Interlinkage, limited liability and strategic interaction

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Abstract

The literature on interlinkage is inconclusive regarding the strict superiority of this arrangement. We analyze a landlord and a moneylender as two players making non-cooperative decisions regarding the terms of their respective contracts with a tenant. In the sequential game where the landlord moves first and the tenant has limited liability, we demonstrate that there exist circumstances in which interlinkage is superior, even with nonlinear loan contracts, a result that carries over when there is moral hazard. The incorporation of risk aversion yields strict superiority in general. The main result is unaffected by changes in the seniority of claims, but is sensitive to changes in the order of moves: limited liability ceases to ensure the strict superiority of interlinked contracts if the principal who provides the variable factor of production moves first, even if he has junior claims to the output. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Interlinkage, or the practice of offering contracts that combine transactions across multiple markets, is a phenomenon that prevails in the agrarian sector of many less-developed economies. What explains the prevalence of such contracts in underdeveloped rural sectors? Clearly, any attempt to answer this fundamental question must involve the identification and analysis of the factors that create conditions for the superiority of interlinked contracts over non-interlinked ones (for a general survey, see Bell, 1988).

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At first glance, such a task may seem simple. Consider (as we shall in this paper) the well-worn example of a landlord, a moneylender and a tenant, where the landlord has access to finance on the same terms as the moneylender. The landlord can always offer an interlinked tenancy-cum-credit contract whose terms are identical to those offered when the landlord and moneylender are separate and independent principals. The question that arises at once is the following: can the landlord do strictly better with interlinkage? Newbery (1975) shows that in the case where there is no uncertainty and the tenant is subject to unlimited liability, interlinkage is not necessary for achieving a first-best allocation. In an uncertain world in which enough variables are observable, Ray and Sengupta (1989) show that the ability to impose sufficiently nonlinear contracts neutralizes the superiority of interlinkage. In other papers, the strict optimality of interlinked contracts arises from the presence of some sort of informational asymmetry, either in the form of moral hazard (Braverman and Stiglitz, 1981), or adverse selection (Banerji, 1995).

In this paper, we offer a different reason for the prevalence of interlinkage — one that is valid even when informational asymmetries or gains from risk-sharing are absent — namely, the tenant’s limited liability. The theoretical literature on agrarian organization has not paid much attention to limited liability (in the context of tenancy, see Kotwal, 1985; Basu, 1992), despite its significant incidence in less-developed economies; but it plays a central role in our analysis. We demonstrate that limited liability can have an adverse effect on allocative efficiency in the non-cooperative game between two independent principals, and that coordination through interlinking, in contrast, can bring about a first-best outcome.

Given that default is possible, the seniority of claims to the tenant’s output assumes central importance. We initially assume that the landlord has ‘first rights’. He usually lives in the same village, and if the rent depends on the level of output, the crop is harvested and threshed under his watchful eye. The moneylender is usually less well situated to exploit such advantages of proximity. In addition, we first focus on the case, where, in the absence of interlinkage, the landlord plays the role of a Stackelberg leader, since a landless and assetless tenant will, typically, not qualify for a loan without a tenancy contract. The sequential, non-cooperative game between the two principals in such a setting is analyzed in Section 4, after we have set out the model (Section 2) and established the first-best case as a benchmark (Section 3). As extensions, we examine the consequences of introducing risk-aversion and moral hazard (Section 5).

With increased commercialization and greater availability of formal sector finance, the above described customary assignment of ‘first rights’ may begin to weaken. Changes in lending practices may also render it unnecessary for the tenant to close contracts in a particular sequence. We, therefore, extend our analysis in Section 6 to allow for changes in the seniority of claims and in the order of moves in the game, and examine the robustness of our results to such alterations. The paper concludes with a brief discussion of the results.

2. The model

The tenant-cultivator is assumed to apply his endowment of an indivisible unit of labor to an indivisible unit (a plot) of land. The output produced is stochastic, with two possible outcomes: a high yield (denoted by $y_H$), and a bad harvest, or low yield (denoted by $y_L$). The
probability of the good outcome is an increasing, strictly concave and twice differentiable function of $K$, $\pi(K)$, where $K$ is the amount of investment in non-labor inputs inputs, with $\pi'(K)>0$ and $\pi''(K)<0$. We additionally assume that $\pi(K)$ satisfies both Inada conditions, that is, $\lim_{K \to 0}\pi'(K) = +\infty$ and $\lim_{K \to +\infty}\pi'(K) = 0$, to ensure the existence of interior solutions.

The landless and assetless tenant acquires a unit of land and the entire investible funds from the land and credit markets in the informal sector. Let $\beta_i$ be the rent in state $i$ (with $i=H, L$). If the landlord has access to funds, he may offer credit on terms which are interlinked with the rental agreement. If he does not, or chooses not to lend, then the source of the loan is a professional moneylender. The opportunity cost of funds is the same for the landlord and the moneylender, and is a constant $(1+m)$ per dollar loaned.

All three agents are assumed to be risk neutral. The reservation income of the tenant is $\tilde{y}$, which is assumed to be strictly greater than $y_L$. The investment $K$ is made essential for participation in cultivation by assuming that $\pi(0)=0$.

If the output produced by the tenant is less than the combined amounts of the rent and credit obligations, there will be a shortfall in repayment of dues. The question arises as to how this shortfall will be shared. As indicated in Section 1, we initially assume that the landlord has the ‘first rights’ to the harvest, due to either the nature of property rights in the countryside, or his greater power and proximity to the tenancy, and thus eliminate the possibility of situations which involve bargaining strategies. With this assumption, it is easy to see that we do not sacrifice any generality by confining our attention to those rental contracts where $\beta_i \leq y_i$, for $i=L, H$; for demanding a state-contingent rent that is larger than the output in that state is pointless whatever be the landlord’s bargaining power. It is also assumed that the monopoly power of the lender (or the landlord-cum-lender) over the tenant enables him to offer ‘all-or-nothing’ loans which specify both the rate of interest and the amount of the loan.

We analyze the landlord’s optimum under interlinkage first, and then compare it to the outcomes when the landlord and the moneylender make separate and independent decisions. From this comparison, we establish the conditions under which the interlinking of the land and the credit markets leads to allocations that cannot be reproduced when the landlord and the moneylender play non-cooperatively. In the latter case, we assume that the tenant contracts with the moneylender after he has accepted the terms of the rent as specified by the landlord. This is a realistic assumption, which naturally makes the landlord a first mover, and the lender a follower, in the sequential game that is developed in Section 4.

