The paper analyzes the decision made by firms to issue one-time coupons as a means of attracting new deal-prone customers. Given the structure of the market and the share of loyal customers, we derive boundaries for the value of the coupon, as well as the optimal face value of the coupon. The main variables which determine the coupon value are: the size of deal-prone and loyal market segments, the initial profit margin and the coupon’s processing cost. We show that the optimal share of discount out of the profit margin per customer should never exceed the customer share of the deal-prone segment. © 1999 Elsevier Science Inc.

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I. Introduction

The phenomenon of free coupons as a means of increasing sales is well-known and widely practiced in the market place. Price discounts allow firms to reach new customers who are currently not buying the product. There is, however, a difficulty in differentiating between loyal and new customers, thus coupons are used as a means of inducing voluntary price discrimination. One purpose of this price discrimination is to limit the loss which results from the extensive use of the deal by the original full-price customers. Therefore, the firm may use multipart pricing in conjunction with its coupon distribution, limiting the number of coupons per customer.
We start with a very intuitive and simple approach to multipart pricing and illustrate the use of coupons and limited numbers of coupons. Consider two consumer segments that are potential users of the product. Group A, called the loyal group, is ready to pay the full price, $p^*$, and each consumer purchases at least one unit of the product, while for consumers of Group B, the full price, $p^*$, is too high, i.e., $p^*$ is above their reservation price.

Clearly, an optimal policy for the firm is to offer a low price or a coupon only to consumers in the second segment, as long as the discount is lower than the net profit margin from this group. A simple example of this practice of price discrimination is marketing half-price theater tickets in big cities. After the full-price customers purchase their ticket for the show in advance, half-price tickets are offered to deal-prone theater lovers on the day of the performance by special outlets.

Another example is discrimination by age, offering products and services for seniors based on identity cards issued by special associations. In the case where the administrative discrimination is easily enforced and inexpensive, and products are not transferable, there is no need to offer a coupon. For most products, administrative price discrimination is not feasible, or too costly. There, the firm may use coupons as a price discrimination tool, counting on the correlation between the tendency to redeem coupons and segment memberships.

A firm would tend to restrict the number of coupons, say to one, when the average number of units purchased by a full-price customer is expected to be much larger than one. Restricting the number of coupons per customer makes special traveling to the store for the sole purpose of purchasing the product unattractive. The cost of transportation may not be offset by the limited discount. From this, it follows that when the main purpose of the firm is a quick shifting of inventory to the customer, it will offer a price discount on unlimited use of coupons. Quick inventory shifting is more important in those cases where the supply and demand at normal full price are highly imbalanced. This is typical of seasonal goods, especially when these are nonstorable, perishable or suffer from obsolescence, or goods with high storage cost.

On the other hand, when the main purpose of the discount is to attract new customers (or customers who are not buying the product at a full price), it is desirable for the firm to offer a restricted number of coupons. This is especially relevant when the purpose of the firm is to induce a massive trial of a new product. The reservation prices of the potential customers and their quantity demanded may be considerably lower when compared with those who already have tried the product and found it satisfactory. To reach these new customers, a considerably higher discount than the ordinary must be offered.

Restricting the number of items per coupon enables the firm to offer the required deeper discount and increase diffusion. Furthermore, in many of the cases where the coupon is offered by the retailer in order to attract new customers to its store, a restricted number of coupons can be justified by two more considerations. First, an unrestricted number of coupons may require too much shelf space. Second, by restricting the number of coupons, we may deter the extreme type of cherry-pickers, deal-prone customers, who are not interested in purchasing other goods, and may not be ready to travel to the store from afar for a limited bargain.

We consider the form of the deal as secondary. However, we do not consider trade discounts because, in this expository paper, we ignore channel structure effects. Straight discounts, rebates and coupons generate similar search, purchasing and administrating
effects. We may consider a straight discount as a coupon without the clipping, dropping and processing costs, and a rebate as a coupon without the retailer processing cost. Therefore, we have decided to model the dealing as the somewhat more complex and encompassing couponing practice. It must be recognized at the outset that only a few features of the couponing phenomena are critical for the unified modeling of dealing with a limited number of coupons per customer. Yet, there is a long list of couponing variables which typically appear in strictly empirical (rather than analytical) couponing studies. We refer to several of these studies in the next section.

