On the formation of industry lobby groups

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Abstract

This paper provides a step towards a more complete theory of lobbying, extending the menu-auction model of Grossman and Helpman [Grossman, G.M., Helpman, E., 1994. American Economic Review 84, 833–850]. A new explanation is proposed for why more concentrated industries more easily overcome the free-rider problem inherent in political collective action. Instead of focusing on transactions costs as Olson [Olson, M., 1965. The Logic of Collective Action, Harvard University Press, Cambridge, MA], we show that more collusive industries with higher collusive profits have a greater incentive to form lobby groups and to contribute to industry lobbying. Moreover, more polluting industries also have a greater incentive to form and contribute to a lobby group.

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1. Introduction

It is well known that defection is the dominant strategy in one-shot prisoner’s dilemma games as in Olson (1965) analysis of the collective action problems facing special interest groups. On the other hand, it is also well established that in infinitely repeated games, collective action can be sustained over time (see, e.g., Friedman, 1971; Axelrod, 1981; Hardin, 1982; Radner, 1986). In these games the discount factor plays a crucial role in determining whether collective action is feasible.

In this paper we propose two novel factors — collusion and collusive profits — that influence the ability to overcome free-rider problems in the formation of political lobby groups in repeated games. We argue that more collusive industries, with greater collusive...
profits, have a greater incentive to lobby. The incentive to form and sustain a lobby group derives from the impact of lobbying on the level of profits. In concentrated industries lobbying may, or may not, occur depending on the level of collusion, the effects of lobbying on collusion, and the discount rate.

The literature contains a number of empirical studies that test whether concentrated industries are able to more easily organize because of lower transaction costs, as argued by Olson. Several authors have, in addition, explored the effects of the potential rents available. The results are inconclusive. Pittman (1988) shows that the level of federal regulations (primarily measured as the level of capital expenditures on pollution abatement induced by EPA-regulations) significantly determines the level of campaign contributions, but only in concentrated industries. Andres (1985), Masters and Keim (1985) and Heywood (1988) find positive effects of industry concentration and the degree of regulation on the probability of making political contributions.1 Salamon and Siegfried (1977) provide evidence which suggests that larger industries are less successful in their political activities (i.e., they did not study the level of lobby group formation), confirming Olson’s theory of free riding. They also find a positive relationship between industry political success (the ‘tax avoidance rate’) and firm size, but a negative relationship between political success and concentration. Restricting himself to politically active firms, Zardkoohi (1985) finds no effect of concentration per se on PAC contributions, but shows a significant positive relationship between the interaction of the latter variable and potential rents.2 Esty and Caves (1983) find that seller concentration increases political activity. They argue that this occurs not because of firms’ ability to overcome free-rider problems, but because of more unified interests in concentrated industries. One possible explanation for the divergence of results may be a non-monotonic relationship between concentration, available rents, and lobbying. Grier et al. (1991) find an inverted-U shaped relationship between the level of PAC formation and industry concentration. The maximum political participation rate occurs at a four-firm concentration ratio around 0.45.

In sum, the existing empirical work does not bring clarity to the question of the relationship between concentration, potential rents, and lobby group activities. One objective of the present paper is to further our understanding of the interactions involved by developing a theory that can serve as a base for further empirical work.

We focus on the incentive for two polluting firms to jointly lobby the government for a more favorable pollution tax policy. We consider a duopoly which produces a homogenous good. The firms are assumed to interact for an indefinite period of time. It is well established that this provides an incentive to collude tacitly by restricting output levels. Collusion, however, gives rise to the familiar problem that each firm has an incentive to defect when its rival colludes. We assume that such defection is deterred by the threat of reversion to the one-shot Cournot–Nash equilibrium. It is further supposed that production results in

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1 See McKeown (1994) for some factors relating to managment organization affecting the likelihood of PAC formation.
2 Zardkoohi (1985, 1988) interprets this result as evidence that in competitive industries the incentive for political activity is greater because the market offers no rents, and therefore, firms have a greater incentive to undertake lobbying.
emissions which adversely affect all consumers in the economy. In order to control the level of pollution the government imposes a pollution tax on the firms.

Lobbying is introduced into this framework by drawing on the model of political competition developed by Bernheim and Whinston (1986), Grossman and Helpman (1994), and extended by Fredriksson (1997), Aidt (1998) to pollution taxation. In this literature, lobby group formation is simply assumed rather than explained. It is further supposed that a self interested government cares not only about aggregate welfare, but also about political contributions from one (or many) organized lobby group(s). The lobby group is assumed to offer the incumbent government prospective contribution schedules that are contingent on the policy implemented. The government in turn selects the policy that maximizes its own welfare.

We see the two firms as organized into a lobby group when they both offer the government separate contribution schedules which are contingent upon the emission tax chosen by the government. The industry lobby group’s contribution schedule is the sum of the two firms’ schedules, and depends on the benefit that each derives from a reduction in the emission tax rate. The formation of a lobby group may not be feasible because each firm has an incentive to free-ride on its rival’s lobbying contributions. However, we show that in an infinitely repeated game, considerations of long-term profitability may render political lobbying incentive compatible. Specifically, this occurs when the discounted payoffs from lobbying exceed those from free-riding. Under certain weak restrictions on the discount factor, lobbying is found to be the individually rational strategy for each firm. A strength of the analysis is that we demonstrate that noncooperative behavior can sustain lobbying.

The analysis is based upon the following sequence of moves. First, the firms decide whether to offer contingent contributions. If both firms decide to do so, an industry lobby group is organized, and it offers the government a contribution schedule. In the second stage, the government determines its optimal pollution tax, and collects the associated lobbying contribution(s). In the final stage, the two firms determine the optimal output level, given the pollution tax. The model is solved by backward induction. The game is assumed to be repeated over an infinite horizon.

In order to explore the impact of variations in the degree of collusion on lobbying, we focus on equilibria in which there is constrained tacit collusion in the output market. It is demonstrated that the degree of tacit collusion (competition) in the output market, as well as the level of collusive profits, have a critical effect on the incentive to lobby. In particular, firms in more collusive industries (with greater collusive profits) have a greater incentive to offer independent contribution schedules to the government (i.e., to form a lobby group), ceteris paribus. Intuitively, this result reflects the fact that as collusive profits rise, the opportunity cost of free-riding on lobby group contributions also increases. Hence it is demonstrated that the incentive to free ride on the rival firm’s lobby group contributions declines as the degree of collusion in an industry rises.

3 In our model, there is for simplicity no abatement technology and the pollution-output coefficient is fixed. In this paper, we also assume that the environmentalists do not form a lobby group.

4 A firm can of course also undertake lobbying by itself, which can be seen as the formation of a separate Political Action Committee (PAC). However, this is not seen as the organization of a lobby group in this paper.
We find that lobbying for a lower pollution tax has two opposing effects on profits. Lobbying for a lower pollution tax increases the industry’s profit level by lowering pollution tax expenditures. However, a lower pollution tax also reduces the ability of the firms to collude in the output market. Because the former (tax) effect is unambiguously greater than the latter (anti-collusive) effect, and thus profits are decreasing in the tax rate, firms always lobby for a lower tax, when able to organize a lobby group.

It is also shown that pollution intensive industries have a greater incentive to form and contribute to a lobby group. In the absence of lobbying these industries would confront higher pollution tax expenditures. Thus, the change in profits from a change in the tax policy is increasing in the emissions of the firms.

