The Economics of Factoring Accounts Receivable

Ben J. Sopranzetti*

A moral hazard problem develops when a factor cannot contract upon a seller’s ex-post level of credit management. Because of the deleterious price impact of the moral hazard problem, sellers with a sufficiently high bankruptcy risk may be unable to factor their entire accounts receivable pool, even though they offer recourse. The structure of the equilibrium factoring contract is empirically tested using new factoring-specific data. It will be found that the credit quality of the seller’s accounts receivable pool and the seller’s probability of bankruptcy both have a negative impact upon the seller’s propensity to factor with recourse. © 1998 Elsevier Science Inc.

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I. Introduction

Factoring companies are financial institutions which specialize in the business of accounts receivable management. If a factor chooses to purchase a firm’s receivables, then it will pay the firm a pre-negotiated discounted amount of the face value of the invoices. Several recent papers have attempted to examine the motivations behind a firm’s decision to factor its accounts receivable: Mian and Smith (1992, 1994) provided a comprehensive empirical examination of several cross-sectional explanations of receivables policy determinants (with a special emphasis placed upon the formation of captive subsidiaries.) However, because of a scarcity of factoring observations, their tests provided only weak evidence with respect to the variables that motivate a firm’s decision to factor its accounts receivable. Sopranzetti (1997b) formally argued that the sale of accounts receivable can

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mitigate the underinvestment problem; Smith and Schnucker (1994) took a different approach and derived testable implications about factoring from the industrial organization literature, which focuses on vertical integration and transaction costs. They found that factoring is expected when the seller has very sparse specialized investment in its customers and when the seller’s cost of monitoring is high, i.e., firms factor their accounts receivable in order to better manage their credit risk. If Smith and Schnucker (1994) are correct and factoring is indeed used to manage credit risk, then why is it that firms are only able to factor their highest quality receivables? 

Although factoring companies are in business to purchase receivables, there is a systematic tendency for factors to restrict their without recourse purchases to a seller’s highest quality (or least risky) receivables. Why isn’t there some sufficiently low market clearing price such that any receivable, regardless of credit quality, could be purchased without recourse? Stiglitz and Weiss (1981) demonstrated that, in the presence of adverse selection or moral hazard, loan markets can break down so that a market clearing price may not exist. Like Stiglitz and Weiss (1981), this paper also examines the effect of agency problems on market prices; however, here the focus is on the moral hazard problem and its impact on the factoring of accounts receivable. We analyze the impact of the moral hazard problem on the factoring market by deriving and then testing the structure of the optimal factoring contract. The contract is derived in a one-period model which extends the technology that was originally developed in the loan sales literature, e.g., Pennacchi (1988) and Gorton and Pennacchi (1995). This paper differs from that literature by: 1) permitting both the seller and the factor to monitor the credit quality of the underlying receivable, and 2) by utilizing a new factoring-specific data set to test its empirical implications.

A moral hazard problem develops when the seller’s credit management efforts are unobservable to the factor: once the entire receivable is sold (factored), the seller has no incentive to monitor, as it no longer bears any credit risk. A rational factor would anticipate the suboptimal monitoring effort and would reflect this belief in the equilibrium price that it offers. The model demonstrates that under certain conditions, the moral hazard problem will not affect the seller’s decision to factor its accounts receivable. In this case, 

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2 In addition to the aforementioned papers, Papadimitriou et al. (1994) provided a comprehensive discussion of the history of factoring, the factoring industry, and the differences between factoring and commercial bank lending. They also explained how niche and community-based factors can help alleviate credit rationing in depressed communities. With regard to the broader category of asset sales, Hite et al. (1987) claimed that the principal reason that firms choose to sell their fixed assets is to promote operating efficiencies. John and Ofek (1995) found that although firms which sold their assets experienced an improvement in their operating performance, the improvement, if any, came mostly from an increase in focus. Lang et al. (1995) offered conflicting evidence. They argued that firms sell their assets because asset sales provide the cheapest source of funding rather than because of improvements in operating efficiencies alone. And other papers, such as Shleifer and Vishny (1992), Brown et al. (1994), and Sopranzetti (1995a) have shown that firms sell their assets to generate liquidity when they are in financial distress.

3 If a receivable is sold without recourse, then the seller is not liable for any default on the part of the customer. On the other hand, if a receivable is sold with recourse, then the seller may be responsible for a portion or even all of the uncollected amount, depending upon the terms of the factoring agreement. The recourse guarantee is, in essence, a put option. The factor will be able to put the delinquent receivable back to the firm if the realized payoff is less than the promised amount.

4 See Papadimitriou et al. (1994) and, especially, Naitove (1969) for a discussion of the relevance of the moral hazard problem on the factoring decision. Adverse selection may also be an important consideration in the design of factoring contracts. As pointed out by an anonymous referee, the existence of an adverse selection problem might easily explain the factor’s preference for high quality receivables; however, it is our intent to demonstrate that the three-tiered structure of the factoring contract will exist even in the absence of adverse selection.
the equilibrium will be characterized by the seller factoring all of its accounts receivable without recourse and performing zero credit monitoring. However, if the conditions do not hold, then a seller with a sufficiently low probability of bankruptcy will sell its highest credit quality receivables without recourse, but must resort to the use of recourse in order to sell both its intermediate and poor credit quality receivables. The equilibrium will be slightly different for sellers which have a high probability of bankruptcy. A seller in this category will still sell its highest credit quality receivables without recourse, but will be able to sell only its intermediate quality receivables through the use of recourse. Because of the deleterious impact of the moral hazard problem on the equilibrium price offered by the factor, even the promise of recourse will be insufficient to motivate the sale of the seller’s lowest quality receivables.

The paper is organized as follows. Section II outlines the assumptions and motivates the model. Section III models the seller’s decision when it keeps its accounts receivable. Section IV models the seller’s decision to factor its accounts receivable. Section V provides a discussion of the data set, testable implications, and empirical methodology. Section VI presents the empirical results, and Section VII concludes the paper.

II. The Assumptions and A Generalized Form of the Seller’s Profit Function

Assume that a risk neutral seller has an order on its books to manufacture a product (or to provide a service) which it will sell for a promised payment of \( S_L \). The cost to manufacture the good is \( L \), which the seller must pay at time \( t = 0 \). The seller finances its initial investment at its internal cost of capital, \( r_i \). Assume that the manufacturing process is instantaneous and that the seller extends trade credit to its customers. \(^5\) Thus, at time \( t = 0^+ \), the seller delivers the finished good, but the customer need not pay for the good until the end of the credit extension period, which is assumed to be \( \tau \) periods.