Dealing with a limited number of coupons per customer is a form of multipart pricing which has received attention in the economics and business literature [see, for example, Alchian and Allen (1977); Hirshleifer (1984)]. Yet, the only analytical model of multipart pricing we are aware of is that of La Croix (1981, 1983). La Croix’s purpose was confined to that of deriving welfare implications. He thus ignored several concerns which are critical when advocacy of managerial action is the purpose, but which are secondary when deriving general welfare implications. Specifically, he [La Croix (1981, 1983)] was not concerned with the role of coupons in inducing voluntary third-degree price discrimination [see Levedahl (1984, 1986) and Sweeney (1984)]. He did not derive explicit optimal coupon values formulae, and his analysis of changes in the optimal coupon value as a function of the shape of the market response function was limited.

The plan of our paper is as follows: In the next section, we review some relevant literature affecting our modeling approach. It is followed by Section III where we develop the model of optimal discount couponing. In Section IV, we conclude with implications and directions for future research.

II. Selected Couponing Literature

Typically, dealing studies have emphasized the role of coupons in inducing voluntary third-degree price discrimination, based on factors such as differential inventory cost [see Blattberg et al. (1981)], differential coupon processing costs [Narasimhan (1984); Levedahl (1986)], or both differential reservation prices and inventory costs [Jeuland and Narasimhan (1985)].

If a higher coupon value is directly related to the size of the order quantity, then, in the post-deal period, we should expect a lower probability of repurchase. The empirical findings of several studies have supported this hypothesis and, in turn, the inventory-shifting hypothesis [Shoemaker and Shoaf (1977); Dodson et al. (1978); Jones and Zufryden (1981); Guadagni and Little (1983)]. Other studies have emphasized differential exposure to information [Varian (1980)], differential shopping tendencies [White (1983)], taste or quality differences [Narasimhan (1988)], or brand loyalty through differential reservation prices [Raju et al. (1990)]. Empirical support for the hypothesis of negative correlation between brand loyalty and coupon usage has been given by Webster (1965), Montgomery (1971) and Bawa and Shoemaker (1987a). Several explanations were offered as support for this observation. They range from the effect of couponing on reference pricing [Monroe (1973); Winer (1986); Shindler (1992)], to cognitive dissonance [Doob et al. (1969)], to self-perception theory [Dodson et al. (1978)].

Bawa and Shoemaker (1987b) and Krishna and Shoemaker (1992) further clarified the intricate relationships between brand loyalty and coupon usage. They found that customers who are brand loyal to a specific brand possess a higher probability of redeeming a
coupon for that brand than customers who are not loyal to that brand. Accordingly, suppose the segment of full-price, brand-loyal customers is sufficiently large, and the degree of price discrimination is mild. The firm may not be able to rely on voluntary price discrimination induced by coupon distribution to bar a sufficient number of old customers from entering the market, thus encountering a loss, unless it restricts the number of coupons per customer.

Following this traditional modeling emphasis, we extended the function set of the coupon beyond that of strict multipart pricing considered by La Croix, and allowed for coupon inducement of price discrimination. Our approach thus emphasizes complementarity rather than substitution between the role of the coupon in multipart pricing and its role in price discrimination.

Although the results of our analysis can be quite general, for simplicity of exposition, the mathematical presentation has been simplified by restricting our analysis to the case where the number of coupons per customer is limited to one. In practice, one can observe periodical general discounts for all customers, and cases where the discount is limited to new customers only or to all customers who are restricted by the number of coupons per customer. In this paper, we analyze the last case.

**Coupon Value Formulae**

The prime decision variable of a marketer administering couponing policy is the optimal coupon value. Hence, the derivation of an explicit expression for the coupon value as a function of other parameters is highly desirable. Most previous couponing models have provided only implicit expressions for the optimal coupon value, and their analysis of the change of the value is limited. The only exception we are aware of is that of Blattberg et al. (1981), who also derived an explicit solution for the optimal coupon value, but only for the case where the new customer share is insensitive to the coupon value. In addition, Jeuland and Narasimhan (1985) also provided an approximation of the optimal deal value. In our study, we have derived implicit solutions for the coupon value, thus providing the marketer with a clearer instrument for evaluating the couponing policy. Our approach is shared with Shaffer and Zhang (1995), who also explicitly calculated coupon values and performed some comparative statics in a competitive spatial framework with a linear market response function to coupon value. We have extended their analysis to some other questions, such as the question of how the coupon value is sensitive to the distribution of customers.\(^1\)

**Aggregate Demand and Market Composition**

Most dealing studies have not formally restricted the number of coupons per customer in any manner; studies by La Croix (1981, 1983) being the exception. Hence, the traditional objects of analysis are the aggregate demands of the different segments.

