Financial returns and efficiency as seen by an artificial technical analyst

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Abstract

We introduce trading rules which are selected by an artificially intelligent agent who learns from experience — an Artificial Technical Analyst. These rules restrict the data-mining concerns associated with the use of ‘simple’ technical trading rules as model evaluation devices and are good at recognising subtle regularities in return processes. The relationship between the efficiency of financial markets and the efficacy of technical analysis is investigated and it is shown that the Artificial Technical Analyst can be used to provide a quantitative measure of market efficiency. We estimate this measure on the DJIA daily index from 1962 to 1986 and draw implications for the optimal behaviour of certain classes of investors. It is also shown that the structure of technical trading rules commonly used is consistent with utility maximisation for risk neutral agents and in a myopic sense even for risk-averse agents. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In the last few years, increasing evidence that technical trading rules can detect non-linearities in financial time series has renewed interest in technical analysis (see e.g. LeBaron, 1998a; Levich and Thomas, 1993; Brock et al., 1992; Neftci, 1991). Based on this evidence, much research effort\(^1\) has also been devoted to examining whether trading rules can be used to evaluate and create improved time-series and theory driven returns models (Hudson et al., 1996; Kho, 1996; Gencay, 1996).

In this type of empirical studies, the term technical analysis is used to refer to the practice of investing according to well-known technical trading rules. However, in other areas of financial theory technical analysis is sometimes defined to be any conditioning of expectations on past prices (Brown and Jennings, 1989; Treynor and Ferguson, 1985). Indeed, noisy asymmetric information models in which rational agents condition on past prices because they reflect (a noisy signal of) private information provide one explanation of why technical analysis is observed. Unfortunately, theoretical models which lead to conditioning consistent with the precise form of observed technical trading rules are currently unavailable: there is as yet no positive model of investment behaviour consistent with the behaviour of real Technical Analysts.

This paper models Technical Analysts as agents whose actions are de facto consistent with observed technical rules. Our terminology therefore will be consistent with that of researchers examining empirical aspects of technical analysis and as such will be more narrow than that of theoretical models. The objective however is not the modelling of Technical Analysts per se; rather, it is to use our model of a Technical Analyst to derive a more sophisticated approach to examining the statistical properties of trading rules. It is somewhat surprising that some studies have found a single arbitrarily selected rule to be ‘effective’ over long periods (e.g. Brock et al.) given that real Technical Analysts use different rules in different times and in different markets. In order to truly evaluate the effectiveness of technical analysis as implemented we need a model of how analysts adapt to the market environment.

We provide such a model by introducing Technical Analysts who are artificially intelligent agents (see e.g. Marimon et al., 1990). In Section 2 technical analysis is introduced in the simple case where agents are fully informed and circumstances in which it may be a rational activity are derived. This is a necessary step for the development of a model of a Technical Analyst who learns from his environment – an Artificial Technical Analyst. This agent chooses amongst technical trading rules and his actions are the outcome of an

\(^{1}\) An exhaustive bibliography for academic research on technical analysis can be found at http://www.econ.com.ac.uk/faculty/skouras/tablibio.htm.
explicit decision problem which formalises the loose notion of what it means for a rule to be ‘good’ or optimal (examples of informal uses of this term are Allen and Karjalainen, 1996; Neely et al., 1996; Pictet et al., 1996; Taylor, 1994; Allen and Phang, 1994; Chiang, 1992; Pau, 1991. As we discuss in Section 3, this formalisation is important because it indicates that an explicit measure of rule optimality can and should be derived from a specific utility maximisation problem and that a rule which is optimal for investors with different objective functions will not typically exist.

A standard application of artificially intelligent agents is to design them so that their actions can reveal interesting aspects of the environment in which they are placed (see e.g. Sargent, 1993, pp. 152–160). In this vein, we will use our Artificial Technical Analysts to reveal certain regularities in financial data. In particular, in Section 4 they will be used to characterise financial series as in Brock et al., and we will show that they can provide sharper characterisations than those based on simple technical trading rules.

In Section 5 we propose a quantitative notion of weak market efficiency which admits measurement of the degree to which a market is weak form efficient. Furthermore, we show that such measurements can be based on the returns obtained by the Artificial Technical Analyst and illustrate with a measurement of the efficiency of the Dow Jones Industrial Average index, interpreted as a proxy for the market portfolio. This is a step in addressing the relationship between market efficiency and the profitability of technical analysis, an issue that has appeared in some of the theoretical literature (e.g. Brown and Jennings, 1989) but is absent from many empirical investigations of technical analysis.

We conclude this paper with a synopsis of our results. The main contribution is the introduction of the Artificial Technical Analyst, a theoretical tool which is shown to be useful in providing corroborating empirical evidence for the view many Technical Analysts hold of econometric returns models and market efficiency: that the models are inadequate for the purpose of making investment decisions and that markets are not always efficient.

2. Technical analysis with full information

At some level of abstraction, technical analysis can be viewed as a methodology for selecting decision rules which determine (conditional on certain events) whether a position in a financial asset will be taken and whether this position should be positive or negative. One important difference between an analyst and a utility maximising investor is that the decision rules the analyst uses do not specify the magnitude of the positions he should take.
These observations lead us to the following description of technical analysis:

Definition 1. Technical analysis is the selection of a function \( d: I_t \rightarrow \Omega \) which maps the information set \( I_t \) at time \( t \) to a space of investment decisions \( \Omega = \{ -s, 0, l \}, l, s \geq 0 \) specifying the size of short, neutral and long positions respectively.

This assumption captures the main structure of technical analysis as traditionally practiced\(^2\) (see e.g. Kaufman, 1978; Murphy, 1986 – the so-called ‘classics’) in that the magnitude of these positions is not specified as a function of \( I_t \). As mentioned in the introduction, this is the definition to technical analysis used in the economics literature on empirical properties of trading rules. However, a different definition is used in the literature on financial equilibrium with asymmetric information where any agent who conditions on past prices is a technical analyst (e.g. Brown and Jennings, 1989; Treynor and Ferguson, 1985). We are not apologetic about this: the reason we do not allow a more general definition is because we wish to examine the properties of observed trading rules which at first sight seem very different to the investment behaviour we would expect from utility maximising agents. Despite their restrictive nature, these rules include the ‘market timing’ rules which have been extensively studied in the literature, particularly since Merton (1981) developed an equilibrium theory applicable to their evaluation.

The information sets on which these rules condition on are usually the realisations of some random variable such as prices, volatility measures, or the volume of trading (Blume et al., 1994) of an asset. Our focus here will be on rules which are defined on the realisation of a truncated history of past prices \( P_t, P_{t-1}, P_{t-N+1} \in \mathbb{R}_+^N \) (this is expedient only in that it simplifies the exposition – our proofs do not depend on it).

Assumption 1. The information set \( I_t \) on which Technical Analysis is based is restricted to a truncated history of past prices \( P_t \). For notational convenience, we use \( E_t(\cdot) \) to refer to \( E(\cdot|P_t) \).

2.1. Technical trading rules and rule classes

Technical analysts change the mappings \( P_t \rightarrow \Omega \) (or technical trading rules) they use and not all of them use the same rules. Nevertheless, rules used are often very similar and seem to belong to certain families of closely related rules, such

\(^2\)In recent years investors have used increasingly sophisticated ways of conditioning their decisions on past prices. Such conditioning is technical analysis in the sense of, for example, Treynor and Ferguson (1985) but not in the ‘traditional’ sense which is the focus of this study.
as the ‘moving average’ or ‘range-break’ family (see Brock et al.). These families belong to even larger families, such as those of ‘trend-following’ or ‘contrarian’ rules (see, for example, Lakonishok et al., 1993). Whilst it is difficult to argue that use of any particular rule is widespread, certain families are certainly very widely used. The distinction between a rule and a family is formalised as a distinction between technical trading rules and technical trading rule classes.

**Definition 2.** A technical trading rule class is a parametric function

\[ D : \mathcal{P}_t \times B \rightarrow \Omega \]

which for each parameter \( c \) in a parameter space \( B \subseteq \mathbb{R}^k \) maps past prices into investment positions.

**Definition 3.** A technical trading rule is an element of a technical trading rule class (indexed by a parameter \( c \in B \))

\[ d_t \equiv D(P_t, c) : \mathcal{P}_t \rightarrow \Omega \]

which determines a unique investment position as a function of past prices.\(^3\)

Notice that the trivial technical trading rule which always has a position as large as possible in the asset (\( d_t = l \)), is identical to the ‘Buy-and-Hold’ strategy which specifies that once some quantity of an asset is bought, this quantity should not be altered and the value of the investment should be allowed to evolve without any form of intervention.