3. The optimal interlinked contract

When the landlord offers interlinked loans, the terms of payment for both the land and the loan can be subsumed under a contract that appropriately specifies the state-contingent dues $\beta_L$ and $\beta_H$, together with the amount of funds advanced to the tenant. The optimal values of these variables can be derived from the following program:

$$\begin{align*}
\text{Maximize} & \left[ \pi(K)\beta_H + (1 - \pi(K))\beta_L - (1 + m)K \right] \\
\text{subject to} & \beta_L, \beta_H, K
\end{align*}$$
subject to the tenant’s participation constraint
\[ \pi(K)(y_H - \beta_H) + (1 - \pi(K))(y_L - \beta_L) \geq \tilde{y}. \]  

(2)

It is assumed that there exists a solution involving non-negative values of \( \beta_H, \beta_L, \) and \( K \) such that the landlord’s expected income is positive.\(^3\) It is clear that Eq. (2) will be binding at the optimum, \( \beta_H \) and \( \beta_L \) being lump-sum transfer instruments. Hence, the objective function of the landlord can be expressed as
\[ \pi(K)y_H + (1 - \pi(K))y_L - (1 + m)K - \tilde{y}. \]

The assumptions on \( \pi(\cdot) \) ensure that there is an interior solution with respect to \( K \), given by the associated first-order condition
\[ \pi'(K)(y_H - y_L) = 1 + m, \]  

(3)

Let the optimal value of \( K \) be denoted by \( K^o \), where it should be noted that \( K^o \) is the amount of investment that maximizes the expected output net of the opportunity cost of producing it, and is independent of the amount of rent. Substituting for \( K^o \) in Eq. (2), and recalling that Eq. (2) is strictly binding in the optimal solution and that \( 0 \leq \beta_i \leq y_i \), it is seen that with \( \tilde{y} > 0 \), any pair \((\beta_H, \beta_L)\) in the non-negative region of the line segment so described will, together with \( K^o \), induce a fully efficient allocation of resources.

4. A separate moneylender

We now consider the alternative situation where the tenant borrows the necessary funds from a separate, monopolistic moneylender following the rental agreement with the landlord. In the sequential game to be analyzed, the lender moves second, and chooses the size of the loan, and the state-contingent terms of repayment \( R_H \) and \( R_L \), as the best responses to \((\beta_H, \beta_L)\) chosen by the landlord, so as to extract any remaining surplus from the tenant. The moneylender’s program can be represented as
\[ \text{Maximize} \left( \pi(K)\min(R_H, y_H - \beta_H) + (1 - \pi(K))\min(R_L, y_L - \beta_L) - (1 + m)K \right) \]

(4)

subject to the tenant’s participation constraint, which takes the form
\[ \pi(K)(y_H - \beta_H) + (1 - \pi(K))(y_L - \beta_L) - \pi(K)\min(R_H, y_H - \beta_H) \]
\[ + (1 - \pi(K))\min(R_L, y_L - \beta_L) \geq \tilde{y} \]

(5)

Noting that the participation constraint of the tenant will bind at the optimum, inspection of Eqs. (4) and (5) reveals that the problem of the moneylender reduces to maximizing the

\(^3\) We assume that, in the all cases considered in this paper, \( y_H \) and \( y_L \) are sufficiently high, and \( \tilde{y} \) is sufficiently low, to make non-negative rents feasible. This rules out the possibility that a principal may find it optimal to supplement the tenant’s income in any state of nature.
residual surplus after the claims of the other parties have been satisfied. That is, he chooses \( K \) in order to

\[
\text{Maximize} [\pi(K)(\gamma_H - \beta_H) + (1 - \pi(K))](\gamma_L - \beta_L) - (1 + m)K] - \tilde{y}
\]  

(6)

The associated first-order condition is

\[
\pi'(K)[(\gamma_H - \gamma_L) - (\beta_H - \beta_L)] = 1 + m.
\]  

(7)

Let \( K^m \) represent the optimal choice of the moneylender. \( K^m \) will be the solution to Eq. (7) as long as

\[
\pi(K^m)(\gamma_H - \beta_H) + (1 - \pi(K^m))(\gamma_L - \beta_L) - (1 + m)K^m \geq \tilde{y},
\]

that is, the lender’s resulting payoff (subject to the satisfaction of the tenant’s participation constraint) is non-negative. Otherwise, \( K^m = 0 \).\(^4\) Provided that \( \beta_H \) and \( \beta_L \) are not so high as to render lending activities unprofitable, it is clear from Eq. (7) that \( K^m \) will depend on the terms of the rental contract, being a decreasing function of \( \Delta \beta \equiv \beta_H - \beta_L \). As is evident from Eqs. (3) and (7), \( K^m \neq K^0 \), except in the case where the rent is independent of the state of nature (i.e., \( \beta_H = \beta_L \)).

Now, not only is a uniform rent both necessary and sufficient to induce the lender to advance a loan of \( K^0 \), but it will also be optimal from the landlord’s point of view if it can be chosen such that the moneylender and the tenant obtain exactly their reservation alternatives \( m \) and \( \tilde{y} \), respectively. This is the case if there exists a \( \beta^0 \in (0, \gamma_L) \) that satisfies

\[
\pi(K^0)\gamma_H + (1 - \pi(K^0))\gamma_L - (1 + m)K^0 - \beta^0 - \tilde{y} = 0.
\]  

(8)

The lender’s best response is to choose \( R_H \) and \( R_L \) such that

\[
\pi(K^0)[\min\{R_H, \gamma_H - \beta^0\}] + (1 - \pi(K^0))[\min\{R_L, \gamma_L - \beta^0\}] = (1 - m)K^0.
\]

Such an outcome, if it exists, is fully efficient, so that there will be no gains from interlinking in this case.\(^5\)

If, however, the \( \beta^0 \) that solves Eq. (8) has the property that \( \beta^0 > \gamma_L \), such a uniform rent is infeasible, and the landlord’s optimum involves rents that differ across the states of nature. As demonstrated in Appendix A, it will be optimal for the landlord to appropriate the entire output when the yield is low (i.e., \( \beta_L = \gamma_L \)), and \( \beta_H \) will be strictly greater than \( \beta_L \). It follows at once from Eq. (7) that \( K^m < K^0 \), so that the outcome in the sequential game is inefficient (in the sense of production efficiency) and there will be gains from interlinkage. This is likely in practice, for droughts, floods and pestilence ensure that the value of \( \gamma_L \) is small.