\(^1\) We will inquire into the relationship between the shape of the market response functions and the optimal coupon value. We thus will admit any general market response function that is monotonic in the coupon value, derive some explicit coupon solutions for non-linear cases and demonstrate that the shape of the market response function carries immediate and critical consequences for the feasibility, value, and sensitivity of the optimal coupon value. [Ben-Zion et al. (1990, 1992, 1993)].
Generally, previous analytical normative models have assumed that each segment is comprised of consumers with identical demand of some particular shape. This assumption has been only somewhat relaxed in competitive models [Narasimhan (1988); Raju et al. (1990)]. However, these competitive models have assumed the consumer demand at the product category level to be perfectly inelastic.

A study by Levedahl (1986) is an exceptional normative study, which did not restrict parametrically the demand shape of the individual consumer. Rather, he studied the effect of a general aggregate demand of the deal-prone segment on the optimal coupon value. However, Levedahl (1986) assumed no restriction on the number of coupons per customer and did not deal with the customer composition of the aggregate demand.

In contrast with most prior models, we used a different approach. Our study allowed for a continuous distribution of reservation prices in the deal-prone segment. This allowed us to endogenize the number of deal-prone customers as a function of the coupon’s discount value.

As is common, we assumed that a consumer whose reservation price is above the discounted price purchases the product. Thus, the cumulative number of consumers in the deal-prone segment whose reservation price is above the discounted price is defined by us as the market response function. It is a function of the coupon value.

Our modeling focuses on the market response rather than on individual consumer motivation for coupon usage, and follows La Croix (1981) and the econometric models of Ward and Davis (1978a) and Neslin (1990). The shape of the market response function critically affects couponing feasibility and coupon value. A prime aim of this paper is to study this effect.

III. The Model

Assumptions

We will consider an industry with one or more firms, each offering its differentiated brand or product in the market. The number of firms in the industry is constant (no entry or exit of firms). We will deal with the behavior of a single firm, assuming that the price of its rival firms is fixed during the couponing period.

We will assume that the firm strategically determines the optimal list price, $p^*$, of its product, to which we refer as full price. Optimality of $p^*$ takes into account expected promotional activity, even though it may be that as time passes, $p^*$ ceases to be optimal because it is not changed as a result of price stickiness. In this paper, we do not analyze the properties of $p^*$. The coupon value, on the other hand, is determined for the optimally given $p^*$. The purpose of the analysis is to focus on insight regarding the optimal coupon face value, $D^*$, which may be a short-horizon decision. By that, we are following Blattberg et al. (1981), and La Croix (1981, 1983). We will assume one coupon per customer, which applies only toward purchasing one product unit. Furthermore, the coupons are not transferable between consumers.

For clarity, we will postulate direct marketing. The firm sells its product directly to the consumers (disregarding the role of the intermediate distributors). We assume two segments of consumers, according to their product loyalty, following traditional modeling. The first segment of $N_0$ loyal customers [called “original customers” by La Croix (1983, pp. 850, 851)] buys, on the average, $X$ units of the product at the list price, determined by the firm. We will refer to this group as the full-price market segment. We would like to
emphasize that we do not assume that loyal customers have identical demands; we only assume that each loyal customer purchases at least one unit of the product at the full price.

Thus ignoring an income effect, the marginal net price of the product is not affected by the coupon discount policy. Therefore, the number of units purchased by each loyal customer is not affected.

The second segment consists of \( M \) potential new customers, some of whom may purchase the product but only at a lower price through use of discount coupons. The demand of a deal-prone customer may be assumed stochastic as long as his/her reservation price is below \( p^* \). The larger the discount of each coupon, the larger the number of new purchasers of the product. We will refer to this group as the deal-prone market segment.

Given this setting, the profit-maximizing firm decides on the level of discount, \( D \), of its cent-off coupon.