The seller has the capability of credit monitoring its customer at a level \( c \), where \( c \in [0, \infty) \). Credit monitoring is defined to be the ongoing process of expediting payment on an outstanding invoice so that the invoice will be paid off promptly. The seller’s cost of credit monitoring is given by \( u \times c \), where \( u \) is some positive constant. The cost is incurred at the end of the period (time \( t = \tau \)). A greater \( c \) implies that the seller is exerting more expediting pressure on its customer.

The seller knows the payoff distribution of the trade receivable. The random variable \( \tilde{x} \), the realized payment by the customer, is assumed to have a probability distribution which is an increasing function of the level of credit monitoring and the receivable’s credit quality, \( \alpha \). \( \alpha \in [1, \infty) \), and represents the reciprocal of the perceived a priori probability that the customer will not pay off the receivable within the allocated credit extension period. Thus, \( \alpha \) provides an indication of the customer’s creditworthiness: the greater the \( \alpha \), the greater the likelihood that the receivable will be paid off in full.

It is further assumed that the distribution for the payoff satisfies a strict convexity of distribution function constraint with respect to the level of credit monitoring, \( c \), and the level of receivable credit quality, \( \alpha \). Thus, the distribution function, \( F \), has the following property:

\(^5\) See Petersen and Rajan (1997) for a detailed discussion and empirical test of the existing theories which attempt to explain the existence of trade credit.
\[ F(x|\alpha, \lambda c + (1 - \lambda)c') < \lambda F(x|\alpha, c) + (1 - \lambda) F(x|\alpha, c') \]

and

\[ F(x|\lambda \alpha + (1 - \lambda)\alpha, c) < \lambda F(x|\alpha, c) + (1 - \lambda) F(x|\alpha', c). \]

The strict convexity of distribution function constraint implies that the expected payoff will be increasing and **strictly concave** in both \( \alpha \) and \( c \).\(^6\) If the seller increases its monitoring/expediting efforts (as \( c \) increases) or the customer’s perceived creditworthiness increases (as \( c \) increases), the probability that a given receivable will be paid promptly increases. In turn, as the probability that the receivable will be paid off in a timely fashion increases, so does the receivable’s expected value. Note that the strict concavity of the expected payoff with regard to \( \alpha \) and \( c \) implies that there will be strictly decreasing returns to scale in both the receivable’s credit quality and in the seller’s level of monitoring.

At time \( t = 0^+ \), the seller may sell a fraction \( b \) of its receivable to a risk neutral factor, who will pay the seller a discounted amount of the receivable’s face value.\(^7\) Assume that the market for the purchase of receivables is competitive. Both the seller and the factor can costlessly observe \( \alpha \), the receivable’s credit quality, at time \( t = 0 \). The factor also knows the receivable’s payoff distribution, but is not able to accurately perceive what the seller’s level of credit monitoring, \( c \), will be. In some cases, the seller may choose to sell its receivable with recourse. Let \( \zeta \) be an indicator function that equals 1 if the seller offers recourse, and 0 otherwise. It is assumed that if the seller offers recourse, then it must offer full recourse (i.e., in the event that the seller’s customer defaults on the receivable, the seller will be liable for the *entire* delinquent amount).\(^8\)

Sopranzetti (1997a) argued that the seller may have a higher cost of internal funding than the factor because of tax incentives, underinvestment, or costs related to financial distress, so we assume that the factor has a cost of internal funding equal to \( r_f \) which is less than \( r_i \).\(^9\)

The factor undertakes a given level of monitoring, \( m \), where \( m \in [0, \infty) \) and is determined optimally by the factor given his cost and efficiency parameters. The cost of monitoring will be \( w \) times \( m \), where \( w \) is some positive constant that will be paid at the end of the period. The level of monitoring, \( m \), represents the amount of effort that the factor expends on expediting prompt payment by the customer on the outstanding receivable. It is assumed that once the factor commits to a given level of monitoring, it will perform as promised, and that the factor must pay the seller for the receivable at the beginning of the period. The expected payoff when the factor monitors is \( \int_0^\infty x dF(x|\alpha, c, m) \). Lastly, the factor is able to accurately and costlessly observe \( p \), the seller’s probability of solvency.

The model, as presented above, yields essentially no closed-form, general results; so, in order to obtain useful results, it will be necessary to invoke a specific functional form for the receivable’s expected payoff. Our chosen functional form for the expected payoff is \( \int_0^\infty x dF(x|\alpha, c, m) = L(1 - 1/\alpha e^{-\beta (1 - \zeta - bc - \gamma m)}). \) Although the paper’s theoretical findings depend crucially upon the chosen functional form, this representation for the receivable’s

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\(^6\) See Hart and Holmström (1987) for a proof of this claim.

\(^7\) The factor pays the seller at time \( t = 0^+ \), when the receivable is actually sold.

\(^8\) The case of partial recourse can be handled by allowing \( \zeta \in [0, 1] \).

\(^9\) See Section V.B for empirical justification for this claim.
expected payoff seems sensible, because as the credit quality of the receivable increases (as \( \alpha \) increases), the expected payoff converges to \( L \) (the face value) regardless of how much credit monitoring is done by the seller or the factor. Also, if zero credit monitoring is performed by both the seller and the factor (\( c = 0 \) and \( m = 0 \)), then the expected payoff will be bounded below by \( L(1 - 1/\alpha) \). However, as the level of the seller’s or the factor’s credit monitoring effort increases, then the expected payoff on the receivable converges at the rate \( \beta \) or \( \delta \), respectively, to the promised payment, \( L \).\footnote{The monitoring efficacy parameter, \( \beta \), is a measure of the marginal increase in the receivable’s expected return from additional credit monitoring.} Also, as \( c \in [0, \infty) \) and \( m \in [0, \infty) \), it is necessary to constrain the parameters such that \( L\beta > u \) and \( L\delta > w \).

### The Generalized Form of the Seller’s Expected Profit Function

The general form of the seller’s expected profit function is:

\[
e^{-rT} \int_0^L \left[ (1 - b)p x - \xi bp(L - x) \right] dF(x|\alpha, c, m) - e^{-rT}upc - 1
\]

\[
+ e^{-rT}\beta \left\{ \int_0^L \left[ x + \xi p(L - x) \right] dF(x|\alpha, c, m) - w \right\},
\]

where the first term is the seller’s expected payoff from keeping a fraction \( (1 - b) \) of the receivable minus the seller’s expected liability due to the recourse guarantee; the second term is the seller’s monitoring cost; the third term is the initial investment; and the last term is the price that is paid to the seller by the factor.