A special case, where it is always in the landlord’s interest to offer an interlinked deal, occurs if \( \gamma_L = 0 \), that is, the bad outcome is associated with no output. This case yields our result with particular transparency. Since \( \beta_L = R_L = 0 \), we may denote \( \beta_H \) by \( \beta \), \( R_H \) by \( R \) and \( \gamma_H \) by \( \gamma \) without risk of confusion.

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\(^4\) This is so in the sense of the reservation payoff of the moneylender (net of the opportunity cost of funds) being zero.

\(^5\) A higher uniform rent is infeasible, given that the participation of the lender is essential for production by virtue of the assumption that \( \pi(0) = 0 \).
Once $\beta$ is known, the moneylender’s problem is to choose $R$ and $K$ so as to maximize $[\pi(K)\min\{R, y-\beta\}-(1+m)K]$ such that $\pi(K)(y-\beta-R)\geq \bar{y}$. Since $\bar{y} \geq 0$, the constraint requires $y-\beta > R$. Hence, the maximand can be simplified accordingly, and we get the first-order condition

$$\pi'(K) = \frac{1 + m}{y - \beta}$$

Since efficiency requires $\pi'(K) = (1+m)/y$, inefficiency occurs whenever $\beta > 0$.

Given that the landlord moves first, his problem is to choose $\beta$ so as to maximize $\pi(K)\beta$, keeping in mind that $K$ is implicitly defined by $\pi'(K) = (1+m)/(y - \beta)$. It is immediately evident that the chosen value of $\beta$ exceeds zero. It follows that if the landlord displaces the moneylender by offering interlinked contracts to his tenant, he will do better.

The results of the preceding analysis are summarized in the following proposition:

**Proposition 1.** Given the assumptions of Section 2, the sequential game between the landlord and the moneylender, where the landlord is the first mover and has ‘first rights’ to the tenant’s harvest, yields the following outcome:

(i) $\beta_L = \min\{\beta^0, y_L\}$, where $\beta^0$ is defined by Eq. (8).

(ii) If $\beta^0$ is strictly greater than $y_L$, the nature of the tenant’s ‘limited liability’ makes it optimal for the landlord to charge non-uniform rents, with $\beta_H > \beta_L = y_L$.

Interlinkage results in a higher expected yield, and a higher payoff to the landlord, in this case.

**Proof.** The proof of (i) is provided in Appendix A. To show that $\beta_H > \beta_L = y_L$ when $\beta^0 > y_L$, consider the effect of an increase in $\beta_H$ on the landlord’s expected payoff. Let

$$\Omega(K; \beta_H, \beta_L) = \pi(K)\beta_H + [1 - \pi(K)]\beta_L$$

Then,

$$\Omega(K; \beta_H, \beta_L) = \pi'(K)(\beta_H - \beta_L) \frac{\partial K}{\partial \beta_H} + \pi(K)$$

from which it follows that $\Omega(K; y_L, y_L) > 0$. By part (i), $\beta^0 > y_L$ implies that $\beta_L = y_L$. Clearly, in this case, a small increase in $\beta_H$ from the uniform rent of $y_L$, while not violating the participation constraints in the ensuing subgame, would result in an expected payoff which is strictly greater than $y_L$. Thus, the optimal rents are non-uniform, with $\beta_H$ strictly greater than $y_L$. As discussed earlier, this results in $K^m < K^0$, which proves the rest of part (ii).

It is important to stress that if the equilibrium in the sequential game involves non-uniform rents, the participation constraint of the moneylender need not necessarily bind. In the situation corresponding to part (ii) of Proposition 1, let $(\beta_H^{\max}, y_L)$ be the pair of state-contingent rents such that, with $K$ being given by Eq. (7), the participation constraint of the lender is binding. As long as $\beta_H$ is strictly less than $\beta_H^{\max}$, increases in $\beta_H$ above $y_L$ decrease the
amount of investment. Clearly, it is possible for $\beta_H^*$, the interior solution to $\partial \Omega / \partial \beta_H = 0$ to be strictly less than $\beta_H^{\text{max}}$, in which case the moneylender chooses $\beta_H^*$ as the optimal rent, and the moneylender’s equilibrium payoff will be strictly positive.

Eqs. (3) and (7) reflect the fundamental difference between the incentives of the landlord and the moneylender with respect to advancing credit to the tenant. Since, under interlinkage, the landlord receives the entire output (and therefore, any increase therein) net of $y$, the LHS of Eq. (3) represents the change in both the expected output and the landlord’s payoff that results from an increment in $K$. If $\beta_H$ is strictly greater than $\beta_L$, it follows from Eq. (7) that the tenant will obtain a smaller loan in the absence of interlinkage. This is because, the independent lender does not get the entire increase in the expected yield. In fact, with the lender receiving the residual surplus after the rent has been appropriated by the landowner, an increase in $K$, while raising $\pi(K)$, also increases the ‘leakage’ (in the form of the higher expected rent) from the residual that is available to the lender (over and above the reservation income of the tenant). As evident from Eq. (7), an increase in $\Delta \beta$ increases the leakage to the landlord, and reduces the optimal size of the moneylender’s loan.

Observe that while we allow the landlord and the moneylender to write state-contingent contracts, we do not allow the former to write a contract contingent on the amount borrowed by the peasant. If we did allow such contracts, the landlord would specify two functions $\beta_H(K)$ and $\beta_L(K)$, which denote the state- and loan-contingent rents, such that

$$\beta_H(K^0) = y_H = \frac{\Delta_H}{\pi(K^0)}$$

and,

$$\beta_L(K^0) = y_L = \frac{\Delta_L}{1 - \pi(K^0)}$$

Where $\Delta_H$ and $\Delta_L$ are non-negative amounts that satisfy $\Delta_H + \Delta_L = (1+m)K^0$. For all other $K$, the landlord punishes the lender by charging a high enough rent (one which extracts all the output, say). This makes any amount of credit, other than the one that is optimal from the landlord’s point of view, infeasible. On the other hand, the participation constraints of the moneylender and the tenant bind with a loan of size $K^0$. Thus, $\beta_i(K)$ as specified above, together with $R_i = (1+m)K^0$ for $i = L, H$, and $K = K^0$, constitute a sequential equilibrium which produces a first-best outcome even in the absence of interlinkage.

