi}^{(0)}_{t,x})^2
\]

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<td>(1.9)</td>
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*The endpoint of the after-tax nominal rate, \((1 - t_x) r^{(i)}_x\), is defined as the sum of the long-run real interest rate, $\rho_x$, and of a weighted average of inflation endpoints used by competing forecast newsletters, $\pi^{(i)}_{t,x}$. The weights are the probabilities of choosing a forecast newsletter and, as noted in the text, the choice probabilities, $n_{i,t}$, are generated by a multinomial logit model. Estimates are based on the monthly sample, 1955m-1997m. The estimated precision is: $\gamma = 3.28$. Estimates of remaining parameters are listed in the table, along with $t$-ratios in parentheses.

clustered, on average, around newsletters 7–11, which use minimum observation samples from 48 to 72 months. Thus, the implied aggregate ‘real time’ lag in detecting past shifts in inflation endpoints appears to be relatively lengthy with a minimum detection lag of around five years. However, the sizeable standard deviations of the relative subscription shares of each newsletter suggest substantial variation over time in the aggregate detection lag. To illustrate this time variation, Fig. 5 plots the aggregate minimum detection lag, defined as the weighted average of minimum observation periods used by each newsletter, $\sum_{i=1}^{m} n_{i,t} S_t$. The estimated aggregate minimum detection lag ranges from lows of about three years (36 months) to highs of around six years (72 months). The adjustment pattern in Fig. 5 suggests that a sizeable and sustained change in the level of recent inflation leads to an initial increase in the aggregate minimum detection lag followed by a relatively rapid decrease. This reversing movement in the aggregate detection lag is driven by the penalty on deviations of newsletter inflation endpoints from the consensus aggregate endpoint. That is, agents initially cancel subscriptions to the newsletters with short detection lags whose long-run forecasts appear to be overreacting to recent inflation observations.
Table 4
Distribution of agents over inflation forecast newsletters

\[ \hat{\pi}_t^{(i)} = \sum_{i=1}^{15} n_{i,t} \hat{\pi}_t^{(i)}, n_{i,t} \geq 0, \sum_{i=1}^{15} n_{i,t} = 1 \]

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<th>(i)</th>
<th>(s_{i}^{b})</th>
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<th>(\sigma(n_{i,t})^{d})</th>
<th>(i)</th>
<th>(s_{i}^{b})</th>
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*The aggregate inflation endpoint, \(\hat{\pi}_t^{(0)}\), is a weighted average of inflation endpoints used by simulated forecast newsletters, \(\hat{\pi}_t^{(i)}\), where the weights are the estimated choice probabilities of newsletters, \(n_{i,t}\), over the 1955–1997 sample.

\(s_{i}^{b}\) Minimum number of monthly observations since last changepoint, \(s_{i} = 6(1 + i)\).

\(\hat{n}_{i,t}^{c}\) Average share of agent subscriptions to the \(i\)th newsletter.

\(\sigma(n_{i,t})^{d}\) Standard deviation of subscription shares.

However, if additional forecast newsletters also revise their endpoint estimates in the same direction, the aggregate endpoint is eventually dominated by revised endpoint estimates, and subscribers abruptly move away from the most conservative newsletters whose endpoints have not yet been revised.
The last two autoregressions shown in Table 1 provide evidence against the hypothesis that deviations of monthly inflation from either the aggregated calendar time endpoints series or the aggregated ‘real time’ endpoints series are I(1).\(^{20}\) Because the purpose of the VAR is to generate agent expectations that approximate information available to markets when the historical data are recorded, the assumption of symmetric access to policy target information implicit in the fixed and moving average endpoints models is dropped, and the ‘real time’ inflation endpoint is selected as a more realistic estimate of historical market perceptions of the long-run policy target for inflation. This selection is supported in the bottom panel of Fig. 3, which shows that the predictions of 10 yr inflation rates based on the shifting endpoints VAR resemble surveys of expected long-run inflation.