### 2.2. Technical analysis and rationality

Having defined the main concepts required to describe technical analysis, we now attempt to identify investors who would choose to undertake this activity. In particular, we find restrictions on preferences of utility maximising investors which guarantee they behave as if they were Technical Analysts. The purpose of this is to clarify the meaning of ‘optimal technical analysis’ in a full information setting. This concept can then be applied to the more interesting case where optimal behaviour must be learned from experience.

Consider the following simple but classic investment problem. An investor with utility function \( U \) can invest in two assets: A risky asset paying interest \( R_{t+1} = (P_{t+1} - P_t)/P_t \) (random at time \( t \)) and a riskless asset (cash) which pays no interest. He owns wealth \( W_t \) and his objective is to maximise expected utility

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\(^3\) Notice that any set of rule classes \( \{D_i\} \) can be seen as a meta-class itself, where the parameter vector \( c' = \{i, c\} \) determines a specific technical trading rule.
of wealth at the end of the next period by choosing the proportion of wealth $\theta$ invested in the risky asset. We will assume that borrowing and short-selling are allowed but only to a finite extent determined as a multiple of current wealth, so that $\theta$ is constrained to lie in the compact set $[-s, l]$. Expectations $E_t$ are formed on the basis of past prices $P_t$, as dictated by A1.

Formally, the problem is

$$\max_{\theta \in [-s, l]} E_t U(W_{t+1})$$

subject to $W_{t+1} = \theta W_t (1 + R_{t+1}) + (1 - \theta) W_t$,

or equivalently,

$$\max_{\theta \in [-s, l]} E_t U(W_t (1 + \theta R_{t+1})), \quad (1)$$

the solution $\theta^*$ of which can be denoted as a function

$$\theta^*: P_t \rightarrow [-s, l] \quad (2)$$

satisfying

$$\theta^* \equiv \arg\max_{\theta \in [-s, l]} E_t U(W_t (1 + \theta R_{t+1})). \quad (3)$$

In a general utility maximisation setting therefore, the proportion of wealth invested is a function that depends on the conditional distribution of returns on past prices, utility and wealth. Since by definition a technical trading rule may only take three distinct values and cannot be a function of wealth, rational investment behaviour generally differs from technical analysis because it cannot be described by a trading rule.\(^4\)

In the special case that investors are risk-neutral however, their optimisation problem has a bang-bang solution which depends on $\text{sign} \left[ E_t(R_{t+1}) \right]$. Denoting $\theta^*_\text{rn}$ the solution to the risk-neutral investor’s problem and assuming that when expected returns are zero the investor stays out of the market ($E_t(R_{t+1}) = 0 \iff \theta^*_\text{rn} = 0$), then

$$\theta^*_\text{rn}: P_t \rightarrow \{-s, 0, l\},$$

which is consistent with the definition of a technical trading rule.

We have therefore shown that only a risk-neutral investor conditioning on past prices will choose technical trading rules of the form typically encountered

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\(^4\) The inclusion of transaction costs in the objective function cannot easily reverse this fact. Whilst it can make $\theta^*$ a discontinuous function, $\theta^*$ is only a technical trading rule under very unrealistic joint assumptions on utility, the conditional distribution of returns and the nature of the transaction costs. (Except in the trivial case where large transaction costs make the buy-and-hold strategy an optimal decision for a rational investor).
and as defined here. As we shall see in Section 5, technical trading rules are behavioural rules that have attractive properties for certain classes of *boundedly rational* risk averse agents. These properties suggest an explanation of why technical trading rules are used by risk-averse agents.

Our current result is summarised in the following proposition:

**Proposition 1.** For all past prices, wealth and short-selling or borrowing constraints, the solution of (1) for a risk-neutral investor is a technical trading rule.

A direct corollary of this proposition is that expected utility maximisation and technical analysis *are compatible*. This allows us to define a technical analyst as an expected utility maximising investor:

**Definition 4.** A *Technical Analyst* is a risk-neutral investor who solves

\[
\max_{d \in D} d(P_t) \cdot E_t(R_{t+1}),
\]

where \(D\) is a technical trading rule class.

We will let \(R_{t+1}^d \equiv d(P_t) \cdot R_{t+1}\) denote the returns obtained by a technical analyst who uses a rule \(d\). Clearly, different trading rules lead to different expected returns.

### 3. Artificial technical analysts

Having specified what is meant by technical analysis in the case of full information, let us assume henceforth that the technical analyst does not know \(E_t(R_{t+1})\) but has a history of observations of \(P_t\) on the basis of which he must decide his optimal action at time \(t\). This is a similar amount of information to that possessed by econometricians and hence a technical analyst learning his optimal actions in this environment can be modeled as an *artificial intelligent agent* in the sense of Sargent (1993) or Marimon et al. (1990). In this section, we will propose a ‘reasonable’ model for how a Technical Analyst might try and learn his optimal actions. The term *Artificial Technical Analyst* will refer to an agent who attempts to make the decisions of a Technical Analyst but who is equipped with an explicit ‘reasonable’ mechanism for learning optimal actions rather than rational expectations.

#### 3.1. Parametrising Analysts’ learning

Typically, the learning technique of an artificial agent is similar to that of an econometrician. In the context of this paper, the agent might learn the solution
to (4) by selecting a forecasting model for $E_t(R_{t+1})$ from some parametric class (e.g. GARCH-M). This selection is typically made according to some standard statistical estimation method such as least squares or quasi-maximum likelihood. The artificial adaptive agent then chooses an action which would be optimal if $E_t(R_{t+1})$ were in fact what the forecasting model predicts.

Whilst for some applications this may be a useful approach, we are forced to depart from this methodology somewhat due to the fact that there is significant empirical evidence that statistical fitness criteria can be misleading when applied to decision problems such as that of the Technical Analyst. For example, Kandel and Stambaugh (1996) show that statistical fitness criteria are not necessarily good guides for whether a regression model is useful to a rational (Bayesian) investor. Taylor (1994) finds that trading based on a channel trading rule outperforms a trading rule based on ARIMA forecasts chosen to minimise in-sample least squares because the former is able to predict sign changes more effectively than the latter.\(^5\) More generally, Leitch and Tanner (1991) show that standard measures of predictor performance are bad guides for the ability of a predictor to discern sign changes of the underlying variable.\(^6\)

These empirical considerations suggest that any reasonable model of analysts’ learning must take his loss function into account. One way to achieve this would be to create a Bayesian Artificial Technical Analyst but this would require specification of a prior on the conditional distribution of returns which might be very difficult. Instead, we confine the Artificial Technical Analyst to a frequentist perspective and calculate an estimator $\hat{d}$ for the trading rule solving (4) given a specified technical trading rule class $D$. This can be viewed as a decision

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\(^5\) Under some assumptions on the underlying processes, the technical analyst is interested primarily in the sign of $R_{t+1}$ rather than in its actual value. In particular, the value of $R_{t+1}$ is irrelevant for his decision problem if $\text{sign}(R_{t+1})$ is known with certainty, or if $\text{sign}(R_{t+1})$ and $|R_{t+1}|$ are independent. In these circumstances, a prediction which is formulated to take into account the purpose for which it will be used is likely to be accurate in terms of a sign-based metric. Satchell and Timmerman (1995) show that, without severe restrictions on the underlying series, least square metrics are not directly related to sign-based metrics.

\(^6\) A number of studies of technical trading implicitly or explicitly assume away the possibility that there exists a non-monotonic relationship between the accuracy of a prediction in terms of a metric based on least squares and a metric based on the profit maximisation. Examples are Allen and Taylor (1990), Curcio and Goodhart (1991) and Arthur et al. (1996) who reward agents in an artificial stockmarket according to traditional measures of predictive accuracy. When the assumption is made explicit its significance is usually relegated to a footnote, as in Allen and Taylor (1990, fn. p. 58), ‘our analysis has been conducted entirely in terms of the accuracy of chartist forecasts and not in terms of their profitability or ‘economic value’ although one would expect a close correlation between the two’. As we have argued however, the preceding statement is unfounded and results of such studies should be interpreted with caution.
theoretic approach in that learning about the underlying stochastic environment is replaced with the task of learning about the optimal decision.\textsuperscript{7}

One convenient estimator of the solution to (4) is given by the solution to the following simple in-sample analogue to (4):

$$\max_{c \in B} \sum_{i=1}^{t-1} D(P_i, c) \cdot R_{i+1},$$

where $B$ is some parameter space determining the choice set of technical trading rules. If $(1/m)\sum_{i=1}^{t-1} D(P_i, c) \cdot R_{i+1}$ converges uniformly to $E[D(P_t, c) \cdot R_{t+1}]$ almost surely as $m \to \infty$, under certain regularity conditions\textsuperscript{8} it is also the case that the maximum on $B$ of the former expression converges to the maximum of the latter almost surely as $m \to \infty$, so this estimator is consistent.

