The contingent nature of the revolution predicted by Marx

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Abstract

A formalisation of Elster’s interpretation of Marx’s theory of revolution is presented in terms of a one sector model with continuous substitution, land as a factor of production and a particular population theory. A justification for calling such a model Marxian is given. Then, it is shown that the revolution may occur only if the drag caused by the fixed supply of land is greater than the force of technical progress. ©2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The most striking feature of Marx’s thought is his prediction that capitalism will end with a revolution. The object of this paper is to present a model in which the occurrence of this revolution, rather than being certain, is contingent upon the values of the parameters. The interpretation of Marx’s theory of revolution is taken from Elster (1985). This is embedded in an expanded version of Marglin’s (1984) one sector neoMarxian model. The model is expanded by adding technical progress, land as a factor of production and Green’s (1991) version of Marx’s population theory. It is then shown that the revolution can be generated if and only if the drag caused by land scarcity is greater than the strength of technical progress.
The paper is structured as follows: Elster’s interpretation of Marx’s theory and the model used to formalise it are discussed in Section 2. The addition of land is extensively justified in Section 3. The model is presented and analysed in Section 4. The basic result on the contingent nature of the revolution is set out in Section 5 and finally, Section 6 explains the importance of the model and the results.

2. Elster’s interpretation of Marx and a description of the model

It is generally conceded that Marx left no precise explanation of how the communist revolution would come about so that any model must be based on an interpretation. I have chosen Elster’s interpretation mainly because its level of complexity is such as to make modelling both possible and interesting. Many readers may feel that his interpretation does not accurately reflect Marx’s thoughts. I think that this is inevitable regardless of the interpretation chosen and should be taken as an invitation to formalise alternative interpretations rather than as a criticism of the present paper.

Elster, 1985, pp. 528–531 held that two conditions would bring about the revolution. First, the wage would have to reach such a low level that the workers would be driven to risk revolution. A falling wage that approaches zero will ensure this condition is satisfied. Second, the rate of profit would have to be falling so that the capitalists would become too demoralised to resist the workers. I want to strengthen this condition. Since it is unlikely that capitalists would be demoralised by a falling but high rate of profit, I will take the second condition to be a falling rate of profit with an arbitrarily small lower bound. When these two conditions are satisfied I will say that the revolution can be generated. The sense of this is that, for arbitrary parameter values, the revolution may not occur, the rate of profit may never fall to a sufficiently low level, but there are parameter values for which the revolution will certainly occur.

Elster’s interpretation will be embedded in Marglin’s one sector continuous substitution neoMarxian model (Chap. 9). This model has a fixed subsistence wage, the condition that the wage be equal to the marginal product of labour, and a variety of savings functions of which I will use the most simple where the capitalists save all.1 This model has two drawbacks as

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1 There are two issues that may bother the reader at this point: he or she may feel first that Marx’s ideas cannot be adequately expressed in terms of a single sector continuous substitution model and/or second that these models are not a valid representation of a many sector world. In regard to the first, the paper relies on Marglin’s justification: He starts with a single sector fixed coefficient model, step by step generalises this to a multi-sector continuous substitution model and shows that the characteristics remain unchanged. In regard to the second issue, there are three points: First, the concern of the paper is with dynamics so that the issues raised in the reswitching debate described by Harcourt (1972), which concerned steady state, are not directly relevant. Second, the use of single capital good models to gain an insight into the behaviour of multi capital good models depends, not on the ability to go from the latter to the former by aggregation, but on the hope that the dynamic properties of the two types of models are similar. While this has not been generally demonstrated, Shell and Stiglitz (1967) among others have shown that with perfect foresight the stability characteristics are, indeed, similar. Third, the assumption of continual equilibrium on the approach to steady state, implicit in the one sector model, has been described by Duménil and Levy (1985) as a special case of the more general classical description of the approach in terms of multi sector disequilibrium. Thus, at least for the concerns of the paper, there is some justification for the one sector treatment.
a framework for Elster’s interpretation: first, the only equilibrium of the model is a steady state in which the rate of profit is constant and second, by assumption, the wage is constant. To remedy these I will first add land which gives the model equilibrium dynamics so that the rate of profit may fall; and second add Green’s interpretation of Marx’s population theory so that the wage may fall.

