Monetary Rules and Stock Market Value

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We examine the wealth effects of two widely-advocated monetary rules. An inflation rule offers lower dividend volatility than a money growth rule, but the latter can provide higher expected dividends. Thus, the real value of the stock market is higher under the inflation rule if and only if the market’s intertemporal elasticity of substitution is sufficiently low. © 1999 Elsevier Science Inc.

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I. Introduction

Motivated by the difficulties of successfully operating a discretionary monetary policy, researchers and policymakers have become increasingly interested in the stabilization properties of a credible precommitment or rule on monetary policy. The theoretical and empirical effects of various monetary rules on real sector variables, such as employment and GNP, have been extensively studied [e.g., Bean (1983); Fischer (1991); Frankel and Chinn (1995)], but their impact on financial markets (and, therefore, aggregate wealth) has received little attention. This is surprising, given the importance of monetary factors for financial markets as documented by, among others, Bailey (1988), Friedman (1988) and Roley (1987).

Recent papers by Svensson (1989) and Boyle and Peterson (1995) have incorporated monetary policy into standard models of asset trade and pricing. In a two-period world, Svensson (1989) identified specific monetary policy rules, and demonstrated that these have significant effects on interest rate risk and the pattern of international trade, but did not examine the implications of these rules for equity values. In an infinite-horizon model, Boyle and Peterson (1995) generalized the monetary policy framework used by Svensson (1989) to allow for imperfect implementation of the chosen policy, but did not address the stock market implications of specific monetary policy targets.
In this paper, we show that one mechanism by which monetary policy can influence equity values is by altering the mean and volatility of future dividends. In particular, we show that an inflation target provides lower dividend volatility than a money growth target, but also lower expected dividends. Consequently, equity values are higher under the inflation target if and only if the market’s intertemporal rate of substitution is sufficiently low, all else equal.

II. The Model

There is a single representative good whose period $t$ output is denoted by $Y_t$, $t \geq 0$. In each period $t$, a fixed proportion, $b_t$, of output is reinvested in a linear production technology. The residual is sold to consumers, with the proceeds paid to investors as a dividend in period $t+1$. Period $t+1$ output is then given by:

$$Y_{t+1} = \lambda_{t+1} \phi(b_Y),$$

where $\phi$ is a constant production technology coefficient, and $\lambda$ is an iid random variable representing a multiplicative shock to the production process.\(^1\) If $\lambda$ also has a normal distribution, then equation (1) implies a geometric random walk process for output, consistent with the evidence of, for example, Bradley and Jensen (1995), Kormendi and Meguire (1990), and Nelson and Plosser (1982).

There is a single representative individual whose period $t$ consumption, $X_t$, is determined by maximization of the expected value of a time-additive, isoelastic, utility function:

$$E_t \left[ \sum_{t=1}^{\infty} \beta^{t-1} X_t^{1-\alpha} \right] \frac{1}{1-\alpha},$$

where $\alpha$ is the reciprocal of the intertemporal elasticity of substitution, and $\beta$ is the subjective discount factor.

Consumption is subject to the standard cash-in-advance constraint of Lucas (1982), so all prices are in monetary units. This constraint is binding if the nominal interest rate is strictly positive, which we henceforth assume. The money supply, $M$, evolves according to:

$$M_t = \omega_t M_{t-1},$$

As in Svensson (1989) and Boyle and Peterson (1995), monetary policy is incorporated by supposing that the money growth factor, $\omega_t$, satisfies:

$$\omega_t = k \theta_t \lambda_t^e,$$

where $k > 0$ is a constant; $\theta_t$ is an iid random variable, and $e = \partial \ln \omega_t / \partial \ln \lambda$ is the monetary response to the realization of the real state of the economy. Thus, in each period, the monetary authority adjusts the money growth factor in response to the realization of

\(^1\) The assumed output process was chosen for its simplicity. A more general structure would allow for diminishing returns to scale via a loglinear production technology, and for stochastic fluctuations in the reinvestment rate. It is straightforward, but tedious, to show that neither of these features would affect our results.
\(\lambda\), according to its chosen policy, \(\varepsilon\). However, because of institutional rigidities or imperfections in observing and measuring \(\lambda\), there remain unplanned fluctuations, \(\theta\), in monetary growth.\(^2\) We assume that output fluctuations cannot forecast monetary policy errors, i.e., \(\text{cov}(\theta_t, \lambda_{t-k}) = 0, \forall k \geq 0\). This ensures that planned money shocks operate independently of unplanned shocks, thereby enabling us to isolate the impact of the former.