Next we choose $D$ so as to impose some restrictions on the solution to (5) that allow regularities of the in-sample period to be captured. Having no theory to guide us on how to make this choice it is reasonable to use empirically observed rule classes $D^\circ$. We may therefore define an Artificial Technical Analyst as follows:

**Definition 5.** An Artificial Technical Analyst is an agent who solves

$$\max_{c \in B} \sum_{i=1}^{t-1} D^\circ(P_i, c) \cdot R_{i+1},$$

where $D^\circ$ is an empirically observed trading rule class.

Turn now to an example illustrating the mechanics of this agent which will be useful in subsequent sections.

### 3.1.1. Example: Choices of the Artificial Technical Analyst determine the optimal moving average rule

The moving average rule class is one of the most popular rule classes used by technical analysts and has appeared in most published studies of technical analysis. For these reasons, we will use it to illustrate how an Artificial Technical Analyst might operate if this is the set from which he chooses rules. Let us begin with a definition\textsuperscript{9} of this class:

\begin{itemize}
  \item \textsuperscript{7} Brandt (1999) recently proposed a related approach for estimating optimal investment decisions but which is applicable only to risk-averse agents since it relies on non-parametric estimation of smooth first-order conditions.
  \item \textsuperscript{8} Such conditions are provided in Skouras (1998).
  \item \textsuperscript{9} As defined, the moving average class is a slightly restricted version of what Brock et al. (1991, 1992) refer to as the ‘variable length moving average class’ (in particular, the restriction arises from the fact that the short moving average is restricted to have length 1).
\end{itemize}
U was discretised to
U unh 0, 0.005, 0.01, 0.015, 0.02
N. This discretisation allowed us to solve (7) by trying all dim (N) \* dim (\( \Phi \)) = 1000 points composing the solution space in each of the 6157 recursions. More sophisticated search methods could lead to more intelligent Artificial Technical Analysts but such niceties do not seem necessary when \( D^o \) is as narrow as it is in this example.

These data correspond to the third subperiod used by Brock et al. and to most of the data used by Gencay (1996).

Definition 6. The Moving Average technical trading rule class \( MA(P_t, c) \) is an (empirically observed) technical trading rule class such that

\[
MA(P_t, c) = \begin{cases} 
  l & \text{if } P_t \geq (1 + \phi) \frac{\sum_{i=0}^{m} P_{t-i}}{m+1} \\
  0 & \text{if } (1 - \phi) \frac{\sum_{i=0}^{m} P_{t-i}}{m+1} \leq P_t \leq (1 + \phi) \frac{\sum_{i=0}^{m} P_{t-i}}{m+1} \\
  -s & \text{if } P_t < (1 - \phi) \frac{\sum_{i=0}^{m} P_{t-i}}{m+1}
\end{cases}
\]

(7)

where

\[
P_t = [P_t, P_{t-1}, \ldots, P_{t-N}],
\]

\[
c = (m, \phi),
\]

\[
B = \{M, \Phi\},
\]

\[
M = \{1, 2, \ldots, M\} \text{ is the 'memory' of the rule,}
\]

\[
\Phi = \{\phi: 0 \leq \phi \leq \Phi\} \text{ is the 'filter' (or bandwidth) of the rule.}
\]

Now if \( D^o = MA(P_t, c) \), (6) becomes

\[
\max_{m \in M, \phi \in \Phi} \sum_{i=t-N}^{t-1} MA(P_i, m, \phi) R_{i+1}.
\]

(8)

An Artificial Technical Analyst learning technical trading rules by solving (8) uses \( N \) daily observations of \( MA(P_t, m, \phi) R_{i+1} \) derived from \( N + M + 1 \) observations of prices \( P_t \). Let us assume \( M = 200 \) (a 'standard' value for the longest moving average of interest), \( N = 250 \) (approximately a year's worth of data is used for estimation), that \( s = l = 1 \) (position size cannot exceed current wealth) and \( \Phi = 0.02 \) (another 'standard'). How do the trading rules used by this Artificial Technical Analyst behave when \( P_t \) are draws from the Dow Jones Industrial Average index? Fig. 1 plots the parameters \((m, \phi)\) indexing trading rules estimated according to this method,\(^{10}\) where the parameters were estimated recursively on a daily level from \( t = 1/6/1962 \) to \( 31/12/1986.\(^{11}\)

\(^{10}\) \( \Phi \) was discretised to \( \Phi = \{0, 0.005, 0.01, 0.015, 0.02\} \). This discretisation allowed us to solve (7) by trying all \( \text{dim (N)} \times \text{dim (\( \Phi \))} = 1000 \) points composing the solution space in each of the 6157 recursions. More sophisticated search methods could lead to more intelligent Artificial Technical Analysts but such niceties do not seem necessary when \( D^o \) is as narrow as it is in this example.

\(^{11}\) These data correspond to the third subperiod used by Brock et al. and to most of the data used by Gencay (1996).
It is difficult to interpret the sharp discontinuities observed in the sequence of estimated parameters. However, they strongly suggest there is additional structure in the series which a ‘more intelligent’ Artificial Technical Analyst (one with a more sophisticated learning mechanism) might be able to identify.

4. Artificial Technical Analysts and the distribution of returns in financial markets

Much of the literature on technical trading rules has asked whether popular types of rules will yield returns in excess of what would be expected under some null hypothesis on the distribution of returns (e.g. Brock et al., 1992; Levich and Thomas, 1993; Neftci, 1991, etc.). The rules considered are typically selections that are meant to be ‘representative’ for a plausible and widely used rule class. However, the fact that they are chosen according to non-rigorous and often implicit criteria makes results drawn from them subject to standard data-mining criticisms which diminish their forcefulness. This is a problem that is avoided if the rules considered are the choices of an Artificial Technical Analyst which are
by construction explicit and can be expected to be robust with respect to reasonable variations in the agent’s design.\textsuperscript{12}

In Section 4.1 we show with reference to the study of Brock et al. that such criticisms are \textit{not trivial}. In Section 4.2 we show that the Artificial Technical Analyst can be used to construct more powerful tests of certain hypotheses regarding the distribution of returns.

4.1. The variation of returns across rules

Clearly, we must rely on empirical evidence to see whether rule returns are correlated closely enough within a class to justify using a few rules as proxies for the behaviour of the class as a whole.\textsuperscript{13} Fig. 2 shows the returns accruing to each rule belonging to the moving average class if it were applied on the DJIA index throughout the period considered (1962–mid-1986).

This figure illustrates the inadequacies of the ad hoc approach whereby specific rules are used as proxies for the distribution of expected returns of a whole class. Notice in particular two highly prominent facts evident in this figure.

Firstly, while all rules earn positive returns, the mean return of the best rule in the class is \textbf{1270} times larger than that of the worst rule. Since the means are taken from samples with more than 6000 observations, it is unlikely that sampling uncertainty can account for these differences. We must conclude that returns accruing to rules \textit{within the same class} vary very significantly.

Secondly, the expected returns of rules display significant variation even \textit{within local regions} of the parameter space. The best rule is the three period average with no filter $MA(3,0)$ and the worst is the four period moving average with a 2\% filter $MA(4, 0.02)$. This is important because most researchers choose to calculate returns for a few rules sampled evenly from the space of

\textsuperscript{12} Of course, a degree of arbitrariness remains in our selection of the rule class to be tested. However, we have already mentioned that there exists much stronger empirical evidence on the basis of which to choose a rule class than for any specific rule. The arbitrariness involved in the specification of learning schemes may be an additional problem, but overall such choices are generally considered to be robust and are certainly more robust than choices of arbitrary rules.