In this context, the result of the paper is the following. If technical progress is stronger than the drag caused by land then the model approaches a steady state with a rising wage and a constant rate of profit so that the revolution does not occur. But if technical progress is weaker, then, on the approach to steady state, the wage falls toward zero, the rate of profit falls and there are parameters such that the lower bound for the rate of profit can be made arbitrarily close to zero; that is, in this case, the revolution can be generated.

3. The justification of land

The majority opinion is that Marx did not think the rate of profit would fall because of land scarcity so that this addition must be extensively justified both as a cause of the falling rate of profit and as a cause of revolutions. As a cause of the falling rate of profit a group of five writers (Meek, 1967, pp. 129–142; Rosdolsky, 1977, pp. 405–408; Lebowitz, 1982, Perelman, 1985, 1987 chap. 2 and Clarke, 1994, pp. 216–219), independently (except for Meek) have argued that the combination of sections of Theories of Surplus Value with Marx’s central argument in volume III of Capital leads to a consistent theory of the falling rate of profit based on resource scarcity. Since these arguments have not been rebutted by the majority I think that, at least provisionally, one is entitled to have the rate of profit fall because of land scarcity and still call the model Marxian.

As a cause of the communist revolution Marx specifically identified land scarcity as one of the major reasons that the peasants would unite with the proletariat against the bourgeoisie. This position is referred to in ‘The Civil War in France’ (1871, pp. 337–338), his analysis of the Paris Commune which he took to be a prototype of the transition from capitalism to socialism (Berlin, 1963, pp. 256–258). There he reiterates the point he made earlier at greater length in his analyses of the 1848–1850 events, ‘The Class Struggles in France’ (1850, pp. 487–492) and ‘The Eighteenth Brumaire of Louis Bonaparte’ (1852, pp. 119–124 and ‘The Eighteenth Brumaire of Louis Bonaparte’ (1852, pp. 487–492). He states that the peasants are oppressed by bourgeois taxes and further more, since their income has been falling due to population pressure on the land, they have become

\[ \text{For example see (Robinson, 1941, p. 241), (Samuelson, 1957, p. 898), (Steadman, 1977, p. 129) (who qualifies this by a counter reference to Theories of Surplus Value) and (Roemer, 1981, p. 87).} \]

\[ \text{Clarke’s argument is given as an example. The problem with Marx’s (1959) volume III argument for the falling rate of profit is that he gives no reason why the direct negative effect of an increase in capital should be stronger than the indirect positive effect that works through an increase in output. Clarke notes that this is explained in certain sections of Theories of Surplus Value by growing resource scarcity and that Theories of Surplus Value was meant to be the first part of Capital and claims that Marx supposed that the reader would refer back to his earlier statements.} \]

\[ \text{This is not to say that the reason for the fall in the rate of profit in the model is an exact formalisation of Marx’s intuition. For example, his starting point, that productivity in industry grows faster than in agriculture, cannot be set out in the context of a one sector model.} \]
deeply indebted and pay a good part of their income to the bourgeoisie as debt service. For these reasons, said Marx, the peasants are beginning to see the proletariat as their natural allies. If one interprets the labour of the model as a proletarian-peasant composite, then one has an explicit textual justification for the use of land scarcity in the model to provoke the revolution.

Finally, although it does not concern an interpretation of Marx, it is worth noting that population pressure on land is invariably associated with revolutions. This has been the case with Chiapas in southern Mexico and the former Zaire (Renner, 1997), Vietnam (Popkin, 1988), and all major Latin American revolutions (Wickham-Crowley, 1992). But of most relevance, Skocpol in her now classic work on revolutions (1979) both agrees with Marx’s analysis of population pressure on the land as playing an important role in the French revolution (1979, p. 56) and identifies this pressure as one of the important causes of the two great communist revolutions, the Russian (p. 132) and the Chinese (1979, p. 74). Thus, interpreting Marx as saying that population pressure on land is a root cause of revolution puts him in harmony with what has actually happened.

4. The model

4.1. The basic model

The production function is Cobb-Douglas \( K^\alpha L^\beta M^{1-\alpha-\beta}e^{\gamma t} \), where \( K \), \( L \), and \( M \) are capital, labour and land, \( \alpha \) and \( \beta \) are positive constants, \( \alpha + \beta < 1 \) and \( \gamma > 0 \) is the rate of technical progress. \( M = 1 \) by choice of units and disappears from the analysis. The conditions that the wage \( W \) is equal to the marginal product of labour and that capitalists save all are given by

\[
W = \beta K^\alpha L^\beta -1 e^{\gamma t} \tag{1}
\]

5 The population pressure on the land was brought about by political and ecological changes as well as pure population growth. (See Wickham-Crowley, 1992, pp. 239–240) for an example of the latter.

