Dynamic consistency in incentive planning
with a material input

Hugh M. Neary
Department of Economics, University of British Columbia, Vancouver, BC V6T 1Z1, Canada

Received 4 February 1999; received in revised form 21 March 2000; accepted 29 March 2000

Abstract

An incentive model of a planner and two firm-types is presented, in which the planner assigns
material input to the firms as part of the organization’s production plan. The optimal incentive-
compatible one-period plan is described. This static plan is then shown to be not incentive-compatible
in a dynamic (two-period) setting if the firm’s discount factor is not too low, and if it can hoard
the material input between periods. The incentive-compatible dynamic problem is presented and
analyzed. This incentive problem is related to hoarding and dysfunction in the Soviet planning
system. © 2001 Elsevier Science B.V. All rights reserved.

JEL classification: D71; D81

Keywords: Incentive compatible planning; Hoarding; Material input planning

1. Introduction

The most fundamental problem faced by the principal of a hierarchy is the need to
secure information from subordinates and to motivate them in the organizational interest in
the presence of asymmetric information and moral hazard. Incentive models that address
this problem emphasize the trade-off that the principal faces between rent-extraction and
efficiency. To induce a particular subordinate to correctly reveal his hidden productivity
parameter the principal must pay an information rent. The amount of the rent required can
be reduced by imposing less than efficient plans on less-productive types.

Simple one-period incentive models establish a best-case baseline on how well the princi-
pal can do, given that a rent must be paid to subordinates to induce separation of types. The
optimum payoff to the principal typically decreases with the addition of any institutional or systemic feature that constrains the principal or that gives the subordinates additional flexibility. One example considered by Tirole (1986), Laffont and Tirole (1991) and Laffont and Martimort (1997) arises when subordinates can collude with each other to hide information from the principal. A second example occurs in a multi-period context when the principal cannot commit to a long-term incentive scheme, as shown by Freixas et al. (1985) and Laffont and Tirole (1988, 1993). A final example is provided by Litwack (1993), who shows that a coordination cost, incurred if actual output differs from a planned output level, weakens the benefit of type separation to the principal.

The present paper contributes to this literature by elaborating on a further source of constraint which involves subordinates’ use in production of a storable material input that is allocated by the principal. Storability and the consequent possibility of input hoarding gives the subordinate flexibility over the inter-temporal allocation of effort in a dynamic (two-period) context; this flexibility can result in the undermining of static incentive compatibility constraints, reducing the principal’s ability to extract net output from the system.

Provision by the principal of material input for subordinates’ use is a ubiquitous feature of productive organizations. A good example is the late Soviet economy. Industrial production was based on a system of materials balance planning that assigned to each enterprise a comprehensive input–output plan. Input hoarding was widespread at all levels of the system. Classic analyses of Soviet planning such as Levine (1966) interpret input hoarding as a response by enterprises to taut plans — high output targets and low input assignments. Planners in turn formulated taut plans to put pressure on enterprises to make use of hidden reserves. Plan tautness and hoarding thus coexist as mutual responses.

More recent interpretations of Soviet input hoarding have to do with corruption and the illegal use of inputs diverted from the state system for use in the second economy. The incentive model with material input hoarding presented here is relevant to these ideas of illegality and corruption. In the optimal tax literature the failure of an individual to reveal type (personal ability) unless provided with a rent is interpreted as an acceptable exercise of self-interest. In the present model, however, type is misrepresented in order to secure planner-provided inputs with the intention of using these inputs for personal gain; thus the exercise of self-interest is fraudulent. In the model the personal gain from input hoarding takes the form of effort reduction; however, a remodelling of the manager’s payoff function could allow the gain to be production for the second economy.

This observation is relevant to some aspects of the failure of the Soviet economic system from the mid-1960s onwards. Input hoarding is dysfunctional for the planner since it results in rents and plan inefficiencies additional to those that occur in the static incentive model. The manager’s incentive to hoard depends on his cost of hoarding and on his discount factor. In the Brezhnev era cronyism and the blunting of accountability within the regime worked to reduce hoarding costs and to increase managerial discount factors. Thus, trends in the wider system can be linked directly to the performance of the production system through the incentive model of material input planning and the dynamic hoarding problem considered here.

Section 2 presents a simple incentive model of cost reduction with a planner and two firm types. The benchmark static optimum, of independent interest for the way in which the

---

2 Hunter (1961) and Keren (1972) provide early models of the use of taut plans to elicit additional effort.
planner’s assignment of material input to the less productive type varies with the population frequency of firm types, is examined. Section 3 analyzes the dynamic consistency problem that can arise when material input hoarding is possible. Section 4 presents the solution to the dynamic hoarding problem. Section 5 summarizes the results and provides some additional comments.

2. Model

The cost-reimbursement model from Laffont and Tirole (1993, Chapter 1) is extended to allow for use of a material input allocated by the planner. There is a project of fixed value $S$. The technology is represented by the cost function

$$c = g(\beta, m) - e,$$  

(2.1)

where $e$ is managerial effort, $m$ is the level of material input used, and $\beta$ is an efficiency parameter that characterizes the firm type. A higher-$\beta$ firm has higher costs: $g_{\beta} > 0$. Material input reduces cost, but at a decreasing rate: $g_m < 0$ and $g_{mm} > 0$. The marginal cost of $\beta$ falls as $m$ increases: $g_{\beta m} \leq 0$. Finally, managerial effort is cost-reducing. 3

Managerial utility is given by

$$U = t - \psi(e),$$

where $t$ is the reward paid by the planner, and the effort disutility function $\psi()$ is increasing and convex, with a non-negative third derivative, $\psi'' \geq 0$, and with $\psi'(0) = 0$. Reservation utility is normalized to zero.

Suppose only two firm types, $[\beta_1, \beta_2]$, with $\beta_2 > \beta_1$. These will be referred to as the high and low (cost) types, respectively. The planner’s payoff is the sum of the project’s net surplus and managerial utility. There is a shadow cost, $\lambda > 0$, of raising the public funds needed to finance the project. The financial cost of the project is $C = C \left( \frac{r}{C} \right)$, where $r$ is the price of material input. The planner’s payoff from a manager of type $\beta$ is

$$W_{\beta} = S - (1 + \lambda)(c + rm + t) + U$$

$$= S - (1 + \lambda)(g(\beta, m) - e + rm + \psi(e)) - \lambda U.$$  

(2.2)

2.1. Complete information

When $\beta$ is directly observable, the planner will choose $e$, $m$ and $U$ to maximize $W_{\beta}$. The full information solutions are given by

$$\psi'(e^*) = 1, \quad g_m(\beta, m^*) + r = 0, \quad U = 0.$$  

(2.3)

---

3 Consider a production function $S = f(\beta^{-1}, m, e + x)$, where $x$ is a local input with price $w$ chosen by the manager. Then, $e = wx(\beta, m, e, S) = w f^{-1}(\beta^{-1}, m, S) - e$, or (2.1), where $g(\beta, m) := w f^{-1}(\beta^{-1}, m, S)$. Then, $g_{mm} > 0$ if $f()$ is concave in $(x, m)$. And $g_{\beta m} \leq 0$ if and only if $(\frac{\partial(-f_m/f_x)}{\partial \beta} + \frac{(\partial(-f_m/f_x))}{\partial x}(\partial x/\partial \beta)) \leq 0$. The second term is negative for $f_{xx} \geq 0$. Hence, a sufficient condition for $g_{\beta m} \leq 0$ is $\frac{\partial(-f_m/f_x)}{\partial \beta} \leq 0$, which is assumed.
Optimal effort, $e^*$, is independent of type, while the optimal assignment of material input is non-decreasing in $\beta$: the higher type receives more material input because $m$ achieves a higher marginal reduction in cost the larger is $\beta$ ($g \beta m \leq 0$). Because funds are socially costly ($\lambda > 0$) the planner will set managerial utility to zero: $r^* = \psi(e^*)$.