We will assume that through its couponing process, the firm may administer price discrimination which may be both involuntary and voluntary. It may selectively distribute its coupons through different listings, media and channels, thereby deliberately reaching only a lower portion of its loyal full-price customers. In addition, many of the loyal full-price customers will not make use of the coupon because of the high costs of time, storage and effort involved.

We admit any degree of price discrimination. A parameter, \( k, 0 \leq k \leq 1 \), denotes the degree of price discrimination. One can also interpret this as the degree of leakage in a coupon targeting promotion.

\( N = kN^0 \) denotes the number of original customers who purchase the product with a coupon. When \( k = 0 \), there is perfect price discrimination. When \( k = 1 \), there is zero price discrimination. In the latter case, we will assume that the seller has no way or no intention to identify and therefore to discriminate between different types of customers. The coupon package (in terms of the number and the value of each coupon) is identical for old original (loyal) and potential new customers.

Our analysis is performed for a given coupon dropping. Therefore, we will assume a given fixed coupon dropping cost, \( C^1 \). Optimizing the size of coupon dropping and its mixture is outside the focus of this paper. The effect of differential coupon dropping was empirically exemplified by Ward and Davis (1978b). The initial marginal production and marketing cost of the firm is assumed to be constant at level, \( m^0 \), and the processing cost per coupon is assumed constant at a level \( a \). Thus, when couponing is practiced, the fixed cost is \( C = C^1 + aN \), and the marginal cost is \( m = m^0 + a \).

The model was designed to be a parsimonious one in order to serve as a robust prototype for generating elementary building blocks in future extension models, where more complex markets and channel structures are assumed.

**Feasibility and Optimality of Coupon Offering**

In order to determine the optimal discount level of a coupon, \( D \), we compare the initial profit (before issuing coupons), \( \Pi_0 \), to the profit, \( \Pi_1 \), after issuing coupons:

\[
\Pi_0 = (p^* - m^0)XN^0; \quad (1)
\]

\[
\Pi_1 = (p^* - m^0)XN^0 - (D + a)N + (p^* - m^0 - D - a)M(D) - C^1. \quad (2)
\]

Let us emphasize that the firm’s losses from offering a coupon to the full-price customers is \( (D + a)N \) rather than \( (D + a)N^0 \). Through careful price discrimination, the
firm may be able to restrict the coupon distribution to old customers, thus limiting its
losses \((D + a)N \leq (D + a)N \) \(0 \). In equation (2) above, \( M \) is the number of new customers
which depends on the level of discount, \( D \). We will refer to \( M(D) \) as the market response
function (to the coupon value). An admissible \( M(D) \) is assumed to be a bounded,
non-negative, non-decreasing function of the coupon value, satisfying the initial condition
\( M(0) = 0 \). For convenience, we will consider only \( M(D) \)s that are continuous in the
domain \([0, p - m] \) and possess continuous first- and second-order derivatives there:

\[ M(D) \] obtains its maximum, \( M^* \) at \( (p - m) \), i.e.,

\[ M^* = M(p - m). \]  

We assume further that:

\[ M'(D) \geq 0. \]  

The difference in profit, \( \Delta \Pi \), is given as:

\[ \Delta \Pi = (p - m^0 - a - D)M(D) - DN - aN - C = (p - m - D)M(D) - DN - C, \]  

where the first term represents the added profits from attracting additional customers due
to the coupon promotion; the second term is the loss in revenue as a result of the discount
given to old customers, and the third term is the fixed cost of couponing. Note, that as \( N = kN^0 \), the second term vanishes when either \( N^0 = 0 \) or \( k = 0 \), i.e., when there are no
original customers or when the firm discriminates perfectly.

**Feasibility and Optimization Conditions**

Given an admissible \( M(D) \), we denote the optimal discount as \( D^* \). For simplicity of
notation, when a function \( f(D) \) is evaluated at \( D^* \), we may omit its arguments, writing:

\[ f(D^*) = f^* \]

e.g., \( M(D) = M^* \), \( M'_D(D^*) = M^* \), \( M''_D(D^*) = M''_D. \)

Maximizing the objective, \( \Delta \Pi \), a feasible optimal solution requires:

\[ \Delta \Pi > 0; \]  

\[ D^* \geq 0; \]  

\[ \Delta \Pi_D = (p^* - m - D^*)M^*_D - (M^* + N) = 0; \]  

\[ \Delta \Pi_D^* = -2M^* + (p^* - m - D^*)M^*_D < 0; \]  

\[ M^*_D = \frac{M^* + N}{p^* - m - D^*}. \]  

Optimal \( D^* \) satisfies:

\[ D^* = (p^* - m) - \frac{M^* + N}{M^*_D}. \]
It is clear that for couponing feasibility, $C$ must not be too large, $N$ must be relatively small, and $(p^*-m)$ relatively large.