6 The correspondence is not exact since Skocpol refers to the condition of the peasants pre 1789 and Marx to the pre 1848 and 1871 periods.

7 This may be interpreted as a net production function. In this case, the gross function would be \( K^\alpha L^\beta e^{\gamma t} + \delta K \) where \( \delta \) is the rate of depreciation. When \( \delta = 1 \), the model corresponds exactly to Marglin’s circulating capital model. When \( \delta < 1 \), the model can be interpreted in the standard neoclassical way with depreciating fixed capital or as a combination of circulating and fixed capital with \( \delta \) as the average rate of depreciation.

8 Marx (1972) in Theories of Surplus Value, part III, p. 368, emphasised the importance of exhaustible resources. In my paper, on the other hand, land is modelled as a fixed stock which is necessary for production and does not deteriorate. I think that the model could be extended easily to make land a renewable resource and, with much more difficulty using the techniques of Stiglitz (1974), to make land an exhaustible resource as well. However, Stiglitz’s formulation has aroused considerable criticism which has culminated in the Georgescu-Roegen versus Solow/Stiglitz forum organised by Daly (1997). The basic point is that the neoclassical production function is not valid because output consists of processed exhaustible resources which implies that increases in capital or labour alone cannot increase output. Although, the issue admittedly is contentious, I find the response first given by Ayres, 1978 (Chap. 3) a convincing answer. Essentially, he pointed out that a unit of aggregate output is defined in terms of the utility it gives and that it is possible, by changing the mix of goods, to substitute capital and labour for resources in the production of utility.
\[ \dot{K} = K^\alpha L^\beta e^{\gamma t} - WL \]  
(2)

where it is assumed that capitalists own the land and where \( \dot{x} \) is the time derivative of \( x \).\(^9\)

It is important to note that the assumption that capitalists save all implies that the rate of growth of capital is equal to the rate of profit \( \Pi \), that is

\[ \Pi = \dot{K} \]

where \( \dot{x} \) is the percentage change in \( x \) with respect to time.

The supply of labour, according to Green, is affected by the wage and the value of labour power \( V \). The latter reflects the cost of reproduction of labour in some form. Population growth is determined by a normal growth rate \( n \) and also affected positively by the level of the wage relative to the cost of reproduction,

\[ \dot{L} = n + a \left( \frac{W}{V} - 1 \right) \]  
(3)

where \( a \) is a positive constant. Since one wants low wages to be able to cause a decline in the population, it is assumed that \( a > n \).

Next consider how the value of labour power changes. In line with Marx’s statement that it is fixed for any given period but in general is determined by moral and historical factors, Green defines it as a weighted average of past values of the wage:

\[ V(t) = \frac{\int_{-\infty}^{t} W(s)e^{b(s-t)}ds}{\int_{-\infty}^{t} e^{b(s-t)}ds} \]

where \( e^{b(s-t)} \) is the weight for \( s \). Cancelling \( e^{-bt} \), multiplying both sides by the RHS denominator, integrating it and finally taking the derivative gives

\[ \dot{V} = b \left( \frac{W}{V} - 1 \right) \]  
(4)

where \( b \), a positive constant, is a measure of the sensitivity of the change in the value of labour power to the relative level of the wage.

The model consists of the three differential equations

\[ \dot{K} = (1 - \beta) K^{\alpha-1} L^\beta e^{\gamma t} \]  
(5)

\[ \dot{L} = n + a \left( \frac{\beta K^\alpha L^{\beta-1} e^{\gamma t}}{V} - 1 \right) \]  
(6)

\[ \dot{V} = b \left( \frac{\beta K^\alpha L^{\beta} e^{\gamma t}}{V} - 1 \right) \]  
(7)

where these are gotten by solving (1) for \( W \) and substituting this into (2)–(4).