\textsuperscript{13} That this is the case is suggested by Brock et al. who write that ‘Recent results in LeBaron (1990) [now available in LeBaron (1998b)] for foreign exchange markets suggest that the results are not sensitive to the actual lengths of the rules used. We have replicated some of those results for the Dow index’ (p.1734). The ‘recent results’ to which Brock et al. refer are a plot of a certain statistic of 10 rules. Apart from the fact that 10 rules constitute a small sample, the minimum statistic is almost half the size of the maximum statistic – so it is not entirely clear that these results support the claim made.

On the other hand, the conclusions Brock et al. draw happen to be valid since the rules they chose generated returns which were slightly \textit{lower} than the average of the class they considered.
Fig. 2. Mean returns (over 6157 periods) of each rule as a function of its parameters $m$ and $\phi$.

all rules, reflecting the unfounded implicit assumption that rules are ‘locally’ representative.

Taken together these two observations imply that ad hoc rules cannot be the basis for convincing tests of specifications of models for returns. Rules must be selected according to an explicit procedure (such as that of the Artificial Technical Analyst) which is justifiable on theoretical grounds.

4.2. Artificial Technical Analysts’ rules generate ‘above average’ profits

We now show that ‘representative’ mean returns are likely to be smaller than those obtained by an Artificial Technical Analyst and hence will be less effective in rejecting hypotheses on the distribution of returns. The reason for this is that a well designed Artificial Technical Analyst who chooses a technical trading rule from some class should have learned to make a better-than-average choice of $d$. Imposing the use of a ‘representative’ rule by an Artificial Technical Analyst would be the analogue of estimating a parametric model for a time series by choosing the parameters which have average rather than minimum least-squared errors.

Table 1 serves to empirically confirm this reasoning. Utilising some of the information in Table V of Brock et al. (1991)\(^{14}\) it indicates that the results reported there on the basis of various fixed rules are much weaker than those

\(^{14}\) Note that Brock et al. (1992) reproduce only a part of this table.
Table 1
The first column of this table indicates which rule is being used. The parameters in parentheses constitute specifications of the memory and filter of the moving average rule used. The second and third columns indicate the number of days in which the rule was long or short respectively and the fourth and fifth the mean return on those days. The sixth column reports the proportion of days on which the rules made profits conditional on being long and the seventh column provides a similar statistic for days when they were short. t-statistics applying a test used in Brock et al. to check significance are provided for some estimates.

<table>
<thead>
<tr>
<th>Rule</th>
<th>N (buy)</th>
<th>N (sell)</th>
<th>Buy</th>
<th>Sell</th>
<th>Buy &gt; 0</th>
<th>Sell &gt; 0</th>
<th>Buy-sell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always long</td>
<td>6157</td>
<td>0</td>
<td>0.00023</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Brock et al. (m, φ)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(50, 0)</td>
<td>3468</td>
<td>2636</td>
<td>0.00036</td>
<td>− 0.0004</td>
<td>0.5167</td>
<td>0.4879</td>
<td>0.00041</td>
</tr>
<tr>
<td>(50, 0.01)</td>
<td>2782</td>
<td>1985</td>
<td>0.00053</td>
<td>0.00003</td>
<td>0.5230</td>
<td>0.4861</td>
<td>0.00049</td>
</tr>
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<td>(150, 0)</td>
<td>3581</td>
<td>2424</td>
<td>0.00037</td>
<td>− 0.00012</td>
<td>0.5205</td>
<td>0.4777</td>
<td>0.00049</td>
</tr>
<tr>
<td>(150, 0.01)</td>
<td>3292</td>
<td>2147</td>
<td>0.00035</td>
<td>− 0.00018</td>
<td>0.5216</td>
<td>0.4742</td>
<td>0.00052</td>
</tr>
<tr>
<td>(200, 0)</td>
<td>3704</td>
<td>2251</td>
<td>0.00037</td>
<td>− 0.00016</td>
<td>0.5173</td>
<td>0.4780</td>
<td>0.00053</td>
</tr>
<tr>
<td>(200, 0.01)</td>
<td>3469</td>
<td>2049</td>
<td>0.00038</td>
<td>− 0.00018</td>
<td>0.5189</td>
<td>0.4763</td>
<td>0.00056</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>0.00037</td>
<td>− 0.00011</td>
<td></td>
<td></td>
<td>0.00048</td>
</tr>
<tr>
<td>ATA</td>
<td>3313</td>
<td>2650</td>
<td>0.00095</td>
<td>− 0.00067</td>
<td>0.5337</td>
<td>0.4675</td>
<td>0.00162</td>
</tr>
<tr>
<td>(d (m^<em>, φ^</em>))_{j=1}</td>
<td></td>
<td></td>
<td>(3.95033)</td>
<td>(− 4.57949)</td>
<td>(7.34848)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

which can be drawn by using the time-varying estimated rule $\hat{\alpha}_t$ derived in Section 3.1.1. That the rules chosen by Brock et al. are ‘average’ across the space of all rules can be verified by inspection of their position in Fig. 2.

The table indicates that all t-ratios are much higher for the optimal rule we have developed than for the simple rules used by Brock et al. This implies that the Artificial Technical Analyst is ‘better’ at detecting the dependencies in the DJIA series since it can be used to reject the hypothesis that the returns are independent draws from some distribution with much greater confidence than that offered by the Brock et al. analysis. We expect that the Artificial Technical Analyst’s rule will be equally powerful as a specification test for particular

---

15 The table also contains information which is sufficient to show that the Cumby and Modest (1987) test for market timing would, if the riskless interest rate were zero, confirm the ability of a technical analyst learning optimal rules to conduct market timing.
models for the time dependency in returns, such as those considered by Brock et al. (AR, GARCH-M, EGARCH). However, we must leave confirmation of this conjecture for future research.

As we have already mentioned, it is inevitable that the creation of an Artificial Technical Analyst involves certain design decisions which themselves raise data-mining issues. We have noted however that such an agent is likely to be robust with respect to such decisions. Furthermore, there is often a very natural way to make these decisions in the context of a specific application. Here for example we have avoided most problems of this form by anchoring our selection of the trading rule class and the data set to the set of Brock et al. Whilst the selection of the length of the optimisation period $N$ is still under our control we set $N = 250$ on the a priori basis that it is the standard rounded approximation to the number of trading days in a year. In the appendices we replicate some of our results for other values of $N$ and thereby provide evidence that this choice has only a marginal effect on our results.\(^{16}\)

In summary, the Artificial Technical Analyst provides a powerful and robust tool for examining the performance of technical trading rules; we therefore propose the use of Artificial Technical Analysts’ rules as the basis for model specification tests. This decreases the probability of obtaining misleading results whilst at the same time delivering more powerful conclusions relative to those obtainable from an analysis of ‘representative’ rules (which are also difficult to identify).

5. Market efficiency and technical trading

It is often stated that ‘If markets are efficient, then (technical) analysis of past price patterns to predict the future will be useless’, (Malkiel, 1992). In this section, we attempt to analyse the relationship between the efficiency of markets and the efficacy of technical analysis, with a view to a formal assessment of this statement.

Currently, only a model-specific notion of efficiency is available, deriving from successive refinements on the definition of Fama (1970). The latest element in this sequence of definitions is that of Latham (1986) according to which a market is E-Efficient (‘E’ for equilibrium) with respect to an information set if and only if its revelation to all market participants would leave both equilibrium prices and investment decisions unchanged. However, there seems to be little consensus as

\(^{16}\)This is hardly surprising given that this parameter operates exactly like the choice of sample size in a recursively estimated econometric model (which also does not usually affect results if – as in this case – it is reasonably large).
to what empirical properties an efficient market should display, partly due to the lack of an accepted equilibrium model for financial markets (LeRoy, 1989; Fama, 1991). Furthermore, market efficiency is only testable in the context of such a model.

We now propose a definition of efficiency which has the advantage that its testability does not hinge on the assumption of a specific equilibrium model, but rather on a model for the behaviour of at least some agents in the market. Our definition is a necessary condition for the market to be E-efficient if we accept any model in which some agents are modelled in this way. It is also consistent with the weak requirement implicit in almost all efficiency notions according to which conditioning on publicly available information does not increase utility. In its very weakest forms, this is interpreted as meaning that once transaction costs are included, no risk-averse agent can increase his utility by attempting to ‘time’ the market. This statement is so weak that some authors (for example LeRoy, 1989; p. 1613 fn) consider it to be non-testable. However, if we assume that the time series of prices is the market clearing equilibrium of an economy with a single risky asset or that we know the market portfolio and have a time series of prices for it, we show that the performance of technical trading rules can be used to construct precisely such a test. For simplicity, the analysis is restricted to the case where the risk-free interest rate is zero and past prices are the information set with respect to which we evaluate efficiency (weak form).