2.2. Incomplete information

With incomplete information the planner cannot observe $\beta$ or $e$, he knows only the possible $\beta$ values and the probability, $v$, that a firm is of type $\beta_i$. The planner will offer a pair of contracts, $[c_i, m_i, t_i], i = l, h$ to maximize payoff subject to managerial individual rationality constraints, and subject to the incentive compatibility constraints

$$t_i - \psi(g(\beta_i, m_i) - c_i) \geq t_j - \psi(g(\beta_i, m_j) - c_j) \quad \text{for } i, j = 1, 2, \ i \neq j,$$

(2.4)

here we have substituted for $e_i$ from (2.1).

At given $m$ the cost advantage of the low type is $\zeta(m) := g(\beta_h, m) - g(\beta_l, m)$. The minimum effort required by the low type to achieve the high type’s cost target is $e_h - \zeta(m_h)$, or zero if this term is negative. Define this minimum effort formally as

$$z(e, m) := \max[0, e - \zeta(m)].$$

Misrepresentation of type brings the low type a utility benefit of $\psi(e_h) - \psi(z(e_h, m_h))$, an information rent of this amount must be given to induce correct revelation of type. Denote this rent by

$$\phi(e_h, m_h) := \psi(e_h) - \psi(z(e_h, m_h)).$$

Its derivatives are

$$\phi_{e_h} = \begin{cases} 
\psi'(e_h) - \psi'(z) \frac{\partial z}{\partial e_h} > 0 & \text{for } e_h - \zeta(m_h) > 0, \\
\psi'(e_h) > 0 & \text{for } e_h - \zeta(m_h) < 0,
\end{cases}$$

(2.5)

$$\phi_{m_h} = \begin{cases} 
-\psi'(z) \frac{\partial z}{\partial m_h} < 0 & \text{for } e_h - \zeta(m_h) > 0, \\
0 & \text{for } e_h - \zeta(m_h) < 0.
\end{cases}$$

(2.6)

The rent function is kinked at $e_h = \zeta(m_h)$, depending on effort but not material input when $e_h < \zeta(m_h)$. Higher effort for the high type increases the information rent. Higher material input reduces the cost advantage $\zeta$ and so reduces the information rent. The planner will seek to economize on rent paid to the low type by reducing the high type’s effort, and by increasing its material input, relative to efficiency levels. The rent-reducing role for material input occurs only if $e_h - \zeta(m_h) > 0$; to allow for this effect we assume that $e_h^* > \zeta(m_h^*)$. 
The planner will set managerial rewards as low as possible, subject to the individual rationality and incentive compatibility constraints. The high type will receive zero utility and the low type will receive the information rent, getting \( U_l = \Phi(e_h, m_h) > 0 \). Substituting in (2.2), the asymmetric information problem is

\[
\max_{e_h, m_h, e_l, m_l} W = S - v[(1 + \lambda)(g(\beta_l, m_l) - e_l + rm_l + \psi(e_l)) + \lambda \Phi(e_h, m_h)] - (1 - v)(1 + \lambda)(g(\beta_h, m_h) - e_h + rm_h + \psi(e_h)).
\] (2.7)

As usual, the solutions to this program involve an efficient allocation for the low type, with \((\hat{e}_l, \hat{m}_l) = (e^*, m^*_l)\). The solutions for the high type depend in a crucial way on the value of \( v \), the proportion of low types in the population.

**Proposition 1.** There is a critical value \( \hat{v} \in (0, 1) \), defined by the equality \( \hat{e}_h(\hat{v}) = \xi(m^*_h) \), such that the second-best plan assignments for the high type are characterized by

1. for \( 0 < v < \hat{v} \) then: \( \hat{e}_h - \xi(\hat{m}_h) > 0, 0 < \hat{e}_h < e^* \) and \( \hat{m}_h \geq m^*_h \)
2. for \( 1 > \hat{v} \) then: \( \hat{e}_h - \xi(\hat{m}_h) < 0, 0 < \hat{e}_h < e^* \) and \( \hat{m}_h = m^*_h \).

The proposition is illustrated in Fig. 1. The first-order maximization conditions are

\[
\psi' (\hat{e}_h) = 1 - \alpha \Phi_{e_h} (\hat{e}_h, \hat{m}_h) < 1, \quad g_m (\beta_h, \hat{m}_h) + r = -\alpha \Phi_{m_h} (\hat{e}_h, \hat{m}_h) \geq 0,
\] (2.8)

where \( \alpha = v \lambda / (1 - v)(1 + \lambda) \).

Note that as \( v \) (and hence \( \alpha \)) approaches 0 the information-constrained solution becomes efficient: \( (\hat{e}_h(v), \hat{m}_h(v)) \rightarrow (e^*, m^*_h) \). Hence, \( \hat{e}_h - \xi(\hat{m}_h) > 0 \) for \( v \) in a neighborhood above zero. As \( v \) approaches 1 suppose that \( \hat{z} = \hat{e}_h - \xi(\hat{m}_h) \rightarrow 0; \) then the effort equation in (2.8) can be solved only if \( \hat{e}_h = \hat{z} \), an impossibility. Hence, in a neighborhood of \( v = 1 \) the solution involves \( \hat{e}_h - \xi(\hat{m}_h) < 0 \). The critical value \( \hat{v} \) that separates these solution regions solves \( \hat{e}_h(\hat{v}) = \xi(m^*_h) \).

Compare (2.3) with (2.8). When \( e_h - \xi(m_h) > 0 \) the information rent depends on both effort and material input of the high type; to economize on rent effort is distorted below \( (\Phi_{e_h} > 0) \) and material input is distorted above \( (\Phi_{m_h} < 0) \) their efficiency levels. When \( e_h - \xi(m_h) < 0 \) then \( \Phi_{m_h} = 0 \): the information rent depends on effort only. The high type continues to receive a less than efficient effort assignment but receives an efficient material input assignment.