### III. The Seller’s Decision To Keep Its Accounts Receivable

When the seller keeps its accounts receivable, then \( b \) in equation (1) is identically equal to zero. In this case, the seller’s task is to choose a level of credit monitoring such that the present value of its expected profit from keeping the receivable is maximized. By substituting \( b = 0 \) into equation (1), the seller’s problem becomes:

\[
\max_{c > 0} \int_0^L x dF_c(x|\alpha, c) - uc = 1.
\]

The first order condition is:

\[
\int_0^L x dF_c(x|\alpha, c) = u.
\]

Using the specific functional form for \( \int_0^L x dF(x|\alpha, c) \), the first order condition can be rewritten as:
\[ \frac{L \beta e^{-\beta c}}{\alpha} = u, \]  
\[ c^k = \max \left\{ \frac{1}{\beta} \ln \frac{L \beta}{u \alpha}, 0 \right\}. \]

IV. The Seller’s Decision To Factor Its Accounts Receivable

This section focuses on modeling the optimal contract between the seller and a factor. Because factoring contracts usually involve the sale of the entire receivable, \( b \) will be identically equal to 1. The model will demonstrate that, similar to Stiglitz and Weiss (1981), the existence of a moral hazard problem on the part of the seller plays an active role in causing lower quality receivables to remain unsold or to be sold with recourse.

The Seller’s Decision to Factor the Receivable Without Recourse

If the seller factors its receivable without recourse, then \( b \) is equal to 1 and \( \zeta \) is equal to 0. Because the seller’s level of credit monitoring is unobservable to the factor, a moral hazard problem can develop.

Lemma 1.

If the seller’s level of credit monitoring is unobservable to the factor, then the seller will monitor at a level \( c = 0.11 \)

Proof: See the Appendix.

Intuitively, it is easy to see that in equilibrium, the seller’s level of credit monitoring will be zero. Once the entire receivable is sold, the seller no longer bears the consequences of inefficient monitoring. Because the factor cannot ex-post verify the seller’s level of credit management effort, once the seller receives the contracted payment from the factor, it will not derive any marginal benefit from further credit monitoring. A rational factor will expect the zero level of monitoring by the seller and will, consequently, offer a price which reflects this knowledge. Thus the seller’s expected profit when it factors its receivable without recourse will be:

\[ e^{-rT} \left[ \int_0^L x dF(x|\alpha, 0, m) - wm \right] - 1. \]  

In the above analysis, the factor pays the seller at time zero and will select a level of credit management, \( m(\alpha) \), such that it solves \( \max_{m(\alpha)} \int_0^L x dF(x|\alpha, 0, m) - wm \).

\[ 11 \text{ It is important to note that Lemma 1 depends crucially upon the model’s assumption of a one-period, no-reputation contract. As noted in John and Nachman (1985), the moral hazard problem may be partially mitigated in multi-period models which include a reputation feature.} \]
Proposition 1.

If

\[
e^{-rt}p \left[ L \left( 1 - \frac{u}{L\beta} \right) - \frac{u}{\beta} \ln \frac{L\beta}{u} \right] - e^{-rt} \left[ L \left( 1 - \frac{w}{L\delta} \right) - \frac{w}{\delta} \ln \frac{L\delta}{w} \right] < 0,
\]

then the seller will factor all of its accounts receivable without recourse.

Proof: See the Appendix.

The inequality in Proposition 1 is derived by comparing the seller’s profit when it factors a given receivable without recourse to its profit when it keeps the receivable. When the parameterization is such that the inequality is satisfied, then for any given value of receivable credit quality, \( a \), the seller’s profit will be greater when it factors the account receivable than when it keeps the receivable. Thus the seller will prefer to factor all of its accounts receivable regardless of their credit quality.

Proposition 1 demonstrates that one possible motivation why firms choose to factor their accounts receivable may be to take advantage of the factor’s superior information technology. This reason was suggested by Mian and Smith (1992) and empirically tested by Smith and Schnucker (1994). According to these two studies, firms which do not have a customer specific sunk cost in information will have a higher propensity to factor their accounts receivable. The intuition behind Proposition 1 is that if the factor’s information advantage is sufficiently large—in terms of the model’s parameters, if \( w \) is much smaller than \( u \) and/or if \( \delta \) is much larger than \( \beta \)—then the price impact of the moral hazard problem on the seller’s decision to factor its accounts receivable will be negligible. Consequently, if the inequality is satisfied, then the seller’s profit will always be higher when it sells all of its accounts receivable and transfers the responsibility (and the credit risk) of monitoring the receivables to the factor, rather than retaining the receivables and credit monitoring itself.

Now, assume that the inequality is not satisfied. Then for some value of \( a \), the seller’s expected profit from factoring its receivables must equal the expected profit from keeping the receivables.\(^{12}\) Let \( a_n \) be the receivable quality level such that

\[
\int_0^L xDF (x|\alpha_n, c = 0, m) - \int_0^L xDF (x|\alpha_n, c_n^k, m = 0) = e^{-rt} \left[ L \left( 1 - \frac{w}{L\delta} \right) - \frac{w}{\delta} \ln \frac{L\delta}{w} \right] \]

Note that the left hand side is the expected equilibrium payoff to the seller from factoring the receivable, and the right hand side is the expected equilibrium payoff from keeping the receivable.

\(^{12}\) In other words, the two expected profit functions must intersect for some value of \( a \). The existence and uniqueness of the intersection point will be demonstrated in the proof of Proposition 2.
Proposition 2. If the inequality in Proposition 1 is not satisfied, then there exists a unique level of receivable credit quality, \( \alpha_n \), such that for \( \alpha > \alpha_n \) the seller will factor the receivable without recourse and perform zero credit monitoring, and for \( \alpha \leq \alpha_n \) the seller will keep the receivable and credit monitor at a level \( c = c^k \).

Proof: See the Appendix.

The impact of the moral hazard problem on the price will be least severe for high credit quality receivables, as their expected payoffs will not depend greatly on the seller’s monitoring efforts. On the other hand, the moral hazard problem will have a larger deleterious impact on the expected payoff to the intermediate and poor quality receivables; consequently, the lower price offered for these receivables will be insufficient to motivate the seller to sell. Thus, in equilibrium, the seller will keep its intermediate and poor credit quality receivables.

Proposition 3. The breakpoint level of receivable credit quality such that the receivable will be factored without recourse, \( \alpha_n \), will be: decreasing in the seller’s cost of internal funding, \( r_i \); increasing in the factor’s cost of funding, \( r_f \); increasing in the seller’s monitoring efficacy parameter, \( \beta \); decreasing in the factor’s monitoring efficacy parameter, \( \delta \); and increasing in the factor’s cost of monitoring, \( w \).

Proof: The proof consists of taking the derivative of \( \alpha_n \) with respect to \( r_i, r_f, \beta, \delta, \) and \( w \).