While the analysis in this paper develops a theory for the case of a pollution tax, the results directly apply to other similar forms of regulation. The empirical evidence cited above suggests that we may have identified a more general and hitherto unidentified phenomenon which influences the degree of lobbying. Thus, the model can be recast to explain the formation of special interest groups seeking, e.g. trade protection in collusive industries.

The remainder of this paper is organized as follows. Section II outlines the structure of the output market, i.e., stage three of the game. Section III studies the second stage of the game where tax rates are determined in a political equilibrium. Section IV analyses the first stage, in which the industry lobby group is formed. Sections V and VI describe the equilibria and their properties. Section VII concludes the paper. All proofs are given in the appendix.

2. Stage three: the output market

Modeling the demand side of the economy as in Singh and Vives (1984), we consider an economy with two sectors: a competitive, numeraire sector and a duopoly which produces a homogenous good. Production of good $Q$ results in pollution emissions, denoted $E$, which adversely affects consumers (also called environmentalists) in the economy. The pollution damage suffered by consumers is defined by the damage function $D(E)$, with $D_E > 0$ and $D_{EE} > 0$; subscripts denote partial derivatives. Consumers are assumed to disregard the effect on pollution emissions of their own consumption. The utility function of the consumer group is separable in the numeraire good and pollution damage:  

$$U = x + u(Q) - D(E) - PQ,$$

where $x$ is their consumption of the numeraire good, $Q$ is the total output of the duopolists, and $P$ is the price of good $Q$. It is assumed that $u_Q > 0$ and $u_{QQ} < 0$. 

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5 This simple separable utility function is based on Singh and Vives and implies that consumers derive utility from consumption of goods and disutility from pollution and payment for goods. It implies that utility declines linearly which price $P$. Separability in the numeraire good implies that we can ignore income effects and perform partial equilibrium analysis. Singh and Vives further demonstrate that when $u(Q)$ is quadratic the resulting direct demand functions are linear. A similar utility function is used by, e.g. Tirole (1990).
The duopolists, labelled 1 and 2, compete using quantities as the strategic variable in an infinitely repeated game. From maximization of (1) it follows that
\[
\frac{\partial P(Q)}{\partial Q} < 0 \tag{2.1}
\]
While Eq. (2.1) asserts that the demand function is negatively sloped, condition Eq. (2.2) serves to ensure that a one-shot stable Cournot-Nash equilibrium exists (see Shapiro, 1990). It implies that a firm’s marginal revenue does not rise with its rival’s output:
\[
\frac{\partial P(Q)}{\partial q_i} + q_i \frac{\partial^2 P(Q)}{\partial q_i^2} < 0, \quad i = 1, 2, \quad i \neq j. \tag{2.2}
\]
As noted, production of good \(Q\) is assumed to result in pollution emissions which are related to production levels, i.e.,
\[
E = \theta Q, \tag{3}
\]
where \(\theta\) is the emission coefficient of output. For simplicity, we assume that there is no pollution abatement.

The government is assumed to tax emissions at a rate \(t^s \in T\), where the superscript shows that this is the tax selected when the aggregate political contribution schedules offered by the firms are given by \(S\), where \(S = S_i + S_j\), and \(S_i\) denotes the political contribution schedule offered by firm \(i, i = 1, 2\). Taking the tax and contribution schedules as given, the profits of firm \(i, i = 1, 2\), are defined as
\[
\pi_i(q_i, q_j, t^S_i, S_j) \equiv (P(Q) - \theta t^S)q_i - S_i, \quad i = 1, 2, \quad i \neq j, \tag{4}
\]
where \(q_i\) denotes output of firm \(i\), and \(Q = q_i + q_j\). For simplicity, we assume that marginal production costs equal zero.

Firms are assumed to interact over an indefinite period of time. It is well established that under certain conditions firms may have an incentive to tacitly collude. Tacit collusion, however, gives rise to the familiar problem that each firm has an incentive to defect, given that its rival sets some collusive output level. More specifically, let \(q_c^j\) denote some collusive output level set by firm \(j\). Then the defection output level of its rival \(i\), \(q_d^i\) is determined by
\[
q_d^i \in \text{Arg max} \pi_i^d(q_d^i, q_c^j, t^S), \tag{5}
\]
where
\[
\pi_i^d(q_d^i, q_c^j, t^S) \equiv (P(q_d^i, q_c^j) - \theta t^S)q_i - S_i.
\]

6 Observe that Eq. (2.2) implies that \(\frac{\partial^2 P(Q)}{\partial q_i^2} < -\frac{(\partial P(Q)/\partial q_i)}{q_i}\). It, therefore, places restrictions on the curvature of the demand function.

7 A sufficient condition for the results in this paper to hold is that the emissions-output function \(E(Q)\) be convex. However, if \(E(Q)\) is concave then the convexity of \(D(E)\) must outweigh the concavity of \(E(Q)\). In essence, the results rely on the assumption that pollution damage is non-decreasing in output and emission levels.

8 This assumption merely simplifies the analysis without altering any of the conclusions.

9 Introducing positive marginal costs would not alter the results, as long as marginal costs do not exceed price over the relevant range. Then, production is feasible in both the Cournot equilibrium and the tacitly collusive equilibrium.
It is well established that defection can be prevented if firms adopt a credible and severe threat of retribution. In what follows it is assumed that firms employ the familiar ‘grim trigger strategy’ to deter defection. 10

Specifically, both firms abide by the tacit agreement and produce at some collusive output level as long as there is no defection. However, if a firm defects, its rival immediately reverts to the Cournot–Nash equilibrium output level denoted \( q^*_i \) so that the collusive agreement is dissolved. 11 This strategy will serve to deter defections if the following incentive compatibility constraint is satisfied:

\[
\pi_i^d(q_i^d, q_j^d, r^S) - \pi_i^c(q_i^c, q_j^c, r^S) \leq \frac{\pi_i^c(q_i^c, q_j^c, r^S) - \pi_i^c(q_i^c, q_j^c, r^S)}{r}, \tag{6}
\]

where \( r \) is the discount rate, \( \pi_i^d(q_i^d, q_j^d, r^S) \) are profits of firms \( i = 1,2 \) at the Cournot–Nash equilibrium output level \( (q_i^d, q_j^d) \), and \( \pi_i^c(q_i^c, q_j^c, r^S) \) are collusive profits of firms \( i = 1,2 \).

Note that the left-hand side of Eq. (6) represents the one period gains from defection, while the right-hand side defines the discounted loss of collusive profits which result if collusion is abandoned, and the punishment is implemented. When Eq. (6) is satisfied, firm \( i \) has no incentive to deviate from the collusive output level, \( q_i^c \), given \( r^S \).

In determining its own tacitly collusive output level each firm must consider the impact of its production decision on its rival’s incentive compatibility constraint. More specifically, this requires that firm \( i \)’s collusive output levels are determined by the solution to the constrained maximization problem: 12

\[
\text{Max } \pi_i^c(q_i^c, q_j^c, r^S), \quad i = 1, 2; \quad i \neq j, \tag{7.1}
\]

s.t. \( \Phi_j \leq 0, \tag{7.2} \)

where

\[
\Phi \equiv \pi_j^d(q_j^d, q_i^d, r^S) - \pi_j^c(q_j^c, q_i^c, r^S) \leq \frac{\pi_j^c(q_j^c, q_i^c, r^S) - \pi_j^c(q_j^c, q_i^c, r^S)}{r} \tag{7.3}
\]

Observe that Eq. (7.3) is obtained from the incentive compatibility constraint in Eq. (6). Eqs. (7.1)–(7.3) assert that each firm \( i \) sets output levels to maximize collusive profits, subject to the constraint that its rival does not defect from the tacitly collusive agreement. The constraint reflects the fact that variations in firm \( i \)’s output levels influence its rival’s incentive to defect. The precise manner in which firm \( i \)’s output levels affect its rival’s incentive compatibility constraint is described in Lemma 1.