\(^9\)The interpretation of the production function as net does not affect these equations.
4.2. Steady state

Steady state is where the three variables grow at constant rates \( k \), \( l \), and \( v \), that is

\[
\dot{K} = k, \quad \dot{L} = l, \quad \dot{V} = v
\]

Substitute for \( \dot{K} \) etc. in (5) and (6). Then logarithmically differentiating the equations that result from (5) and (6) gives

\[
\begin{align*}
(\alpha - 1)k + \beta l + \gamma &= 0 \\
\alpha k + (\beta - 1)l + \gamma &= v
\end{align*}
\]

while the equations that result from (6) and (7) together imply

\[
(l - n) \frac{1}{a} = \frac{v}{b}
\]

Eqs. (8)–(10) can be solved for the steady state growth rates.

4.3. Stability

Define the normalised variables

\[
K = K e^{-kt}, \quad L = L e^{-lt}, \quad V = V e^{-vt}
\]

which are constant in steady state. Then

\[
\dot{K} = \dot{K} + k, \quad \dot{L} = \dot{L} + l, \quad \dot{V} = \dot{V} + v
\]

Substituting these expressions into (5)-(7) and remembering that the exponents cancel by (8) and (9) gives

\[
\begin{align*}
\dot{K} &= (1 - \beta)K^{a-1}L^{b-1} - k \\
\dot{L} &= a \left[ \left( \frac{\beta K^a L^{b-1}}{V} - 1 \right) - \frac{l - n}{a} \right] \\
\dot{V} &= b \left[ \left( \frac{\beta K^a L^{b-1}e^{-vt}}{V} - 1 \right) - \frac{v}{b} \right]
\end{align*}
\]

This can be reduced to a two variable system as follows: (12), (13) and (10) imply that

\[
\dot{V} = \frac{b}{a} \dot{L}
\]

Integrating this from 0 to \( t \) and taking both sides of the resulting equation as exponents gives

\[
V(t) = CL(t)^{b/a}
\]

where \( C = V(0)L(0)^{-b/a} \). Substituting this into (12) and rewriting (11) gives

\[
\dot{K} = (1 - \beta)K^a L^b - kK
\]
The steady state of the system defined by these equations $L^*, K^*$ is globally stable, that is:

**Lemma 1.** Let $L(0) > 0, K(0) > 0$. Then $\lim_{t \to \infty} L(t) = L^*$, $\lim_{t \to \infty} K(t) = K^*$.

**Proof.** Since the argument is simple only a sketch is given. The phase diagram that arises from (14) and (15) is given in Fig. 1. If the initial point is in either areas A or D, then one can easily show that either the system converges to $L^*, K^*$ or it crosses into either areas B or C. In this latter case, one can easily show that the system cannot escape from either of these areas and must converge to $L^*, K^*$. If the initial point is in B or C it must converge as well.

4.4. Comparative statics of the growth rates

Solving (8)–(10) for the growth rates gives

$$k = \frac{b(n\beta + \gamma)/a + \gamma}{1 - \alpha - (b/a + (1 - \alpha - \beta)/(1 - \alpha))}$$

$^{10} a > n$ ensures the $1 + (l - n)a > 0$. 

Fig. 1. The Phase diagram for (14) and (15).

\[
\dot{L} = a \left[ \beta \frac{K^{\alpha} L^{\beta - 1 - b/a}}{C} \right] - \left( 1 + \frac{l - n}{a} \right) L 
\]
Two central conclusions follow directly from these equations:

Lemma 2. $v > (<) 0$ as $\gamma > ( <) (1 - \alpha - \beta)n$.

Lemma 3. $\lim_{b \to 0, \gamma \to 0} k = 0$.

It is important to understand the intuition of these results. Consider Lemma 2. Note that, from Eq. (3) or Eq. (4), $W/V$ is constant in steady state. Furthermore $(1 - \alpha - \beta)$ is the exponent on land so that $(1 - \alpha - \beta)n$ is a measure of the ‘drag due to land’. Thus Lemma 2 states that both the value of labour power and the wage will be rising if technical progress is stronger than the drag due to land. Suppose initially that technical progress and the drag are equal so that the wage and the value of labour power are constant. Now raise the rate of technical progress. We seek the new value of $W/V$ that will re-establish steady state. At the old value $V$ is constant but the increase in the rate of technical progress means that $W$ will start to rise. If $W/V$ is raised this will speed the rate of growth of labour which will slow the rise in the wage, and at the same time it will cause $V$ to start to rise. The correct choice of $W/V$ will mean that they both rise at the same rate so that the system is again in steady state, but this time with a rising wage and value of labour power. This is the intuition behind the non-occurrence of revolution.