The evaluation of financial market efficiency has an interesting role to play in building useful dynamic asset pricing models. There is now a technology for determining broad conditions such a model must satisfy in order to be consistent with a given sample of data (Hansen and Jagannathan, 1991; Hansen and Richard, 1987). For some purposes it may be crucial that the model used also replicates the efficiency properties of actual markets (e.g. when the model will be used to address issues relating to the role of financial markets in allocating resources or to evaluate the investment performance of various types of agents). We can use the results of this section to judge whether a market is efficient and evaluate whether it is necessary to design models that replicate this feature. Whilst the relationship between technical analysis, market efficiency and dynamic asset pricing models is interesting, it lies beyond the scope of this paper and we must leave it for future research.

We will refer to the version of the efficient market hypothesis that we have alluded to as the Lack of Intertemporal Arbitrage (LIA) Hypothesis and discuss its implications for technical trading rules. It will become evident that this efficiency notion is formulated so that it is consistent with the idea that if technical analysis ‘works’, loosely speaking, then markets must be inefficient. In this sense it formalises the efficiency notion to which many empirical analyses of trading rule returns allude, yet typically leave undefined. A desired property efficiency notions have failed to deliver is a way of quantifying near efficiency. The following definition of LIA allows precisely such a quantification.
Definition 7. The Lack of Intertemporal Arbitrage (LIA) Hypothesis holds for all investors with objective functions in some space $\mathcal{U}$ if their optimal decisions do not depend on past prices.

The size of the space $\mathcal{U}^e$ for which LIA holds can be viewed as a measure of the efficiency of a market. Here we focus on the simple portfolio decision (1) and therefore treat the space $\mathcal{U}$ as a way of imposing restrictions on the form $U$ can take. LIA is confirmed for a class of agents solving (1) if knowledge of past prices does not affect their optimal actions. This is consistent with the idea that current prices reflect all information in past prices that might be of relevance. In this case, LIA requires that for all $P_t$:

$$\arg\max_{\theta \in [-s,1]} E\{U[W_t(1 + \theta R_{t+1})|P_t]\} = \arg\max_{\theta \in [-s,1]} E\{U[W_t(1 + \theta R_{t+1})]\}.$$ (9)

Clearly, LIA imposes restrictions on the joint distribution of returns and past prices. While it does not require these to be independent, it requires that knowledge of past prices does not affect an agent's optimal investment decision. For example, suppose only the third- and higher-order moments of the conditional distribution of returns depend on past prices; then in a market with mean-variance agents, actions will not be affected by knowledge of past prices (LIA holds) even though in a market with more general agents, it might (LIA does not hold). Our definition implies that market efficiency is defined with respect to a class of agents and that the size of this class can therefore be interpreted as a measure of the degree of the market's efficiency. Formally, the degree of efficiency is determined by the 'size' of the space:

$$\mathcal{U}^e \equiv \left\{ \begin{array}{ll} \arg\max_{\theta \in [-s,1]} E\{U[W_t(1 + \theta R_{t+1})]|P_t]\} & \forall P_t \\ U \in \mathcal{U}: & \\ = \arg\max_{\theta \in [-s,1]} E\{U[W_t(1 + \theta R_{t+1})]\} & \end{array} \right\}. $$ (10)

We may interpret the size of $\mathcal{U}^e$ as a measure of near-efficiency: as it increases, fewer agents (distinguished by their utility functions) find past prices useful and hence the market becomes more efficient (with respect to past prices). In particular if for two markets A and B we know that $\mathcal{U}^e_A \subset \mathcal{U}^e_B$, LIA provides a well defined sense in which market A is less efficient than B. Such comparisons are relevant if there exist markets which may be treated as separate on a priori grounds or if we wish to compare the efficiency of a single market during different time periods.

A dual way of describing the degree of LIA efficiency is in terms of the set of distributions $\mathcal{F}_{\mathcal{U}}$ for which LIA efficiency with respect to $\mathcal{U}$ holds. Denoting a joint
distribution of \((R_{t+1}, P_t)\) as \(F\), a marginal of \(R_{t+1}\) as \(F_R\) and a conditional of returns on prices as \(F_R|P\), this set is given by

\[
\mathcal{F}_U \equiv \left\{ \begin{array}{l}
F : \\
\forall P_t, U \in \mathcal{U} \\
= \arg \max_{\theta \in [-s, t]} \mathbb{E}[W_t(1 + \theta R_{t+1})] dF \end{array} \right\}.
\]

(11)

To decide empirically whether a market is efficient with respect to a particular set of objective functions \(\mathcal{U}\) of interest, we can conduct the following hypothesis test:

\[
H_0: F \in \mathcal{F}_U \quad \text{(LIA)},
\]

versus,

\[
H_1: F \notin \mathcal{F}_U \quad \text{(not LIA)}.
\]

As we have already mentioned, we do not interpret a rejection of the null as an indication that the market cannot be modelled as an equilibrium of a standard model. Such a rejection would simply provide evidence against the particular notion of efficiency according to which prices should not be 'useful' to any agent (implicit, for example, in Malkiel's statement with which we opened this section). As we have discussed, this is a very weak notion of efficiency which in a well-defined sense is a necessary condition for E-efficiency.

5.1. Technical trading rules and LIA

5.1.1. The Artificial Technical Analyst provides a condition on rule returns for testing LIA

If for some market the true joint distribution of returns and the past prices \(F^*\) is known, we can check whether LIA holds for utility functions in \(\mathcal{U}\), in some cases even analytically. If \(F^*\) is unknown, one test of the hypothesis described by (12 and 13) could be based on checking whether an estimated model for \(F^*\) is in \(\mathcal{F}_U\). Here we propose an alternative test based on the implications of the null hypothesis for the returns obtained by an Artificial Technical Analyst. This approach is based on the fact that a sufficient condition for LIA to be rejected is that technical trading is preferred over a position that is optimal when the agent does not condition on the history of past prices.\(^{17}\) That this is the case is almost trivial, but is shown formally below:

\(^{17}\)This is stronger than the condition that the technical trading rule is preferred over the buy-and-hold strategy because this strategy in addition to not conditioning on prices does not condition on wealth either.
Proposition 2. Let $\theta^*$ be an investment (as a fraction of wealth) which maximises an investor’s (unconditional) expected utility when the joint distribution of returns and past prices is given by $F$:

$$\theta^* = \arg \max_{\theta \in [-s,1]} \int U[W_t(1 + \theta R_{t+1})] \, dF.$$ 

Assume this investor can increase his expected utility by investing this fraction $\theta^*$ according to a trading rule $d$:

$$\max_{\theta \in [-s,1]} \int U[W_t(1 + \theta^* d(P_t) R_{t+1})] \, dF > \max_{\theta \in [-s,1]} \int U[W_t(1 + \theta R_{t+1})] \, dF.$$ 

Then LIA does not hold, i.e.

$$F \notin \mathcal{F}_u$$

Proof. See Appendix A.

We can use our proposition to reject LIA for a class $\mathcal{U}$ in which $U$ belongs by showing that there exists a technical trading rule $d$ with associated returns $R^d_{t+1}$ such that for all $\theta$:

$$\int U[W_t(1 + \theta R^d_{t+1})] \, dF > \int U[W_t(1 + \theta R_{t+1})] \, dF. \quad (14)$$

Clearly, markets may be inefficient even when technical trading rules do not perform well, but we are not concerned with this here.

5.1.2. The risk-neutral case

In order to implement a test of the null hypothesis (11), we must specify $\mathcal{U}$. Let us begin with the very narrow specification requiring $\mathcal{U}$ to contain only linear functions. Then (14) is equivalent to

$$\mathbb{E}(R^d_{t+1}) > \mathbb{E}(R_{t+1}). \quad (15)$$
Table 2
Note that the last column was calculated conditional on the (false) assumption that returns of rules were normal i.i.d.