10 There are other retribution strategies such as those described by Abreu (1986). However, we focus here on reversion to the Cournot equilibrium because of its simplicity and because it is widely adopted. More importantly, however, more severe punishments do not alter the fundamental qualitative properties described here when there is constrained collusion.

11 This follows from the fact that when firm \( i \) moves to the Cournot–Nash output level its rival is also forced to produce at this level.

12 This approach to modeling tacit collusion has been widely adopted (see, Davidson and Martin, 1985; Shapiro, 1990). It appears to have been first suggested by Friedman.
Lemma 1. An increase in firm j’s output lowers its rival’s incentive to defect, i.e.,

$$\frac{d\Phi_i}{dq_j} < 0, \quad i = 1, 2; \quad i \neq j.$$

Lemma 1 reveals that when firm j’s collusive output level rises both the defection and collusive profits of its rival decline. However, the profits that the rival firm i obtains from defection decline more rapidly than do its profits from collusion. This result suggests that a firm can influence its rival’s incentive to tacitly collude by varying its own output level. Hence, in determining its collusive output level each firm must take account of the impact of its production decisions on its rival’s incentive to defect. The properties of the resulting equilibria depend critically upon the prevailing discount rate. Accordingly, rearranging Eq. (6) we define a threshold discount rate:

$$r_q \equiv \frac{\pi_i^c(Q^c, t^S) - \pi_i^a(q_i^0, q_j^0, t^S)}{\pi_i^d(q_i^0, q_j^0, t^S) - \pi_i^c(Q^c, t^S)}, \quad (8)$$

where $Q^c = q_i^c + q_j^c$. Let $r$ be the prevailing discount rate. There are three distinct cases.

Case (A1). Assume $r = r_q$. The constraint (7.2) holds with equality which implies that we have constrained collusion. In this case output levels are determined by the solution to the incentive compatibility constraint, which is termed a ‘balanced temptation equilibrium’ (see Friedman, 1971). In a balanced temptation equilibrium, the industry is neither as collusive as in the joint profit maximizing outcome, nor as competitive as in the Cournot–Nash equilibrium. Thus, collusive output levels lie in the intermediary range between the joint profit maximizing level and the Cournot–Nash equilibrium level.

Case (A2). Assume $r < r_q$. The incentive compatibility constraint (7.2) holds with slack, and collusive output levels are determined by unconstrained joint profit maximization. In this case each firm produces half the monopoly output level, denoted by $q_i^m$, $i = 1, 2, i \neq j$.

Case (A3). Assume $r > r_q$, \forall $q_i \in [q_i^m, q_i^p)$. The incentive compatibility constraint is violated so that collusion is not sustainable. Hence output levels are at the Cournot–Nash equilibrium.

In what follows our main focus will be Case (A1). Lemma 2 describes the impact of the pollution tax on the incentive to collude.

Lemma 2. Assume that Eq. (7.2) binds. Ceteris paribus, the incentive to collude is increasing in the emission tax, i.e.,

$$\left. \frac{d\Phi_i}{dr} \right|_{dq_i^c = dq_j^c = 0} < 0, \quad i = 1, 2, \quad i \neq j.$$

Intuitively, this result arises because under Cournot competition, output, and therefore the emission tax expenditures, are both higher. Defection therefore increases firms’ tax liability. When the emissions tax is high, not only are there gains to be had by restricting output and therefore increasing the price, but there is the additional benefit of paying lower taxes. To see the significance of this, suppose that the prevailing discount rate is such that constraint (7.2) binds. This implies that firms are in a balanced temptation equilibrium,
producing at some tacitly collusive output level \( q_i^c \in (q_i^m, q_i^n) \). Consider a decrease in the emission tax. Lemma 2 reveals that this increases the incentive to defect, and the original tacitly collusive output level can no longer be sustained. However, Lemma 1 suggests that this greater incentive to defect can be negated by raising collusive output levels. Thus, under constrained tacit collusion lower emission taxes induce increases in output levels and as a consequence higher emission levels.\(^{13}\) Note also that it can be shown that when the incentive compatibility constraint is non-binding, and output levels are at the joint profit maximizing outcome, \( q_i^m \), an emission tax has the effect of raising production costs and thus inducing a reduction in output and emission levels. These results are summarized in the following proposition.

**Proposition 1.** Let \( q_i^c \) be the solution to the maximization problem defined in Eqs. (7.1)–(7.3) when constraint problem (7.2) binds. Let \( q_i^m \) be the solution to problem (7) when constraint (7.2) holds with slack. Then, an increase in the emission tax reduces \( q_i^c \) and \( q_i^m \), i.e.,

\[
\frac{dq_i^c}{dt} < 0 \quad \text{and} \quad \frac{dq_i^m}{dt} < 0, \quad i = 1, 2.
\]

Having characterized the output stage of the game we now turn to the interaction between firms and the government in the political equilibrium.

### 3. Stage two: the political equilibrium

This section explores the effects of lobbying by firms on the level of the emission tax which eventuates in the political equilibrium. It is assumed that in stage one, the two firms have each independently offered the government a political contribution schedule. This is deemed to be equivalent to forming a lobby group. The political contribution schedule of the industry lobby group, \( S(t) \), is contingent on the tax rate chosen by the government (see Grossman and Helpman). The government is assumed to maximize a weighted sum of the aggregate political contributions it receives and aggregate social welfare, gross-of-contributions. Social welfare gross-of-contributions is given by the sum of profits, consumers’ surplus, pollution tax revenues and the damage suffered from emissions.\(^{14}\)

\[
W(t) \equiv \int_0^{Q^c} P(Q^c) \, dQ - \theta t Q^c - D(E) + \theta t Q^c.
\]

For future reference we define the welfare maximizing level of emission taxes as\(^{15}\)

\[
t^w \in \arg\max W(t).
\]

\(^{13}\) Note that this result is sensitive to, but does not depend on, the assumption of no pollution abatement.

\(^{14}\) Social welfare is the sum of profits \( Pq^c - \beta tQ^c \), consumers’ surplus \( \int_0^{Q^c} P(Q^c) \, dQ - P Q^c \), pollution damage \(-D(E)\), and government revenues \( \theta t Q^c \). Summing these yields Eq. (9).

\(^{15}\) There are two distortions in the economy: oligopoly and pollution. It is, therefore, possible that \( t^w < 0 \). We are grateful to an anonymous referee for highlighting this fact.
Following Grossman and Helpman, the government’s objective function is given by

$$G(t) = S(t) + \alpha W(t),$$

(11)

where $\alpha$ is the weight given to aggregate social welfare relative to political contributions.