Viewed in this manner, our model may be interpreted as one that demonstrates interlinkage to be a consequence of the landlord’s inability to make the rents contingent on the volume of loan when there is a separate moneylender. In this connection, we would like to argue that the volume of loan is typically unobservable. In collecting field data, it is well known that credit information is very hard to get, since it is such an intangible transaction. Moreover, in our model the value of $K$ cannot be deduced from output since output is unaffected by $K$. This point carries over to cases where $K$ affects the volume of output, but there are other unobservable inputs (such as the tenant’s effort) as well.

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6 In this case, there is an interior solution to the moneylender’s program. His optimal choice of $K$ is given by Eq. (7), from which it is easy to derive $\partial K / \partial \beta_H = (\pi(K) / \pi^\prime(K)(y_H - \beta_H)) < 0$. 
To conclude this section, it should be evident from Eqs. (4)–(6) that if the lender is unable to observe the state of nature, and so must stipulate a uniform interest rate, then all of the above argument goes through as before. The tenant’s limited liability simply implies that the lender may be unable to collect the entire amount of the principal plus the interest in the bad state.

5. Risk-sharing and moral hazard

If the tenant is risk averse, or his effort is prohibitively costly to monitor, then the sharing of risk and the provision of incentives, respectively, become central considerations in the determination of the contractual terms. Since the state-contingent payments cease, in general, to operate as lump-sum transfer instruments in this case, one might expect coordination over instruments through interlinkage to offer advantages over independent deals under a wider range of conditions than those described in Proposition 1. We begin by demonstrating that it is indeed so in the case of a risk-averse tenant, even when his effort is not variable.

The tenant is assumed to have preferences over lotteries that can be represented by the von Neumann–Morgenstern utility function \( u(x) \) for a risk-averse agent, where \( x \) denotes his realized income. Thus, the participation constraint becomes

\[
\pi(K)u(y_H - \beta_H) + (1 - \pi(K))u(y_L - \beta_L) \geq u(\bar{y}).
\]

Suppose, as in Section 3, the landlord is in a position to impose an interlinked contract. Replacing Eq. (2) by Eq. (11) in the landlord’s problem when there is interlinkage yields the same first-order condition with respect to \( K \), Eq. (3), as before. Those associated with \( i \) \((i=H, L)\) yield \( y_H - \beta_H = y_L - \beta_L \), that is, we have the standard result that the tenant receives a fixed payment \( \bar{y} \), with the landlord bearing all risk, a result that holds for any value of \( K \). The allocation is fully efficient, as expected.

Matters are otherwise in the absence of interlinking. We can, indeed, go further than in Proposition 1.

**Proposition 2.** If the tenant is risk averse, then in any equilibrium of the sequential game in which an independent moneylender moves second, the tenant will bear some risk. Hence, the associated allocation is always less efficient than that under interlinkage.

**Proof.** Since the tenant’s only means of repaying a loan is his income from cultivation, he will bear no risk if and only if

\[
y_H - \beta_H = y_L - \beta_L > R_H = R_L
\]

Suppose, therefore, that the landlord pays the tenant the fixed sum \( w \). If the tenant is to bear no risk, the loan contract must take the form of a fixed repayment \( R \) that is feasible in both states of nature. Thus, the lender’s problem is

\[
\text{Maximize } [R - (1 + m)]K
\]

Subject to

\[
R, K \in [w]
\]
subject to the participation constraint

\[ w - R \geq \bar{y}, \quad (14) \]

from which it follows that Eq. (12) is indeed satisfied, with \( w = \gamma_H - \beta_H = \gamma_L - \beta_L \). It is clear that there can be no equilibrium in which the lender participates unless \( w > \bar{y} \). If \( w > \bar{y} \), however, the moneylender’s problem has the solution \( K = 0 \) and \( R = w - \bar{y} \). Since the lender moves second, there is nothing the landlord can do to avoid such an outcome. If finance is essential in production, in the sense that \( \pi(0) = 0 \) and \( \bar{y} > \gamma_L \), there will, moreover, be a positive lower limit on \( K \) such that the landlord finds it profitable to offer a tenancy. It follows that the sequential game will not possess an equilibrium if the landlord absorbs all the risks of production. If there does exist an equilibrium, therefore, its associated allocation of resources will be inefficient.

It should be emphasized that in this setting, there will be with gains from the interlinking of contracts even without an appeal to the infeasibility of uniform rental payments when the bad outcome is a sufficiently miserable level of output, as is necessary when the tenant is risk-neutral (see Proposition 1).

To complete this section, we introduce moral hazard by assuming that observing and monitoring \( e \), the tenant’s effort, is now prohibitively costly. We initially continue with the assumptions of the indivisibility of \( e \), and a production technology that requires that land and labor be employed in fixed proportions. Thus, the tenant’s choice is confined to the two alternatives of applying one indivisible unit of effort, or none at all, to the given unit of land. Our results are then extended to the case where the tenant may choose among all non-negative levels of \( e \).

Specifically, we suppose that effort, as well as \( K \), now has a positive effect on the probability of producing the higher level of output that is represented in the following manner

\[
\pi(K, e) = \begin{cases} 
\pi(K) & \text{if } e = 1 \\
0 & \text{if } e = 0 
\end{cases} \quad (15)
\]

where \( \pi(K) \) has the same properties as in the previous section. Let \( x_i \) be the tenant’s income in state \( i \). For simplicity of exposition, the tenant’s utility function is assumed to be separable in \( x_i \) and \( e \), and is represented as

\[ V(x_i, e) = u(x_i) - e \]

where \( u(x_i) \) is concave and twice differentiable in its argument. The reservation utility of the tenant is \( \bar{u} (= u(\bar{y})) \). After entering into all the necessary contractual agreements, the tenant chooses \( e \in \{0, 1\} \) to

\[ \max\{\pi(K, e)u(x_H) + (1 - \pi(K, e))u(x_L) - e\} \]