The implicit equation for the coupon value in equation (11) expresses a sensitivity of the coupon value to the shape of the market response function, $M(D)$. It also enables us to derive some explicit formulae for a face value of a coupon in some important reference cases.

Consider the constant elasticity $M(D) = AD^a$, when $A > 0$ and $\alpha > 0$. We will exhibit the explicit solutions for $D^*$ in equation (11) for $\alpha = \frac{1}{2}$, $\alpha = 1$, and $\alpha = 2$.

For $\alpha = \frac{1}{2}$,

$$D^* = \frac{2N^2}{9A^2} + \frac{1}{3} (p^*-m) - \frac{2N}{3A} \sqrt{\frac{N^2}{9A^2} + \frac{1}{3} (p^*-m)}.$$  

At $N = 0$, the discount reaches its maximum value, $D^* = (p^*-m)/3$. The discount gradually decreases as $N$ increases, diminishing to an infinitesimal value as $N$ approaches infinity.

For $\alpha = 1$,

$$D^* = \frac{p^*-m}{2} - \frac{N^2}{2A}.$$  

If $N$ is sufficiently large, the discount disappears. The minimum level of discount is zero, when $N = A(p^*-m)$ and the maximum $(p^*-m)/2$.

For $\alpha = 2$,

$$D^* = \frac{(p^*-m) + \sqrt{(p^*-m)^2 - \frac{3N}{A}}}{3}.$$  

When $N$ is zero, i.e., there are no loyal customers, the first resorts to monopoly pricing and the optimal discount is at its maximum, $D^* = \frac{1}{3} (p^*-m)$. When $N$ is sufficiently large, the term under the root becomes imaginary and there is no solution for the $D^*$, i.e., no discount is offered. However, if a discount is to be offered, its minimum value is $\frac{1}{3} (p^*-m)$, obtainable when $N = [A(p^*-m)^2]/3$.

The results of the above examples are easily generalizable for other values of $\alpha$, as follows:

- The maximal discount is always obtained for $N = 0$, i.e., when $k = 0$ or $N^0 = 0$. It is the monopoly price, $D^* = [a/(\alpha + 1)](p^*-m)$.
- For concave market response functions, $(0 < \alpha < 1)$, $M_D = \alpha AD^{\alpha-1}$ is infinite at $D = 0$, and $M_{DD} = \alpha (\alpha - 1)AD^{\alpha-2} \leq 0$ at $(0, p^*-m)$. Hence, for any values of $N$ and $p - m$, the firm always offers a discount, provided the fixed couponing cost, $C$, is sufficiently small. However, as $N$ approaches infinity or $(p^*-m)$ approaches 0, the value of the coupon becomes negligible.
- In contrast, for convex market response functions, $(\alpha > 1)$, $M_D(0) = 0$, and $M_{DD} > 0$. By continuity, $M_D = N/(p^*-m)$ is strictly negative for $De[0, b]$, for some $b$. 


Hence, for any \( N \), and \((p^* - m)\), the minimal feasible discount must be strictly positive. No coupon would be profitable for a sufficiently large \( N \) or a sufficiently small \( p^* - m \), no matter how small \( c \) is.

Two immediate additional results can be derived from equations (8) and (9) above. By applying the implicit function theorem to equation (8), we get:

\[
[(p^* - m - D^*)M_{DD}^* - 2M_B^*]dD^* - dN^* = 0.
\]

Substituting this result in equation (9) gives:

\[
\frac{dD^*}{dN} = \frac{1}{-2M_B^* + (p^* - m - D^*)M_{DD}^*} = \frac{1}{\Delta \Pi_{DD}^*} < 0.
\]

The implication of this result can be extended further.