As the population fraction of low types increases from zero to one, the high type’s effort plan declines. This is expected. As \( v \) increases, the rent burden increases (more low types) and the inefficiency burden decreases (fewer high types) ceteris paribus; the regulator lowers

[4] The low type’s incentive constraint in (2.4) is rewritten as

\[
U_l \geq t_h - \psi(g(\beta_l, m_l) - c_h) = U_h + \psi(e_h) - \psi(e_h - \xi(m_h)).
\]

With \( U_h = 0 \), then incentive compatibility requires \( U_l = \Phi(\cdot) > 0 \). The high type’s incentive constraint is \( U_h \geq U_l + \psi(e_l) - \psi(e_l + \xi(m_l)) \). Substituting for \( U_h = 0 \) and \( U_l = \Phi \) gives the requirement \( 0 \geq \psi(e_h) - \psi(e_h - \xi(m_h)) - \psi(e_l + \xi(m_l)) - \psi(e_l) \). The inequality will be strict for \( e_l > e_h \) and \( \xi(m_l) \geq \xi(m_h) \). Since \( g_{\min} \leq 0 \), this latter requires \( m_l \geq m_l \). Both \( e_l > e_h \) and \( m_l \geq m_l \) can be verified in the solution.

[5] \( \hat{v} \) is explicitly solved as \( \hat{v} = (1 + \lambda)(1 - \psi'(\xi(m^*_h)))/(1 + \lambda)(1 - \psi'(\xi(m^*_h)) + \lambda \psi'((\xi(m^*_h))) < 1. \) Since for all \( v > \hat{v}, \delta e_h(v)/\delta v \) is negative and \( \hat{m}_h = m^*_h \), it follows that \( \hat{v} \) is unique.
\( \hat{e}_h \) in response, reducing rent at the expense of additional inefficiency. The decline in \( \hat{e}_h \) continues through \( \hat{v} \) and \( \hat{e}_h \) goes to zero as \( v \) goes to unity.

By contrast, over the interval \((0, \hat{v})\) where second-best material input targets are larger than efficient levels, material input is an inverted-U shaped function of \( v \). The absolute magnitude of the marginal impact of material input on the information rent, \( \Phi_{m_h} \), is larger the larger is minimum required effort, \( z \). For low values of \( v \), \( \hat{z} \) and hence \( \Phi_{m_h} \) are relatively large and the planner is willing to increase inefficiency to reduce rent as \( v \) increases: \( \partial \hat{m}_h / \partial v > 0 \). However, close to \( \hat{v} \) where \( \hat{z} \) and \( \Phi_{m_h} \) are small, the planner becomes willing to reduce inefficiency by reducing material input as \( v \) increases, \( \partial \hat{m}_h / \partial v < 0 \), albeit at the cost of additional rent. Eventually, when effort falls to the point where \( \hat{z} = 0 \) the material input becomes irrelevant to the rent trade-off and so is chosen efficiently for \( v \geq \hat{v} \).\(^6\)

### 3. Dynamic consistency

Now consider a dynamic two-period environment. The optimal static allocation is also the optimal allocation in each period if the parties can sign a long-run contract. Suppose that the planner is committed to a long-run contract, but that the firm is free in each period to choose whichever contract it wishes. The dynamic consistency problem to be highlighted here is most easily identified in this case. Alternative specifications of the model’s information structure, in particular those where the planner might learn something about the

\(^6\)Three final notes on the static solution. First, if \( g_{m_h} \equiv 0 \), then \( \hat{m}_h(v) = m^* \) for all \( v \). Second, since \( \hat{e}_h < \hat{e}_l \) and \( \hat{m}_h > \hat{m}_l \) (because \( \hat{m}_h > m^*_h > m^*_l = \hat{m}_l \)), the incentive constraint of the high type is satisfied in the solution to (2.2). Finally, if we assume that \( S > (1 + \lambda)(g(\beta_h, m^*_h) + r m^*_h) \) the planner will never wish to shut down the high type.
firm’s cost parameter before period 2 begins, can then be analyzed with reference to this benchmark case.

3.1. Material input hoarding and intertemporal effort shifting

Denote a plan assignment in period \( i \) by \( \theta_i = (e_{hi}, m_{hi}, U_{hi}, e_{li}, m_{li}, U_{li}) \) and a two-period plan by \( \Theta = (\theta_1, \theta_2) \). With commitment the planner will offer firms the optimal static contract twice: \( \bar{\theta}_i = (\hat{e}_{hi}, \hat{m}_{hi}, 0, e_{i}^*, m_{i}^*, \Phi(\hat{e}_{hi}, \hat{m}_{hi})) \) for \( i = 1, 2 \). Is this set of contracts incentive-compatible over the two periods? Under two plausible conditions the answer is no. First, suppose that the material input is storable between periods: it can be hoarded albeit at some cost. And second, suppose that managers do not discount the future too heavily. Then at the repeated static solution the low cost firm may have an incentive to misrepresent its type in period 1 while declaring its true type in period 2.

In the repeated static solution misrepresentation of type makes the low type no better off directly, since he is indifferent between the low and high plan in period 1. But misrepresentation enables him to save on effort in period 1, giving a sequence of unequal effort levels over the two periods. This, together with convexity of the effort-disutility function, raises the possibility of an intertemporal utility gain if some of the period-1 effort reduction is transferred to period 2. Hoarding material input allows such a transfer. By working a little harder in period 1, the low type can achieve the high type’s cost target without using all the material input received. Carrying a positive hoard of input into the second period allows the manager to enjoy a relatively slack effort level in period 2 when the low-type assignment is chosen. The only question is whether the disutility of additional effort undertaken in period 1 plus the cost of hoarding is sufficiently compensated by the discounted utility benefit of lesser effort in period 2. This depends on the convexity of the effort-disutility function, the magnitude of the discount factor and the cost of hoarding.

To formalize the argument, let \( \delta \in (0, 1) \) be the firm’s discount factor, and let \( (1 + k)m_o \) be the gross amount of input hoarded, where \( k \geq 0 \) reflects the cost of hoarding. The amount hoarded must be non-negative, and cannot be larger than the amount available in period 1: \( 0 \leq (1 + k)m_o \leq m_{h1} \); the firm retains \( m_o \) units for use in period 2. The low type’s aggregate utility from choosing the high type’s contract in period 1, his own contract in period 2, and hoarding between periods is

\[
U_{I_A}(m_o; \Theta) = t_{h1} - \psi(g(\beta_1, m_{h1} - (1 + k)m_o) - c_{h1}) + \delta[t_{l2} - \psi(g(\beta_1, m_{l2} + m_o) - c_{l2})]
\]

\[
= t_{h1} + \delta t_{l2} - [\psi(z(e_{h1}, m_{h1})) + x(m_{h1}, m_o, k)] + \delta \psi(e_{l2} - y(m_{l2}, m_o)),
\]

(3.1)

where \( x(m, m_o, k) = g(\beta_1, m - (1 + k)m_o) - g(\beta_1, m) \) and \( y(m, m_o) = g(\beta_1, m) - g(\beta_1, m + m_o) \).