As the seller’s cost of internal financing, \( r_i \), increases (decreases) relative to the factor’s financing cost, \( r_f \), the seller will find it relatively more (less) expensive to internally finance its trade credit. Sopranzetti (1997a) argued that one of the possible motivations for why a seller might choose to sell its accounts receivable is that the factor may be able to more cheaply fund the trade credit if its cost of internal funding is less than the seller’s cost of internal funding. If this is true, then one would expect the comparative static result reported in Proposition 3: as the seller’s internal cost of capital increases (decreases) relative to that of the factor, then the firm’s propensity to factor its accounts receivable would also increase (decrease) and, consequently, \( \alpha_n \) would decrease (increase).

As the firm’s monitoring efficacy parameter, \( \beta \), increases (decreases) relative to the factor’s efficacy parameter, \( \delta \), the firm will have a lesser (greater) incentive to factor its accounts receivable, consequently, \( \alpha_n \) will increase (decrease). This result is consistent with the findings of Smith and Schnucker (1994) who empirically demonstrated that firms have a greater tendency to internalize the credit management function (and hence to not factor their accounts receivable) when they have a specific sunk investment in the customers/vendor relationship—such firms would have a larger relative spread between \( \beta \) and \( \delta \).

As the factor’s cost of monitoring, \( w \), increases, the committed level of monitoring by the factor will decrease and with it the equilibrium price which will be offered for the receivable. The result will be a decrease in the proportion of the seller’s receivable pool that will be sold, i.e., an increase in \( \alpha_n \). Q.E.D.

\[13\] Unfortunately, the comparative static result with respect to the seller’s cost of monitoring, \( u \), on the break-point level of receivable credit-quality \( \alpha_n \), is ambiguous: the derivative \( \partial \alpha_n / \partial u \) cannot be signed.
The Seller’s Decision to Factor the Receivable With Recourse

In this section, the seller is permitted to offer recourse if it wishes to do so; thus, in the event that the receivable’s actual payoff is less than its face value, the seller must reimburse the factor for the delinquent amount. The value of this recourse guarantee is contingent upon the seller’s solvency at the end of the period. As the seller is made contingently liable when the receivables are sold with recourse and, thus, bears some of the credit risk, it is reasonable to expect that the seller will choose some positive level of credit monitoring in equilibrium.

Substituting $b = 1$ and $\zeta = 1$ into equation (1) yields the seller’s profit function when it factors with recourse. Thus the seller’s equilibrium level of credit monitoring, $c_r$, will solve:

$$\begin{align*}
\text{Max}_{c \geq 0} & \quad e^{-rT} \int_0^L x F(x|\alpha, c, m) + e^{-rT} \int_0^L [p(L-x)] dF(x|\alpha, c, m) - e^{-rT}upc \\
& - e^{-rT} \int_0^L [p(L-x)] dF(x|\alpha, c, m) - e^{-rT}wm - 1. \quad (8)
\end{align*}$$

Equation (8) can be rewritten as:

$$\begin{align*}
\text{Max}_{c \geq 0} & \quad e^{-rT} \int_0^L x F(x|\alpha, c, m) + (e^{-rT} - e^{-rT}) \int_0^L [p(L-x)] dF(x|\alpha, c, m) \\
& - upc - e^{-rT}wm - 1. \quad (9)
\end{align*}$$

The first order condition is:

$$e^{rT} \left[ \left( \frac{1}{p} - 1 \right) e^{-rT} + e^{-rT} \right] \int_0^L x F(x|\alpha, c’, m) = u. \quad (10)$$

**Proposition 4.** There exist a value, $p^*$, and levels of receivable credit quality, $\alpha_{\text{keep}}$ and $\alpha_{\text{recourse}}$, such that when the seller’s probability of solvency $p$ is less than $p^*$, the seller will keep receivables for which $\alpha < \alpha_{\text{keep}}$, sell with recourse receivables for which $\alpha_{\text{keep}} \leq \alpha < \alpha_{\text{recourse}}$, and sell without recourse receivables for which $\alpha \geq \alpha_{\text{recourse}}$.

Proof: See Appendix.

**Corollary to Proposition 4.** When $p \geq p^*$, then the seller will sell with recourse receivables for which $\alpha < \alpha_{\text{recourse}}$, and sell without recourse receivables for which $\alpha \geq \alpha_{\text{recourse}}$.

Proof: See the Appendix.

The implications of Proposition 4 and its corollary are very interesting. Because of the moral hazard problem inherent when the seller sells its accounts receivable, only the highest quality receivables will be factored without recourse. For receivables the credit quality of which is above $\alpha_{\text{recourse}}$ (i.e., the highest quality receivables), the seller’s expected profit will be maximized when it sells them without recourse, because for these receivables the price impact of the moral hazard problem is not very significant. The credit quality of these receivables is excellent, so even if the seller were to perform zero credit...
management, there would still be a high probability that the factor would collect the receivable’s full face value at the maturity date.

Because of the larger impact of the moral hazard problem on the expected payoff to the intermediate and poor quality receivables, sellers must offer recourse in order to sell them. As is delineated in the corollary to Proposition 4, sellers which have a high likelihood of being able to satisfy their recourse guarantee (i.e., sellers with \( p \geq p^* \)) will be able to sell all of their intermediate and poor quality accounts receivable through the use of recourse. On the other hand, Proposition 4 demonstrates that sellers who have a low probability of being able to satisfy their recourse guarantee (i.e., sellers with \( p < p^* \)) will factor their intermediate credit quality receivables with recourse, but will keep their poor quality receivables because the equilibrium price offered by the factor for these receivables will be too low to motivate the sellers to sell.

V. Data and Empirical Methodology

Data

The model’s testable implications were examined using a new data set which combines the National Automated Accounting Research System database (NAARS) with the COMPUSTAT and CRSP databases. NAARS is a compilation of the annual reports to shareholders of over 4,000 publicly-traded firms from 1972–1993. The database is offered jointly by the American Institute of Certified Public Accountants (AICPA) and Mead Data Central, Inc. The NAARS database was searched for footnotes to the annual reports which indicate whether the firm factored any of its accounts receivable. The character string “factor” was applied as a preliminary screening filter. When the set of initial candidates was located, the annual reports were analyzed, and candidates which had superfluous occurrences of the character string were discarded. Data extracted from the surviving candidates’ annual reports include the name of the firm, the date of the annual report, the SIC code, whether or not the firms factored with recourse, the amount factored, and (when available) the terms and conditions of the factoring agreement. In order for a candidate to be considered as a recourse firm, specific mention of the existence of the use of recourse or of a contingent liability on the firm’s behalf must have been made in the annual report.