Lemma 3 states that the rate of growth, which is equal to the rate of profit, will be zero if and only if the rate of technical progress and the sensitivity of movements in the value of labour power to the relative wage, $b$, are both zero. If technical progress is positive this can be viewed as a continual increase in the quantity of land which clearly makes growth possible. But even when there is no technical progress, growth is possible if the value of labour power is sensitive to the wage. In this case, the fixed supply of land forces down the wage, but the falling value of labour power permits the continuous expansion of the population which sustains growth. This is the intuitive reason why low values of both the rate of technical progress and the sensitivity of changes in the value of labour power to the wage are needed to generate the revolution.

5. The triumph of capitalism and the road to revolution

5.1. The triumph of capitalism

The situation in which the revolution does not occur is called the triumph of capitalism. Formally:

Definition 1. Capitalism is said to triumph when the system approaches a steady state with a growing wage and a positive constant rate of profit, i.e. $\lim_{t \to \infty} \dot{V} = v > 0$ and $\lim_{t \to \infty} \Pi = k > 0$. 

\[
\begin{align*}
I &= \frac{nb/a + \gamma/(1 - \alpha)}{b/a + (1 - \alpha - \beta)/(1 - \alpha)} \\
V &= \frac{\gamma - n(1 - \alpha - \beta)}{a(1 - \alpha - \beta)/b + (1 - \alpha)}.
\end{align*}
\]
Proposition 1. If the rate of technical progress is greater than the drag of land, i.e. $\gamma > (1-\alpha-\beta)n$, then capitalism triumphs.

Proof. $\lim_{t \to \infty} \dot{W} = v$ and $\lim_{t \to \infty} \Pi = k$ by Lemma 1, $v > 0$ by Lemma 2 and $k > 0$ by (16).

5.2. The road to revolution

Remember that $k$ is the steady state rate of profit. The situation in which the revolution may occur is given formally as:

Definition 2. It is said that the revolution can be generated if

(a) $\lim_{t \to \infty} \Pi = k$ and the parameters of the model can be chosen so that $k$ is arbitrarily close to 0; and

(b) there is a $\rho^*$ such that $t^* > t > \rho^*$ implies that $\dot{W}(t) < 0$ and $\dot{\Pi}(t) < 0$ where $t^* \leq \infty$ is the moment at which the system reaches steady state.

Proposition 2. Let $0 < L(0) < L^*$ and $0 < K(0) < K^*$. If the rate of technical progress is less than the drag due to land, i.e. if $\gamma < (1-\alpha-\beta)n$, then the revolution can be generated.

Sketch of the proof: Lemma 1 and Lemma 3 show that (a) of Definition 2 is satisfied. Since, by Lemma 2, $\dot{W} < 0$ in steady state and steady state is approached, it is reasonably clear that $\dot{W} < 0$ for $t > \rho^*$ for some $\rho^*$, so that the first part of (b) is satisfied as well.

The second part of (b) can be seen with the aid of the phase diagram of Fig. 2. Along the line $\dot{\Pi} < 0$, the arrows point as they do at the point a. This means that once the system is in the area A it cannot escape. Thus, if the initial position is in area A, the system remains in A and $\dot{\Pi} < 0$ until the steady state is reached. If the initial position is in $B_1$, it must enter $B_2$ since this is the only way to arrive at the stable equilibrium $K^*, L^*$. Finally once it is in $B_2$, $\Pi > k$ because $\Pi = k$ on the line $K = 0$ and the increase in $L$ to enter $B_2$ will raise $\Pi$. Thus, it must enter $A$ where $\dot{\Pi} < 0$ since when it arrives at $K^*, L^*$ it must have $\Pi = k$. But once it enters $A$ it cannot escape so there is a $\rho^*$ such that $t^* > t > \rho^*$ implies $\dot{\Pi}(t) < 0$.

The details of the construction of the phase diagram are set out in the Appendix. The justification of the limits imposed on $K$ and $L$ is that Marx’s theory refers to a growing economy.

6. Conclusions

The following points are treated: the compression of Marx’s theory of revolution, its relation to capital accumulation, Marx’s theory of population and, finally, the contingent nature of the revolution.