<table>
<thead>
<tr>
<th>Returns</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Probability under null</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always long</td>
<td>0.0002334</td>
<td>0.008459</td>
<td>—</td>
</tr>
<tr>
<td>ATA</td>
<td>0.000801</td>
<td>0.008335</td>
<td>8.887e-5</td>
</tr>
</tbody>
</table>

We can use the rules \( \{d_i\}^{6157}_{i=1} \) which were optimal for the Artificial Technical Analyst in Section 3.1.1 and their corresponding returns to test the null hypothesis for the Dow Jones Industrial Average. Referring to Table 2, we find that the probability LIA is not rejected is extremely low.

Hence we can conclude with great confidence that conditioning on past prices is desirable (LIA is rejected) for risk-neutral agents investing for the DJIA index. The set \( \mathcal{U}^e \) of utility functions with respect to which the market is efficient (in the LIA) sense cannot include any linear functions.

5.1.3. The mean-variance case

Let us now consider whether the market is efficient with respect to quadratic utility functions. This may be the case if rules satisfying (15) involve a sufficiently larger variance than market returns to make them less desirable to a mean-variance agent. For example, LeRoy (1989) argues that

\[ \ldots \text{even though the existence of serial dependence in conditional expected returns implies that different formulas for trading bonds and stock will generate different expected returns, because of risk, these alternative trading rules are utility-decreasing relative to the optimal buy-and-hold strategies.} \]

To check whether this is the case, consider the class of quadratic utility functions

\[ \left\{ U: U(W) = aW^2 + bW + c, a \leq 0, -\frac{b}{2a} \geq W \right\}. \]

For all \( U \) in this class, if wealth \( W \) is a random variable, it is the case that \( EU(W) \) is increasing w.r.t. \( E(W) \), and decreasing w.r.t. \( \text{Var}(W) \).

The following proposition shows the somewhat surprising result that if there exists a rule that mean-dominates a long position, then it will also variance dominate it and hence LeRoy’s statement is a logical impossibility in rather general circumstances. The proposition is crucial because it establishes broad circumstances in which the mean-variance case collapses to the risk-neutral case.
Proposition 3. If the trading rule $d_1$ leads to a larger expected return than another rule $d_2$ with positive expected returns, i.e.

$$E(d_1(P_t)R_{t+1}) > E(d_2(P_t)R_{t+1}) \geq 0,$$

and

(a) the second rule is always long (i.e. is the ‘Buy and Hold’ strategy) and long positions are not smaller in absolute value than short positions

$$d_2(P_t) = I \text{ all } P_t,$$

$$l \geq s,$$

or

(b) trading rules have a binary structure and position sizes are symmetric

$$d_1, d_2 \in \{-s, l\},$$

$$l = s.$$

Then the returns from rule 1 have a smaller variance than the returns from rule 2

$$V(d_1(P_t)R_{t+1}) < V(d_2(P_t)R_{t+1}).$$

This proposition is useful for establishing the following corollary, but also because it provides a shortcut to ranking rules by performance in terms of Sharpe Ratios—an exercise attempted by many researchers and practitioners, supposedly as an alternative to ranking by mean returns. In the circumstances indicated however, Sharpe Ratios are inversely related to mean returns and such exercises are often redundant.

Corollary 1. Consider a market in which the unconditional expected returns are positive $E(R_{t+1}) \geq 0$. Suppose agents solving (1) have a tighter constraint on short positions than long positions $l \geq s$. Suppose also that (15) holds so the market is not LIA-efficient for risk neutral agents. Then it will also not be LIA-efficient for mean-variance agents.

Proof. See Appendix A.

Under the very plausible conditions of this corollary the risk-neutral case implies the mean-variance case and therefore mean-variance investors in the DJIA would find knowledge of past prices useful. Indeed, note that Table 2 confirms the implications of this corollary.

5.1.4. The risk-averse case

It is substantially more complicated to use technical trading rules to provide evidence that LIA does not hold when $H$ is a concave class of functions. An
exception arises when market and trading rule returns are normally distributed in which case second order stochastic dominance can be expressed as a function of means and variances.

Proposition 4. Consider a market in which the unconditional returns are normally distributed with positive mean \((E(R_{t+1}) \geq 0)\). Suppose agents solving (1) have a tighter constraint on short positions than long positions \((l \geq s)\). Suppose also that (15) holds, so the market is not LIA-efficient for risk neutral agents and furthermore that this is the case for a trading rule with normally distributed returns. Then it will also not be LIA-efficient for risk-averse agents.

Proof. See Appendix A.

If the assumptions of Proposition 4 are not satisfied, we can reformulate the test for LIA in terms of a second-order stochastic domination criterion of trading rule returns over market returns. This is shown in Proposition 5 below:

Proposition 5. Let \(f_{R_{t+1}}, f_{R'_{t+1}}\) be the marginal densities of market and trading rule returns respectively. Define the function

\[
M(\gamma) = \int_{-\infty}^{\gamma} [f_{R_{t+1}}(x) - f_{R'_{t+1}}(x)] \, dx.
\]  

Suppose agents solving (1) have a tighter constraint on short positions than long positions \((l \geq s)\). If unconditional market returns are positive \((E(R_{t+1}) \geq 0)\) and there exists a trading rule the returns of which are distributed so that

\[
M(\gamma) \geq 0 \quad \forall \gamma,
\]

and \(M(\gamma) > 0\) for at least one \(\gamma\) then the market is not LIA-efficient for any risk-averse agent.

Proof. See Appendix A.

Clearly, if we reject the hypothesis that \(M(\gamma) \leq 0\) which is an implication of LIA, we can also reject LIA. However, the available tests for this hypothesis are not generally applicable or involve huge computational costs (Tolley and Pope, 1988). This obstacle forces us to offer only an informal evaluation of whether LIA can be rejected. Such an evaluation can be conducted by inspecting a plot of the sample version \(\hat{M}(\gamma)\) of \(M(\gamma)\) for the returns of the Artificial Technical Analyst and the Dow Jones Industrial Average.

Observing Fig. 3, we notice that for small \(\gamma\), \(M(\gamma) < 0\), indicating that the minimum returns from the Artificial Technical Analysts’ rule resulted in smaller returns than the minimum market return. This implies that an agent with
a utility function which greatly penalizes very low returns would prefer not to use the trading rule. Hence, even without taking account of sample uncertainty we are unable to reject LIA in the risk-averse case. Taking sample uncertainty into account using a formal statistical procedure cannot reverse this result, since it could only weaken the case against LIA. We conclude that there are risk-averse agents who may be indifferent to information contained in past prices so Malkiel’s statement can be justified albeit in a narrow sense: The set of utility functions $\mathcal{U}$ with respect to which the DIJA is efficient does not include any mean-variance utility functions but includes at least some strictly concave utility functions.

5.2. Efficiency with transaction costs

We now define LIA for the case where an investor faces proportional symmetric transaction costs. In this situation, the set of distributions consistent with LIA is

$$\mathcal{F}_\mathcal{U}^c = \left\{ \begin{array}{l}
\arg \max_{\theta \in [s-t,1]} \int U [W_t (1 + \theta R_{t+1}) - c |\theta - \theta_{-1}|] dF_R |P \\
F: \forall P_t, U \in \mathcal{U},
\end{array} \right\}
\arg \max_{\theta \in [s-t,1]} \int U [W_t (1 + \theta R_{t+1}) - c |\theta - \theta_{-1}|] dF
\right\}
\right\}
\right\}
\right\}
\right\}$$

where $\theta_{-1}$ is the size of yesterday’s position.
Consider an Artificial Technical Analyst choosing rules to maximise the type of objective function in (17). As before, we let \( s = l = 1 \) and the Artificial Technical Analyst uses rules given by (compare to (8)):

\[
\max_{\phi \in \Phi, \theta \in \{ -1, 0, 1 \}} \sum_{i=1}^{t-1} [MA(P_i, m, \phi)R_{i+1} - c|MA(P_i, m, \phi) - \theta_{i-1}|],
\]

(18)

where \( c \) are proportional transaction costs and \( \theta_t \in \{ -1, 0, 1 \} \) is the position at time \( t \).

Beginning with a neutral position \( (\theta_{t-m-1} = 0) \), we can obtain the estimated parameters, estimated trading rules and out-of-sample trading rule returns \( R_{t+1}^* \) at each date for various levels of proportional transaction costs \( c \). Our objective will be to find a test statistic which can be used to reject LIA. With this, we can then determine the level of transaction costs \( c \) for which LIA cannot be rejected at the 5% significance level.\(^{18}\)

This exercise is conceptually similar to that of Cooper and Kaplanis (1994) who try to estimate the level of ‘deadweight costs’ that would explain the home bias in international equity portfolios. Notice that allowing for transaction costs changes our near-equilibrium notion in that we now seek pairs \( \{ U^e, c^e \} \) rather than just \( U^e \) for which LIA holds.