The data set is comprised of 269 observations distributed over 98 unique, publicly-traded firms. Table 1 provides a breakdown of the firms in the sample. Of the 98 firms, 26 manifested evidence of factoring with recourse and were consequently classified as recourse firms, while the other 72 were classified as non-recourse firms.\(^{14}\) The largest proportion (47%) of firms in the sample are involved in the textile and apparel manufacturing industries: industries which historically have made heavy use of factors’ services.\(^{15}\) Figure 1 presents a graphical depiction of the number of factoring observations per year, classified by whether or not it was a recourse observation. Interestingly, there was a drop in the reported number of factoring instances in the late Seventies. It is difficult to predict whether or not the decrease is attributable to a drop in the amount of factoring which actually occurred or, instead, to lax reporting requirements.\(^{16}\)

\(^{14}\) There were no instances where with-recourse firms later became without-recourse firms, or vice-versa.

\(^{15}\) The use of factoring in the apparel industry was common in Colonial American times.

\(^{16}\) See Mian and Smith (1992) for a detailed explanation of why the use of factoring is systematically understated.
Of the 98 firms in our sample, 41 appear only once, 16 appear twice, 11 appear three times, 11 appear four times, 10 appear five times, and 9 (all are apparel manufacturers) appear more than five times. Also, during the period under consideration, of the 98 firms in our sample, 8 firms were acquired and 13 went bankrupt. COMPUSTAT and CRSP data were available for approximately 65% of the firms in the sample. The balance of the data was hand collected from both the ISL bound copies of daily stock prices and from the NAARS annual reports.

### Table 1. Firms Which Factor Their Receivables Sorted by Industry Classification

<table>
<thead>
<tr>
<th>Industry Classification</th>
<th>Number of Firms that Factored</th>
<th>Number of Firms that Factored with Recourse</th>
<th>Number of Firms that Factored Without Recourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Textile</td>
<td>26</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>Apparel Mfg.</td>
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<td>5</td>
<td>16</td>
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<td>Wood Products</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Furniture &amp; Fixtures</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Chemicals</td>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Petroleum &amp; Coal</td>
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<td>1</td>
</tr>
<tr>
<td>Plastics</td>
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<td>1</td>
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<tr>
<td>Footwear</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Metal Products</td>
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<td>3</td>
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<tr>
<td>Machinery</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Electronics</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Medical Instruments</td>
<td>7</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Sundry Items/Games</td>
<td>7</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Trucking/Storage</td>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Radio, TV, Cable</td>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Retail Stores</td>
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<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Business Services</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Hospitals/Medical</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Service Industries</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Dry Goods</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Totals</td>
<td>98</td>
<td>26</td>
<td>72</td>
</tr>
</tbody>
</table>

Descriptive statistics of the data set sorted by industry classification and by whether or not the firm factored its receivables with recourse over the period 1972–1993. The sample consists of 269 pooled cross-sectional time-series observations on a total of 98 publicly-traded firms which factored their accounts receivable. The sample firms were identified by examining the footnotes of their annual reports (which were located on NEXUS) for the existence of the character string “factor.” The annual reports were then examined to determine whether or not the firms factored their receivables with recourse.

Of the 98 firms in our sample, 41 appear only once, 16 appear twice, 11 appear three times, 11 appear four times, 10 appear five times, and 9 (all are apparel manufactures) appear more than five times. Also, during the period under consideration, of the 98 firms in our sample, 8 firms were acquired and 13 went bankrupt. COMPUSTAT and CRSP data were available for approximately 65% of the firms in the sample. The balance of the data was hand collected from both the ISL bound copies of daily stock prices and from the NAARS annual reports.

### Testing the Structure of the Optimal Factoring Contract

The empirical test of the structure of the optimal factoring contract focused on answering the following question: given that a particular firm factors its accounts receivable, what are the determinants which influence its decision to do so with recourse? In this case, the decision variable was binary: either the seller factored with recourse or it did not. It was assumed that there is some latent variable, $y_t$, that represents the firm’s decision to factor with recourse. Assuming a given receivable credit quality-class, if $y_t < y_{r_t}^*$ then the firm factored without recourse, and if $y_t \geq y_{r_t}^*$ then the firm factored with recourse. Because $y_t$ is unobservable, we created a dummy variable called $RECOURSE$ which equals 1 if the
Figure 1. Factoring observations sorted by year and by type. A graphical depiction of the number of factoring observations per year, classified by whether or not the observation represents factoring with or without recourse. The vertical axis represents the number of observations in any given year. The horizontal axis represents the year that the factoring occurred. The light bars represent observations of factoring with recourse, and the dark bars represent factoring without recourse. The sample consists of 269 pooled cross-sectional time-series observations on a total of 98 publicly-traded firms which factored their accounts receivable over the period 1972–1993. The sample firms were identified by examining the footnotes of their annual reports (which were located on NEXUS) for the existence of the character string “factor.” The annual reports were then examined to determine whether or not the firms factored their receivables with recourse.
firm used recourse in its factoring agreements, and 0 otherwise. Probit analysis was employed to assess the power of the independent variables in explaining the structure of the equilibrium factoring contract. Our empirical model is given by:

\[ y_t = a_0 + a_1DREC_t + a_2PBANK_t + a_3TEXAPP_t + a_4FUNDCOST_t + \epsilon_t, \]

where \( DREC \) is a proxy for the credit quality of the firm’s account receivable pool; \( PBANK \) is a proxy for the firm’s probability of bankruptcy; \( TEXAPP \) is a dummy variable that equals 1 if the firm is in either the textile or apparel manufacturing industry, and \( FUNDCOST \) is a proxy for the relative spread between the firm and the factor’s cost of funds.

The contract delineated in Proposition 4 predicts that the use of recourse will allow the seller to sell intermediate credit quality receivables which would not have been sold otherwise. Thus, one would expect that sellers who have a lower credit quality receivable pool or more credit risk exposure would have a greater propensity to factor their accounts receivable with recourse. Standardized lagged doubtful receivables (\( DREC \)) is a proxy for the credit quality of the seller’s receivable pool. \( DREC \) is calculated by dividing lagged doubtful accounts receivables by lagged total accounts receivable, where lagged total accounts receivable includes the amount due, if any, from the factor. Sellers who have a relatively high \( DREC \) will have a higher percentage of receivables which are believed to be uncollectable and therefore of lower credit quality. Thus, the seller’s level of \( DREC \) should be positively related to the seller’s propensity to factor with recourse.

