Measuring Input Substitution in Thrifts: Morishima, Allen-Uzawa, and Cross-Price Elasticities

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This paper examines the behavior of U.S. thrifts in the 1990s, and compares the less known, but theoretically appropriate, Morishima elasticity of substitution to the more common Allen-Uzawa elasticity. The Morishima estimates show substitution between all inputs, are clearly asymmetric, and suggest that inputs are most responsive to changes in the price of capital. In comparison, the Allen-Uzawa estimates substantially overstate substitution between capital and labor. Despite large changes in the thrift industry, the input substitution parameters varied little in the 1990s. Estimation of a thick cost frontier for 1996, however, shows important differences across efficiency levels as low-cost thrifts are more responsive to input price changes. © 1999 Elsevier Science Inc.

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I. Introduction

The 1990s has been a dynamic period for the U.S. thrift industry. After the debacle of the 1970s and 1980s, the regulatory and operating environment of thrifts changed dramatically and a core of over 1,200 healthy institutions emerged with improved performance in the 1990s.

This paper examines the behavior of these surviving thrifts in the 1990s with two distinct purposes in mind. The first is methodological and compares alternative measures of the elasticity of substitution between inputs. The second is policy-oriented and explores how changes in the production behavior of a common set of healthy firms responded to changing market and economic conditions.
Multi-input, multi-output cost functions have been estimated for a balanced sample of 1,202 thrift institutions for 1991 and 1996. Two estimates of elasticity of substitution—Morishima (MES) and Allen-Uzawa (AES)—were then compared across time and efficiency quartiles. Although the AES is more common in empirical work, Blackorby and Russell (1989) showed that the MES is the theoretically correct measure in the multi-input case.

The MES and AES estimates from a three-input cost function analysis (capital, labor, and interest-bearing liabilities) show this distinction to be important. The MES are clearly not symmetric, while the AES are symmetric by definition. Furthermore, the AES substantially overstated the capital/labor elasticity of substitution, particularly in response to a change in the price of labor. Previous research on the behavior of financial institutions which relied on the AES likely overstated the degree of input substitution.

Both the MES and the AES showed little variation between 1991 and 1996. Although the null hypothesis of a common cost function was rejected, the differences in the substitution patterns were not large or economically meaningful. For example, the capital/labor MES in response to a change in the price of capital fell from 0.89 in 1991 to 0.87 in 1996. Despite large changes in the regulatory and operating environment, the surviving thrifts operated in much the same way in 1996 as they did in 1991. The improved performance of the industry can likely be attributed to other factors, such as continued consolidation, improved scale economies, and technical progress.

A thick cost frontier analysis revealed behavioral differences between the most efficient thrifts (low cost quartile) and the least efficient thrifts (high cost quartile) in 1996. In particular, the most efficient thrifts showed larger own-price AES, less asymmetry in the MES, and a larger MES (except for capital/labor substitution). An important characteristic of efficient thrifts is their ability to substitute between inputs.

II. Cost Functions and Elasticities of Substitution

There is a large literature on the behavior of financial institutions. Berger and Humphrey (1997), for example, surveyed over 130 studies that had analyzed commercial banks, savings and loans, and insurance firms from 21 countries. These studies examined a host of questions regarding scale economies, scope economies, technological progress, and x-inefficiency using many techniques, such as data envelopment analysis, stochastic cost frontier evaluation, thick cost frontiers, and nonparametric methods.

This study uses a traditional firm-level cost function to address a very specific question for U.S. thrifts. As the industry regulatory structure and economic environment changed in the 1990s, how did the production behavior and substitution patterns between inputs respond? By focusing on changes in thrift behavior over this period, one can examine the impact of the regulation and gauge the flexibility of U.S. thrifts.