A firm for which the ceteris paribus number of loyal customers in the coupon market increases (decreases) should offer lower a (higher) coupon discount, i.e.,

\[
\frac{dD^*}{dN} = \frac{1}{-2M_B^* + (p^* - m - D^*)M_{DD}^*} = \frac{1}{\Delta \Pi_{DD}^*} < 0.
\]

From equations (9) and (4), we obtain:

\[
\frac{dD^*}{d(p^* - m)} = \frac{-M_B^*}{\Delta \Pi_{DD}^*} > 0.
\]

This result indicates that, ceteris paribus, a firm which experiences an exogenous increase in profit margin per customer, should offer a higher discount. The intuitive explanation can be that the increase in profit margin also increases the marginal benefit from attracting additional (deal-prone) customers. Therefore, one should increase the coupon value to attract more consumers. Moreover, for uniform distribution of the reservation prices of the deal-prone consumers, i.e., when \( M_{DD}^* = 0 \), we would expect that one-half of the increase in the profit margin should be allotted to an increase in the coupon’s discount value. This result supports, and is supported by, the comparative statics found in Shaffer and Zhang (1995).
Inequalities of $M_D^*$

Combining feasibility and optimization conditions, we will develop related inequalities on the size of $M_D^*$. See Appendix II.

$$\frac{(M^* + N)^2}{N(p^* - m)} > \frac{(M^* + N)^2}{C + N(p^* - m)} > \frac{M^* + N}{(p^* - m)} > \frac{N}{(p^* - m)}. \tag{12}$$

These inequalities are used to derive bounds for $D^*$ (see Appendix I).

Specifically, we obtain the following proposition:

**Proposition 1.**

The discount offered by a profit-maximizing firm may or may not approach, but never exceeds:

$$D^* = \left( \frac{M^d}{M^d + N} \right) (p^* - m) - \frac{C}{M^d + N}, \tag{13}$$

or, alternatively, the discount share in the profit margin does not exceed the following:

$$\frac{D^*}{(p^* - m)} = \left( \frac{M^d}{M^d + N} \right) - \frac{C}{(M^d + N)(p^* - m)}. \tag{14}$$

**PROOF.** Substituting the upper and lower bounds for $M_D^*$ from equation (12) in the expressions for $D^*$ in equation (11), we obtain:

$$\frac{(p^* - m) - \frac{M^* + N}{(M^* + N)^2}}{N(p^* - m) + C} > D^* > \frac{(p^* - m) - \frac{(M^* + N)}{(M^* + N)}}{(p^* - m)} = 0, \tag{15}$$

or, alternatively:

$$\frac{(p^* - m) - \frac{N(p^* - m) + C}{M^* + N}}{M^* + N} > D^* > 0. \tag{16}$$

Hence,

$$\frac{M^d}{M^d + N} (p^* - m) - \frac{C}{M^d + N} > \frac{M^*}{M^* + N} (p^* - m) - \frac{C}{M^* + N} > D^* > 0. \tag{17}$$

From continuity and regularity arguments applied to both $M^*(D)$ and $M(D)$, it follows that the upper bound is a supremum. This means that the sum of the discount and average fixed cost of couponing cannot exceed, but can approach, the maximal market share of the deal-prone segment.

The meaning of the results in equation (14) is as follows: The discount share in the profit margin cannot exceed the maximal customer share when all the profit margin is totally discounted. Furthermore, the upper limit of the discount coupon will be higher:

1. The larger the initial profit margin of the firm $(p^* - m)$;
2. The larger the share of deal-prone (new) customers that use the coupons, or the lower the number of loyal customers who use the coupons;
3. The lower the average cost per all customers (new and old) who may use the coupons.

Firms which face a high cost of coupon processing and have lower profit margins or higher percentages of loyal customers in the population, may decide either not to offer coupons at all or to issue their coupons with a lower face value.

IV. Implications, Qualifications, and Directions for Future Research

Let us begin by classifying the implications of our study.

1. **Conditions for Feasibility of Coupon Offering.** The feasibility conditions imply that a firm is likely to offer a coupon when its initial profit margin is sufficiently high, when it is easy to implement an involuntary price discrimination, or to induce a voluntary one, and where the customer share of the deal-prone segment is larger. Clearly, the firm must also be able to cover its coupon dropping and processing cost.