Proposition 4 also predicts that as a seller’s probability of bankruptcy increases, its propensity to factor its receivables with recourse will decrease. As the probability of bankruptcy increases, the value of the recourse guarantee decreases, and the equilibrium price which will be offered by the buyer decreases accordingly. The lower price will cause the seller to keep some accounts receivable which it would have otherwise chosen to sell with recourse. The seller’s probability of bankruptcy (\( PBANK \)) is calculated by using a methodology (described in the Appendix) which is similar to that of Marcus and Shaked (1984) and Gorton and Pennacchi (1994), who employed an option pricing framework.

---

17 Although it might be more informative to use a continuous dependent variable, for example the proportion of receivables which are actually factored with recourse, unfortunately this data was only available for about 5% of the observations. However, for those observations where the proportion of receivables factored with recourse was available, the average proportion of accounts receivable factored with recourse was only 11%.

18 This method of distinguishing those firms which factor with recourse from those which do not might be prone to errors of omission. For example, some of the sellers who factor with recourse might routinely neglect to mention the use of recourse in the footnotes to their annual reports. This may be especially problematic, as the error arises only for the non-recourse observations. Thus, it might be correlated with the independent variables. Unfortunately, better quality data was unavailable in the annual reports. So, in order to validate the accuracy of our sample data, we telephoned a random subsample of twenty firms that were classified as non-recourse firms (that is, a random sample of those non-recourse firms which are still in business) and asked them whether or not they ever used recourse in their factoring arrangements. In only one case out of twenty did a non-recourse firm report that it had actually factored some of its receivables (around 5%) with recourse.

19 Doubtful receivables is a category on the firm’s balance sheet which reports the dollar value of accounts receivable that the firm believes will be uncollectable. By the time the annual report is compiled (and the footnotes created that describe whether or not the firm factors with recourse), the firm may have already factored some of its receivables which would have otherwise been reported as doubtful receivables. Thus, there may be a problem with endogeneity between recourse and doubtful receivables if contemporaneous doubtful receivables are used. To circumvent the endogeneity problem, we used lagged doubtful receivables.
first developed by Merton (1974). Again, as PBANK increases, Proposition 4 predicts that the seller’s propensity to factor its receivables with recourse will decrease.\textsuperscript{20}

As a disproportionately large percentage of firms in our sample (47%) are in either the textile or apparel manufacturing industry, we believed that it would be prudent to control against the possibility that these industries might be driving our empirical results. We included a dummy variable called TEXAPP which equals 1 if the seller is in either industry, and 0 otherwise.

Lastly, although the model does not make any explicit prediction with respect to the relationship between the firm’s propensity to factor with recourse and the relative cost of funds for the factor and the firm, it does make an explicit prediction with respect to factoring without recourse; thus it might be important to control for the possibility that the relative cost of funds may be a determinant of the firm’s propensity to factor with recourse.\textsuperscript{21} The proxy for the firm’s cost of funds is the firm’s weighted average cost of debt.\textsuperscript{22} The proxy for the factor’s cost of funds is determined by first obtaining the factor’s Moody’s bond rating and then finding the appropriate cost of debt associated with that particular bond rating. The relative cost of funds proxy, FUND\textsubscript{COST}, is then calculated by subtracting the factor’s cost of funds from the firm’s cost of funds. FUND\textsubscript{COST} is greater than 0 for every firm in our sample, so the model’s assumption that $r_i > r_f$ seems to be valid. The average spread between the costs of debt for the firms and factors in our sample (i.e., the mean value of FUND\textsubscript{COST}) is 3.54%.

### VI. Empirical Results

The results of the probit test for the determinants which motivated the seller to factor with recourse are presented in Table 2.\textsuperscript{23} The coefficient on the lagged receivable quality variable $DREC$ had a value of 2.32, and was statistically significant at the 5% level. The 2.32 coefficient on $DREC$ shows the impact of a one unit change in $DREC$ on the index, $y_t$, and not on the propensity to factor. However, the fact that the coefficient is positive implies that sellers with a higher percentage of poor quality receivables, i.e., sellers with a greater exposure to credit risk, will have a greater propensity to factor with recourse. The implication is consistent with the prediction made in Proposition 4, which states that the highest quality receivables can be sold without recourse while the intermediate quality receivables can be sold, but only through the use of recourse.

The coefficient on the probability of bankruptcy variable $PBANK$ was $-5.54$, and was statistically significant at the 5% level. That is, sellers with a high probability of bankruptcy will be less likely to factor with recourse, because their recourse guarantee will often be worthless. This empirical result provides strong support for the optimal contract delineated in Proposition 4 which implies that as the seller’s probability of solvency increases, so will its propensity to factor with recourse.

\textsuperscript{20} In order to check for robustness of the bankruptcy variable, the probit analysis was redone using an Altman-Nammacher (1987) $Z$-score statistic in the calculation of the bankruptcy proxy. The results were qualitatively identical.

\textsuperscript{21} We thank an anonymous referee for suggesting not only this possibility but also the relative cost of funds proxy.

\textsuperscript{22} The weighted average cost of debt is determined by using the information contained in the footnotes to the annual reports.

\textsuperscript{23} To test the sensitivity of the empirical result to outliers, we eliminated the extreme 5% of observations for each of the non-dummy independent variables. The empirical results remained qualitatively unchanged.
The coefficients on the two control variables, TEXAPP and FUNDCOST, were both negative (−0.82 and −0.11, respectively), although only the coefficient on TEXAPP was significantly different from zero at the 5% level. The negative and statistically significant TEXAPP coefficient indicates that firms which are employed in either the textile or apparel manufacturing industry have a lower propensity to factor their receivables with recourse. Lastly, the insignificant coefficient estimate for FUNDCOST indicates that, at least for the firms in our sample, the spread between the firm and the factor’s cost of funds was not a relevant consideration in the firms’ decisions to factor with recourse.

VII. Conclusion

Mian and Smith (1992) and Smith and Schnucker (1994) argued that firms factor their accounts receivable in order to better manage their exposure to credit risk. If so, why is it that factors typically only purchase a seller’s highest quality accounts receivables? This paper has shown that when the seller’s level of credit monitoring is unobservable to the factor, a rational factor will expect a moral hazard problem and will reflect this expectation in the equilibrium price they offer the seller. Sellers with a sufficiently high bankruptcy risk may be unable to factor their entire accounts receivable pool. Such sellers will factor their highest credit quality receivables without recourse, their intermediate quality receivables with recourse, and choose not to factor their lowest quality receivables. Empirical evidence obtained using a new factoring-specific data set supports the model.

This paper has analyzed the factoring decision in isolation from other financing possibilities. An interesting area of further research would be to examine factoring as an alternative to other forms of financing, including the securitization of receivables, bank or finance company asset-backed loans, and conventional loans that are secured by accounts receivable.