Several previous papers have examined input substitution in thrift institutions and commercial banks. Pantalone and Platt (1994), Noulas et al. (1990), Mester (1987), and Humphrey (1981) all examined firm-level cost functions, and estimated the substitution elasticities between capital, labor, and various types of financial inputs. A common characteristic of these papers, however, is that all reported Allen-Uzawa elasticities and not the theoretically appropriate Morishima elasticities.
The elasticity of substitution is a well-known concept which has received much attention in the economics literature. Originated by Hicks (1932), this concept has become a backbone of applied microeconomic theory. In the case of only two inputs, e.g., the traditional capital and labor production function, the elasticity of substitution is clearly defined as the elasticity of the input quantity ratio with respect to the input price (or marginal product) ratio. Variables are net substitutes as determined by the sign of the income-compensated demand derivative, and with two inputs there is no ambiguity. That is, if the production function is \( Y = f(K, L) \), then the substitution elasticity, \( \sigma \), is clearly defined as:

\[
\sigma = \frac{\partial \ln(K/L)}{\partial \ln(P_K/P_L)},
\]

where \( P_K \) is the price of capital and \( P_L \) is the price of labor.

With more than two inputs, however, the elasticity of substitution becomes more difficult. A common definition is the Allen-Uzawa concept of the elasticity of substitution (AES) from Allen (1938) and Uzawa (1962). This partial elasticity of substitution defines the elasticity of substitution for each pair of inputs, say \( j \) and \( k \), as:

\[
\sigma_{jk} = \frac{C_j(p,q)C_{jk}(p,q)}{C_j(p,q)C(p,q)},
\]

where \( C(p,q) \) is the cost function; \( p \) is a vector of \( J \) input prices; \( q \) is a vector of \( I \) output quantities, and a subscript on the cost function refers to the partial derivative with respect to that input price.

The AES turn out to be a simple function of the familiar cross-price elasticities, \( \varepsilon_{jk} \), and factor shares, \( S_k \), as:

\[
\sigma_{jk} = \frac{\varepsilon_{jk}}{S_k},
\]

where

\[
\varepsilon_{jk} = \frac{\partial \ln x_j}{\partial \ln p_k} = \frac{\partial x_j}{\partial p_k} \frac{p_k}{x_j}
\]

and

\[
S_k = \frac{\partial \ln C(p,q)}{\partial \ln p_k} = \frac{p_k x_k}{C(p,q)}.
\]

In a compelling paper, however, Blackorby and Russell (1989) showed that the AES is an appropriate measure of substitution only in specific cases, and provides no additional information relative to the cross-price elasticities and the factor shares. As an alternative, they examined a substitution concept first defined by Morishima (1967), and showed that this statistic preserves the important characteristics of the two-input elasticity in equation (1).

The Morishima elasticity of substitution (MES) is defined as:

\[
1 \text{ See Berndt (1991) for a brief history of the empirical estimation of the elasticity of substitution and more recent examples.}
\]
where the cross-price elasticity is defined in equation (4), and $\varepsilon_{ij} = \partial \ln x_j / \partial \ln p_j$.

Blackorby and Russell (1989) showed that the MES has several advantages over the AES: the MES measures the curvature of an isoquant; the MES is a sufficient statistic for evaluating changes in relative prices and quantities; the MES is the log derivative of the input quantity ratio with respect to the input price ratio (as in equation (1)). These characteristics were in the original formulation of Hicks, but do not apply to the AES. The MES, therefore, is the more natural extension to the multi-input case.

An important characteristic of the MES is the inherent asymmetry. Blackorby and Russell (1989) showed that asymmetry is natural, as the partial derivative must be evaluated in the direction of the input price that actually changes. For any cost function with more than two inputs, the MES is symmetric only in the special case where the cost function is of the constant elasticity of substitution (CES) variety. The AES, on the other hand, is symmetric for all input pairs by definition.

Consider the MES in equation (6) which describes the percent change in the input quantity ratio, say $x_j / x_k$, with respect to a percent change in the corresponding price ratio, $p_j / p_k$. A change in $p_j$, holding $p_k$ constant, has two effects on the quantity ratio: the first term, $k_j$, shows the effect on $x_k$, and the second term, $j_j$, shows the effect on $x_j$. In contrast, a change in $p_k$, holding $p_j$ constant, has two different effects on the input quantity ratio which are given as $M_{jk} = e_{jk} - e_{kj}$. There is no a priori reason that these effects should be the same. Blackorby and Russell (1981) showed that the MES is symmetric if and only if the production function is CES-Cobb-Douglas.