Estimated VAR coefficients are provided in the bottom panel of Table 2. Coefficients in columns labeled ‘A’ correspond to the sum of coefficients on the lagged levels of the relevant regressor variable. Interestingly, in most instances, the sum of coefficients in the shifting endpoints VAR is insignificantly different from the corresponding sum of coefficients in the other two VAR specifications. This feature leads one to expect that short-horizon forecasts from the three VARs will be similar, even though long-run point forecasts may differ considerably due to the competing specifications of forecast endpoints.

As emphasized by Rudebusch (1998), the interest rate equation in the VAR can be interpreted as a policy reaction function, although some additional calculations are required to compare results in Table 2 with other reported policy rules. For example, the point estimate of the total response (accounting for lagged responses) of the level of the nominal short-term interest rate to a unit change in inflation (or capacity utilization) can be estimated as the ratio of the sum of coefficients on inflation (or capacity utilization) to the negative of the sum of coefficients on lags of the level of the short rate. Under this calculation, the total response of the short rate to a one percentage point increase in inflation is 0.42 percentage points and the total response of the short rate to a one unit increase in capacity utilization is 0.46 percentage points. Thus, the implied total response coefficients for inflation and output over the 1966–1997 sample are remarkably close to the parameter values (0.5) selected by Taylor (1993).

Representative multiperiod inflation forecasts of the shifting endpoints specification are shown in the bottom panel of Fig. 2. Although the inflation forecast in 1979m\(_{10}\) initially declines, somewhat like the beginning contour of the corresponding fixed endpoints forecast, the forecast eventually edges up to

\(^{20}\) Although most simulated agents are long-run rational in the sense that an arbitrary fixed endpoint will be identified in large samples, there is no reason to expect deviations of historical inflation from the evolution of perceived inflation endpoints to appear stationary in a finite sample with shifting endpoints.
a higher endpoint, in this instance about 6%, as the contributions of the initial conditions of the forecast model diminish and the estimate of the inflation endpoint perceived in 1979m10 becomes more influential in distant periods of the forecast horizon.

Predictions of 10 yr inflation rates based on the shifting endpoint specification are shown in the bottom panel of Fig. 3. The predicted 10 yr inflation rates are relatively smooth, lag historical inflation rates, and gradually trend downward through the 1980s and 1990s. These predictions come much closer to matching the movements of survey perceptions of long-run inflation than the predictions of the competing VAR models. As seen in the remaining panels of Fig. 3, the 10 yr inflation predictions of the fixed endpoints VAR model do not appreciably differ from the sample mean, and the 10 yr inflation rate predictions of the moving average endpoints VAR closely track recent monthly inflation rates.

4. Term structure implications of VAR specification

This section explores the roles of expected short rates and term premium in longer maturity bond yields. In particular, the apparent term structure role for short rate expectations is shown to hinge on the VAR specification used to proxy agent expectations. Following the derivation in Section 2, predictions of the expected short rate component can be constructed as

\[
\frac{1}{n} \sum_{i=0}^{n-1} E_{t-i} r_{t+i} = \frac{1}{n} \sum_{i=0}^{n-1} \left[ \hat{H}^{i+1} x_t + (I - \hat{H}^{i+1}) \mu_x^{(t)} \right]
\]

(25)

where \( \hat{H} \) is the companion form of an estimated VAR forecasting system. The first subsection examines the contribution of the expected short rate component to bond yields for each of the three VAR specifications introduced in Section 3. Also analyzed are the implications of VAR specification for predictions of ex ante real rates, constructed according to

\[
(1/n) \sum_{i=0}^{n-1} (t_1 - t_\pi) [\hat{H}^{i+1} x_t + (I - \hat{H}^{i+1}) \mu_x^{(t)}]
\]

(26)

where \( t_\pi \) is a selector vector that identifies the position of inflation, \( \pi_t \), in \( x_t \).