A subgame perfect Nash equilibrium for this game is a set of contribution schedules \( \{S^*_i\}_{i=1,2} \) and a tax policy \( t^* \) such that: (i) each contribution schedule is feasible, i.e., contributions are non-negative and less than the aggregate firm profits; (ii) the policy \( t^* \) maximizes the government’s welfare, \( G(t) \), taking the contribution schedules as given; and (iii) no firm \( i \) has a feasible strategy yielding a net payoff greater than the equilibrium net payoff, given the strategy of firm \( j \), and the government’s anticipated decision rule.

From Lemma 2 of Bernheim and Whinston and Proposition 1 of Dixit et al. (1997), the following two necessary conditions yield a subgame perfect Nash equilibrium \( \{S^*_i, t^*\}_{i=1,2} \) when both firms offer the government political contribution schedules:

\[
\begin{align*}
\text{(KII)} & \quad \frac{\partial \pi^g_i(Q, t) + \pi^g_j(Q, t) + G(t)}{\partial t} \quad \text{on } T \\
\text{(KI)} & \quad t^* \text{ maximizes } G(t) = S^*(t) + \alpha W(t) \quad \text{on } T
\end{align*}
\]

The first condition (KI) asserts that the equilibrium tax \( t^* \) must maximize the government’s payoff, given the offered contribution schedules. The second condition (KII) requires that \( t^* \) must also maximize the joint payoff of the firm lobby group and the government. If this condition is not satisfied, the firms have an incentive to alter their strategies to induce the government to change the tax rate, and capture close to all surplus. Maximizing (KI) and (KII) and performing the appropriate substitutions reveals that in equilibrium the contribution schedule of the industry lobby group (when both firms participate in the lobbying process) satisfies

\[
\frac{\partial \pi^g_i(Q, t) + \pi^g_j(Q, t)}{\partial t} = \frac{\partial S^*(t)}{\partial t},
\]

(12)

where \( \pi^g_i \equiv (P(Q) - \theta_i S)q_i \) are profits gross of contributions. Eq. (12) suggests that in equilibrium the change in the lobby group’s contribution equals the effect of the tax on the utility (profits) of the lobby group. Thus, as noted by Grossman and Helpman, the political contribution schedules are locally truthful. As in Bernheim and Whinston and Grossman and Helpman, this concept can be extended to a contribution schedule that is globally truthful. This type of schedule accurately represents the preferences of the special interest group at all policy points. As discussed by Grossman and Helpman, with one active lobby group the equilibrium contribution to the government is given by the difference in social welfare when the government sets the emission tax at \( t^w \), and at the political equilibrium \( t^* \). Specifically, the necessary contribution for the industry lobby group (or the one lobbying firm) is defined by

\[
S^m = \alpha[W(t^w) - W(t^*)],
\]

(13)

which thus perfectly compensates the government for the welfare loss associated with the participation of the industry lobby group in the political process. In a symmetric equilibrium
the firms are assumed to offer identical contribution schedules to the government, i.e., $S^m_i = S^m_j$. The welfare loss is weighted by the factor $\alpha$ in order to adjust for its importance in the government’s objective function. Having defined the optimal level of contributions for the industry, it is now necessary to determine whether firms have an incentive to lobby rather than to free ride.

4. Stage one: the firm decision to lobby

Since firms are assumed to interact over an indefinite period, considerations of long term profitability will influence each firm’s decision to offer a contribution schedule at each point in time, i.e., to join the lobby group. In this section we explore the conditions under which it is individually rational for firms to offer a contribution schedule and participate in the lobbying process rather than to free ride. In what follows, it is assumed that tacit collusion is sustainable in the output stage of the game, so that output levels are given by the solution to problem (7). We begin by considering the symmetric one-shot equilibrium at some given collusive output level $Q^c = q^c_i + q^c_j$. First, suppose that firm $j$ (in the symmetric case) offers the contribution schedule $S^m_j > 0$ to the government, then the one-shot best response of its rival firm $i$ is defined by

$$S^d_i \in \text{Arg max } \pi^{ed}_i(Q^c, t^{sd}, S^m_j),$$

where

$$\pi^{ed}_i(Q^c, t^{sd}, S^m_j) = (P(Q^c) - \theta t^{sd} - S^m_j)q^c_i - S^d_i, \quad i = 1, 2, \quad i \neq j,$$

defines the profits that firm $i$ obtains by free riding on its rival’s contribution, given that the rival firm contributes $S^m_j$ in equilibrium. Free-riding is a dominant strategy for firm $i$ if the one shot gains from free-riding exceed the costs of contributing to the lobby group.

Let $\pi^{ed}_i(Q^c, t^{0, S^m_j})$ define firm $i$’s profits when it colludes in the output stage but free-rides in the lobbying stage by offering a zero contribution (for any $t$), while its rival firm $j$ both colludes in the output stage and contributes the equilibrium amount $S^m_j > 0$.

Define $\pi^{eg}_i(Q^{eg}, t^{S^m_i, S^m_j})$ as firm $i$’s profits, gross of its political contribution when neither firm free-rides and they both offer contribution schedules $S^m_i = S^m_j$ to the government. Complete free riding in the lobbying stage is then always a dominant strategy for firm $i$ if

$$\pi^{ed}_i(Q^c, t^{0, S^m_j}) - \pi^{eg}_i(Q^{eg}, t^{S^m_i, S^m_j}) > S^m_i(t^e) > 0.$$  

Note that the attempt to free-ride by firm $i$ will necessitate an increase in the equilibrium contribution paid by firm $j$ if the welfare loss to the government from the new policy choice is larger than half the welfare loss in the original (symmetric) equilibrium. Firm $j$ must now compensate the government for the deadweight loss due to the rival firm. The larger is the additional welfare loss, the more likely is it that Eq. (15) will hold, i.e., the larger the free-rider problem. Define $\pi^{rn}_i(Q^c, t^w)$ as the equilibrium profits when there is no lobbying and taxes are set at the welfare maximizing level $t^w$. Then unilateral lobbying by firm $j$ is unprofitable if

$$\pi^{co}_j(Q^c, t^{0, S^m_j}) < \pi^{rn}_j(Q^c, t^w),$$
where $S_j^k$ is the equilibrium contribution level. Clearly a sufficient condition for each firm to free ride is that a unilateral increase in contributions lowers profits, i.e.,

$$\frac{d\pi_i^c}{dS_i} \bigg|_{dS_j=0} < 0.$$  

This, in turn, requires that $\varepsilon_iS > -S_i/E_i$, where $\varepsilon_iS = (\partial t/\partial S_i)(S_i/t)$ is the elasticity of the tax with respect to the contribution of firm $i$.\(^{16}\) Since we wish to focus on lobby group formation when the incentive to free-ride is present, we assume that Eq. (15)–(16) always hold and complete free-riding is assumed to be the dominant strategy in any given one-shot game.