2. **Computation of Optimal Coupon Value.** We expressed the optimal value of coupons (equation (11)). This result can be used by firms to determine the optimal coupon value, based on the profit margin, the market response of the deal-prone segment, and the number of loyal customers.

3. **Comparative Static Results Concerning Coupon Pricing.** The coupon value is higher for a lower coupon processing cost and for a higher initial profit margin. It is also higher for a lower number of old customers, and a higher degree of price discrimination, associated with a coupon distribution.

It would be desirable to expand our model to include horizontal price-mix competition. We must further suggest that such future models should treat the decision to employ coupon restrictions endogenously, while considering the cost structure of administrating alternative coupon policies. This proposed effort was inspired by the game development in marketing by Narasimhan (1988), Raju (1987), Raju et al. (1990) and Rao (1991), investigating the existence and properties of the equilibrium price-mixed strategy under full information.

Another possible extension would be to consider intertemporal effects of discount coupons in a two-period model. Some deal-prone customers in the first period who used the coupon may become loyal customers, and other loyal customers may shift to the deal-prone group. (See Appendix I).

All in all, we hope that this and similar papers will prove helpful in expanding managerial dealing capabilities as firms are developing their customer databases, and shifting to marketing to the non-anonymous customers.

**Appendix I**

**The Optimal Couponing Policy with Learning by Consumption (Multi-Period Model)**

In this section, we present simple extensions of the above model to multi-period models with product learning and adoption. There are no exact guidelines for modeling the effect
of learning on the pattern of purchases by the deal-prone segment in the post-promotion period. However, it is reasonable to assume that the full-price purchases by the deal-prone customers are directly related to the level of their purchases during the promotion period. We will focus here on a class of multi-period models which enable us to obtain an analytical solution principle that is easily linked with solutions of the model developed in the paper for the single-period case.

We will consider models where total purchases of the deal-prone segment in future periods are proportional to the total coupon purchases by the deal-prone segment in the promotion period. We present here only a single model of this class.

For simplicity, we will ignore the cost of coupon dropping, and assume first that the deal-prone customers learn fast, and learning is completed in the first period. In future periods, they purchase the quantity \( \lambda M(D) \), where \( \lambda > 0 \) is a proper proportionality factor. Formally, we can write the present value of future purchases by the deal-prone segment as:

\[
(p^* - m) \left\{ \frac{\lambda M(D)}{1 + \delta} + \frac{\lambda M(D)}{(1 + \delta)^2} + \cdots + \frac{\lambda M(D)}{(1 + \delta)^n} \right\}, \tag{A.1}
\]

\[
= (p^* - m) \frac{\lambda \left[ 1 - \left( \frac{1}{1 + \delta} \right)^n \right]}{\frac{\delta}{1 + \delta}} M(D) = (p^* - m) v M(D), \tag{A.2}
\]

where

\[
v = \frac{\lambda \left[ 1 - \left( \frac{1}{1 + \delta} \right)^n \right]}{\frac{\delta}{1 + \delta}} \tag{A.3}
\]

is a proportionality factor, \( 1/(1 + \delta) \) the one-period discount rate, and \( n \) the number of future periods. We derive the optimal couponing values for this simple case from a more general principle.

**A Solution Principle Under Proportional Learning**

In general, for any model in which the present value of future purchases of the deal-prone market is of the form \((p^* - m) v M(D)\), the incremental profit function, \( L \), is:

\[
L = \Pi_2 - \Pi_1 \tag{A.4}
\]

The expressions, \( \Pi_2 \) and \( \Pi_1 \), are as follows: \( \Pi_2 \) is the present value of the firm’s profit when coupons are used in the first period.

\[
\Pi_2 = (p^* - m) N^0 X - DN + (p^* - m - D) M(D) \]

\[
+ \frac{(p^* - m)}{1 + \delta} \left[ \frac{\lambda M(D)}{1 + \delta} \right] \frac{\left[ 1 - \left( \frac{1}{1 + \delta} \right)^n \right]}{\frac{\delta}{1 + \delta}} N^0 X + (p^* - m) v M(D). \tag{A.5}
\]