The expression in brackets on the last line shows clearly the disutility tradeoff involved in hoarding. The minimum effort required by the low type to achieve the high type’s cost target in period 1 is \( z_1 := z(e_{h1}, m_{h1}) \). If the low type chooses to hoard input in period 1 his effort must increase by the amount \( x \). However, this hoarding allows him to decrease his effort in period 2 by an amount \( y \). Even with a discount factor below 1 and a positive
hoarding cost this intertemporal effort shifting can reduce the aggregate disutility of effort because of the convexity of \( \psi() \).

Aggregate utility is strictly concave in \( m_o \). The first-order condition for maximization is

\[
\frac{\partial U_A(m_o; \theta)}{\partial m_o} = \frac{\partial x(m_{h1}, m_o, k)}{\partial m_o} + \delta \psi'(e_{t2} - y) \frac{\partial y(m_{t2}, m_o)}{\partial m_o} \leq 0
\]

with strict equality when \( m_o > 0 \).

Define the critical discount factor \( \delta_o \) as that which would solve (3.2) as an equality at \( m_o = 0 \)

\[
\delta_o = (1 + k) \xi,
\]

where \( \xi = \psi'(z_1) g_m(\beta_t, m_{h1}) / (\psi'(e_{t2}) g_m(\beta_t, m_{t2})) \). The derivative in (3.2) is then strictly positive whenever the manager’s discount factor is above \( \delta_o \). In this case hoarding material by choosing \( m_o > 0 \) strictly increases the low type’s aggregate utility.

It is straightforward to identify circumstances in which the manager’s discount factor will be larger than the critical discount value in the repeated static solution. Write \( \hat{\xi}(v) \) as the value of \( \xi \) in (3.3) evaluated at the static assignments \( \hat{\theta}_1(v) \). Since \( 0 \leq \hat{z}_1(v) < e^* \), and \( \hat{m}_{h1}(v) > m_o \), \( \hat{\xi}(v) \) is strictly less than 1 for all \( v \), and is zero (since \( \psi'(\hat{z}_1(v)) \) is) for all \( v \) above \( \hat{v} \). First then, in this latter region hoarding will occur at any positive value of the manager’s discount factor irrespective of the cost of hoarding. Second, for \( v \) below \( \hat{v} \) hoarding will occur so long as the cost of hoarding is not too large, and the manager’s discount factor is close to 1. Conversely, however, for a discount factor of 1 hoarding can always be choked off by a large enough hoarding cost when \( v < \hat{v} \).

Note that \( U_A(0; \hat{\theta}) \) is the low type’s aggregate utility at the repeated static plan whether or not there is misrepresentation of type in period 1. Whenever maximization of (3.1) has a positive hoarding solution then \( U_A(\hat{m}_o, \hat{\theta}) > U_A(0; \hat{\theta}) \) and the low type strictly prefers to misrepresent type in period 1 and hoard input. Incentive compatibility of the repeated static optimum may therefore be lost when the low type has the opportunity to carry forward material input between periods. We summarize this in the following proposition.

**Proposition 2.** The optimal static incentive contract is not dynamically incentive compatible if hoarding is possible and if the low type’s discount factor is greater than a critical value, given by \( \delta_o(k, v) := (1 + k) \hat{\xi}(v) \). In this case the low type has an incentive to misrepresent type in period 1. The high type has no incentive to misrepresent itself.

Socialist firms have been characterized as ‘input-hungry’, demanding unlimited input quantities because of the planner’s constant pressure for output production, combined with the soft budget constraint that does not penalize high input demands. This proposition suggests an additional source of input hunger, motivated by the desire of the low cost firm to build slack into the system by securing an inappropriately large input stock in period 1, to be consumed as reduced effort in period 2. This interpretation is consistent with the existence of corruption and ‘second economy’ production, which reached a sizeable fraction of Soviet national output in the later Brezhnev years.

---

7 Use \( \partial x / \partial m_o = -(1 + k) g_m(\beta_t, m_{h1}) > 0 \) and \( \partial y / \partial m_o = -g_m(\beta_t, m_{t2} + m_o) > 0 \).
3.2. Anti-hoarding rent

The planner must redesign the incentive scheme to take account of the above dynamic self-selection problem. This involves paying an additional anti-hoarding rent to the low type in period 1 to compensate it for revealing its type. Define \( m_0 = m_0(\Theta; \delta, k) \geq 0 \) to be the hoarding solution that maximizes aggregate utility (3.1). The planner takes this function as given when choosing plan assignments. The incremental utility benefit of first-period misrepresentation and hoarding by the low type for any dynamic planning assignment \( \Theta \) is

\[
\Gamma(e_{h1}, m_{h1}, e_{l2}, m_{l2}; \delta, k) = [U_{1A}(m_0(\cdot), \Theta) - U_{1A}(0, \Theta)]
\]

\[
= \psi(z) - \psi(z + x(m_{h1}, m_0(\cdot), k))
\]

\[
+ \delta[\psi(e_{l2}) - \psi(e_{l2} - y(m_{l2}, m_0(\cdot)))].
\]

To induce truth-telling an anti-hoarding rent of amount \( \Gamma(\cdot) \), must be paid to the low type in period 1 in addition to the information rent \( \Phi(\cdot) \).

Some qualification is necessary here. This additional anti-hoarding rent addresses only the incentive for misrepresentation by the low type described above. In a consistent solution that uses \( \Gamma(\cdot) \) as defined here it must be verified that other potential misrepresentations do not occur. First, it must be verified that in period 2 the low type with a positive input hoard prefers to reveal its true type rather than claim to be the high type. With \( m_o \) units of input in period 2 the low type enjoys a utility gain of \( \psi(e_{l2}) - \psi(e_{l2} - y(m_{l2}, m_o)) \) with the low assignment, and \( \psi(z(e_{h2}, m_{h2})) - \psi(z(e_{h2}, m_{h2}) - y(m_{h2}, m_o)) \) with the high assignment. Since, the marginal utility from reducing effort is larger the higher the effort level, a given level of hoarded input is more valuable to the low type if it truthfully reveals itself in period 2; this must be checked.