Table 2. Probit Analysis of the Firm’s Propensity to Factor with Recourse

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>−0.61*</td>
</tr>
<tr>
<td></td>
<td>(−3.02)</td>
</tr>
<tr>
<td>DREC</td>
<td>2.32*</td>
</tr>
<tr>
<td>Standardized doubtful receivables</td>
<td>(2.07)</td>
</tr>
<tr>
<td>PBANK</td>
<td>−5.54*</td>
</tr>
<tr>
<td>Probability of bankruptcy</td>
<td>(−1.98)</td>
</tr>
<tr>
<td>TEXAPP</td>
<td>−0.82*</td>
</tr>
<tr>
<td>Dummy for textile or apparel</td>
<td>(−3.03)</td>
</tr>
<tr>
<td>FUNDCOST</td>
<td>−0.11</td>
</tr>
<tr>
<td>Relative cost of funds</td>
<td>(−0.32)</td>
</tr>
<tr>
<td>Likelihood ratio test</td>
<td>27.9*</td>
</tr>
</tbody>
</table>

Estimates of the coefficients of a probit regression for the firm’s propensity to factor its accounts receivable using recourse based upon a pooled sample of 269 observations from 98 publicly-traded firms in the time interval 1972–1993. The sample firms were identified by examining the footnotes of their annual reports (which were located on NEXUS) for the existence of the character string “factor.” The annual reports were then examined to determine whether or not the firms factored their receivables with recourse. The independent variables are DREC, PBANK, TEXAPP, and FUNDCOST. DREC, standardized doubtful receivables, is a proxy for the credit quality of the firm’s account receivable pool, which was measured by dividing last period’s doubtful receivables by last period’s total receivables. PBANK is the firm’s probability of bankruptcy. TEXAPP is a dummy variable which equals 1 if the firm is in either the textile or apparel manufacturing industry. FUNDCOST is a proxy for the relative spread between the firm and the factor’s cost of funds. * statistics appear in parentheses.

* Significant at the 5% level.
Appendix

Proof of Lemma 1.
If the firm factors its accounts receivable without recourse, then $\xi = 0$. Substituting $\xi = 0$ into equation (1) yields the firm’s profit function. The firm’s problem is

$$\max_c e^{-r_f t} \int_0^L (1 - b) px dF(x|\alpha, c, m) - e^{-r_f t} upc - 1$$

$$+ e^{-r_f t} \int_0^L bx dF(x|\alpha, c, m) - e^{-r_f t}$$

such that

$$\int_0^L (1 - b) px dF_c(x|\alpha, c, m) = up$$

$c > 0$.

By doing the necessary Kuhn-Tucker algebra, it is straightforward to demonstrate that $[e^{-r_f t} \int_0^L px dF_c(x|\alpha, c, m) - up e^{-r_f t}]_c = 0$. The first term in the brackets is the marginal cost of one additional unit of monitoring, while the second term is the marginal benefit of an additional unit of monitoring. Because the factor is unable to ex-post verify the firm’s level of credit monitoring, the second term will equal zero. The reasoning is as follows: the factor cannot ex-post verify the firm’s level of credit monitoring effort, so once the contracted price has been agreed upon, the firm’s payoff from the factor will be independent of the firm’s level of credit monitoring. Thus the derivative of the expected payoff with respect to $c$ will be zero. Thus, in order for the Kuhn-Tucker condition to hold, $c$ must equal zero. Q.E.D.

Proof of Proposition 1.
The firm would find it desirable to always factor its receivables without recourse if

$$e^{-r_f t} \left[ L \left(1 - \frac{w}{L\delta}\right) - \frac{w}{\delta} \ln \frac{L\delta}{w\alpha}\right] - 1 < e^{-r_f t} p \left[ L \left(1 - \frac{u}{L\beta}\right) - \frac{u}{\beta} \ln \frac{L\beta}{u\alpha}\right] - 1$$
for all values of $a$, where the righthand side represents the payoff to keeping the receivable and credit monitoring optimally, and the lefthand side represents the payoff to factoring the receivable without recourse. Simplifying the inequality by factoring out the $a$ terms yields:

$$e^{-r\tau}p \left[ L \left( 1 - \frac{u}{L\beta} \right) - \frac{u}{\beta} \ln \frac{L\beta}{u} \right] - e^{-r\tau} \left[ L \left( 1 - \frac{w}{L\delta} \right) - \frac{w}{\delta} \ln \frac{L\delta}{w} \right] < \ln a.$$ 

Now, recall that $a \in [1, \infty)$. So, if the above equation holds for the lowest possible value of $a$, it will hold for all values of $a$. Therefore, parameterizations such that

$$e^{-r\tau}p \left[ L \left( 1 - \frac{u}{L\beta} \right) - \frac{u}{\beta} \ln \frac{L\beta}{u} \right] - e^{-r\tau} \left[ L \left( 1 - \frac{w}{L\delta} \right) - \frac{w}{\delta} \ln \frac{L\delta}{w} \right] < 0$$

imply that the firm will always factor its accounts receivable without recourse, regardless of their credit quality. Q.E.D.

**Proof of Proposition 2.**

In order to prove Proposition 2, it will be necessary to invoke the specific functional form for the expected payoff. Substituting $f(x) \cdot dF(x|\alpha, c, m)$ into equation (7) yields:

$$e^{-r\tau} \left[ L \left( 1 - \frac{1}{\alpha_n} e^{-bm} \right) - wm_n \right] - 1 = e^{-r\tau}p \left[ L \left( 1 - \frac{1}{\alpha_n} e^{-b\beta_n} \right) - uc_n \right] - 1$$

where

$$c_n^k = \frac{1}{\beta} \ln \frac{L\beta}{u\alpha_n} \quad \text{and} \quad m_n = \frac{1}{\delta} \ln \frac{L\delta}{w\alpha_n}.$$ 

Thus

$$\alpha_n = \text{EXP} \left[ \frac{e^{-r\tau}p \left[ L \left( 1 - \frac{u}{L\beta} \right) - \frac{u}{\beta} \ln \frac{L\beta}{u} \right] - e^{-r\tau} \left[ L \left( 1 - \frac{w}{L\delta} \right) - \frac{w}{\delta} \ln \frac{L\delta}{w} \right]}{\frac{w}{\delta} e^{-r\tau} - \frac{up}{\beta} e^{-r\tau}} \right].$$

The proof consists of demonstrating that the expected payoff to the firm from selling the receivable is higher than from keeping it when the quality of the receivable is at its highest, and that the opposite is true when the quality of the receivable is at its lowest. Let $K(\alpha)$ denote the firm’s expected profit from keeping the receivable; $R(\alpha)$ the expected profit from selling the receivable with recourse, and $N(\alpha)$ the expected profit from selling the receivable without recourse. It can be shown (by examining the first and second derivatives of $K(\alpha)$ and $N(\alpha)$ with respect to $\alpha$) that the two payoff functions are
monotonically increasing and concave in $\alpha_n$; thus, there will only be a single crossing point and therefore a unique $\alpha_n$. Q.E.D.