If free riding is to be prevented, then firms must adopt some credible and severe punishment strategy which renders such behavior unprofitable. As in the previous section we consider the possibility of deterring free riding by employing the familiar grim-trigger strategy. Specifically, it is assumed that each firm abides by a tacitly collusive strategy which requires it to contribute half the amount implied by Eq. (13) to the lobby group as long as its rival does not defect (i.e., free-rides). If, however, the rival deviates from this strategy the firm immediately reverts to the one-shot Nash equilibrium if Eq. (16) holds.\(^{17}\) Under the assumption that conditions (15) and (16) hold there is no lobbying in the one-shot Nash equilibrium and both firms make a zero contribution to the industry lobby group. In this case, we know from Eq. (10) that the government sets the tax rate at $t^w$ to maximize social welfare. This strategy will serve to deter free riding if the following incentive compatibility constraint is satisfied:

$$\pi_i^{cd}(Q^{cd}, t^0, S_j^m) - \pi_i^{cc}(Q^{cc}, t^{sm}, S_j^m) \leq \pi_i^{cn}(Q^{cn}, t^w) - \pi_i^{cn}(Q^{cn}, t^w)/r,$$  

where $\pi_i^{cd}(Q^{cd}, t^0, S_j^m)$ are the profits of firm $i$ when it colludes in the output stage but free-rides on its rival’s contribution given that its rival contributes a positive amount $S_j^m$, and $\pi_i^{cc}(Q^{cc}, t^{sm}, S_j^m)$ are profits when both firms abide by the collusive agreement in the output and lobbying stages by contributing amounts $S_j^m$, $S_j^n$, respectively. Finally, $\pi_i^{cn}(Q^{cn}, t^w)$ are profits when firm $i$ colludes in the output stage, but there is no lobbying so that the tax is set at the welfare maximizing level, $t^w$.\(^{18}\)

The left-hand side of Eq. (17) defines the one period gains to firm $j$ when it free-rides, while the right-hand side defines the profits foregone when lobbying ends and the tax is set at $t^w$. When Eq. (17) is satisfied, contributing to the lobby group is the individ-

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\(^{16}\) To see this note that $d\pi_i^c/dS_i = -E_i \partial t/\partial S_i - 1$ Rearranging it can be verified that $d\pi_i^c/dS_i < 0$ if $\varepsilon_iS > (\partial t/\partial S_i)(S_i/t)$ since $\partial t/\partial S_i < 0$.

\(^{17}\) If Eq. (16) does not hold, lobbying is sustained by one firm which compensates the government for the welfare loss.

\(^{18}\) The notation $Q^{k}$ ($k = c, d, n$) reflects the fact that by Proposition 1 output levels depend on the tax rate, amongst other things. The tax rate (which is determined in Stage 2) in turn is a function of the overall level of lobby group contribution (determined in Stage 1). This depends on whether firms free-ride (defect) or contribute in the lobbying stage of the game.
ually rational strategy for each firm\(^{19}\). Rearranging Eq. (17) defines a critical discount rate:

\[
R_L = \frac{\pi^{cc}_i(Q^{cc}, i_s^{m}, s^m_i) - \pi^{cn}_i(Q^{cn}, i_w)}{\pi^{ed}_i(Q^{cd}, i_0, s^m_i) - \pi^{cc}_i(Q^{cc}, i_s^{m}, s^m_i)}.
\]

(18)

Observe that from Eq. (17) it follows that equilibrium contributions of \(S^{m}_i, i = 1, 2, i \neq j\), can be sustained by this strategy if the prevailing discount rate is \(r \leq R_L\).

For future reference, Lemma 3 describes the manner in which each firm’s contribution affects its rival’s incentive to free ride.

**Lemma 3.** Each firm’s incentive to free ride is increasing in its rival’s contributions.

Lemma 3 is the lobbying game counterpart of Lemma 1. It simply reveals that an increase in contributions by one firm renders free riding more attractive to its rival. Lemma 4 shows the effect of the pollution tax on collusive profits.

**Lemma 4.** Ceteris paribus, collusive profits are decreasing in the pollution tax.

Lemma 4 shows that a lower pollution tax unambiguously increases profits, although from Proposition 1 the level of collusion decreases. Thus, the direct effect on profits of the tax dominates the indirect effect via collusion.

Having defined the conditions necessary to make lobbying feasible in an infinitely repeated game, we now turn to the equilibrium properties.

5. **Equilibrium properties**

We now explore the factors which influence the incentive to form a lobby group. We, therefore, begin by assuming that lobbying is feasible which by Eqs. (17) and (18) requires of the prevailing discount rate that \(r \leq R_L\). The properties of the resulting equilibria depend not only on the level of the prevailing discount rate but also on the relative positions of the critical levels \(r_q\) and \(R_L\) as defined in Eqs. (8) and (18), respectively. We distinguish three cases (corresponding to Cases (A1)–(A3) above).

**Case (B1):** Assume \(r = r_q \leq R_L\). Lobbying is incentive compatible and tacit collusion in the output stage is sustainable. Since \(r = r_q\), the incentive compatibility constraint in Eq. (7.2) holds with equality, thus (as noted in Section II) output levels lie between the Cournot and the joint profit maximizing levels.

**Case (B2):** Assume \(r \leq r_q\) and \(r < r_q\). In this case lobbying and tacit collusion in the output stage are sustainable. Since \(r < r_q\), the incentive compatibility constraint in Eq. (7.2) holds with slack, and output is at the joint profit maximizing level.

**Case (B3):** Assume \(r \leq r_q\) and \(r > r_q\). From Eq. (17), lobbying is incentive compatible since \(r \leq r_q\). However, since \(r > r_q\), tacit collusion in the output stage of the game cannot be sustained. Thus, output levels are set at the Cournot–Nash equilibrium. In this case lobbying takes place even in the absence of tacit collusion in the output market.

\(^{19}\)The incentive compatibility constraint in Eq. (17) is based on the implicit assumption that tacit collusion in the output stage is feasible, even in the absence of lobbying. Eschewing this assumption would merely strengthen the results outlined below without altering the qualitative conclusions.
In what follows, we are interested in analyzing the manner in which changes in the degree of collusion in the output market affect the incentive to contribute to the lobby group. Hence, we focus only on those equilibria in which lobbying is feasible and there is constrained tacit collusion.\textsuperscript{20} This is equivalent to assuming that $r = r_1 \leq r_L$, i.e., Case (B1).

6. Lobby group formation

In this section we explore the properties of the lobbying equilibria when there is constrained tacit collusion in the output market. We begin by determining whether the incentive to offer a contribution schedule is affected by variations in the degree of collusion, and the level of collusive profits, in the output market.\textsuperscript{21} Next, we investigate whether variations in pollution intensity influence the incentive to undertake lobbying activities.

To assess the effects of collusion on the incentive to lobby we investigate the impact of an exogenous change in collusive output levels in the lobbying equilibrium. In order to do so, define

$$
\Psi_i = \pi_i^{cd}(Q^{cd}, r^{S_i^{cd}}) - \pi_i^{cc}(Q^{cc}, r^{S_i^{cc}}) \left(1 + \frac{1}{r}\right) + \frac{\pi_i^{cn}(Q^{cn}, r^{w})}{r}
$$

as the incentive compatibility constraint in the lobbying stage of the game. The main results of this paper are stated in the following propositions. We assume that the effect of $Q^{cc}$ on the aggregate discounted collusive profits is greater than the effect on one shot free-riding profit.\textsuperscript{22}

**Proposition 2.** Ceteris paribus, when lobbying is feasible: (i) The incentive to offer a political contribution schedule is increasing in the level of collusive profits (i.e., $d\Psi_i/d\pi(Q^{cc}) < 0$); (ii) The incentive to offer a political contribution schedule is increasing in the degree of collusion in the output stage (i.e., $d\Psi_i/dQ^{cc} > 0$).