The second subsection discusses the interpretation and properties of residual estimates of term premium. Residual estimates are constructed as the difference between observed bond yields and VAR-based constructions of expected future short rates:

\[
\hat{\theta}^R_{n,t} = r_{n,t} - \frac{1}{n} \sum_{i=0}^{n-1} i'_1 [\hat{H}^i x_t + (I - \hat{H}^i) \mu_x^{(t)}]
\]

(27)

By construction, for VAR proxies of agent expectations that imply a relatively small term structure role for expected future short rates, movements of
This approach, also followed by Bernanke et al. (1997), excludes term premium from ex ante real rate constructions. An important empirical issue discussed in this section is the extent to which movements in residual estimates of term premiums are driven by variation in actual term premiums rather than by time variation in specification errors.

4.1. Predictions of the expected short rate component of yields

The effects of alternative specifications of endpoints will often be negligible in short-horizon forecasts. This is demonstrated in Fig. 6, where the three panels display model predictions of expected short rate components of 3-month bond yields under each of the three VAR specifications. The plotted series are concatenations of predictions generated for each month in the 1966–1997 sample. That is, in each month, the relevant VAR model is conditioned on information available at the beginning of that month. Predicted bond yields are constructed as an average of the VAR model predictions of expected future short rates over the maturity of the bond. As shown, predictions of 3-month rates under the alternative VAR specifications are visually indistinguishable.

The corresponding predictions of expected short-rate components of 10 yr bond yields under the three VAR specifications are shown in Fig. 7. The influential roles of the alternative endpoint specifications are more apparent here than in the 3-month predictions examined above. In the top panel, the effect of the fixed endpoints for variables is that the forecast movements of the 10 yr expected short rate component are excessively damped relative to historical movements in 10 yr bond yields. Conversely, in the middle panel, the problem is reversed where the predicted short-rate component of the 10 yr bond yield is more volatile than historical data on 10 yr bond yields, because the endpoint of the short-term nominal rate is a weighted moving average of recent movements in the short rate. By contrast, the bottom panel of Fig. 7 indicates that movements of the 10 yr expected short-rate component predicted by the shifting endpoints model most closely track the contours of the historical data on 10 yr bond yields.

As noted in the introduction, an important role of expectations in many descriptions of the transmission of monetary policy is the market translation of current and anticipated policies into real rates on long-term assets, associated with market valuations of tangible wealth and the cost of borrowed funds for durable expenditures. Ex ante 10 yr real rates predicted by the three VAR models are shown in Fig. 8. Here, tax rates on earnings from Treasuries are ignored and the predicted 10 yr inflation rate is subtracted from the predicted 10 yr average of expected short rates.21

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21 This approach, also followed by Bernanke et al. (1997), excludes term premium from ex ante real rate constructions.
‘Stylized facts’ based on the properties of fixed endpoint VARs are often encountered in discussions of monetary policy.\textsuperscript{22} To illustrate the fragile nature of regularities based on mean-reverting VARs, scaled plots of the predicted

\textsuperscript{22}See, for example, Christiano et al. (1996) and Evans and Marshall (1997).
3-month bond rate are also included in the panels of Fig. 8. In an intriguing interpretation of the Bernanke and Blinder (1992) finding of strong effects of the short-term nominal interest rate on output, Fuhrer and Moore (1995) suggest that the short-term nominal rate is a close proxy for the long-term real rate. Indeed, this correspondence is reproduced based on the fixed endpoints VAR in the top panel of Fig. 8; the simple correlation between the predicted 10 yr real
rate and the predicted 3-month nominal rate is 0.95. However, this close relationship is not maintained in the predicted real rates generated by the remaining VARs. The correlation of the 3-month rate with the real rate predicted by the moving endpoints specification falls to 0.42, and the correlation with the 10 yr real rate predictions of the shifting endpoints specification is 0.43.

The movements of the ex ante 10 yr real rates in the middle and lower panels of Fig. 8 are also much more volatile than those of the real rate predicted by the fixed endpoints specification. For example, the scale of the vertical axis for the real rate predicted by the moving average endpoints model in the middle panel is
three times the scale of the real rate predicted by the fixed endpoints model in the top panel.