In Table 3 we have tabulated the returns from a rule used by the Artificial Technical Analyst solving (18). The level of costs \( c^e \) below which we can reject the hypothesis that \( F^* \in \mathcal{F}_U \) for \( \forall U \) including risk-neutral or mean-variance utility functions is represented by the line dividing Table 3. Notice that this table incorporates the special case of zero transaction costs which was reported in Table 2.

The table indicates that at the 5% level of significance, LIA will not be rejected for \( c \geq c^e = 0.06\% \). The mean return of the optimal rule remains larger for \( c \leq 0.09\% \) (but not for the usual confidence margin).\(^{20}\) These levels of costs make it tempting to argue that with today’s cost conditions\(^{21}\) LIA is rejected for a broad class of investors. However, costs were certainly larger at the beginning of the sample we have considered. How large the decrease in transaction costs

\(^{18}\) Note that as defined, the cost of switching from a long to a short position and vice versa is \( 2c \).

\(^{19}\) It is important to note that Proposition 3 can be extended to the case of transaction costs if these are small enough. The same is not true for Proposition 1 if transaction costs are proportional.

\(^{20}\) Note that Proposition 3a can be extended to the case with transaction costs if these are ‘small enough’. Table 3 indicates a mean-variance investor should probably prefer the ATA’s rule.

\(^{21}\) An investor will access to a discount broker, e.g. via e-mail, can purchase 1000 shares of a company listed on the NYSE for a $9.95 fee. However, micro-structure frictions such as bid-ask spreads may be more important.
Table 3

The first column indicates which rule is under consideration. The next two columns indicate the empirical mean and the standard deviation of the rules’ returns. The fourth column shows the probability (under the assumption of normal distributions) that the mean returns from a specific rule were smaller or equal to the mean market returns. The final column shows the cumulative returns from each strategy during the whole time period.

<table>
<thead>
<tr>
<th>Rules</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Prob. under null</th>
<th>( P \frac{\mu_1 - \mu_2}{\sqrt{\text{Var}_1 + \text{Var}_2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always long</td>
<td>0.0002334</td>
<td>0.008459</td>
<td>—</td>
<td>2.378</td>
</tr>
<tr>
<td>c = 0</td>
<td>0.000801</td>
<td>0.008335</td>
<td>8.887e-05</td>
<td>110.7</td>
</tr>
<tr>
<td>c = 0.0001</td>
<td>0.0007304</td>
<td>0.008328</td>
<td>0.0005109</td>
<td>71.41</td>
</tr>
<tr>
<td>c = 0.0002</td>
<td>0.0006911</td>
<td>0.008321</td>
<td>0.001238</td>
<td>55.88</td>
</tr>
<tr>
<td>c = 0.0003</td>
<td>0.0006196</td>
<td>0.008318</td>
<td>0.00533</td>
<td>35.63</td>
</tr>
<tr>
<td>c = 0.0004</td>
<td>0.0005508</td>
<td>0.008311</td>
<td>0.01788</td>
<td>22.99</td>
</tr>
<tr>
<td>c = 0.0005</td>
<td>0.0004893</td>
<td>0.008305</td>
<td>0.0452</td>
<td>15.44</td>
</tr>
<tr>
<td>c = 0.0006</td>
<td>0.0004707</td>
<td>0.00829</td>
<td>0.058</td>
<td>13.67</td>
</tr>
<tr>
<td>c = 0.0007</td>
<td>0.0003741</td>
<td>0.00828</td>
<td>0.1755</td>
<td>7.101</td>
</tr>
<tr>
<td>c = 0.0008</td>
<td>0.0003099</td>
<td>0.008273</td>
<td>0.3061</td>
<td>4.456</td>
</tr>
<tr>
<td>c = 0.0009</td>
<td>0.0002763</td>
<td>0.008205</td>
<td>0.3876</td>
<td>3.453</td>
</tr>
<tr>
<td>c = 0.001</td>
<td>0.0002191</td>
<td>0.008215</td>
<td>0.5379</td>
<td>2.13</td>
</tr>
</tbody>
</table>

has been and how it has affected different types of investors is a question which is beyond the scope of this paper and which we do not attempt to answer. It is probably fair to say that few agents would face transaction costs that were so low during most of the sample.

Bessembinder and Chan (1998) have independently performed a similar exercise on the same data to find the maximum level of costs for which a certain investment strategy beats Buy-and-Hold and arrive at a much higher estimate. This is due to the fact that they allow their strategy to take long positions double the size of that allowed for what they call the Buy-and-Hold strategy. Since the strategy is usually long and mean returns in the market are positive, clearly the strategy does better than Buy-and-Hold but this is merely a reflection of the larger positions their strategy takes. Their results are likely to be very different if their Buy-and-Hold strategy were defined in a more standard way and if they took sampling uncertainty into account.

5.3. Qualifications and further comments on the results

In this section, we have developed a test of our version of the weak efficient market hypothesis based on the profits of an Artificial Technical Analyst. We have used this test to characterise a broad class of investors (defined by their preferences and transaction costs) who would find it desirable to condition on
past prices; as long as we believe such investors exist, we should reject our version of the efficient market hypothesis and hence E-efficiency for which our version is a necessary condition. Note that there are numerous asymmetric information models which generate equilibria for which LIA is rejected (e.g. Hussman, 1992; Brown and Jennings, 1989; Treynor and Ferguson, 1985) so some of the available models may describe the data generating process accurately (at least in this respect).

The methodology proposed is useful because it relates efficiency to a set of objective functions rather than an equilibrium of some model. It allows efficiency comparisons across time and markets by comparison of the generality of \{M^e, c^e\} for which LIA is not empirically rejected. It thus provides a quantifiable measure of near efficiency. Equally importantly, it formalises a sense in which markets can be characterised as inefficient when empirical studies find trading rules to be profitable. It therefore formalises the popular notion of efficiency according to which agents solving simple investment problems such as (1) should not need to condition on past prices (or use technical trading rules).

The empirical exercise we conduct using the DJIA is intended to illustrate the implications of rule efficacy for market efficiency. A more accurate quantification of market efficiency using the Artificial Technical Analyst would require data on something closer to the market portfolio than the DJIA and a relaxation of the assumption that the riskless interest rate is zero. We chose to focus on the DJIA data in order to make our results comparable to those of other researchers who have used it (Brock et al.; Gencay, 1996; Sullivan et al., 1999; Bessembinder and Chan, 1998 among others) and to diminish the susceptibility of our analysis to data-mining criticisms. However, the use of these data by us and them is unfortunate because the DJIA is not a traded asset and as such suffers from the complications raised by nonsynchronous trading (Scholes and Williams, 1977). Some authors attempt to correct for this (for example by requiring positions to be taken one day after a signal for them is generated) but there is no corrective procedure that does not raise other equally serious complications arising from the microstructure of markets. Further complications also arise from the fact that transaction costs for the DJIA during 1962–1986 are likely to have varied and may not even be proportional as we have assumed. Finally, the series in question is not adjusted for dividends which may bias our results against LIA.

Use of recent ultra high-frequency data for options on market indexes would probably mitigate many of these problems and should be the subject of future investigations. Ultimately, the only indisputable empirical analysis of these issues would require the Artificial Technical Analyst to become Real: if actual trades based on this agents’ decisions turned out to be profitable, this would constitute prima facie evidence against the belief that ‘technical analysis is useless’.
6. Conclusions

Utility maximising best responses derived from investment decisions depend on utility functions, wealth, transaction costs and positions in the market. Technical trading rules are a class of behavioural rules which impose restrictions on the functional form these best response functions may take. However, these restrictions are only binding for strictly risk-averse investors and therefore utility maximising investors will ‘be’ Technical Analysts only if they are risk-neutral (Proposition 1). Nevertheless, if for some reason risk-averse agents restrict themselves to the use of technical analysis (e.g. because more sophisticated investment rules are costly to derive, learn and implement) we can provide circumstances in which the choices of a Technical Analyst are also optimal for risk-averse agents in this constrained sense (Propositions 3–5).