Second, it must be verified that the additional anti-hoarding rent is not sufficient to tempt the high type to misrepresent itself as low in period 1, gain the rent, and then behave as a high type in period 2. The reward that the high type would receive by misrepresenting itself as low in period 1 is the low type’s total period-1 rent, \( \Phi + \Gamma \). The period 1 cost of this misrepresentation to the high type can be expressed in effort disutility terms as \( \psi(e_{11} + \xi(m_{11})) - \psi(e_{11}) \). The required incentive compatibility constraint is that the cost of misrepresentation be at least as large as the benefit

\[
\psi(e_{11} + \xi(m_{11})) - \psi(e_{11}) \geq \Phi(e_{h1}, m_{h1}) + \Gamma(e_{h1}, m_{h1}, e_{l2}, m_{l2}; \delta, k). \tag{3.4}
\]

These requirements are satisfied for \( \delta \) in a neighborhood above \( \delta_0(k, v) \) and so they are ignored here.\footnote{ First, sufficient for \( \psi(e_{l2}) - \psi(e_{l2} - y(m_{l2}, m_o)) \geq \psi(z(e_{h2}, m_{h2}))-\psi(z(e_{h2}, m_{h2}) - y(m_{h2}, m_o)) \) are the inequalities (1) \( e_{l2} \geq z(e_{h2}, m_{h2}) \), and (2) \( y(m_{l2}, m_o) \geq y(m_{h2}, m_o) \) (which holds if and only if \( m_{l2} \leq m_{h2} \)). As \( \delta \to \delta_0(k, v) \), then \( m_o \to 0 \), and the solution values for the program that includes the anti-hoarding rent, denoted by the ‘tilde’ accent, satisfy \((\tilde{e}_{l2}, \tilde{m}_{l2}) \to (e^*, m^*_o) \) and \((\tilde{e}_{h2}, \tilde{m}_{h2}) \to (\tilde{e}_h, \tilde{m}_h) \). But then \( \tilde{e}_{l2} > \tilde{e}_{h2} > z(\tilde{e}_{h2}, \tilde{m}_{h2}) \) and \( \tilde{m}_{l2} < \tilde{m}_{h2} \). This verifies the inequality. Second, analogous to Footnote 4, dynamic incentive compatibility for the high type requires \( U_{h1} \geq U_{l1} + \psi(e_{l1}) - \psi(e_{l1} + \xi(m_{11})) \), or, setting \( U_{h1} = 0 \) and \( U_{l1} = \Phi + \Gamma \), inequality (3.4). But as \( m_o \to 0 \) this constraint reduces to the high type’s static incentive-compatibility constraint, which is strictly non-binding. Hence, neither of these potential constraints holds in a neighborhood above \( \delta_0(k, v) \).}
Proposition 3. To ensure truth-telling in period 1, it is necessary that the low type be paid a total rent \( \Phi(e_{h1}, m_{h1}) + \Gamma(e_{h1}, m_{h1}, e_{l2}, m_{l2}; \delta, k) \) in that period. This rent is sufficient for truth-telling when the firm’s discount factor is in a neighborhood above \( \delta_0(k, v) \).

3.3. Properties of the anti-hoarding rent

The anti-hoarding rent depends in particular on the high type’s period-1 plan \((e_{h1}, m_{h1})\), and on the low type’s period-2 plan \((e_{l2}, m_{l2})\). Because it depends on the low type’s plan, the planner will reduce the rent by distorting this plan away from efficiency. The nature of the distortions depends on the derivatives of \( \Gamma \). Because \( m_o(.) \) maximizes aggregate utility (3.1), \( d\Gamma/d\theta = \partial(U_{I/A}(m_o(\cdot, \Theta) - U_{I/A}(0, \Theta)))/\partial\theta \) by the envelope theorem. Then simplification gives derivatives with respect to the low type’s period 2 plan

\[
\Gamma_{e_{l2}} = \delta(\psi'(e_{l2}) - \psi'(e_{l2} - y)) > 0, \tag{3.5}
\]

\[
\Gamma_{m_{l2}} = \delta \psi'(e_{l2} - y) \frac{\partial y}{\partial m_{l2}} < 0. \tag{3.6}
\]

The signs here indicate that the planner can economize on anti-hoarding rent \textit{ceteris paribus} by giving the low type a smaller effort and a larger material input assignment in period 2 than are called for by efficiency considerations. That is, a slacker-than-efficient plan in period 2 makes period-1 hoarding less attractive to the low type.

With respect to the high type’s plan, the anti-hoarding rent is kinked at \( e_{h1} = \zeta(m_{h1}) \); it depends on material input but not effort for \( e_{h1} < \zeta(m_{h1}) \). The rent derivatives are

\[
\Gamma_{e_{h1}} = \begin{cases} 
\left[ \psi'(z_1) - \psi'(z_1 + x) \right] \frac{\partial z_1}{\partial e_{h1}} \leq 0 & \text{for } e_{h1} > \zeta(m_{h1}), \\
0 & \text{for } e_{h1} < \zeta(m_{h1}). 
\end{cases} \tag{3.7}
\]

\[
\Gamma_{m_{h1}} = \begin{cases} 
\left[ \psi'(z_1) - \psi'(z_1 + x) \right] \frac{\partial z_1}{\partial m_{h1}} - \psi'(z_1 + x) \frac{\partial x}{\partial m_{h1}} & \text{for } e_{h1} > \zeta(m_{h1}), \\
-\psi'(x) \frac{\partial x}{\partial m_{h1}} > 0 & \text{for } e_{h1} < \zeta(m_{h1}). 
\end{cases} \tag{3.8}
\]

There are two cases here. When \( e_{h1} > \zeta(m_{h1}) \) (3.7) shows that the anti-hoarding rent can be reduced \textit{ceteris paribus} by an increase in the high type’s period-1 effort assignment; this makes the high contract less attractive to the low type because an increase in \( e_{h1} \) raises \( z_1 \), the minimum effort that the low type must expend to achieve the cost target \( c_{h1} \).

Change in material input has two offsetting effects on the rent. A decrease in input directly reduces the size of the hoarding ‘budget’ available to the low cost firm; this increases \( x \), the amount of additional effort required by the low type to achieve the high type’s cost target, making misrepresentation more costly for the low type. Less rent is needed as seen in the positive second term in (3.8), where \( \partial x/\partial m_{h1} < 0 \). However, just as with an increase in effort, an increase in material input increases the minimum effort \( z_1 \) making misrepresentation less attractive to the low type; increased input can reduce the anti-hoarding
rent as seen in the negative first term in (3.8). It is unclear in general which of these offsetting effects will dominate, so the high type’s period 1 material input assignment may be higher or lower than in the static solution depending on whether the anti-hoarding rent is reduced by an increase or by a reduction in input.

In the second case, when \( e_{h1} < \zeta(m_{h1}) \), the minimum effort function \( z_1 \) is constrained at zero. Then, the anti-hoarding rent is independent of effort, and it increases with increases in material input since only the budget effect mentioned above operates.

Note that the derivatives in (3.7) and (3.8) are discontinuous at \( e_{h1} = \zeta(m_{h1}) \). Here, \( z_1 = 0 \) but \( \psi'(0 + x) > 0 \), and whereas the left-hand side derivatives of \( z_1 \) with respect to either effort or material input are strictly positive the right-hand side derivatives are zero.