An important determinant of differences among the real rates predicted by the three VAR specifications is the implied responsiveness of the perceived inflation target and, thus, expected long-run inflation to actual inflation. Of course, in the first panel, the perceived inflation target corresponds to the inflation endpoint which is constant regardless of the course of actual inflation. This implausible assumption is replaced in the middle panel by the moving average inflation endpoint. This inflation endpoint closely tracks actual inflation. Thus, the ex ante real rate is quite low in the high-inflation years of the 1970s and remains relatively high throughout much of the 1980s and 1990s as expected long-run inflation quickly falls in the early 1980s, tracking the relatively prompt reduction in historical inflation. By contrast, expected long-run inflation in the shifting endpoints model remains elevated in the early 1980s, as supported by available survey evidence on long-run expectations. Consequently, the shifting endpoints model indicates several episodes of relatively low real rates in the 1980s when nominal bond rate predictions fell faster than the cautious downward revisions of long-run inflation expectations.

As shown earlier, the long-run forecasts of nominal interest rates and inflation rates based on a fixed endpoints VAR appear to have little correspondence with the historical movements of available measurements of agents’ long-horizon forecasts, such as long-maturity bonds and surveys of long-horizon inflation and market expectations embedded in long-maturity bond yields. This is not to suggest that fixed endpoints VARs are not useful descriptors, especially as atheoretic summaries of short-term dynamic associations, but use of fixed endpoints models in long-horizon predictions of nonstationary behavior is likely to generate misleading regularities.

4.2. Residual estimates of term premium

The term premium component of bond yields, also known as the ‘liquidity’ or ‘risk’ premium, is the component of bond yields not accounted for by movements in expected short rates. In empirical studies employing the expectations hypothesis, such as Bernanke et al. (1997), it is not uncommon to extract ‘residual’ estimates of term premiums by the difference between observed bond yields and predictions of the expected short rate component of bond yields, where, as in Section 4.1, the latter are formed as an average of expected future short rates approximated by forecasts from an empirical model of short rates. As illustrated in this section, these estimates are sensitive to the specification of the particular model used to generate short rate predictions.

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23 A rapid fall of expected inflation in the early 1980s also appears to be consistent with results in Fuhrer (1996) where agents infer the policy target for inflation without learning lags.
If the model in question does not produce forecasts that match those of agents, then the ‘residual’ term premiums also contain misspecification errors between the agent expectations and model-based proxies of the agent expectations. In other words, large movements in residual estimates of term premiums may not be indicative of variation in actual term premiums but misspecifications of the model used to describe agent forecasts of expected future short rates.

The residual estimate of the term premium is based on (6) but with market expectations of short rates replaced by forecasts of short rates from an estimated VAR describing dynamics of the vector of state variables. For notational convenience, let $\hat{r}_{t+i}$ represent the VAR-based conditional prediction of $r_{t+i}$. Let $\hat{\theta}^R_{n,t}$ denote the residual estimate of the term premium and $\theta^T_{n,t}$ denote the theory-based term premium as defined in Eq. (7).

$$\hat{\theta}^R_{n,t} = r_{n,t} - \frac{1}{n} \sum_{i=0}^{n-1} \hat{r}_{t+i}$$

$$= \left[ \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i} - \frac{1}{n} \sum_{i=0}^{n-1} \hat{r}_{t+i} \right] + \theta^T_{n,t}. \quad (28)$$

Thus, the residual estimate contains two components: one component is a specification error reflecting the gap between market expectations of future short rates and empirical predictions of the expected short-rate component of the bond yield and the second component is the theory-based term premium, $\theta^T_{n,t}$. As shown earlier, although near-term forecasts of short rates from a representative collection of empirical specifications may be similar, the same is not true for long-horizon forecasts. Thus, small variations in the specifications of forecast models can result in large differences in long-horizon forecasts. Furthermore, if empirical forecasts do not replicate market expectations, residual estimates of the term premium will contain a potentially large component of specification error in addition to the actual term premium.