The structure described thus far contains eight exogenous variables \(\{\phi, b, \lambda, \beta, \alpha, k, \theta, \varepsilon\}\). By assuming goods and asset market clearing, we can express the endogenous goods and asset prices as functions of these variables. We first examine the goods market. With \(bY_t\) of period \(t\) output reinvested in the production process, only \((1-b)Y_t\) is available for consumption. In equilibrium, period \(t\) real consumption therefore equals \((1-b)Y_t\). Moreover, a binding cash-in-advance constraint implies that period \(t\) nominal consumption equals the period \(t\) money supply. The representative good price, \(P_t\), is therefore given by:\(^3\)

\[P_t = \frac{M_t}{(1-b)Y_t}.\]  

Combining equations (1)–(4) implies that the inflation factor, \(\Pi_t = P_t/P_{t-1}\), satisfies:

\[\Pi_t = \frac{\omega_t}{\phi b \lambda_t} = \frac{k \theta \lambda_t^{\varepsilon-1}}{\phi b}.\]  

Turning to the financial sector, let \(q_t\) be the period \(t\) real price of an equity claim to the stream of future aggregate dividends, \(d_{t+s}\). In equilibrium, this satisfies (see the Appendix for Proof):

\[q_t = \beta E_t[(d_{t+1} + q_{t+1})Y_{t+1}^{-\alpha}]/Y_{t}^{-\alpha}.\]  

Solving equation (6) forward in the usual manner gives the equity price as the expected present value of the sum of future dividends, \(d_{t+s}\), weighted by the marginal rate of substitution between future and current consumption, \((Y_{t+s}/Y_t)^{-\alpha}\). The cash-in-advance constraint, together with equations (1)–(4) and the stochastic structure of \(\theta\) and \(\lambda\), imply that, in equilibrium, these weighted dividends grow at a constant expected rate, so the equilibrium equity price can be simplified to:

\[q_t = \frac{\beta(1-b)(\phi b)^{1-\alpha}Y_tE[1/\theta][E[\lambda^{1-\alpha-\varepsilon}]\]  

\[= G_t H(\varepsilon),\]  

\(^2\) This construct is intended to capture the broad spirit of a cyclical monetary policy framework with low \((\text{var}(\theta)\text{ large})\) or high \((\text{var}(\theta)\text{ small})\) credibility. As our objective is to analyze the implications of cyclical money rules for the stock market, we do not address the reasons for adopting, or the difficulties of implementing, such policies. For a recent discussion of these complex issues, see Canzoneri et al. (1997).

\(^3\) Velocity is therefore constant in this model. Although the theoretical importance of the form of money demand and velocity is undeniable, it seems to be less important empirically. For example, Hodrick et al. (1991) found that calibration of cash-in-advance models predicts a constant velocity even when the underlying theoretical model allows for a variable velocity. As a result, there seems to be relatively little loss of generality in adopting the constant velocity fiction.
where \( G_t = \beta(\phi b)^{1-\alpha}(1 - b)Y_t E[1/\theta]/k(1 - \beta(\phi b)^{1-\alpha}E[\lambda^{1-\alpha}]), \) and \( H(\epsilon) = E[\lambda^{1-\alpha-\epsilon}]. \)

Equation (8) isolates the two components of \( q_t \): the component \( G_t \), which is a function of preference and production parameters, the distributions of \( \lambda \) and \( \theta \), and the current state, but is independent of monetary policy; and the component \( H(\epsilon) \), which is a function of intertemporal substitution, the distribution of \( \lambda \), and monetary policy, but is independent of the current state. It follows that monetary policy affects equity prices only through the term \( H(\epsilon) \).

To compare the effects of alternative monetary policies on \( q_t \), we need to be able to evaluate the exact form of \( H(\epsilon) \). This depends on the distribution of the growth factor, \( \lambda \). As output cannot be negative, we assume that \( \lambda \) is lognormal, so that \( \ln \lambda \) is normally distributed with mean, \( \mu_{\ln \lambda} \), and variance, \( \sigma_{\ln \lambda}^2 \). Using the properties of the lognormal distribution, we have:

\[
q_t = G_t \exp\{(1 - \alpha - \epsilon)\mu_{\ln \lambda} + (1 - \alpha - \epsilon)^2(\sigma_{\ln \lambda}^2/2)\}. \tag{9}
\]

III. Monetary Rules and Aggregate Wealth

As discussed by Frankel and Chinn (1995, p. 319), a monetary rule is generally characterized by the selection of a nominal variable that “... the central bank pledges to keep close to a constant or predetermined path.” In our framework, this can be modeled by supposing that at some initial date, the monetary authority chooses the value of \( \epsilon \) that insulates future realizations of the selected nominal variable from real production shocks, \( \lambda \). Note that this does not preclude stochastic fluctuations in the targeted variable: the actual realization is still random, due to the residual shocks \( \theta \).