provided that, with this optimal choice, his expected utility from participation is no less than \( \bar{u} \). Since he is the player who moves last of all, his state-contingent payoffs are the residual amounts that are left after the claims of the decision-maker in the previous stage of the game have been satisfied. Irrespective of the identity of the penultimate player, the \( x_i \)'s that induce unit effort from the tenant must satisfy the following participation and incentive compatibility conditions.
Following fairly standard arguments, it is easy to see that, provided $e = 1$ is desirable for the penultimate player, Eqs. (16a) and (16b) will bind at his optimum, whereupon $x_H^*$ and $x_L^*$ satisfy $u(x_H^*) = \bar{u} + [\pi(K)]^{-1}$, and $u(x_L^*) = \bar{u}$. The landlord-cum-moneylender will then choose $\beta_H$, $\beta_L$, and $K$, such that $\beta_i = y_i - x_i$ for $i = H, L$, and $K$ maximizes

$$
\pi(K)[y_H - x_H^*] + (1 - \pi(K))[y_L - x_L^*] - (1 + m)K
$$

Keeping in mind that $x_H$ depends on $K$, the first-order condition for an interior solution is

$$
\pi(K)[y_H - x_H^*] + (1 - \pi(K))[y_L - x_L^*] - (1 + m)K \geq 0
$$

An important special case is one where the tenant is risk neutral, so that $V$ specializes to $V(x_i, e) = x_i - e$. The fact that Eq. (16b) is binding in the optimal solution then yields

$$
\pi(K)(x_H^* - x_L^*) = 1
$$

Consequently, Eq. (17) becomes identical to the corresponding first-order condition Eq. (3) in Section 3 which yields $K^*$ as the optimal interlinked loan. With risk aversion, however, the optimal $K$ that is derived from Eq. (17) will be strictly less than $K^*$: the compromise between risk-sharing and incentives takes a toll on efficiency.

The absence of interlinkage causes a further deterioration of allocative efficiency, unless the landlord is able to extract all surplus by charging a uniform rent. To show this, let $K^*$ satisfy Eq. (17), and let $\beta^*$ be such that

$$
\pi(K^*)[y_H - x_H^*] + (1 - \pi(K^*))[y_L - x_L^*] - \beta^* - (1 + m)K^* = 0.
$$

If $\beta^*$ is strictly greater than $y_L$, then following the procedure of the previous section, it is easy to show that the landlord will find it optimal to charge $\beta_H > \beta_L$. This distorts the returns to the moneylender, net of the payments of $x_H^*$ and $x_L^*$, and results in the latter providing an amount of funds lower than $K^*$. Observe that if the tenant is risk neutral, the incentive problem can be solved without reference to risk-sharing, so that we are back in the world of Sections 3 and 4, where Proposition 1 holds.

For a brief demonstration of the robustness of our results when the tenant may choose any $e \in [0, \infty)$, we proceed in a manner similar to Grossman and Hart (1983). That is, we suppose that the penultimate player first determines his minimum cost (in terms of the expected amount of residual output that the contract leaves to the tenant) of inducing a particular level of effort from the tenant for any given $K$. This yields the cost function $C(e, K)$. He, then finds the levels of induced effort and the size of the loan that maximize his expected payoff, net of the total costs $(1 + m)K$ and $C(e, K)$. Accordingly, the penultimate player chooses $x_H$ and $x_L$, given $K$, to

$$
\min[\pi(K, e)x_H + (1 - \pi(K, e))x_L]
$$

subject to the tenant’s participation constraint

$$
\pi(K, e)u(x_H) + (1 - \pi(K, e))u(x_L) - e \geq \bar{u}
$$
and her incentive compatibility constraint

\[ e \in \arg\max_{e'} \{ \pi(K, e')u(x_H) + (1 - \pi(K, e'))u(x_L) - e' \} \]  

(21)

Under the assumptions of strict concavity of \( u(\cdot) \) and \( \pi(\cdot, \cdot) \) with respect to their arguments, the minimization program Eqs. (19)–(21) will yield \( x^*_i(e, K) \) as the unique optimal solution, when

\[ C(e, K) = \pi(K, e)x^*_i(e, K) + (1 - \pi(K, e))x^*_h(e, K) \]

The cost function \( C(e, K) \) will be the same whether the player concerned be the landlord (with interlinkage) or the moneylender (as in the sequential game of Section 4). The difference lies in the nature of the benefit function of the penultimate player (i.e., his payoff net of \((1 + m)K\) but gross of \(C(e, K)\)). In the case of an interlinked contract, this player is the landlord, who provides the loan and appropriates the surplus from the tenant. His benefit function is

\[ B^e(e, K) = \pi(e, K)\gamma_H + (1 - \pi(e, K))\gamma_L - (1 + m)K \]

(22)

and he can be regarded as choosing \( e \) and \( K \) to maximize \( B^e(e, K) - C(e, K) \). Let \( e^o \) and \( K^o \) represent the amount of inputs that are optimal from the landlord’s point of view under interlinkage. Once these are determined, his optimal state-contingent rents follow from the corresponding values of \( x^*_i = x_i(e^o, K^o) \), and from the relationship \( x_i = \gamma_i - \beta_i \) for \( i = L, H \).

The procedure for uncovering the equilibrium decisions in the sequential game is similar, except that, in this case, it is the moneylender who determines the optimal \( e \) and \( K \) by maximizing the difference between \( B^m(e, K; \beta_H, \beta_L) \) and \( C(e, K) \), where his benefit function is

\[ B^m(e, K; \beta_H, \beta_L) = \pi(e, K)(\gamma_H - \beta_H) + (1 - \pi(e, K))(\gamma_L - \beta_L) - (1 + m)K \]

(23)

Note that the difference between Eqs. (22) and (23) is the expected rent. In fact, we can express Eq. (23) as

\[ B^m(e, K; \beta_H, \beta_L) = B^e(e, K) - [\pi(e, K)\beta_H + (1 - \pi(e, K))\beta_L] \]

(24)

Once again, it is evident from Eq. (24) that if the subgame perfect equilibrium of the sequential game does not involve uniform rents, then a marginal increment in \( K \), by increasing expected rents, will result in leakages (of the same nature as described in Section 4) from the incremental payoff accruing to the moneylender. Consequently, the marginal benefit from increasing the \( i \)-th input, given the level of the \( j \)-th input \((i, j = e, K, \text{with } i \neq j)\), will be less for the moneylender than for the landlord, that is \( B^e_i(e, K) > B^m_i(e, K) \) if \( \beta_H \neq \beta_L \). Suppose there exist interior solutions to the maximization programs under interlinkage and in the sequential game, that uniform rents are not optimal, and that the second-order (sufficiency) are satisfied. Then, it is easy to show, using simple comparative statics, that interlinkage results in higher levels of \( e, K \) and expected output than the non-interlinked case. If the uniform rent that appropriates the entire surplus at the input levels \( e^o \) and \( K^o \) is feasible, it is easy to see that it will be the optimal choice of the landlord in the subgame perfect equilibrium of the sequential game. Interlinkage will not generate any additional advantage in this case.
Thus, the incorporation of moral hazard causes no change in the fundamental conclusion of Section 4. In fact, this is true even if the agent is risk neutral. In this case, following the same procedure as before, it can, once again, be easily demonstrated that, if the limited liability condition renders the optimal uniform rent infeasible, interlinked contracts will be strictly superior in terms of production efficiency.