Finally, note that the information rent is increasing in the discount factor, and decreasing in the cost of hoarding

\[
\frac{\partial \Gamma}{\partial \delta} = (\psi(e_{l2}) - \psi(e_{l2} - y)) > 0 \quad \text{and} \quad \frac{\partial \Gamma}{\partial \delta} = -\psi'(z_1 + x) \frac{\partial x}{\partial \delta} < 0.
\]

The more the firm values the future, and the less costly is hoarding, the larger must be the anti-hoarding rent paid.

4. Consistent plans

The two-period problem faced by the planner can now be specified as the maximization of

\[
\mathcal{W} := W(\theta_1) - v\lambda \Gamma(e_{h1}, m_{h1}, e_{l2}, m_{l2}; \delta, k) + \delta_p W(\theta_2).
\] (4.1)

Here, \( \delta_p \) is the planner’s discount factor. The payoff function \( W(\cdot) \) is defined in (2.7) and includes the information rent \( \Phi(\cdot) \). The distinguishing feature of this overall payoff is the presence of the dynamic anti-hoarding rent \( \Gamma(\cdot) \) paid to the low type in period 1. Without this rent the problem would reduce to the static period-by-period optimization, in which case the resulting plan assignments are not dynamically incentive compatible.

To maximize this payoff the planner must choose an 8-tuple of endogenous variables — an effort/material-input pair for each type for each period, taking the hoarding function \( m_o(\cdot) \) as given. Use the tilde accent to denote solutions.

Note first that, given the solution, the envelope theorem can be invoked to write

\[
\frac{\partial \hat{\mathcal{W}}}{\partial k} = -v\lambda \Gamma_k > 0 \quad \text{and} \quad \frac{\partial \hat{\mathcal{W}}}{\partial \delta} = -v\lambda \Gamma_\delta < 0.
\]

The planner’s maximized payoff decreases either as the cost of hoarding decreases, or as the firm’s discount factor increases. Thus, any change in circumstances that facilitates or encourages hoarding by the firm makes the planner unambiguously worse off. It involves a weakening of the planner’s ability to separate out the types, requiring the payment of additional rent and the introduction of additional inefficiencies, in particular for the efficient type. This is emphasized in the following proposition.

**Proposition 4.** The planner is better off the smaller is the low type’s incentive to hoard.
Hoarding is dysfunctional from the planner’s viewpoint. What the model suggests is the desirability of raising the cost of hoarding and reducing the firm’s discount factor to the point where hoarding is choked off: \( \delta \leq \delta_0(k, v) \). Ways to achieve this include emphasizing accountability of firms for input demands and usage, and strategies such as frequent rotation of managers to reduce managerial weighting of specific futures. \(^9\) If the benefits of future effort reduction cannot accrue to a manager who has been transferred he will not hoard at the cost of present effort. In the Soviet case frequent managerial transfer was an explicit policy of the Khrushchev era but was reversed in the ‘stability of cadres’ policy in Brezhnev’s time. Further, the cronyism and mutual shielding of ‘family group’ members from accountability to the system at large and the outright corruption of the Brezhnev era \(^10\) were features that served to reduce rather than increase the cost of hoarding to firms. This ‘period of stagnation’ was one in which dysfunctional managerial behavior and the conversion of public inputs to private benefit was encouraged by the system’s parameters.

Turning to the solution the optimal values for the high type’s period-2 plan and the low type’s period-1 plan, neither of which are involved in the anti-hoarding rent, are the same as the static solutions \( (\hat{e}_{h2}, \hat{m}_{h2}) = (\hat{e}_{i1}, \hat{m}_{i1}) = (e^*, m^*_l) \). These solutions are not further discussed.

Assuming that \( m_{o}(\theta) \) is positive, both the low type’s period-2 plan and the high type’s period-1 plan are involved in the anti-hoarding rent.

4.1. Low type period-2 solutions

The first-order conditions for the low type’s period-2 plan are

\[
\psi'(e_{l2}) = 1 - \frac{\lambda \Gamma_{l_{i2}}}{\delta_p(1 + \lambda)} < 1, \quad g_m(\beta_{l1}, m_{l2}) + r = -\frac{\lambda \Gamma_{l_{m2}}}{\delta_p(1 + \lambda)} > 0. \quad (4.2)
\]

These conditions indicate immediately that the planner distorts the low type’s period 2 plan away from efficiency.

**Proposition 5.** When \( m_{o}(\theta) > 0 \) the low type gets a lower than efficient effort assignment and a larger than efficient material input assignment in period 2: \( \hat{e}_{l2} < e^* \), and \( \hat{m}_{l2} > m^*_l \).

To reduce the anti-hoarding rent effort is reduced and material input increased relative to efficient levels. The low type is given a less taut overall cost assignment in the second period to reduce its incentive to hoard; this argument contradicts an intuition that suggests that the planner’s best response to hoarding is extra-taut plans.

4.2. High type’s period-1 solutions

The structure of the high type’s period-1 solution is similar to the static case. There is a critical value of the distribution of types, \( \hat{v}(\delta, k) \), above which the minimum effort level

---

\(^9\) See, e.g. Ickes and Samuelson (1987) who explicitly model job transfers as a means of countering the adverse ratchet effect of plan revision.

\(^10\) See, e.g. Gill (1994).
$z_1$ is zero and below which it is positive. Qualitative properties of the solution depend on whether $v$ is above or below this critical value.

**Proposition 6.** For $\delta$ in a neighborhood above $\delta_0(k, v)$, there exists a unique critical value of the population proportion of low types, $\tilde{v}(k, \delta)$, such that the high type's dynamic period-1 plan assignments satisfy

1. for $v > \tilde{v}(k, \delta)$
   (a) $\tilde{e}_{h1} < \xi(\tilde{m}_{h1})$;
   (b) the effort assignment is identical to that in the static case: $\hat{e}_{h1} = \hat{e}_{h1}$;
   (c) the material input is less than the efficient level and less than the static assignment: $m_{h1} < m^*_h \leq \tilde{m}_{h1}$.

2. for $v < \tilde{v}(\delta, k)$
   (a) $\tilde{e}_{h1} > \xi(\tilde{m}_{h1})$;
   (b) the effort assignment is higher than the static level but still less than the efficient level: $\hat{e}_{h1} < \hat{e}_{h1} \leq e^*$;
   (c) the material input assignment may be either above or below the static assignment and may be above or below the efficient level: $m_{h1} \geq \tilde{m}_{h1}$ and $\tilde{m}_{h1} \geq m^*_h$ are possible.