### III. U.S. Thrifts in the 1990s

The 1990s has been a period of substantial change for the U.S. thrift industry and can be characterized by steady consolidation and improved performance. The Office of Thrift Supervision (OTS) reports that the number of OTS-regulated thrift institutions fell from 2,359 in 1990 to 1,543 in 1994 to 1,238 in September of 1997, while industry-wide return on average assets (ROA) increased from 0.35% in 1990 to 0.56% in 1994 to 0.84% (annualized) for the first three quarters of 1997. Other performance indicators like Tier 1 Risk-Based Capital Ratios, asset quality, and expense ratios also show considerable improvement in the 1990s.²

These improvements occurred immediately after the massive restructuring of thrift regulation due to The Financial Institutions Reform, Recovery, and Enforcement Act (FIRREA) of 1989. This act transferred primary regulatory power to the Treasury Department, moved deposit insurance under the control of the Federal Deposit Insurance Corporation (FDIC), increased restrictions on thrift lending behavior, and gave regulators substantially more power to deal with problem institutions. An important result of this regulation was the consolidation of the U.S. thrift industry, as many inefficient thrifts were forced from the industry.

From a production viewpoint, FIRREA also affected input choices, as the financing of deposit insurance changed. Higher deposit premiums raised the real price of insured deposits for the solvent firms and gave them an incentive to substitute away from insured

deposits as a source of funds. According to OTS (1996b), thrifts paid 23 cents per $100 over the premiums paid by commercial banks, which led thrifts to shrink their reliance on deposits.

Data
Data were taken from balance sheet and income statements for 1,202 OTS-regulated institutions which operated continuously from 1991 to 1996. These 1,202 thrifts held $650 billion in total assets in 1996, which accounted for 90% of all OTS-regulated institutions and 85% of all OTS-regulated assets. Despite missing a small number of new entrants, this sample adequately represents the industry in 1996. For example, industry-wide ROA for all 1,334 OTS-regulated thrifts in 1996 was 0.62% [OTS (1997)], while ROA for the 1,202 thrifts included here was 0.58%. Tier 1 Risk-Based Capital Ratios were also similar: 14.53% for the industry, and 14.25% for this study.3

Following the intermediation approach, an output vector, \( q \), is defined with \( I = 3 \) types of financial assets: mortgage loans (\( q_1 \)), consumer loans (\( q_2 \)), and cash securities (\( q_3 \)). These assets comprise the bulk of thrift assets and allow the analysis to focus on the traditional role of thrifts in the non-business lending market.

On the input side, \( J = 3 \) inputs are utilized. The inputs are capital, with a price \( (p_1) \) defined as annual office occupancy and equipment expense divided by office premises and equipment assets; labor, with a price \( (p_2) \) defined as annual personnel compensation and expense divided by full-time equivalent employees; and interest-bearing liabilities (IBL), with a price \( (p_3) \) defined as annual interest on deposits plus other interest expense divided by total interest-bearing liabilities. The input quantities \( (x) \) are defined as the total expenditure on each input divided by the appropriate price.4

Table 1 presents summary statistics for total assets, the outputs, total costs, the input prices, and the input quantities for 1991 and 1996.5 The size distribution of the firms is asymmetric with a long right-hand tail, i.e., many small firms and few very large firms. There is little variation in the input prices, as one would expect in a competitive market.

IV. Elasticity Estimates for 1991 and 1996
A common specification in empirical work is the translog functional form. The cost function, if allowed to be nonhomothetic, is:

\[
\ln C = \alpha_0 + \alpha \ln q' + \beta \ln p' + \frac{1}{2} \ln q \gamma \ln q' + \frac{1}{2} \ln p \delta \ln p' + \ln q \theta \ln p',
\]

(7)

where the parameters to be estimated are \( \alpha_0 \) (a scalar), \( \alpha \) and \( \beta \) (vectors), and \( \gamma, \delta, \theta \) (matrices).