**Proof of Proposition 4.**

In order to prove the proposition it will be necessary to invoke the specific functional form for the receivable’s expected value. As, by definition, the expected profit functions, $K(\infty)$, $R(\infty)$, and $N(\infty)$ are strictly increasing and strictly concave in $\alpha$, in order to prove the proposition, it is sufficient to prove three conditions:

1. $K(1) > R(1) > N(1)$.
2. $K(\infty) < R(\infty) < N(\infty)$
3. $\alpha_{keep} < \alpha_{recourse}$

Define $p^*$ by

$$p^* = \frac{L^2 e^{-\delta r} - \left(L^2 - L - \frac{u}{\beta}\right)e^{-\theta r}}{e^{-\delta r}L - \frac{u}{\beta} + \frac{L\beta}{\delta} + \frac{ue^{-\delta r}w}{\delta} - \frac{ue^{-\delta r}w}{\delta}}$$

Stage 1. It is straightforward to show that $K(1) > R(1) > N(1)$ when $p < p^*$.

Stage 2. It is straightforward to show that $K(\infty) < R(\infty) < N(\infty)$.

Stage 3. Demonstrate that $\alpha_n < \alpha_r$.

$\alpha_n$ was defined in the proof of Proposition 2. $\alpha_r$ is defined as the level of credit quality which makes the firm indifferent between selling the receivable with recourse and selling it without recourse. $\alpha_r$ is given by:

$$\alpha_r = \text{EXP} \left[ \frac{e^{-\delta r}(L - \frac{u}{\beta}) + \theta pL^2 - \theta p\left(L - \frac{u}{\beta}\right) - \frac{upe^{-\delta r}L\beta}{\beta} - \frac{ue^{-\delta r}\ln L\beta}{\delta} - e^{-\delta r}\left(L - \frac{w}{\delta}\right) + \frac{ue^{-\delta r}w}{\delta}\ln L\beta}{\frac{ue^{-\delta r}}{\delta} - \frac{upe^{-\delta r}}{\beta}} \right]$$

where $\theta = e^{-\delta r} - e^{-\delta r}$. As $e^{-\delta r}(L - u/\beta) > e^{-\delta r}p(L - u/\beta)$, $\alpha_n$ will be less than $\alpha_r$ if $L^2 > L - u/\beta$. As both $u$ and $\beta$ are, by definition, greater than zero, $L^2$ will be greater than $L$ whenever $L > 1$. As, by assumption, the project has a positive NPV, the project’s promised payoff, $L$, must be greater than the project’s cost of $1$. QED.

**Proof of Corollary to Proposition 4.**

The proof is identical to that of Proposition 4 except that in this case $p > p^*$, so the firm will be able to sell all of its low quality receivables through the use of recourse. QED.
Derivation of the Firm’s Probability of Bankruptcy

Merton (1974) demonstrated that the implied standard deviation of the rate of return to a firm’s assets, \( \sigma_A \), can be obtained by backing it out of the Black and Scholes option pricing model. Expanding upon the work of Merton (1974), Pennacchi (1987) argued that

\[
\sigma_A = \sigma_e \left[ 1 - \frac{D_t e^{-r f t} N(d_2)}{A_0 (1 - e^{-r f t} N(-d_1))} \right]
\]

(A.1)

where

- \( \sigma_e \) is the standard deviation of the rate of return on the firm’s equity;
- \( D_t \) is the market value of the firm’s debt at maturity;
- \( A_0 \) is the market value of the firm’s assets,

\[
A_0 = D + E;
\]

(A.2)

- \( E \) is the market value of the firm’s equity;
- \( r_f \) is the risk free rate of interest;
- \( \delta \) is the firm’s dividend rate per dollar of the firm’s assets;
- \( N(*) \) is the cumulative normal distribution;
- \( \tau \) is the weighted average maturity of the firm’s debt,

and

\[
d_1 = \left[ \ln \frac{A_0}{D_t} + \left( r_f + \frac{\sigma_A^2}{2} - \delta \right) \tau \right] \frac{1}{\sigma_A \sqrt{\tau}}, \quad \text{and} \quad d_2 = d_1 - \sigma_A \sqrt{\tau}.
\]

The standard deviation of the rate of return on the firm’s equity, \( \sigma_e \), is calculated using weekly stock price data. Weekly data are used to mitigate possible upward biases in \( \sigma_e \), which may be attributable to bid-ask spreads.\(^{24}\) Because the market value of the firm’s debt, \( D_t \), is not readily observable for the sample firms, similar to Marcus and Shaked (1984), the book value of long-term plus short-term debt is used as a proxy \( D_t \). The market value of the firm’s equity, \( E \), is calculated by multiplying the number of shares of stock that are outstanding by the price per share. Lastly, the weighted average maturity of the firm’s debt, \( \tau \), is estimated by using information available in the footnotes of the annual reports.

Both \( \sigma_A \) and \( A_0 \) are typically unobservable.\(^{25}\) Equations (A.2) and (A.1) are two equations in two unknowns, so one can substitute equation (A.2) into equation (A.1) and obtain a solution for \( \sigma_A \). Unfortunately, however, an analytical solution for \( \sigma_A \) is unattainable, so \( \sigma_A \) must be determined by numerically backing out the value of \( \sigma_A \) from equation (A.1).

---

\(^{24}\) The calculated variance of small stocks (some of the sample firms are penny stocks) might be significantly upwardly biased by the bid-spread if daily data were to be employed.

\(^{25}\) Although the variance of the return on the firm’s assets is typically unobservable, the variance of the rate of return on the firm’s equity is readily available for publicly-traded firms.
Once $\sigma_A$ and $A_0$ have been determined, it is possible to calculate the probability that the firm will remain solvent for the duration of the trade credit period. This can be accomplished by assuming that the expected rate of return on the firm’s assets and liabilities are both equal to the risk-free rate of return.\footnote{This comes from the assumption of risk neutrality.} The probability that the firm will be solvent at time $t = \tau$ is given by:\footnote{This is the probability under the risk-neutral probability measure.}

$$p = N\left(\frac{\ln \left( \frac{A_0}{D} \right) - \left( \frac{\sigma_A^2}{2} + \delta \right) \tau}{\sigma_A \sqrt{\tau}}\right).$$

The probability of bankruptcy $PBANK$ is then given by $1 - p$.

References