\( \Pi_1 \) is the present value of the firm profit if coupons are not used,
The assumption in this formulation is that the number of loyal customers, \( N^0 \), and their average purchase, \( X \), remains constant. Substituting equations (A.5) and (A.6) in equation (A.4), we obtain:

\[
L = (p^* - m)N^0 \left( 1 + \frac{1 - \left( \frac{1}{1 + \delta} \right)^n}{1} \right)
\]

\[
= (p^* - m)N^0 \left( 1 + \frac{1 - \left( \frac{1}{1 + \delta} \right)^n}{\delta} \right). \tag{A.6}
\]

Denoting,

\[
(p^* - m)^\wedge = (p^* - m)(1 + v), \tag{A.8}
\]

we obtain

\[
L = [(p^* - m)^\wedge - D]M(D) - DN. \tag{A.9}
\]

Ignoring the cost of coupon dropping, \( L \) in equation (A.9) has the same form as \( \Delta \Pi \) in equation (5). The exogenous term \((p^* - m)^\wedge\) has the same features as \((p^* - m)\). Hence, all the results we presented for the single-period case are valid for the multiperiod case with \((p^* - m)^\wedge\) replacing \((p^* - m)\).

Using equation (A.9) for \( L > 0 \), it can be shown that there are cases where a sufficiently large \( M(D) \) would result in offering a discount coupon value, \( D \), so large that it may even exceed the current profit margin \((p^* - m)\). The firm would prefer a loss in the first period, in return for a gain in the future period. The greater the product’s life-span, the larger should be the size of the coupon promotion. In this case, it is also possible to develop a bound on the ratio between the discount and the profit margin. This bound generalizes the results of Proposition 1.

It can be shown that in a case where the market response function exhibits constant elasticity and is either linear or concave, coupons would not be offered unless the profit margin \((p^* - m)\) is sufficiently large, or the number of full-price loyal customers, \( N \), is sufficiently small. In the convex case, coupons will always be offered, regardless of the size of \((p^* - m)\) and/or \( N \). Proofs are available upon request.

**Appendix II**

**Bounds of \( D^* \)**

We will proceed to derive an upper bound for \( D^* \). We are guided by the following observation: A firm must decide how much of its profit margin it is ready to discount in order to maximize its profits. It is obvious that the firm will never offer to discount all its
profit margin, for such an offering eliminates all profits. It is also expected that the larger
the market response to a discount, the larger the discount. If the firm had perfect
knowledge of the demand function of its new customers, it could set the optimal discount
level using equation (11). However, the information possessed by a firm is often limited.
It is reasonable to assume that the firm can estimate some upper bound for the total
number of new customers who are about to enter the market if the firm discounts all of
its profit margin. For simplicity of exposition, let us assume for the moment that the firm
knows \( M^* \), the maximum number of (new) customers who will enter the market when the
profit margin is completely discounted. It follows that the market share of new customers
at \( D = \frac{(p^* - m)(M^* + N)}{p^* - m} \), provides an upper bound for the maximal proportion
of the profit margin discounted by a profit-maximizing firm. When the average fixed
couponing cost per customer is negligible, this upper bound is also a supremum.

Appendix III

Inequalities of \( M^*_D \)

At optimum \( D^* \), from equation (10),

\[
M^*_D = \frac{M^* + N}{p^* - m - D^*} > \frac{M^* + N}{p^* - m}.
\]

(B.1)

At \( D^* > 0 \), from equations (5) and (6),

\[
(p^* - m - D^*)M^* - ND^* > C;
\]

(B.2)

substituting the expression of \( D^* \) from equation (11) in equation (B.2),

\[
\frac{M^* + N}{M^*_D} \left[ M^* - N \left( p^* - m - \frac{M^* + N^*}{M^*_D} \right) \right] > C.
\]

(B.3)

Hence,

\[
\frac{M^* + N}{M^*_D} \left[ M^* + N \right] > (p^* - m) + C,
\]

(B.4)

or,

\[
\frac{[M^* + N]^2}{C + N(p^* - m)} > M^*_D.
\]

(B.5)

Combining equations (B.1) and (B.5), we obtain:

\[
\frac{(M^* + N)^2}{N(p^* - m)} > \frac{(M^* + N)^2}{C + N(p^* - m)} > M^*_D > \frac{M^* + N}{p^* - m} > \frac{N}{p^* - m}.
\]

(B.6)

Note that if \( D^* = 0 \) and \( C = 0 \), then \( \Delta \Pi^* = 0 \), \( M^* = 0 \) and \( M^*_D = N(p^* - m) \).

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References