3 Berger and Mester (1997a) and Bauer et al. (1993) also used balanced panels to analyze the behavior of financial institutions. Note, however, that this sample is not representative of the industry in 1991. Because this paper focuses on changes in the behavior of individual thrifts, it is nonetheless appropriate to analyze a consistent set of thrifts over time. See Stiroh (1997b) for more on the problems of sample selection in the thrift industry.


5 Although data are available for the intervening years, this paper focuses on the endpoints and examines the change over the entire period. Note that 58 institutions were excluded due to data problems, such as negative or implausibly large input prices.
Standard production theory imposes several restrictions on the parameters. Integrability, linear homogeneity in input prices, and product exhaustion require that the matrices $g$ and $d$ are symmetric and $b_1 = 5$, $d_1 = 0$, $u_1 = 0$, (8)

where $1$ is a conformable vector of ones.

To improve the efficiency of the empirical estimation, one can take advantage of Shephard’s Lemma, which implies:

$$S = \frac{\partial \ln C(p,q)}{\partial \ln p} = \beta + \ln p \delta + \ln q \theta$$

where $S$ is the vector of input shares.

The cost function and two share equations form a system of seemingly unrelated regressions (SUR). By appending error terms to each equation and assuming a multivariate normal distribution, one can estimate the SUR system using Zellner’s iterative process.6

### Parameter and Elasticity Estimates

Table 2 reports results from three estimations of the SUR system in equations (7) and (9). In all estimations, the parameter restrictions in equation (8) were imposed throughout, and the implied parameters are not reported, e.g., $\beta_1 = 1 - \beta_1 - \beta_2$ and $\beta_3$ is not reported.7 Casual inspection of the estimated coefficients shows a high degree of statistical significance for most coefficients and little change between 1991 and 1996. On a formal level, however, a log likelihood test rejected the null hypothesis that the two sets of coefficients

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6 The share equations must sum to 1.0, so one share equation was dropped to avoid singularity in the data. The estimates are invariant to which share equation is dropped.

7 In each case, righthand-side variables were scaled by the sample mean, which affected the coefficient estimates but not the estimated substitution elasticities.
are the same. 8 This implies that thrifts operated with different cost functions during the 1990s.

Elasticity Estimates

Given the parameter estimates from the translog estimation, one can calculate the various elasticities of substitution for 1991 and 1996. The following definitions are from Berndt (1991) for the AES and from Blackorby and Russell (1989) for the MES.

The Allen-Uzawa elasticities of substitution (AES) are defined as:

\[
\sigma_{jk} = 1 + \frac{\delta_{jk}}{S_j S_k} \quad k \neq j; \quad \sigma_{jj} = \frac{\delta_{jj} + S_j^2 - S_j}{S_j}.
\]  

The own and cross-price elasticities of substitution are defined as:

\[
e_{jk} = \frac{\delta_{jk} + S_j S_k}{S_j} \quad k \neq j; \quad e_{jj} = \frac{\delta_{jj} + S_j^2 - S_j}{S_j} = \sigma_{jj} S_j.
\]

Table 2. Parameter Estimates

<table>
<thead>
<tr>
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<td>4.19</td>
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</tbody>
</table>

All t statistics were calculated from standard errors which were corrected for heteroskedasticity. The pooled estimation imposed a common cost structure for the 1991 and 1996 observations.

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\]

8 The likelihood test was performed by incorporating a dummy variable into the pooled estimation and allowing every parameter to vary across the 1991 and 1996 observations. This unrestricted model was then compared to the restricted model which imposed common coefficients for both years. The likelihood ratio statistic was 585.3 and the critical value was 41.4 at the .99 level of significance.
The Morishima elasticities of substitution (MES) are defined as:

\[ M_{jk} = e_{jk} - e_{jj}, \]  

(12)

where parameters are taken from Table 2, and the input shares are calculated from equation (9) and sample means.