We consider two target variables for monetary policy: the rate of money growth and the rate of price inflation. Both rules have received considerable attention from academic and policy economists. The U.S. Federal Reserve attempted to target the rate of money growth during the period 1979–1982; the last decade has seen at least six countries (Canada, Finland, New Zealand, Spain, Sweden, the United Kingdom) adopt the mechanism of inflation targeting.

By equation (3), the monetary authority insulates money growth from real shocks by setting \( \epsilon = 0 \). Similarly, by equation (5), inflation is insulated from real shocks by setting \( \epsilon = 1 \). Equation (9) for the equity price depends on the monetary rule, \( \epsilon \), only through the term \( H(\epsilon) \). Thus, stock market values under the above two monetary rules have the same ranking as the following expressions:

\[
H(\epsilon = 0) = \exp\{(1 - \alpha)\mu_{\ln \lambda} + (1 - \alpha)^2(\sigma_{\ln \lambda}^2/2)\};
\]

\[
H(\epsilon = 1) = \exp\{-\alpha\mu_{\ln \lambda} + \alpha^2(\sigma_{\ln \lambda}^2/2)\}.
\]

For \( C = \mu_{\ln \lambda}/\sigma_{\ln \lambda}^2 \), this yields:

**Proposition.** If \( \alpha > (\leq) \frac{1}{2} + C \), then stock market value is higher (lower) under a monetary rule targeting inflation than under a rule targeting money growth.

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4 Note that this solution for the equity price is equal to the Boyle and Peterson (1995) solution multiplied by \( (\phi b)^{1-\alpha}(1 - b)[1 - \beta(\phi b)^{1-\alpha}E[\lambda^{1-\alpha}]/1 - \beta E[\lambda^{1-\alpha}]] \). This additional term can be interpreted as the net marginal value of capital formation. Nevertheless, because this term is constant, the two solutions are proportional to each other, and all the Boyle and Peterson (1995) results also hold in our model.
Thus, if the representative investor’s intertemporal elasticity of substitution is low, the real value of equity claims to future dividends is higher if the monetary authority targets inflation than if it targets money growth, while the reverse is true if intertemporal substitution is high. This result can be elucidated by comparing the impact of the two rules on the mean and the volatility of future real equity dividends. In general, investors desire high expected real dividends and low volatility. The lower the intertemporal elasticity of substitution, the less tolerant investors are of volatility and, therefore, the greater their aversion to fluctuations in the future dividend stream. By neutralizing the effect of real disturbances on future real equity dividends, the inflation target rule induces lower volatility in the future real dividend stream than does the money rule and, therefore, is assigned a greater stock market value by risk-averse investors, ceterus paribus. However, the money rule will generally provide higher expected real dividends (via Jensen’s Inequality effects), which for sufficiently high intertemporal substitution (i.e., sufficiently low \( \alpha \)) can offset the desire for low volatility in dividends.

Some simple empirical context can be provided for our result by estimation of the parameter \( C \). Although the variable \( \lambda \) is not readily observable, we can exploit the model property that consumption growth is a close proxy.\(^5\) To estimate the latter, we will use seasonally-adjusted real per-capita U.S. consumption of non-durable goods (obtained from the Citibase files), a series which is available on a monthly basis from January 1959. Quarterly sampling of the natural log of the growth factors for this series yields \( \mu_{\ln \lambda} = 0.00312 \) and \( \sigma^2_{\ln \lambda} = 0.000091 \) and, therefore, \( C = 34.3 \). Given that most empirical estimates of \( \alpha \) are significantly less than 30, our analysis suggests that stock market value is likely to be higher under a money rule, at least to the extent that monetary policy affects equity values via the mechanisms identified in our model.\(^6\)

**IV. Concluding Remarks**

By altering the supply of money in response to real conditions, the monetary authority affects both the average level and the volatility of future real equity dividends. As a result, monetary rules targeting a particular nominal variable can have implications for aggregate wealth as well as for the real sector. In this paper, we have shown that the stock market response depends on the intertemporal elasticity of substitution of investors, and on parameters of the real growth process. Consumption growth data indicate that the stock market may prefer a money growth rule to an inflation rule.

The research presented here could be extended in at least two directions. First, a more dynamic role for monetary policy could be considered by allowing it to feed back into the real output distribution (perhaps with a lag), thereby permitting more complex effects on the stock market. Second, extension of our model to an international setting would permit analysis of an exchange rate rule, as in Svensson (1989).