Artificial Technical Analysts use past data to choose rules from classes known to be used in practice. We show in Section 4.2 that the chosen rules will be more profitable than ad hoc rules used in previous studies (e.g. Brock et al.) and since they are chosen in a reasonable and robust way, they are also subject to less serious data-mining criticisms (Section 4.1). Taking these facts into account, we suggest that bootstrap based model specification tests based on rule returns as pioneered by Brock et al. (1992) should be augmented with artificially intelligent agents in the spirit of Sargent (1993).

In Section 5 we attempt to formalise the idea that the efficacy of technical analysis and the efficiency of financial markets must be inversely linked. To do this, we begin with the observation that if the equilibrium of a model is such that agents would be better off using technical trading rules, then the equilibrium is not efficient in Latham’s (1986) E-efficiency sense. Using daily data on the DJIA we characterise a class of agents (defined by preferences and transaction costs) who find the choices of an Artificial Technical Analyst valuable. Under appropriate assumptions, the size of this class (which includes mean-variance agents facing transaction costs lower than 0.06%) may be interpreted as a measure of the degree of efficiency of the NYSE (Section 5).

There are many natural extensions of this work so we restrict ourselves to some indicative suggestions. Firstly, the Artificial Technical Analyst could be made ‘more intelligent’ by making his learning more sophisticated and by widening the space of trading rules from which he chooses. For example, it is trivial to extend the information set so that rules can condition on variables other than past prices (indicating that the Artificial Technical Analyst is capable even of fundamental analysis!). Secondly, it would be important to try and find statistical processes describing returns that are consistent with the Artificial Technical Analyst’s profits. Finally, for applications where market efficiency is of central relevance, it would be valuable to develop a way of constructing dynamic asset pricing models which are consistent both with the data.
(e.g. Hansen and Richard, 1987) and with the degree of market inefficiency as quantified by the efficacy of Technical Analysis.

Appendix A. Proofs of propositions and corollaries

Proof of Proposition 2. The assumption may be rewritten as

\[ E\{E[U(W_t(1 + \theta^*(P_t)R_{t+1} + \theta^*(P_t))]|P_t]\} > E\{E[U(W_t(1 + \theta^*R_{t+1})]|P_t]\}, \]

which implies that there exists a set \( \mathscr{A} \) in \( \mathbb{R}^k \) such that \( \Pr(P_t \in \mathscr{A}) > 0 \) and for all \( P_t \in \mathscr{A} \):

\[ E[U(W_t(1 + \theta^*(P_t)R_{t+1} + \theta^*(P_t))]|P_t]\} > E\{E[U(W_t(1 + \theta^*R_{t+1})]|P_t]\}. \]

Now define

\[ \theta^{**} = \begin{cases} \theta^*(P_t) & \text{if } P_t \in \mathscr{A}, \\ \theta^* & \text{otherwise}. \end{cases} \]

Clearly,

\[ E\{E[U(W_t(1 + \theta^{**}R_{t+1})]|P_t]\} \geq E\{E[U(W_t(1 + \theta^*R_{t+1})]|P_t]\} \forall P_t, \]

and the inequality is strict if \( P_t \in \mathscr{A} \) which means \( \theta^* \) cannot be optimal for the conditional investment decision.

Proof of Proposition 3. (a) Define \( d_i = (1/l)d_i, i = 1, 2. \)

The assumption implies \( E(d_1 R_{t+1}) > E(R_{t+1}) \geq 0 \) so

\[ E(d_1 R_{t+1}) > E(R_{t+1}) \geq 0. \]

Notice that \( d_i = \{ -s/l, 0, 1 \} \) and by assumption \( l \geq s \) so \( (d_i)^2 \leq 1. \)

Hence \( (d_i R_{t+1})^2 \leq (R_{t+1})^2. \) Using the fact that \( V(x) = E(x^2) - E(x)^2 \) it follows that

\[ V(d_1 R_{t+1}) < V(R_{t+1}), \]

from which it also follows directly that

\[ V(d_1 R_{t+1}) < V(R_{t+1}). \]
(b) Define $d_i \equiv (1/l)d_i$, $i = 1, 2$. Then the assumption $E(d_1 R_{t+1}) > E(d_2 R_{t+1}) \geq 0$ implies

$$E(d_1 R_{t+1}) > E(d_2 R_{t+1}) \geq 0.$$  

Notice that $d_i = \{-1, 1\}$ so $(d_i)^2 = 1$. Hence $(d_i R_{t+1})^2 = (R_{t+1})^2$, $i = 1, 2$. Using the fact that $V(x) = E(x^2) - E(x)^2$ it follows that

$$V(d_1 R_{t+1}) < V(d_2 R_{t+1}).$$

Hence also

$$V(d_1 R_{t+1}) < V(d_2 R_{t+1}).$$

**Proof of Corollary 3.1.** By assumption the returns of the trading rule are larger than the positive market returns

$$E(R^d_{t+1}) > E(R_{t+1}) \geq 0.$$  

So by Corollary 1,

$$V(R^d_{t+1}) < V(R_{t+1}).$$

These two inequalities imply also that for any $\theta$:

$$E(W_t(1 + \theta R^d_{t+1})) > E(W_t(1 + \theta R_{t+1})),$$

$$\text{Var}(W_t(1 + \theta R^d_{t+1})) < \text{Var}(W_t(1 + \theta R_{t+1}))$$

which implies that for all mean-variance utility functions $U$:

$$U(W_t(1 + \theta^* R^d_{t+1})) > U(W_t(1 + \theta^* R_{t+1})).$$

**Proof of Proposition 4.** In normal environments second-order stochastic domination is equivalent to mean-variance domination (Hanoch and Levy, 1969). This together with Proposition 3a yield the desired conclusion.

**Proof of Proposition 5.** As is well known, the assumption on $M(\gamma)$ is a sufficient condition for

$$EU(R^d_{t+1}) > EU(R_{t+1}) \forall \text{concave } U.$$  

Notice now that when $E(R_{t+1}) \geq 0$ then the optimal proportion of invested wealth $\theta^*$ is positive for all wealth levels and so $U(W_t(1 + \theta^*x))$ is concave in $x$, since

$$\frac{\partial}{\partial x} U(W_t(1 + \theta^*x)) = W_t \theta^* U' \geq 0,$$

$$\frac{\partial^2}{\partial x^2} U(W_t(1 + \theta^*x)) = (W_t \theta^*)^2 U'' < 0.$$
Compare to Table 1 in the text. We have estimated statistics for the Artificial Technical Analyst using varying sample sizes ($N$) and for a 'restricted' space of rules where the Analyst is constrained to choose only among the six rules of Brock et al. described in Table 1.

<table>
<thead>
<tr>
<th>Rule</th>
<th>$N$ (buy)</th>
<th>$N$ (Sell)</th>
<th>Buy</th>
<th>Sell</th>
<th>Buy &gt; 0</th>
<th>Sell &gt; 0</th>
<th>Buy–sell</th>
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<tr>
<td>Always long</td>
<td>6157</td>
<td>0</td>
<td>0.00023</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>BLL Average</td>
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<td></td>
<td>0.00037</td>
<td>0.00011</td>
<td>0.00048</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATA ($N = 250$)</td>
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<td>2650</td>
<td>0.00095</td>
<td>0.00067</td>
<td>0.5337</td>
<td>0.4675</td>
<td>0.00162</td>
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<td></td>
<td></td>
<td></td>
<td>(3.95033)</td>
<td>(4.57949)</td>
<td>(7.34848)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATA restricted</td>
<td>3463</td>
<td>2401</td>
<td>0.0005144</td>
<td>0.00008416</td>
<td>0.5224</td>
<td>0.4794</td>
<td>0.0003221</td>
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<tr>
<td>ATA ($N = 225$)</td>
<td>3372</td>
<td>2697</td>
<td>0.0009558</td>
<td>0.0007419</td>
<td>0.538</td>
<td>0.4598</td>
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<tr>
<td>ATA ($N = 200$)</td>
<td>3374</td>
<td>2703</td>
<td>0.000928</td>
<td>0.0006966</td>
<td>0.537</td>
<td>0.4632</td>
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</table>

Therefore it must also be that

$$EU(W_t(1 + \theta^* R_t^{d+1}))) > EU(W_t(1 + \theta^* R_{t+1})).$$

Appendix B. Auxiliary results

From Table 4 conclude that a wide space of rules from which to choose is important for the performance of the Artificial Technical Analyst but that the precise values of $N$ are not (as long as $N$ is large enough for some stability to exist). These results are unsurprising but we provide them as empirical confirmation of what we suspected was the case.

References