Residual estimates of the term premium for 10 yr bonds, as predicted by the three VAR specifications of the forecast model of agents, are shown in Fig. 9. One important result is immediately obvious. Time variation in residual estimates of term premiums depends critically on the specification of the VAR used to proxy market expectations. The top panel of shows the residual estimate obtained when the VAR specification with fixed endpoints is used to approximate market expectations. For this specification, almost all of the variation in bond yields comes from temporal variation in the term premiums. This view of the term structure was taken by Shiller et al. (1983) who concluded: ‘Variations in risk premiums are so large as to destroy any information in the term structure about future interest rates.’ Researchers that use fixed endpoints VARs to approximate market expectations are likely to conclude that the term structure of interest rates is not a reliable transmission channel of monetary policy. For policy transmission to work through long rates, proponents of fixed endpoint
VARs would have to believe that the transmission mechanism works indirectly, through the effects of short-rate expectations on term premiums.

The middle panel shows the residual estimate based on the VAR specification with moving average endpoints. Time variation in estimated term premiums are cyclical. However, this cyclicality merely reflects the cyclical spread between 10 yr bond yields and short rates. Short rate predictions from the moving
average VAR are very close to current short rates. Thus, the component of bond yields that is based on long averages of expected short rates will track movements in short rates quite closely. Researchers that use moving average endpoint VARs are likely to conclude that long-term interest rates are too smooth if they believe that term premiums should be constant. This view of the term structure was expressed by Campbell and Shiller (1991), who concluded that ‘long rates underreact to short-term interest rates.’ Alternatively, researchers that use moving average endpoint VARs may support arguments that term premiums vary considerably in a countercyclical fashion — in particular, that residual estimates of term premiums move with the long-short yield spread.

The bottom panel shows the residual estimate based on the VAR specification with shifting endpoints. Time-variation in term premiums is much smaller when the shifting endpoints VAR is used to approximate market expectations. Variation in bond yields largely reflects current and expected movements in short rates. With bond traders’ expectations approximated by the shifting endpoints VAR, results support the hypothesis that levels of expected short-term rates are a principal source of time variation in bond rates.

Thus, the shifting endpoints model suggests that the lion’s share of historical movements in yields reflects changing market expectations for future short rates. Many previous studies which attributed substantial variation in bond yields to term premiums were based on VAR specifications with either fixed or moving average endpoints. Although these two VAR specifications have been used frequently in past empirical work, the shifting endpoints VAR comes closer to matching observable long-horizon market expectations. Thus, this VAR specification appears to be a preferred empirical proxy for market expectations.

5. Concluding remarks

In many macroeconomic models, an important transmission channel of monetary policy is variation in the value of wealth and the cost of borrowed funds due to policy-induced changes in long-term interest rates. Because most central banks directly control interest rates only in short-term banking markets, this implies that monetary policy is conducted more by auction market perceptions of current and anticipated policy actions than by recent activities of the central bank trading desk. Consequently, auction market prices are monitored by policy authorities and observers, not only because quotations are available on a more timely basis than other measurements of economic activity but also to discern market expectations.

As noted earlier, this stylized characterization of monetary policy transmission rests on a number of assumptions, of which the most beleaguered is the expectations hypothesis. Not only are various implications of the expectations hypothesis routinely rejected in postwar empirical literature but, as illustrated in
this paper, multiperiod predictions of standard time series models of short-term interest rates provide relative poor tracking estimates of historical bond rates. The difficulty, of course, is that researchers cannot reject the hypothesis that bond yields are nonstationary for samples that include the 1970s and 1980s. This is noteworthy because, by the Fisher equation, an important component of bond rates is expected inflation, and nonstationarity is not rejected also for postwar rates of inflation in consumer price indexes. Further, if the stochastic process of the short rate controlled by the policy authority is sufficient under the expectations hypothesis to explain much of the variation in bond yields, then the model of the short rate process needs to capture the response of the short policy rate to the behavior of inflation. The approach adopted in this paper is to represent bond trader expectations by small VARs that accommodate relationships among expectations of inflation and of interest rates.