The first-order conditions are

$$
\psi'(e_{h1}) = \begin{cases} 
1 - \alpha(\Phi_{e_{h1}} + \Gamma_{e_{h1}}) & \text{for } e_{h1} > \xi(m_{h1}), \\
1 - \alpha \Phi_{e_{h1}} < 1 & \text{for } e_{h1} < \xi(m_{h1}), 
\end{cases} \quad (4.3)
$$

$$
g_m(\tilde{\beta}_h, m_{h1}) + r = \begin{cases} 
-\alpha(\Phi_{m_{h1}} + \Gamma_{m_{h1}}) & \text{for } e_{h1} > \xi(m_{h1}), \\
-\alpha \Gamma_{m_{h1}} < 0 & \text{for } e_{h1} < \xi(m_{h1}). 
\end{cases} \quad (4.4)
$$

To show the existence of the critical value $\tilde{v}(k, \delta)$ suppose that $\tilde{e}_{h1} > \xi(\tilde{m}_{h1})$ for all values of $v$. Then, $\partial z_1 / \partial e_{h1} = 1$ and as $v$ approaches 1 the first case in (4.3) can be maintained only if $\psi'(\tilde{e}_{h1}) = \psi'(\tilde{z}_1 + \tilde{\alpha})$; on substituting $\tilde{z}_1 = \tilde{e}_{h1} - \xi(\tilde{m}_{h1})$ this implies $-\xi(\tilde{m}_{h1}) + x(\tilde{m}_{h1}, m_o) = 0$, or $-g(\beta_h, \tilde{m}_{h1}) + g(\tilde{\beta}_h, \tilde{m}_{h1} - (1 + k)m_o) = 0$ on further simplification. For $m_o$ small as when $\delta$ is in a neighborhood above $\delta_0$ this latter equality is contradicted since the expression is in fact negative. Hence, for $v$ close to 1 the possibility of $\tilde{e}_{h1} > \xi(\tilde{m}_{h1})$ is contradicted. Above some value the solution must involve $\tilde{e}_{h1} < \xi(\tilde{m}_{h1})$.

Any solution with $\tilde{e}_{h1} < \xi(\tilde{m}_{h1})$ is characterized by the latter equations from (4.3) and (4.4). It is easy to verify that effort and material input are each decreasing in $v$ for these conditions and hence that $\tilde{e}_{h1}(v) - \xi(\tilde{m}_{h1}(v))$ is decreasing in $v$. If this term is negative for any value of $v$ it must be negative for all higher values. Hence, $\tilde{v}(\delta, k)$ is unique. It is determined by the equality $\tilde{e}_{h1}(v; \delta, k) - \xi(\tilde{m}_{h1}(v; \delta, k)) = 0$.

Comparing the respective pairs of conditions in (4.3) and (4.4) it is immediate that the right-hand side terms are discontinuous at $v = \tilde{v}$. This is due to the discontinuity in the derivatives of the anti-hoarding rent previously noted. In consequence the solutions for effort and material input are discontinuous downwards at $v = \tilde{v}$.

For case 1 where $v > \tilde{v}(\delta, k)$ and minimum effort $z_1$ is zero, the information rent depends on period-1 effort but not material input or the level of hoarding, while the anti-hoarding rent
depends on period-1 material input and hoarding (through x) but not effort. This decomposition means that period-1 effort and material input can be solved independently of each other. The first-order condition for effort is identical to the static case so \( \bar{e}_{h1} = \bar{e}_h \). Material input is less than both the efficient level and the static level; this reduces the anti-hoarding rent by reducing the potential hoarding ‘budget’. This reduced input level might be considered as evidence of a more-taut plan; note however that this tautness is imposed on the non-hoarding rather than the hoarding firm, and that the firm is always compensated such that it receives the reservation utility whatever the plan targets are.

For case 2, where \( v < \bar{v}(\delta, k) \) and minimum effort \( z_1 \) is positive, effort is increased relative to the static solution while material input can be above or below the static solution. Intuitively, effort is increased above the static level because this reduces the anti-hoarding rent; \( \bar{F}_{e1} < 0 \) in (4.3). The impact of material input on the anti-hoarding rent is ambiguous, having the potential to either raise or lower it. If \( \bar{F}_{m1} > 0 \) then a reduction in material input below the static level is expected and conversely. With respect to efficiency, \( \bar{e}_{h1} \) lies below \( e^* \) since the sum of the rent impacts of effort \( \bar{m}_{h1} + \bar{F}_{e1} \) is positive when \( m_o \) is not too large.

Finally, \( \bar{m}_{h1} \) may be less than the efficient level \( m^*_h \) (and hence also less than \( \bar{m}_{h1} \)) if the expression \( \bar{m}_{o1} + \bar{F}_{m1} \) is negative. This expression can be expanded and simplified as \( g_m(\beta_1, \bar{m}_{h1}) - m_o(1 + k)m_o) - g_m(\beta_1, \bar{m}_{h1}) \) which is positive for \( m_o \) small but may be negative for larger values.

In summary the implication for the high type’s period-1 plan of input hoarding by the low type is a tauter effort assignment than if hoarding were not possible, though the high type never gets a super-efficient assignment, and a material-input assignment that can run the gamut from being looser than the no-hoarding case to being tauter than pure efficiency considerations would dictate. This wide range of possibilities undermines any suggestion that there is a simple mapping from the possibility of hoarding to the tautness of input plans.

Proposition 6 is illustrated in Figs. 2 and 3, which are based on numerical computation of an example that uses \( \psi(e) = \frac{1}{2}e^2 \), \( g(\beta, m) = \beta/m \) and \( k = 0 \). Fig. 2 graphs the material input solution, as a function of \( v \), for three values of \( \delta \): (a) \( \delta = 0 \) which recapitulates the static solution, (b) an intermediate value \( 1 > \delta > \delta_o(v) \), and (c) \( \delta = 1 \).

The figure illustrates the impossibility of making unambiguous solution comparisons for material input in the region where \( v < \bar{v}(\delta) \). The static solution (line (a)) is intersected from

---

11 The right-hand side of (4.4) goes to \(-\infty\) as \( v \) goes to 1. It follows that \( m_{h1} \) goes to zero as \( v \) approaches some value less than or equal 1. If the additional assumption that \( \lim_{m_o \to 0} g(\beta, m) = -\infty \) is made, then \( \bar{m}_{h1} \) approaches zero as \( v \) approaches 1.

12 Briefly, the first-order equations in (4.3) and (4.4) can be solved for \( e_{h1} \) and \( m_{h1} \) as functions of \( m_o \). Treating the latter as a parameter the solutions are \( (e_{h1}, \bar{m}_{h1}) \) when \( m_o = 0 \), and \( (\bar{e}_{h1}, \bar{m}_{h1}) \) when \( m_o = \bar{m}_o > 0 \). Then the dynamic solution values can be compared to their static counterparts by looking at the comparative-static derivatives, \( \partial e_{h1} / \partial m_o \) and \( \partial m_{h1} / \partial m_o \). Computation shows that \( \partial e_{h1} / \partial m_o \) is positive so long as \( m_o \) is ‘small’, as when \( \delta \) is close to \( \delta_o \). Thus, \( \bar{e}_{h1} > \bar{e}_{h1} \). The derivative \( \partial m_{h1} / \partial m_o \) may be either positive or negative as \( m_o \) approaches zero. Hence \( \bar{m}_{h1} \) may be larger or smaller than \( \bar{m}_{h1} \).