Table 3 presents estimates of each elasticity for both 1991 and 1996. The top panel is a lower triangular matrix, as, by definition, the AES are symmetric. The middle panel is a full \( J \times J \) matrix, as both own-price and cross-price elasticities are reported. The third panel excludes the diagonal because MES is defined as a logarithmic derivative of the optimal input quantity ratio with respect to the input price ratio, and the diagonal contains no information.\(^9\)

The results from the AES, the own-price, and cross-price elasticities, are as expected and qualitatively similar to Pantalone and Platt (1994) and Noulas et al. (1990). All inputs were substitutes, the degree of substitution was greatest between capital and labor, all own-price elasticities were negative, and input demands were typically inelastic. The

\(^9\) As the data rejected a single cost function for both years, elasticities were calculated using the individual estimates for 1991 and 1996, but not from the pooled data.
similarity to Pantalone and Platt (1994) in particular is remarkable. For example, Pantalone and Platt (1994, Table 10) reported AES estimates for capital and labor of \(-3.22\) and \(-13.58\) for acquiring thrifts in 1988, while Table 3 reports \(-3.56\) and \(-13.17\) for 1991. As discussed above, however, the MES is a more appropriate measure of input substitution.

The MES in Table 3 are reported so that each row represents a change in the price of a particular input, e.g., the first row shows how capital/labor and capital/IBL ratios responded to a change in the price of capital. For both years, changes in the price of capital generated the largest substitution, which implies that thrifts substituted more in response to a change in the price of capital. This asymmetry was apparent in both years, but narrowed from 1991 to 1996 for capital/labor substitution and capital/IBL substitution.\(^{10}\) The MES were typically smaller and often changed in a different direction from 1991 to 1996 when compared to the AES. For example, both MES for capital/labor were less than 1.0 in 1991 and 1996, while the AES for capital/labor were larger than 1.0 in both years. From the 1991 to 1996, the capital/labor AES fell, while the two MES narrowed.

These results suggest that the AES can give a misleading picture of the substitution behavior and raise questions about how to interpret previous research. The AES estimates in this paper overstate the substitution between capital and labor, but are similar to the AES estimates in other papers, e.g. Pantalone and Platt (1994) and Mester (1987). This suggests that previous estimates may be likewise overstated and should be interpreted cautiously.

**A Comparison of 1991 and 1996**

Observed behavioral differences between 1991 and 1996 may also be a factor that contributed to the improved performance of the industry. If firms show significant differences in how they operate, this suggests that firms responded to changing economic and regulatory conditions. Pantalone and Platt (1997), for example, concluded that deregulation in the 1980s was a “stressful environment” that led to behavioral shifts.

The statistical evidence rejects the null hypothesis of a common cost structure between 1991 and 1996, but the magnitude of the MES estimates varied little over time. At an economic level, there was little difference between a MES for capital/labor of 0.87 in 1996 and 0.89 in 1991. The 1996 response to a change in relative input prices was similar to the 1991 response.\(^{11}\)

As the sample includes the same set of thrifts in both years, this evidence suggests that industry improvements were not due to behavioral changes for these healthy firms. Rather, other factors such as technical change, demand shocks, or the continued consolidation of the industry were the primary cause. Stiroh (1997a), for example, showed that exiting thrifts were considerably less successful than surviving thrifts, and that this contributed to the improved performance of the industry. This core of healthy thrifts, however, does not seem to have substantially changed their production behavior.

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\(^{10}\) Fleissig (1997) also found substantial asymmetry in MES in the context of consumer behavior.

\(^{11}\) See McCloskey and Ziliak (1996) for an interesting discussion of statistical significance versus economic significance.
V. A Thick Frontier Analysis for 1996

The preceding analysis implicitly assumed that all thrifts in each year operated on the same cost frontier with the same underlying efficiency. Several authors, e.g., Bauer et al. (1993), Berger and Humphrey (1992, 1991) and Berger and Mester (1997a, 1997b), however, have questioned this assumption and used a thick cost frontier to address the issue of varying efficiency levels between groups of financial institutions. By recognizing that there are observed cost differences (inefficiency) which do not reflect random error, the thick frontier methodology reduces distributional assumptions on the error term in cost function regressions and provides more precise empirical measures.12

A more general cost function regression decomposes the stochastic error into two parts as:

\[ \ln C = C(p,q) + \pi; \]
\[ \ln C = C(p,q) + \mu + \nu, \]  

where \( C(p,q) \) is a general cost function as above; \( \pi \) is the composite error term; \( \mu \) represents firm-specific inefficiencies (both allocative and technical) which raise costs above best practice, and \( \nu \) is a true random error which represents measurement error or luck.