A significant departure in this paper, however, is to represent market expectations by linear expansions around long-run expectations, termed ‘endpoints’. These endpoints may be fixed, may shift, or may be nonstationary. All models in this class can be written as endpoint-reverting. An important empirical finding, shown in various ways in this paper, is that the estimated eigenvalues of endpoint-reverting VARs are relatively unimportant for long-horizon predictions, such as long-maturity bonds, because all endpoints, fixed, shifting, or nonstationary, are reached relatively early in the forecast horizon.

By contrast, the specification of long-horizon expectations is of pivotal importance. Standard VARs offer two options: that variables are mean reverting or difference stationary. The mean-reverting VAR is the workhorse of modern macroeconomic analysis, and many empirical features of this specification have become stylized facts in analyses of monetary policy. Unfortunately, the endpoints of VAR variables are fixed in this specification, with the unrealistic implication that market perceptions of the long-run inflation target of policy are independent of actual inflation. This ‘fixed’ endpoints specification provides nearly constant predictions of long-horizon inflation and the expected short rate component of long bond yields. Consequently, the implied term premiums of bonds that are required to match historical bond rates swamp the forward short rate predictions of the mean-reverting VAR. Thus, by default under fixed endpoints, the main channel for short rate expectations to influence bond yields is through time-varying term premiums. This channel, however, requires that the level of the short rate is a source of variation in term premiums — such as the Cox–Ingersoll–Ross square-root specification of short rate volatility.

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24 As noted by Kozicki and Tinsley (1997), including long-maturity rates into the state vector, such as proposed by many multi-factor term structure models, need not eliminate the problem of fixed endpoints in long-horizon forecasts because the long-horizon expectations of these variables also remain fixed.
Turning to the other conventional specification, the endpoints of the difference-stationary VAR are moving averages of difference-stationary states and, as a result, are also I(1). As illustrated earlier, these ‘moving average’ endpoints move closely with realizations of state variables and provide poor predictions of long-horizon expectations and long-maturity yields under constant term premium assumptions.

Not all nonstationary variables are I(1), and the preferred specification in this paper is that endpoints of expectations shift if agents detect shifts in the long-run targets of monetary policy. By the Fisher equation, and assuming after-tax real rates are stationary, the tax-adjusted endpoint of nominal interest rates is linked to the long-run policy target for inflation. However, important results of this paper are that bond rates are driven by market perceptions of policy targets and there appear to be lengthy learning lags between historical shifts in indicators of policy targets, such as the inflation changepoints used here, and shifts as detected by market agents in ‘real time’.

An additional issue is whether or not the empirical results presented in this paper are robust to the construction of the shifting endpoints. Other approaches being pursued by the authors include: fixed-weight aggregation over simulated forecast newsletters; Kalman filters of structural time series models; and inference of forecast endpoints from surveys of short-horizon expectations. None of the alternatives appear to undermine several basic conclusions: First, in contrast to the long-horizon forecast implications of fixed endpoint VARs, forecasts of expected short-term interest rates from VARs with shifting endpoints can explain a substantial portion of the variability of long-term bond rates. Second, long-horizon inflation forecasts generated by shifting endpoint VARs provide a much closer match to survey measures of expected long-run inflation than do forecasts provided by the conventional stationary and difference-stationary VAR models. Third, as discussed in Rudebusch (1998), the monetary policy reaction rules in many conventional VAR models are not likely to be stable over lengthy samples, reflecting episodic changes in the stabilization policies of central banks.

For the purpose of capturing episodic changes in long-run equilibria, there are several alternatives to a shifting intercepts specification, including multiple regimes models and stochastic coefficient specifications. However, the shifting endpoints model is relatively simple to implement and can provide useful economic insights into plausible and significant sources of nonstationarity in agent expectations.

Finally, models that represent how rational agents cope with limited and heterogeneous perceptions of the economy, such as Sargent (1993), Brock and Durlauf (1995), and Kurz (1998), provide explicit interpretations of nonstationarity and endogenous uncertainty. The model of ‘real time’ detections of changepoints discussed in this paper is a rudimentary step toward an endogenous description of agent ignorance.
References


Brock, W., Hommes, C., 1996. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. SSRI 9621, University of Wisconsin at Madison.