6. Extensions

The results of the previous section were established within the framework of a sequential game where the landlord was the first mover, and possessed seniority of claims. As mentioned in Section 1, we regard this formulation to be the most appropriate reflection of the arrangements in traditional agriculture, since (i) a landless and assetless tenant will be considered creditworthy by informal sector moneylenders only after he is granted a tenancy contract by the landlord, and (ii) the landlord’s proximity to the tenant gives him the advantage of superior accessibility to output. However, in light of the changes that, with increased commercialization and state-sponsored developmental activity, are rapidly altering the position of advantage enjoyed by landlords in rural society, we extend our analysis to examine how robust our results are to changes in these two assumptions. We also allow the landlord to possess a variable instrument of control, and examine the consequences of such multiplicity of instruments on allocative efficiency. As it turns out, all these changes introduce important qualifications to our previous results.

6.1. Changes in the sequence of decisions and seniority of claims

With two principals, there are four possible ways of combining the assignment of the first rights to output with the designation of the first mover. Consider first the alternative where the moneylender, not the landlord, has the first move and the first rights to harvest. It is easy to see that the loan contract will specify $R_H$, $R_L$ and $K^o$ such that $K^o$ satisfies Eq. (3), and

$$\pi(K^o)(y_H - R_H) + (1 - \pi(K^o))(y_L - R_L) = \hat{\beta} + \bar{\gamma} \quad (25)$$

where $\hat{\beta}$ represents the landlord’s reservation income from the next best use of his plot. The outcome is first best, and interlinkage loses its superiority in this case.

The cases where one principal moves first, but the other enjoys seniority of claims introduce new complications. Irrespective of his identity, the player who moves first can now receive payoffs that differ from the state-contingent claims originally specified in his contract. To start with, suppose that the moneylender moves second. Then, the actual payoff received by the landlord in state $i$ will be given by $\min\{\beta_i, y_i - R_i\}$, for $i = L, H$. Furthermore, for the equilibrium outcome of this game to be first best, the landlord must receive uniform payoffs across the states of nature. Otherwise, as obvious from the previous analysis, the moneylender will not find it optimal to offer $K^o$. The condition that $y_L$ have the appropriate magnitude to sustain uniform rents, while necessary, is, however, no longer sufficient. For a first-best outcome, $\beta^o$ must, additionally, satisfy both the following conditions
\[ y_H - \beta^o < \frac{\bar{y}}{\pi(K^o)} \]  
(26)

and,

\[ y_L - \beta^o < \frac{\bar{y}}{1 - \pi(K^o)} \]  
(27)

Satisfaction of Eq. (27) is ensured, given the assumption \( \bar{y} > y_L \) in our model. Violation of Eq. (26), however, is possible, in which case the equilibrium payoffs to the landlord will be non-uniform across the states of nature, even when \( \beta^o \leq y_L \). Consequently, the lender no longer finds it optimal to provide \( K^o \), the amount of funds that corresponds with the first best outcome. Maintaining allocative efficiency in the non-cooperative game is now more demanding than it was in Section 4.

To see this, suppose, to the contrary, that the magnitude of \( \beta^o \) is such that Eq. (26) is violated, and that there exists an equilibrium of this sequential game where the landlord receives the uniform payoff \( \beta \), while the lender offers \( \{K^o, R_H^o, R_L^o\} \) as the credit contract, where

\[ y_i - \beta \geq R_i^o, \]
\[ \pi(K^o)(y_H - R_H^o) + (1 - \pi(K^o))(y_L - R_L^o) - \beta = \bar{y}. \]

Clearly, satisfaction of all participation constraints implies that \( \beta \leq \beta^o \). Consider the alternative credit contract \( \{K^*, R_H^*, R_L^*\} \) where \( K^* = K^o \) satisfies \( \pi(K^*)(y_H - \beta) \geq \bar{y} \),

\[ R_H^* = y_H - \beta - \frac{\bar{y}}{\pi(K^*)}. \]

and, he lender appropriates all output in state L by choosing

\[ R_L^* = y_L. \]

Following the choice of the uniform rent \( \beta \) by the landlord, it is clear that the lender will do better by offering this alternative contract, provided it is feasible. Given the tenant’s limited liability, she earns nothing in state L, but satisfies her participation constraint by earning exactly \( \bar{y}/\pi(K^o) \) in state H. Note that since \( \pi(K^o)(y_H - y_L) = 1 + m \), we have

\[ \pi'(K^o)(y_H - \beta - y_L) < 1 + m \]

which implies that if \( y_H - \beta^o \) is strictly greater than \( \bar{y}/\pi(K^o) \), the lender will find it feasible (in the sense of satisfying all participation constraints) and preferable to choose some \( K^* \) that is strictly less than \( K^o \). If \( y_H - \beta^o = \bar{y}/\pi(K^o) \), the lender chooses \( K^* = K^o \). Then, since Eq. (26) does not hold, and \( \beta = \beta^o \), \( R_H^o \) is non-negative, and \( \{K^*, R_H^*, R_L^*\} \) constitutes a feasible and strictly profitable deviation by the lender.