The intuition is the following. As the level of collusion in the output stage rises, collusive profits increase. This raises the opportunity cost of free-riding in the lobbying stage of the game, since firms now have more to lose by free-riding on the rival firm’s lobbying effort. From Lemma 4, profits are unambiguously decreasing in the pollution tax. Consequently, the incentive to offer a contribution schedule (i.e., to participate in lobbying) is greater the lower is the pollution tax, although the degree of collusion is falling as a result of lobbying. The greater is the reduction of collusion as a result of lobbying, the lower the incentive to offer a contribution schedule. A further implication of Proposition 2 is that ceteris paribus,

\textsuperscript{20} There are other equilibria in the model with lobbying where output is at the joint profit maximizing outcome (i.e., Case (B2)). For brevity this case has been ignored since marginal variations in contributions have no effect on the degree of collusion in the output market when the constraint (7.2) holds with slack (i.e., the joint profit maximizing outcome is sustainable.

\textsuperscript{21} If the Cournot outcome leads to an output level which is greater than socially optimal because of pollution, the government could set a higher tax and thus increase the incentive to collude, at the same time reducing pollution.

\textsuperscript{22} This is not a necessary condition, but simplifies the discussion. See the proof of Proposition 2. Stated differently, the proof in the appendix demonstrates that $Q^{cc}$ has a large first-order impact on $\pi^{cc}$, but a second-order effect on $\pi^{cd}$ (through $Q^{cd}$). As long as the first-order effect is larger than the second-order impact, then $d\Psi_i/dQ^{cc} > 0$. 

in more collusive industries the critical threshold discount level in the lobbying stage, \( r_L \), is lower.

We now explore the effect of changes in the degree of collusion on the equilibrium level of contributions.

**Proposition 3.** The sustainable equilibrium level of lobbying contributions is increasing in the degree of collusion (i.e., \( dS^c/dQ^{cc} < 0 \), \( \forall S^c \in (0, S^m) \)).

Proposition 3 reveals that greater collusion in the output market (i.e., lower output levels) provides an incentive for firms to increase their lobby group contributions. The higher contributions, in turn, lead to a more favorable tax policy, which increases profits further, although it reduces collusion in the output market. Political contributions made in the earlier stages of the game thus create an environment which leads to greater profits despite being less conducive to collusion.\(^23\)

Having described the impact of collusion on lobbying, we now investigate whether lobby group contributions are affected by variations in pollution intensity. The results are outlined in Propositions 4 and 5.

**Proposition 4.** Firm \( i \)'s incentive to offer a political contribution schedule is increasing in the degree of pollution intensity, \( \theta \), iff the aggregate pollution tax burden in the absence of lobbying, \( \theta q_i^{cn} t_w \), is sufficiently great.

When free-riding implies a relatively large aggregate tax burden (due to a large output and/or a high equilibrium tax), an exogenous rise in \( \theta \) reduces the incentive to free ride because the benefit of defection is reduced. The decrease in the incentive to free ride occurs because the higher tax liability becomes a more credible threat.

For completeness we now explore the impact of increases in pollution intensity on the sustainable level of contributions (\( S^c \)).

**Proposition 5.** If the firms’ incentive to offer political contribution schedules is increasing in the degree of pollution intensity, then the sustainable equilibrium level of lobbying contributions is also increasing in the degree of pollution intensity, \( \theta \) (i.e., \( dS^c/d\theta > 0 \), \( \forall S^c \in (0, S^m) \)).

Intuitively, greater pollution intensity reduces the incentive to free ride because the cost of doing so has increased, and thus the level of contributions increase in the constrained equilibrium. Propositions 4 and 5 thus combine to suggest that industries that confront a high emissions tax liability in the absence of lobbying tend to lobby more intensively for tax policy concessions.

\(^{23}\) Assume that all consumers join an environmental lobby group, and thus all individuals in society are organized. Grossman and Helpman (1994) find that in this case, each lobby must make a contribution that compensates the government for the utility loss the government and the opposing lobby make from its participation, as opposed to it being inactive. As long as the interests of the firms and the environmentalists are the opposite, counter lobbying by the environmental lobby would increase the contributions that the firms must pay, since if the firms do not lobby the tax rate would be more unfavorable to them. The impact on the incentive to form a firm lobby group will depend on the relative change in the necessary contributions and profits.
7. Conclusions

This paper has developed a new theory of lobby group formation. A new mechanism that previously has been overlooked in the literature on lobby group formation was identified. It was shown that the degree of tacit collusion in the output market, and collusive profits, have a critical effect on the incentive to lobby. This contrasts with Olson (1965) who attributes the positive relationship between the number of firms in an industry and the free rider problem to transactions costs. In particular, we find that more collusive industries, with greater collusive profit levels, have a greater incentive to form and contribute to an industry lobby group that seeks a lower pollution tax. Finally, it was demonstrated that more polluting industries have a greater incentive to form and sustain a lobby group. The paper thus extended the menu-auction model of Grossman and Helpman (1994) in which the existence of lobby groups is taken as given.

The results are largely consistent with the empirical evidence reported by, e.g. Andres (1985), Masters and Keim (1985), Heywood (1988), and in particular, Zardkoohi (1985) and Pittman (1988). Zardkoohi found that the potential rents from lobbying are important for the explanation of lobbying intensity in concentrated industries, and Pittman showed that this was the case only in these industries. Moreover, Salamon and Siegfried (1977) find that larger industries are less successful in their political activities, indicating that collusion has a positive effect on lobbying.

Our investigation has focused on a pollution tax and can be directly translated to an output tax. However, the findings are likely to apply to a wider range of policy instruments. For instance, it would be interesting and important to investigate the impact of a tariff on collusion and the ability of a protectionist lobby to form.

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\footnote{Future empirical work in this area should preferably move away from the single equation framework used in much of the previous literature. In order to test our theory, we suggest a simultaneous equation system with (at least) three endogenous variables; PAC contribution levels and/or PAC formation, regulatory standards, and collusive profits. Cross-industry time-series data could be employed to test whether the lobbying effort is greater in more collusive industries, and lower when a change in the (relevant) regulation is accompanied by a relatively greater reduction of concentration or collusion. We would need to adjust for the direct effect of regulation on profits (e.g., a tax reduction), that does not stem from a change in collusion. By including a variable measuring the relative profitability we would also adjust for the effect that available resources may have on lobbying activities, as do Salamon and Siegfried and Masters and Keim. They found no significant effects. The system would also include an equation where the level of regulation is explained by lobbying, and another equation where collusive profits are explained by the level of regulations (we would need to control for several other variables used in the literature described above).}
Appendix A

Proof of Lemma 1. Let \((q_i^c, q_j^c)\) be the output levels at the constrained maximum where the incentive compatibility constraint (7.2) binds. By symmetry we have \(q_i^c = q_j^c = q^c\), and \(\Phi_i = 0\). Suppose that Lemma 1 is not true, then \(d\Phi_i/dq_i^c > 0\). Then by Eq. (7), a fall in the collusive output level, \(q^c\), brings output closer to the unconstrained joint profit maximizing level and raises collusive profits. Since \(d\Phi_i/dq_i^c > 0\), this does not prompt any defection. This, however, contradicts the hypothesis that \(q_j^c\) is the solution to the constrained maximum in Eq. (7). Thus, \(d\Phi_i/dq_i^c < 0\).