Skocpol, 1979 (p. 16), when talking about people concerned with revolutions, notes that they have ‘taken for granted Marx’s economic analysis of the sociohistoric conditions for revolution’ and have concentrated on the strategies to be followed once these conditions are present. However, it is generally conceded by people who have concentrated on the logic of Marx’s position that he left no coherent account of how these conditions would arise,
e.g. Elster, 1985 (p. 528) and Berlin, 1963 (p. 283). Clearly there is a gap here of major proportions. A fruitful way in which to fill it is the construction of simple economic models which show how the conditions for revolution arise. One of the contributions of the present paper is to show that this approach is feasible and that, as will be now pointed out, it leads to interesting insights.

First, the model makes the link between capital accumulation and revolution clear. The traditional way of characterising Marx, Morishima (1973); Steadman (1977); Roemer (1980); Foley (1986) etc. had capital intensity increase because of a sequence of exogenous changes in techniques which, in principle, had nothing to do with the actual accumulation of capital. The present model, by inserting land, shows how both the falling wage and the falling rate of profit are directly linked to capital accumulation by means of capital-labour substitution.

Second, the introduction for the first time of the Marxian population theory into a growth model shows that it is more subtle than either the neoclassical or the classical population theory and occupies a middle position between these. In the classical version, there is a fixed subsistence wage and, when the actual wage reaches this, population expansion and thus growth comes to a halt as in Johansen (1967). In the neoclassical version, there is no concept of subsistence wage and the exogenously determined population expansion completely determines the rate of growth. Finally, in the Marxian version the subsistence
wage is non-constant and endogenous and it is the parameters of the mechanism which determine the rate of growth. 11

Third, the model suggests a new way in which trade union activity may be associated with revolutions. As noted by Lapides, 1987 (pp. xiv and n9), Marx saw their role as fomenting class consciousness. But Green has associated a fall in b with increased union militancy and the model shows how a fall in b may push the country over the edge into revolution. This opens the interesting modelling possibility of adding a bargaining model and showing that the occurrence of revolution depends on union bargaining strength.

Finally, the portrait of the revolution as contingent is important for two reasons. The first is defensive. Marx’s prediction of revolution is the most striking aspect of his ideas but, at the same time, the Achilles heel. Until comparatively recently defenders of Marx like Dobb (1937) 12 and Lange (1935) made the realisation of the prediction the test of Marx’s system. Now with a longer historical perspective Marxists like Bose, 1975 (pp. 136–140) have begun to back away from this criterion. But unfortunately others have begun to use the non-fulfilment as a weapon against Marx and worse still, Samuelson and Nordhaus, 1989 (p. 583, pp. 833–837) has emphasised to generation after generation of students that the rate of profit has not fallen, the workers have not become immiserized and, finally, the revolution has not occurred. The making of the revolution contingent largely safeguards Marx from this type of attack. The worse he can be accused of is getting the parameter values wrong and, furthermore, there is nothing to guarantee that these will not change in such a way that a revolution becomes a very real possibility. 13

The second reason for the importance of contingency is that it leads to a profound question: To what do we owe the continued existence of capitalism? This question cannot be easily approached in the context of mainstream thinking because the mainstream contains no natural alternative. But Marxism does: a revolution followed by socialism. The specific answer given by the present model is that capitalism owes its existence to the fact that, currently, technical progress is stronger than the drag of land. It is not hard to think of different contingency models that produce other, more intriguing answers. 14 But in the end, it is not any specific answer but rather the question itself, that models of contingent revolution are designed to resolve, that attracts one’s thoughts.

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11 One might criticise these three theories by noting the recent negative correlation between income and population growth in industrialised countries. This is related to human capital accumulation as in Becker et al. (1990) and thus does not seem relevant for the countries in which revolutions have actually occurred. Of course, it is relevant to the question of why revolutions have not occurred in industrialised countries (See below).
12 These articles are presented in Horowitz (1968). In that book see pp. 68–87 and 49–67.
13 Although, it is not generally realised, a contingent revolution more closely corresponds to Marx’s true view. The inevitability was a public strategic posture but privately Marx was doubtful as noted by Gottheil, 1986 (pp. 165–168).
14 One concerns human capital.
Appendix