Thus, if Eq. (26) is violated, the lender will exercise seniority of claims and appropriate the entire output in state L if the landlord charges uniform rents. The expected income of the landlord, in such a situation, is \( \pi(K^*) \beta \) (\( \leq \pi(K^o) \beta^o \)). It is easy to see that a rent contract which specifies \( \beta_H = \beta^o + \delta \), and \( \beta_L = \pi(K^o) \beta^o \) is feasible for some appropriately small but strictly positive \( \delta \), and strictly dominates a contract that specifies uniform rents.
With non-uniform rents, the loan offered by the lender will be strictly less that \( K^o \). Thus, unless \( \beta^o \) is strictly less than \( y_L \), and satisfies Eq. (26), interlinkage will be superior.

In the remaining configuration, when the landlord is the second mover, but possesses seniority of claims, the equilibrium outcome may be first best even if the moneylender’s payoffs are non-uniform. If \( \tilde{\beta} < \tilde{y} \), it will be feasible and optimal for the moneylender to offer the contract \( \{ K^o, R^o_L, R^o_H \} \), where \( K^o \) satisfies Eq. (3), and

\[
R^o_L = \max \left\{ y_L - \frac{\tilde{y} - \sigma}{1 - \pi(K^o)}, 0 \right\}
\]

(28)

together with

\[
R^o_H = y_H - \frac{\tilde{\beta} + \max\{\tilde{y} - (1 - \pi)y_L, \sigma\}}{\pi(K^o)}
\]

(29)

for some \( \sigma \in (0, (\tilde{y} - \tilde{\beta})) \). Note that, if \((1 - \pi)y_L \leq \tilde{\beta} \leq \tilde{y}\), the equilibrium is unique, and the payoffs to the moneylender are represented by \( R^o_L = 0 \), and \( R^o_H = y_H - (\tilde{\beta} + \tilde{y} - (1 - \pi)y_L)/\pi(K^o) \). If, on the other hand, \((1 - \pi)y_L \) is strictly greater than \( \tilde{\beta} \), there exists a continuum of equilibria, one for each \( \sigma \in (\sigma_0, (\tilde{y} - \tilde{\beta})) \), where \( \sigma_0 = \max\{\tilde{y} - (1 - \pi)y_L, 0\} \). In each equilibrium in the continuum, the moneylender earns strictly positive payoffs, which are represented by the appropriate forms of Eqs. (28) and (29), in both states. Irrespective of the uniqueness or multiplicity of the equilibrium, \( K^o \) is the optimal amount loaned by the moneylender, who extracts all surplus, and interlinked contracts cease to be strictly superior.

In the case where \( (\tilde{\beta} \geq \tilde{y}) \), the assumption \( \tilde{y} > y_L \) implies that \( \tilde{\beta} > \tilde{y} > (1 - \pi)y_L \). Let the loan contract specify

\[
R^o_L = 0
\]

(30a)

and,

\[
R^o_H = y_H - \frac{\tilde{\beta} + \tilde{y} - (1 - \pi)y_L}{\pi(K^o)}
\]

(30b)

together with \( K^o \) as the optimal amount of the loan. With these specifications, the lender extracts the entire surplus, even though his actual payoff in state L is always zero.

6.2. Multiple instruments

Suppose now that each of the two principals controls a variable instrument that affects expected yield at the margin. In particular, let \( \alpha \geq 0 \) denote a variable input, with price normalized to one, that is supplied by the landlord. It has an increasing effect on expected output in the following manner: \( \pi = \pi(K, \alpha) \) is increasing, concave and smooth in its arguments, with \( \pi(K, 0) = \pi(0, \alpha) = 0 \). Interlinkage will now be strictly superior, and this superiority is immune to the order of moves, or the assignment of seniority, in the game with two distinct principals.

\[\text{Note that any attempt by the landlord to exercise seniority to increase expected rents will violate the tenant's participation constraint.}\]
Consider, first, the optimal interlinked contract. The landlord offering this contract specifies $K^o$, $\alpha^o$, $\beta_H$ and $\beta_L$ to achieve the first-best solution, subject to the participation of the tenant. This implies that $K^o$ and $\alpha^o$ maximize

$$\pi(K, \alpha)y_H + (1 - \pi(K, \alpha))y_L - (1 + m)K - \alpha$$

and $\beta_H$ and $\beta_L$ satisfy

$$\pi(K^o, \alpha^o)[y_H - \beta_H] + (1 - \pi(K^o, \alpha^o))[y_L - \beta_L] - (1 + m)K^o - \alpha^o = \bar{\gamma}$$

The first-order conditions that yield $K^o$ and $\alpha^o$ as interior solutions are, respectively

$$\pi_K(K^o, \alpha^o)[y_H - y_L] - (1 + m) = 0$$

and,

$$\pi_x(K^o, \alpha^o)[y_H - y_L] - 1 = 0$$

If non-interlinked contracts are to specify the same $K$ and $\alpha$, it is evident that the principal who moves first must have the same payoffs in both states of nature. But then, the same principal would have no incentive to provide a positive amount of the costly instrument under his control. Consequently, the equilibrium outcome of this game would differ from that determined by the Eqs. (32)–(34).

6.3. Summary

What are the implications that emerge from the analysis of the above subsections? First of all, we conclude that with limited liability and multiple principals, allocative efficiency is unambiguously guaranteed only if a single principal has direct control of all instruments that affect production decisions at the margin, and, in addition, possesses seniority of claims. Second, first-mover advantages to a ‘passive’ principal — one whose decisions are limited to exercising his property rights to determine his share of the returns — impairs allocative efficiency unless his equilibrium payoffs are uniform. Uniform payoffs for the first-mover are, however, not essential for allocative efficiency if he is the only principal possessing a variable instrument of control. As demonstrated above, apart from instances where the reservation income of the landlord and $(1 - \pi(K^o))y_L$ were ‘too large’, the first best outcome was achieved with the first-mover earning payoffs that varied across the states of nature. Finally, the multiplicity of variable instruments, under control of different principals, results in the unambiguous superiority of interlinkage, irrespective of the assignment of seniority of claims and the order of moves.

7. Conclusion

We have established circumstances under which the interlinking of credit and land contracts yields superior outcomes even when non-linear contracts are available and moral hazard is absent. In particular, we have demonstrated that if the landlord is unable to observe the terms set by a lender and the tenant enjoys limited liability, then sufficiently poor
levels of output in adverse states of nature will compel the landlord to charge a rent that varies with the state of nature. Such variations generate negative contractual externalities that cannot be internalized in the absence of interlinkage, even if the tenant, as agent, is risk-neutral. Consequently, under certain conditions, land and loan contracts that arise from non-cooperative behavior are unable to achieve the ‘first-best’ outcome. By offering an interlinked contract, a landlord could then usurp the role of the moneylender and improve both his payoff and allocative efficiency.