Proof of Lemma 2. The incentive compatibility constraint in the output stage of the game is Eq. (7.3). By Shephard’s Lemma,

\[
\frac{d\Phi_i}{dr} = -E^d + E^c \left(1 + \frac{1}{r}\right) - \frac{E^n}{r} = E^c - E^d + \frac{E^c - E^n}{r},
\]

(A.1)

where \(E^k = 0\) for \(k = c, d, n\). Rearranging Eq. (A.1), observe that

\[
\frac{d\Phi_i}{dr} < 0 \quad \text{iff} \quad r > \frac{E^n - E^c}{E^c - E^d} < 0,
\]

(A.2)

where the last inequality holds since \(E^n > E^c\), and \(E^d > E^c\). Since \(r > 0\) by assumption, it follows that Eq. (A.2) holds, and \(d\Phi_i/dr < 0\), i.e., the incentive to collude is increasing in the emission tax.

Proof of Proposition 1. (i) The incentive compatibility constraint in Eq. (7.3) is given by

\[
\Phi_i = \pi_i^d(q_i^d, q_j^c, t^S) - \pi_i^c(q_i^c, q_j^c, t^S) - \frac{\pi_i^n(q_i^d, q_j^n, t^S)}{r}.
\]

Totally differentiating in a symmetric equilibrium yields

\[
d\Phi_i = \left(-E^d + E^c \left(1 + \frac{1}{r}\right) - \frac{E^n}{r}\right) dt + \left[\frac{\partial \pi_i^d}{\partial q_j^c} - \left(\frac{\partial \pi_i^c}{\partial q_i^c} + \frac{\partial \pi_i^c}{\partial q_j^c}\right) \left(1 + \frac{1}{r}\right)\right] dq_i^c.
\]

(A.3)

where by symmetry \(dq_i^c = dq_j^c = dq^c\). Rearranging yields

\[
\frac{dq_i^c}{dr} = \frac{E^d - E^c(1 + 1/r) + E^n/r}{\partial \pi_i^d/\partial q_j^c - \left(\partial \pi_i^c/\partial q_i^c + \partial \pi_i^c/\partial q_j^c\right) (1 + 1/r)}\]

(A.4)

We know (see Lemma 2) that the numerator of Eq. (A.4) is positive. Moreover, \(\partial \pi_i^d/\partial q_j^c = \partial P(q_i^d, q_j^c)/\partial q_i^c + (P(q_i^d, q_j^c) - \theta t)(\partial q_i^d/\partial q_j^c) < 0\) (since by Eq. (2.1),

Note we also use Shephard’s Lemma: \(\partial \pi^k/\partial t = -E^k\), \(k = c, d, n\).
\(\partial P(q_i^a, q_j^a)/\partial q_j^c < 0\), and from the first-order condition corresponding to Eq. (5), \((P-\theta t) \geq 0\) and \(\partial q_j^a/\partial q_j^c < 0\). Eq. (5) implies that \(\partial \pi_i^d/\partial q_j^d = 0\). By concavity of the profit function, \(\partial \pi_i^d/\partial q_j^d < 0\). Given that \(q_i^d > q_j^c\), then \(0 = \partial \pi_i^d/\partial q_j^d < \partial \pi_i^d/\partial q_j^c\). By similar reasoning, \(\partial \pi_i^c/\partial q_j^c + \partial \pi_j^c/\partial q_j^c > 0\). Hence, \(dq_j^c/dt < 0, \forall q_j^c \in (q_j^{m}, q_j^{m})\). (ii) The first order condition for \(q_j^m\) is:

\[
\frac{d\Psi_i(S_i^c, S_j^c)}{dt} = \frac{2\theta}{Q(\partial^2 P/\partial Q^2)} + 2(\partial P/\partial Q) < 0,
\]

since by Eq. (2.2) the denominator is negative.

**Proof of Lemma 3.** Let \(S_i^c \in (0, S_i^{m}), i = 1, 2, i \neq j\) and define \((S_i^c, S_j^c)\) as the contributions at which the incentive compatibility constraint in the lobbying stage just holds with equality:

\[
\Psi_i(S_i^c, S_j^c) \equiv \pi_i^c(\pi_i^{cd}, t_0, S_j^c) - \pi_j^c(\pi_j^{cc}, t_i S_i^c, S_j^c) \left(1 + \frac{1}{r}\right) + \frac{\pi_i^{ca}(Q^{ca}, t^w)}{r} = 0.
\]

(A.6)

Suppose Lemma 3 is not true, then \(d\Psi_i(S_i^c, S_j^c)/dS_j^c < 0\). Thus, an increase in contributions by firm \(j\) does not induce further free riding by the rival firm \(i\) and is therefore sustainable. Moreover, since \(S_j^c \in (0, S_j^{m}), i = 1, 2, i \neq j\), contributions are below their unconstrained optimum levels, so that an increase in contributions raises the contributing firm’s profits. This, however, contradicts the assumption that \(S_j^c \in (0, S_j^{m})\) is the optimum contribution level at which the incentive compatibility constraint in Eq. (A.5) binds. Thus, \(d\Psi_i(S_i^c, S_j^c)/dS_j^c > 0\)

**Proof of Proposition 2.** (i) The incentive compatibility constraint in the lobbying stage is defined as

\[
\Psi_i \equiv \pi_i^c(\pi_i^{cd}, t_0, S_j^c) - \pi_j^c(\pi_j^{cc}, t_i S_i^c, S_j^c) \left(1 + \frac{1}{r}\right) + \frac{\pi_i^{ca}(Q^{ca}, t^w)}{r},
\]

(A.6')

where Eq. (A.6') equals zero when the incentive compatibility constraint in the lobbying game binds. Differentiation yields

\[
\frac{d\Psi_i}{d\pi_i^c(\pi_i^{cc})} = - \left(1 + \frac{1}{r}\right) < 0.
\]
(ii) Consider an increase in industry collusive output levels in the lobbying equilibrium \((Q^c)\), at a given level of contributions. Differentiation of Eq. (A.6) yields

\[
\frac{d\Psi_i}{dQ^c} = \frac{\partial \pi_i^c(Q^c, r^0, S_i^c)}{\partial Q^c} \frac{\partial Q^c}{\partial Q^c} - \frac{\partial \pi_i^c(Q^c, r^0, S_i^c)}{\partial Q^c} \left(1 + \frac{1}{r}\right) \\
+ \frac{\partial \pi_i^c(Q^c, r^0, S_i^c)}{\partial Q^c} \frac{1}{r}.
\]

(A.8)

To find the sign of Eq. (A.8), we sign each term on the right-hand side separately as follows. First, note that

\[
\frac{\partial Q^c}{\partial Q^c} = \frac{\partial Q^c}{\partial Q^c} \frac{\partial Q^c}{\partial Q^c}.
\]

(A.9)

where

\[
\left.\frac{\partial Q^c}{\partial Q^c}\right|_{S^c=0} = \frac{\partial Q^c}{\partial Q^c} \frac{\partial Q^c}{\partial Q^c}.
\]

(A.10)

Next, taking the first-order condition of \(Q^c\) yields

\[
\frac{\partial \pi_i^c}{\partial Q^c} \frac{\partial Q^c}{\partial s_j^0} - \theta \frac{\partial s_j^0}{\partial s_i^0} - 1 = 0.
\]

(A.11)

Total differentiation of Eq. (A.11) and rearranging yields

\[
\frac{dS_j^0}{dQ^c} = -\left(\frac{\partial^2 \pi_i^c(Q^c)}{\partial s_j^0 \partial Q^c} \frac{\partial Q^c}{\partial s_j^0} - \theta \frac{\partial s_j^0}{\partial s_i^0} \right) + \left(\frac{\partial^2 \pi_i^c(Q^c)}{\partial s_j^0 \partial Q^c} \frac{\partial Q^c}{\partial s_j^0} - \theta \frac{\partial s_j^0}{\partial s_i^0} \right).
\]