The thick cost frontier approach decomposes the composite error and isolates the inefficiency component, \( \mu \), by comparing the average efficiency of large subsamples of the available data. The complete sample is divided into quartiles based on average costs and it is then assumed that variation from expected values within each quartile represents random error, \( \nu \), while variation between quartiles represents more fundamental inefficiencies or exogenous differences, \( \mu \). By estimating a separate regression for each quartile, therefore, the error terms are more likely to meet the statistical assumptions, e.g., \( \nu \) is mean zero and finite variance.13

Following Berger and Mester (1997b), the 1,202 thrifts in 1996 were divided into four quartiles based on the residual that was obtained from the cost function estimation described in Section IV.14 Thrifts with the smallest residuals were assumed to be cost efficient (low \( \mu \)), while large residuals were indicative of cost inefficiency (large \( \mu \)).

Table 4 shows descriptive statistics for the thrifts in each quartile and the total of 1,202 thrifts in 1996. The lowest cost quartile shows the smallest average cost residual (by definition), and there is a steady increase in average costs (total cost divided by total assets) across the four quartiles. The lowest cost quartile also shows the highest return on average assets (ROA), return on average equity (ROE), and Tier 1 Risk-Based Capital Ratio. These 300 thrifts appear to be the most cost efficient and the most successful, as one would expect. There is not, however, any definite trend in average size across the quartiles.15

The system for the SUR analysis, equations (7) to (9), was then reestimated separately for each quartile in 1996. To compare behavioral differences across thrifts with varying

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12 See Berger and Humphrey (1992), especially page 257, for a discussion of the econometric advantages.
13 The data envelopment analysis (DEA) approach is another alternative, which assumes that the random error component is zero, so that all unexplained variation represents inefficiency.
14 As an alternative, Bauer et al. (1993) created quartiles based on total costs per dollar of total assets.
15 To conserve space, the thick frontier analysis is reported only for 1996. The results for 1991 are similar and are available upon request.
levels of efficiency, both the AES and the MES for the lowest cost quartile and the highest cost quartile were calculated using the estimated parameters (not shown) and mean values from each quartile, respectively. This approach compares behavioral differences between the two quartiles that reflect both technology (as determined by the estimated parameters) and behavioral variation (as determined by the mean values). Table 5 reports the results.

The estimated substitution patterns show some similarity between the two groups—all own-price AES were negative as expected, substitution was most responsive to a change

Table 4. 1996 Summary Statistics by Efficiency Quartile

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lowest</th>
<th>Second</th>
<th>Third</th>
<th>Highest</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>576,379.9</td>
<td>451,667.5</td>
<td>523,137.9</td>
<td>611,622.6</td>
<td>540,746.4</td>
</tr>
<tr>
<td>(2,198,571.3)</td>
<td>(1,658,846.8)</td>
<td>(2,629,598.0)</td>
<td>(1,833,243.8)</td>
<td>(2,111,602.8)</td>
<td></td>
</tr>
<tr>
<td>ROA</td>
<td>0.669</td>
<td>0.467</td>
<td>0.434</td>
<td>0.266</td>
<td>0.459</td>
</tr>
<tr>
<td>(0.593)</td>
<td>(0.466)</td>
<td>(0.573)</td>
<td>(1.644)</td>
<td>(0.959)</td>
<td></td>
</tr>
<tr>
<td>ROE</td>
<td>5.793</td>
<td>4.814</td>
<td>4.697</td>
<td>3.686</td>
<td>4.747</td>
</tr>
<tr>
<td>(6.394)</td>
<td>(5.677)</td>
<td>(7.342)</td>
<td>(9.827)</td>
<td>(7.506)</td>
<td></td>
</tr>
<tr>
<td>(14.167)</td>
<td>(9.096)</td>
<td>(10.496)</td>
<td>(15.868)</td>
<td>(12.915)</td>
<td></td>
</tr>
<tr>
<td>Average cost</td>
<td>0.056</td>
<td>0.059</td>
<td>0.061</td>
<td>0.065</td>
<td>0.060</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.021)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>Cost residual</td>
<td>−0.268</td>
<td>−0.066</td>
<td>0.052</td>
<td>0.285</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.144)</td>
<td>(0.038)</td>
<td>(0.037)</td>
<td>(0.177)</td>
<td>(0.232)</td>
<td></td>
</tr>
<tr>
<td>No. obs.</td>
<td>300</td>
<td>300</td>
<td>301</td>
<td>301</td>
<td>1202</td>
</tr>
</tbody>
</table>