1. If \( g < (1 - \alpha - \beta)n \), then there exists a \( t^0 \) such that \( t > t^0 \) implies \( \dot{W} < 0 \).

**Proof.** From (1), the definitions of \( K \) and \( L \), and (9)

\[
W = \beta K^\alpha L^{\beta - 1} e^{[\alpha k + (\beta - 1)\gamma - \gamma]} = \beta K^\alpha L^{\beta - 1} e^{vt}
\]

\[
\dot{W} = \alpha \dot{K} + (\beta - 1) \dot{L} + v
\]

Since \( \dot{K} \) and \( \dot{L} \) approach zero by Lemma 1 and \( v < 0 \) by Lemma 2 the result follows. \( \square \)

2. Let \( \dot{\Pi}(K, L) \) give \( \dot{\Pi} \) as a function of \( K \) and \( L \). Then the line defined by \( \dot{\Pi}(K, L) = 0 \), \( 0 < K < K^* \), \( 0 < L < L^* \) has the following properties:

(a) \( \frac{dK}{dL}|_{\dot{\Pi}(K, L)=0} > 0 \)

(b) The line lies between the lines \( \dot{K} = 0 \) and \( \dot{L} = 0 \).

**Proof.** From (8) and the definition of \( \Pi = (1 - \beta)K^{\alpha - 1}L^\beta \)

\[\dot{\Pi} = (\alpha - 1)\dot{K} + \beta \dot{L} \] (A.1)

Calculating \( \dot{K} \) and \( \dot{L} \) from (14) and (15) gives

\[
\dot{\Pi}(K, L) = (\alpha - 1)[(1 - \beta)K^{\alpha - 1}L^\beta - k] + \beta[(a/C)K^\alpha L^{\beta - 1 - b/a} - (a + l - n)]
\]

\[
\frac{\partial \dot{\Pi}}{\partial K} = a_1 + b_1 > 0, \quad \frac{\partial \dot{\Pi}}{\partial L} = -(a_2 + b_2) < 0, a_1 = (\alpha - 1)^2(1 - \beta)
\]

\[
\times K^{\alpha - 2}L^\beta > 0, \quad b_1 = \beta^2(a/C)aK^{\alpha - 1}L^{\beta - 1 - b/a} > 0, a_2 = (\alpha - \alpha)
\]

\[
\times (1 - \beta)\beta K^{\alpha - 1}L^{\beta - 1} > 0, \quad b_2 = \beta^2(a/C)(1 + (b/a) - \beta)
\]

\[
\times K^\alpha L^{\beta - 2 - b/a} > 0, \quad \frac{dK}{dL} \bigg|_{\dot{\Pi}(K, L)=0} = -\frac{\partial \dot{\Pi}/\partial L}{\partial \dot{\Pi}/\partial K} = \frac{a_2 + b_2}{a_1 + b_1} > 0
\]

which proves (a).

To prove (b) note that \( \partial \dot{\Pi}/\partial L < 0 \) implies that \( \dot{\Pi} > 0 \) to the left of the line and \( \dot{\Pi} < 0 \) to the right. Since (A.2) shows that \( \dot{\Pi} > 0 \) on the line \( \dot{K} = 0 \) and \( \dot{\Pi} < 0 \) on the line \( \dot{L} = 0 \) the line in question must lie between them as had to be shown. \( \square \)

3. \( \partial \Pi/\partial L > 0 \). Proof: This follows from (A.1).

4. When \( (K, L) \) is such that \( \dot{\Pi}(K, L) = 0 \), \( 0 < K < K^* \), \( 0 < L < L^* \), then

\[
0 < \frac{\dot{K}}{L} < \frac{dK}{dL} \bigg|_{\Pi(K, L)=0}
\]

where \( \dot{K}/\dot{L} \) is the slope of the path of the system at that point.
Proof.

\[
0 < \frac{K}{L} = \frac{\beta}{1 - \alpha} \frac{K}{L} < \frac{a_2 + b_2}{a_1 + b_1} = \left. \frac{dK}{dL} \right|_{f(K,L)=0}
\]

where the first equality follows from (A.2) and the second inequality is because

\[
\frac{\beta}{1 - \alpha} \frac{K}{L} = \frac{a_2}{a_1} \quad \text{and} \quad \frac{\beta}{1 - \alpha} \frac{K}{L} < \frac{1 + b/a - \beta}{\alpha} \frac{K}{L} = \frac{b_2}{b_1}
\]

References