13 In terms of the first-order conditions (2.8) and (4.4), whether \( \bar{m}_{h1} \) is larger or smaller than \( \bar{m}_{h1} \) depends on whether \( \psi'(z_1)(\partial z_1 / \partial m_{h1}) \) is larger or smaller than \( \psi'(z_1 + x)(\partial z_1 / \partial m_{h1}) \). Expanding and comparing with (3.8) shows that this condition is equivalent to \( \bar{F}_{m1} \leq 0 \).

14 The sign of \( \bar{e}_{h1} + \bar{F}_{e1} \) is opposite the sign of \( -\zeta(\bar{m}_{h1}) + \bar{m}(\bar{m}_{h1}, \bar{m}_o) = g(\beta_1, \bar{m}_{h1}) + g(\beta_1, \bar{m}_{h1} - (1 + k)m_o) \); this is negative for \( m_o \) small.
Fig. 2. The period-1 dynamic material input solutions for the high type, $\bar{m}_{h1}$, as a function of $\nu$, for several values of $\delta$.

Fig. 3. The period-1 dynamic effort solutions for the high type, $\bar{e}_{h1}$, as a function of $\nu$, for several values of $\delta$. 
below by each of the dynamic solutions (lines (b) and (c)): the static solution is larger than
the dynamic one for low values of \( \nu \), reflecting increasingness of the anti-hoarding rent in
material input (\( \hat{I}_{mh1} > 0 \)), and conversely for higher values of \( \nu \).

Comparing the dynamic solutions to the efficient solution (a horizontal line through \( m^*_h \))
the dynamic solutions are typically larger than the efficient solution for low values of \( \nu \) but
may be smaller for a high discount factor and a high value of \( \nu \) (line (c)). The larger is \( \nu \) the
more important is bad behavior by the low type and the less important is inefficiency of the
high type; further, the larger is \( \delta \) the larger is the low type’s incentive to hoard. Thus, the
planner increasingly relies on reductions in material input to lessen the anti-hoarding rent
by reducing the size of the ‘hoarding budget’ available to the low type: \( \hat{m}_{h1} \) falls below \( m^*_h \)
for high enough \( \nu \) and \( \delta \).

A number of other features are brought out by the numerical solutions. First, despite the
last remark the impact of an increase in the firm’s discount factor on the dynamic input
solution may not be monotone. Since line (b) cuts line (c) from above a given increase in
the discount factor can result in a reduction in material input for lower values of \( \nu \) and in an
increase for higher values — the ‘budget restriction’ intuition is not the only force at work.

Second, the critical value \( \tilde{\nu} \) is increasing in the discount factor. Since this critical value
equals the static value \( \bar{\nu} \) when \( \delta \leq \delta_0(\nu) \) it follows that the dynamic value with hoarding is
always larger than the static one: \( \tilde{\nu} > \bar{\nu} \). This observations cannot be proved or disproved
in the general case.

Finally, although not illustrated the numerical solutions show that both the level of hoard-
ing \( \tilde{m}_o \) and the proportion of the high type’s input assignment hoarded by the low type
\( \tilde{m}_{o}/\tilde{m}_{h1} \) are increasing in \( \nu \) and \( \delta \). Hoarding does not actually occur in the solution of
course, because the low type is indifferent between misrepresentation with hoarding and
accepting the anti-hoarding rent as a truth-teller.

The numerical solutions with respect to effort presented in Fig. 3 are less complex.
In particular, for \( \nu < \tilde{\nu} \) the dynamic effort assignment increases with \( \delta \). Intuitively, the
anti-hoarding rent is reduced by increasing effort above its static value. The larger is \( \delta \) the
larger is the anti-hoarding rent that must be paid ceteris paribus and hence the larger is the
increase in effort that is chosen to offset this rent. While the increase in effort to reduce the
anti-hoarding rent runs counter to the reduction in effort needed to reduce the information
rent, the sum of these two effects is never enough to push \( \bar{e}_{h1} \) above \( e^* \).

5. Concluding remarks

The analysis presented in this paper has established a number of points that are of
relevance to an understanding of dysfunction in the Soviet economy in particular, but
also to weakness in the static incentive model in more general circumstances. When a
material input is used in production it introduces the possibility for firm managers or
organizational subordinates in general to use this input as a storage device to transfer effort savings between periods. This intertemporal transfer destroys the incentive compatibility of static plan assignments and necessitates the payment of additional rent and the introduction of additional deviations from efficiency, in particular for the more productive type, to achieve dynamic incentive compatibility. This result depends on the cost of hoard-
ing to subordinates and their discount factor, which in turn reflect wider features of the environment.

The suggestion, common in the literature on Soviet planning, that the optimal response by the planner to the problem of input hoarding by subordinates is the imposition of extra-taut plans receives some support but is subject to important qualifications. With respect to the more productive type, the planner’s best response is to loosen rather than tighten second period plans. This reduces the first period incentive to hoard.

The less productive type will receive a tighter first period effort plan than in the static case, though looser than full efficiency would dictate. On the other hand, it will receive a material input plan that is tighter than the static optimum (and even lower than the efficient) for a range of both low and high values of the fraction of low types in the population. Ironically this tautness is visited on the less productive high type although it is the more productive low type that is the source of the incentive problem. Of course, the reward received by the high type is adjusted so that it always receives the reservation utility independently of the tautness of its plan.

The analysis has been carried out under the maintained assumption that there is no ratchet effect: the principal is committed to not revising the incentive scheme at the end of period 1. While this has allowed presentation of a relatively simple model in which hoarding per se is highlighted the assumption is unsatisfactory for two main reasons. First, as an empirical matter the ratchet phenomenon is widespread in organizations. Litwack (1991), for example emphasizes its importance in the broader context of the difficulty of securing global reform in Soviet economic institutions. Given this it is of great interest to understand how the hoarding incentives interact with dynamic plan revisions. With plan revision at the end of period 1 the supervisor can ameliorate period 1 hoarding by tautening the period 2 material input assignments. Plan revision is thus useful to the planner in dealing with the problem of hoarding. On the other hand, plan revision is dysfunctional because it introduces the ratchet effect. The literature typically emphasizes this negative consequence of plan revision. It might be, however, that the anti-hoarding benefits of non-commitment could outweigh its ratchet costs, making plan revision organizationally desirable.

Second, and related to this, the prediction of the present model is that no hoarding will be observed in equilibrium. Anticipating its occurrence the principal adjusts the incentive scheme in such a way as to preclude it happening. Yet as noted, hoarding is often observed in practice. It is desirable to have a model in which hoarding is an equilibrium outcome. In models with plan revision the more productive type will typically randomize in period 1 over whether it pools with the less productive type or separates from it. It seems plausible that when material input planning and hoarding are introduced to such a model the first period randomization by the productive type will persist and actual hoarding can be expected to occur whenever this type chooses to pool, since that will be a key motivation for such pooling.

References