Standard deviations are in parentheses.

Quartiles are based on a residual from equation (7), estimated for 1,202 thrifts in 1996.

Table 5. Elasticities of Substitution by Cost Quartile in 1996

<table>
<thead>
<tr>
<th>Allen Elasticities of Substitution (AES) - $\sigma_{jk}$</th>
<th>Lowest Cost</th>
<th>Highest Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>Labor</td>
<td>IBL</td>
</tr>
<tr>
<td>Capital</td>
<td>−12.289</td>
<td>(30.344)</td>
</tr>
<tr>
<td>Labor</td>
<td>0.728</td>
<td>(2.838)</td>
</tr>
<tr>
<td>IBL</td>
<td>0.880</td>
<td>(10.402)</td>
</tr>
<tr>
<td>Morishima Elasticities of Substitution (MES) - $M_{jk}$</td>
<td>Lowest Cost</td>
<td>Highest Cost</td>
</tr>
<tr>
<td>Capital</td>
<td>Labor</td>
<td>IBL</td>
</tr>
<tr>
<td>Capital</td>
<td>0.841</td>
<td>(32.475)</td>
</tr>
<tr>
<td>Labor</td>
<td>0.707</td>
<td>(23.401)</td>
</tr>
<tr>
<td>IBL</td>
<td>0.845</td>
<td>(10.894)</td>
</tr>
</tbody>
</table>

* t statistics are in parentheses.

Elasticities are defined in equations (10)–(12).
in the price of capital, and the MES appeared asymmetric—but there were also several interesting differences between the two quartiles.

The MES asymmetry appeared larger for the highest cost thrifts, which is consistent with their relatively high cost structure. If these thrifts cannot respond as quickly or easily to a change in a particular price, one would expect them to be less efficient and incur higher costs. Second, the own-price AES were generally larger for the lowest cost quartile. Third, the AES overstated the elasticity of substitution between capital and labor for the entire industry, but not for the lowest cost quartile separately. Both of these results imply more flexible substitution patterns for the lowest cost thrifts. Finally, the lowest cost quartile shows larger MES in terms of capital/IBL and labor/IBL, but smaller MES in terms of capital/labor substitution. This suggests that the highest cost thrifts are the most restricted with regard to interest-bearing liabilities.

VI. Conclusions

This paper has reported cost functions for 1991 and 1996, and analyzed substitution behavior of thrifts in the United States. The data showed the largest degree of substitution between capital and labor, and more flexible substitution patterns for thrifts in the lowest cost quartile. Although the operating environment for the industry changed substantially over this period, the substitution patterns, however, were relatively constant between 1991 and 1996.

Morishima elasticities were calculated and were quite different from the more commonly-used Allen-Uzawa elasticities. The data showed clear asymmetries in the substitution elasticities, and implied that any function with constant elasticities, e.g., CES or Cobb-Douglas, is inappropriate for this type of analysis. Research which relies on AES estimates for the analysis may lead to incorrect conclusions about input substitution in financial institutions. Finally, the results suggest that an important characteristic of low cost thrifts is a high degree of input substitution.

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References