(A.12)

where the numerator is unambiguously negative due to the concavity of the profit function, i.e., \(\partial^2 \pi_i^c(Q^c) < 0\); by Lemma 2 \(\partial Q^c / \partial s_j^0 < 0\); by Eq. (13) \(\partial s_j^0 / \partial s_i^0 < 0\); and \(\partial \pi_i^c / \partial Q^c < 0\) since \(Q^c < Q^m, \forall k = c, d, n\). It follows that the sufficient conditions for \(dS_j^0 / dQ^c < 0\) are; (CI) if \(\partial^2 \pi_i^c(Q^c) \partial s_j^0 / \partial Q^c < 0\) then \((\partial \pi_i^c(Q^c) / \partial Q^c)(\partial Q^c / \partial s_j^0) > \theta\), and; (CII) if \(\partial^2 \pi_i^c(Q^c) \partial s_j^0 / \partial Q^c > 0\) then \((\partial \pi_i^c(Q^c) / \partial Q^c)(\partial Q^c / \partial s_j^0) < \theta\). It follows that Eq. (A.9) is positive if either of these two sufficient conditions hold. Since from Proposition 1, \(\partial Q^c / \partial t^0 < 0\), using Eq. (A.10) and the above it follows that

\[
\frac{dQ^c}{dQ^c} = \frac{\partial Q^c}{\partial s_j^0} \frac{\partial s_j^0}{\partial Q^c} < 0,
\]

(A.13)

which implies

\[
\frac{\partial \pi_i^c}{\partial Q^c} \frac{\partial Q^c}{\partial Q^c} > 0,
\]

(A.14)
if one of the sufficient conditions (CI) or (CII) above hold. Turning to the second term of Eq. (A.8), notice that \( \frac{\partial}{\partial Q} Q_c^c < 0 \) since \( Q_c^c > Q_m^c \) where \( Q_m^c \) is the profit maximizing output level. It follows that

\[
- \left( 1 + \frac{1}{r} \right) \frac{\partial \pi_c^c}{\partial Q_c^c} > 0.
\]

(A.15)

Finally, consider the last term of Eq. (A.8) where

\[
\frac{\partial Q_c^c}{\partial Q_c^c} = \frac{\partial Q_c^w}{\partial t_w} \frac{\partial t_w}{\partial Q_c^c}.
\]

(A.16)

From Eq. (10) \( t_w^c \) by definition is the welfare maximizing tax rate, and thus no lobbying takes place in this equilibrium. It then follows that

\[
\frac{\partial t_w^c}{\partial Q_c^c} = 0,
\]

(A.17)

since \( \frac{\partial W(t_w^c)}{\partial Q_c^c} = 0 \) and thus \( \frac{\partial Q_c^w}{\partial Q_c^c} = 0 \). Combining this result with Eqs. (A.14) and (A.15) implies that

\[
- \left( 1 + \frac{1}{r} \right) \frac{\partial \pi_c^c}{\partial t_w} \frac{\partial t_w}{\partial Q_c^c} > 0,
\]

(A.18)

assuming (CI) or (CII) holds. Note that Eq. (A.18) unambiguously holds if

\[
\frac{\partial \pi_c^d(Q_c^d, t_0^{S_i^c})}{\partial Q_c^c} < 0 \quad \text{for all } S_i^c \in (0, S_i^m).
\]

(A.19)

i.e., as long as the effect of \( Q_c^c \) on the aggregate discounted collusive profits (the second term in Eq. (A.19)) is greater than the effect on the one shot free-riding profit (the first term in Eq. (A.19)).

**Proof of Proposition 3.** We must sign \( dS_i^c/dQ_c^c, S_i^c \in (0, S_i^m) \). Suppose that \( dS_i^c/dQ_c^c > 0 \). Then higher output levels in stage 1 induce firms to increase their contributions in the later stage of the game. From Proposition 2 we know that \( d\Psi_i/dQ_c^c > 0 \), thus higher output levels raise the incentive to free-ride. Hence, if \( dS_i^c/dQ_c^c > 0 \) this contradicts Lemma 3 and violates the incentive compatibility constraint in the lobbying stage, Eq. (A.6). Thus, \( dS_i^c/dQ_c^c < 0, \forall S_i^c \in (0, S_i^m) \).

**Proof of Proposition 4.** We wish to show \( d\Psi_i/d\theta < 0 \). Totally differentiating Eq. (19) yields

\[
\frac{d\Psi_i}{d\theta} = \frac{\partial \pi_c^d}{\partial \theta} - \frac{\partial \pi_c^c}{\partial \theta} \left( 1 + \frac{1}{r} \right) + \frac{\partial \pi_c^e}{\partial \theta} \frac{1}{r}.
\]

(A.20)

Taking the partials of each of the profit functions yields

\[
\frac{d\pi_c^d}{d\theta} = \left( t_0^{S_j} + \frac{\partial t_0^{S_j}}{\partial \theta} \right) q_c^d + \frac{\partial \pi_c^d}{\partial \theta} + \frac{\partial q_c^d}{\partial \theta} \frac{\partial t_0^{S_j}}{\partial \theta},
\]

(A.21)
where we denote the second term on the right-hand side by $\Omega_{cd}^2$. Thus, Eq. (A.21') can be rewritten as

$$\frac{d\pi_{cd}}{d\theta} = - \left( \Omega_{cd}^1 + \frac{\partial t_{0,S_j}^0}{\partial \theta} \right) q_{cd}^2 + \Omega_{cd}^2.$$ 

Similarly,

$$\frac{d\pi_{cc}}{d\theta} = - \left( \Omega_{cc}^1 + \frac{\partial t_{S}^0}{\partial \theta} \right) q_{cc}^2 + \Omega_{cc}^2,$$

and

$$\frac{d\pi_{cn}}{d\theta} = - \left( \Omega_{cn}^1 + \frac{\partial t_{w}^0}{\partial \theta} \right) q_{cn}^2 + \Omega_{cn}^2.$$ 

Substituting Eqs. (A.21'), (A.22) and (A.23) into Eq. (A.20), and rearranging yields the following condition for $d\Psi_i/d\theta|_{dS^c=0} < 0$ to hold:

$$\theta q_{cn}^1 t_{w}^1 > \frac{1}{r} \left[ \Omega_{cn}^2 - q_{cn}^1 \left( t_{S}^0 + \frac{\partial t_{S}^0}{\partial \theta} \right) \right] + \left( 1 + r \right) \left[ q_{cc}^1 \left( t_{S}^0 + \frac{\partial t_{S}^0}{\partial \theta} \right) - \Omega_{cc}^2 \right]$$

Proof of Proposition 5. Total differentiation of Eq. (19) yields

$$d\Psi_i = \frac{\partial \Psi_i}{\partial \theta} d\theta + \frac{\partial \Psi_i}{\partial S^c} dS^c = 0,$$

which implies $dS^c/d\theta = \left( -\partial \Psi_i/\partial \theta \right) \left( \partial \Psi_i/\partial S^c \right) > 0$ since from Lemma 3 the denominator is positive, and from Proposition 4 the numerator is negative if Eq. (A.24) holds.

References