In our model, the axiom of limited liability, while necessary, is not sufficient to establish the superiority of interlinkage. As we have shown in Section 6, our results are not completely robust to changes in the sequence in which the tenant contracts with the various principals, and in the assignment of the senior claim to output. The main result is unaffected by changes in the seniority of claims, but is sensitive to changes in the order of moves: limited liability ceases to ensure the strict superiority of interlinked contracts if the principal who provides the variable factor of production moves first, even if he has junior claims to the output.

As noted in Section 1, our initial assignment of both the ‘first rights’ to the harvest and the first move to the landlord reflects his traditionally dominant role in village life. Interlinking would hold no attractions for the lender, however, if these advantages were his instead. Yet in the ‘mixed’ case, when one principal has first rights to output and the other the first move, there are circumstances in which interlinking regains its superiority. These variants are now taking on practical importance. With commercialization and state-sponsored developmental activity fast changing the socio-economic scene, there has been a noticeable weakening of the power and influence wielded by landlords in rural society. Credit, at subsidized interest rates, is sometimes available to landless tenants from cooperatives and banks, which may not require the borrower to have the landlord’s prior sanction in the form of a rental contract. Thus, tenants may apply for loans before settling terms with their respective landlords, whereupon it is no longer appropriate to model the latter as a Stackleberg leader. In the same manner, formal legal institutions may increasingly govern property rights and contractual enforcement, making it difficult for the landlord arbitrarily to assert the senior claim to output. These were the considerations that motivated us to re-examine the question of the superiority of interlinking contracts, not only within the confines of traditional agriculture, but also under the new arrangements that are brought about by the forces of modernization.

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Appendix A

Proof of Proposition 1, part (i). We demonstrate that $\beta_L = \min\{\beta^o, y_L\}$ is the optimal choice of the landowner in the equilibrium of the sequential game analyzed in Section 4. As discussed in that section, $\beta_H = \beta_L = \beta^o$ if $\beta^o \leq y_L$. Consider the case where $\beta^o > y_L$.\n
and assume, to the contrary that $\beta_L$ is strictly less than $y_L$. Note that, with $\beta_H = y_H$, the moneylender maximizes his expected returns by choosing $K = 0$, so that the tenant will not participate in production. This implies that $\beta_H$ must be strictly less than $y_H$ in the equilibrium of the sequential game. Then, the participation constraint of the moneylender is strictly binding, as otherwise it is obvious from Eq. (7) that the landlord can increase expected profits, while avoiding any changes in $K^m$, by increasing both state-contingent rents by appropriately small and equal amounts. Thus, if $\beta_L < y_L$, with Eq. (6) representing the payoff of the moneylender, it follows that

$$\pi(K^m_1) \beta_H + (1 - \pi(K^m_1)) \beta_L = \pi(K^m_1) y_H + (1 - \pi(K^m_1)) y_L - (1 + m) K^m_1 - \bar{y} \quad (A.1)$$

where $K^m_1$ denotes the optimal amount of finance advanced by the moneylender in this case.

Now consider the feasibility and optimality of the landlord choosing the pair of state-contingent rents

$$\beta_H' = y_H - \frac{1}{\pi(K^m_1)} (1 + m) K^m_1 + \bar{y} = \beta_H - \frac{1}{\pi(K^m_1)} (y_L - \beta_L) \quad (A.2)$$

and,

$$\beta_L' = y_L \quad (A.3)$$

It is easy to check that these rents satisfy the moneylender’s and tenant’s participation constraints as strict equalities if the previous decisions of the moneylender ($K^m_1$ and the corresponding interest rate) remain unchanged. If $K^m_2$ is the optimal amount of funds loaned by the moneylender under the new circumstances, he can do no worse than before while still satisfying the participation constraint of the tenant. In addition, since the hypothesis $\beta_L < y_L$ implies that $\beta_H < \beta_H'$ and $\beta_L' < \beta_L$, it follows from Eq. (7) that $K^m_2$ is strictly greater than $K^m_1$.

Denote $\pi(K^m_i)$ as $\pi_i$ for $i = 1, 2$. With the new rents, the expected payoff of the landlord is, using Eqs. (A.2) and (A.3),

$$\pi_2 \beta_H' + (1 - \pi_2) \beta_L' = \pi_2 y_H + (1 - \pi_2) y_L - \frac{\pi_2}{\pi_1} [(1 + m) K^m_1 + \bar{y}] \quad (A.4)$$

With the RHS of Eq. (A.1) representing his payoff under the rents ($\beta_H$, $\beta_L$), we have

$$[\pi_2 \beta_H' + (1 - \pi_2) \beta_L' - [\pi_2 \beta_H + (1 - \pi_2) \beta_L] = \frac{\pi_2 - \pi_1}{\pi_1} [\pi_1 (y_H - y_L) - (1 + m) K^m_1 - \bar{y}] \quad (A.5)$$

Again, using Eq. (A.1), it is easy to show that

$$\pi_1 (y_H - y_L) - (1 + m) K^m_1 - \bar{y} = \pi_2 \beta_H + (1 - \pi_2) \beta_L - y_L \quad (A.6)$$

If $\beta_H$ and $\beta_L$ are optimal rents, the RHS of Eq. (A.6) must be non-negative, since, with $\beta^m > y_L$, a uniform rent of $y_L$ is feasible for the landlord, and results in a strictly positive payoff to the moneylender. Furthermore, as demonstrated in the proof of Proposition 1 in Section 3, a small increase in $\beta_H$ from the uniform rent of $y_L$ is feasible, and would result
in an expected payoff to the landlord that is strictly greater than \( y_L \). Thus, for optimal rents, the RHS of Eq. (A.6) is strictly positive. But then, with \( \pi_2 > \pi_1 \), \( \pi_2 \beta_H + (1 - \pi_2) \beta_L \) is strictly greater than \( \pi_2 \beta_H + (1 - \pi_2) \beta_{L+} \). This is a contradiction that cannot be resolved as long as \( \beta_L < y_L < \beta^* \). Therefore, if \( \beta^* \) is strictly greater than \( y_L \), then \( \beta_L = y_L \), and \( \beta_H > y_L \).

References