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\(^5\) By equation (1), consumption growth \( x_t = X_t/X_{t-1} = Y_t/Y_{t-1} = \phi \delta \lambda_t \), so \( \ln x_t = \ln \lambda_t + \ln (\phi \delta) \). Hence, \( \text{var}(\ln x_t) = \text{var}(\ln \lambda_t) \) and \( E[\ln x_t] = E[\ln \lambda_t] + E[\ln (\phi \delta)] = E[\ln \lambda_t] \).\(^6\) If we were to allow monetary policy to operate with a lag (e.g., \( o_t = k\theta \lambda_{t-1}^n \) or \( o_t = k\theta \lambda_{t-1}^d \)), then the critical value of \( C \) in the Proposition would change, but the nature of the dividend mean-variance tradeoff would remain unaltered.
Appendix

Proof of Equations (6) and (7)

At time $t$, the representative investor purchases money holdings $M_t$, consumption $X_t$ at price $P_t$, and equity holdings $Z_t$ at price $q_t$. Let $J(P_t, q_t, W_t)$ be the discounted sum of the utilities which the investor can attain if he begins with wealth $W_t$, faces prices $P_t$ and $q_t$, and follows the optimal policy. These functions, $J$, are defined recursively by [for a full discussion of this procedure, see Stokey et al. (1989)]:

$$J(P_t, q_t, W_t) = \max_{(X_t, M_t, Z_t)} U(X_t) + \delta E_t \left[ J(P_{t+1}, q_{t+1}, (d_{t+1} + q_{t+1})Z_t + \Delta M_{t+1}) \right]$$

References


subject to: \[ M_t + q_t Z_t \leq W_t; \] \[ P_t X_t \leq M_t, \] \[ \text{(A1)} \]

where \( U(.) \) is the investor’s single-period utility function; \( E_t[.] \) denotes the time \( t \) expectations operator; \( d_{t+1} \) is the time \( t + 1 \) equity dividend, and \( \Delta M_{t+1} = M_{t+1} - M_t \) is the time \( t + 1 \) monetary injection. Given a positive time \( t \) nominal interest rate, the investor chooses \( M_t \) just equal to his planned time \( t \) nominal consumption, \( P_t X_t \), and the cash-in-advance constraint (equation (2)) is binding in all states. Given non-satiation in all periods, this is also true of the wealth constraint (equation (1)). We can therefore substitute out these constraints, simplifying the recursion to:

\[
J(P_t, q_t, W_t) = \max_{Z_t} V(P_t, q_t, W_t, Z_t) \]

\[ + \delta E_t[J(P_{t+1}, q_{t+1}, (d_{t+1} + q_{t+1})Z_t + \Delta M_{t+1})], \] \[ \text{(A3)} \]

where \( V(P, M) = U(X) \) is the representative agent’s one-period indirect utility function. Differentiating equation (A3) with respect to \( Z_t \) and utilizing the Envelope condition, the first-order condition for the optimal choice of equity is:

\[ q_t U'(X_t) = \beta E_t[(d_{t+1} + q_{t+1})U'(X_{t+1})], \]

which states that, at the optimum, the marginal cost of a unit of equity equals the expected marginal benefit. As the utility function is isoelastic, \( U'(X_t) = X_t^{-\alpha} \). Moreover, as \( X_t = (1 - b)Y_t \) in equilibrium, we have:

\[ q_t = \frac{\beta E_t[(d_{t+1} + q_{t+1})Y_t^{-\alpha}]}{Y_t^{-\alpha}}. \] \[ \text{(6)} \]

Solving equation (6) forward in the usual manner yields:

\[ q_t = \beta Y_t^\alpha \sum_{s=0}^{\infty} \beta^s E_t[d_{t+s+1}Y_{t+s+1}^{-\alpha}]. \]

Nominal dividends equal the nominal proceeds from the previous period’s output sales. The cash-in-advance constraint therefore implies that period \( t + s + 1 \) real dividends are given by \( M_{t+s+1}P_{t+s+1} \). By equations (4) and (2), this equals \((1 - b)Y_{t+s+1}/\omega_{t+s+1}\). Therefore:

\[ q_t = \beta(1 - b)Y_t^\alpha \sum_{s=0}^{\infty} \beta^s E_t\left[ \frac{Y_{t+s+1}^{1-\alpha}}{\omega_{t+s+1}} \right] \]

\[ = \beta(1 - b)(\phi b)^{1-\alpha} Y_t^\alpha \sum_{s=0}^{\infty} \beta^s E_t\left[ \frac{\lambda_{t+s+1}^{1-\alpha} Y_{t+s+1}^{1-\alpha}}{\omega_{t+s+1}} \right] \text{ by equation (1)} \]

\[ = \beta(1 - b)(\phi b)^{1-\alpha} Y_t^\alpha \sum_{s=0}^{\infty} \beta^s E_t\left[ \frac{\lambda_{t+s+1}^{1-\alpha} Y_{t+s+1}^{1-\alpha}}{k_0 \theta_{t+s+1}} \right] \text{, by equation (3)} \]
As realizations of $\lambda$ and $\theta$ are serially and jointly independent, this simplifies to:

$$q_t = \frac{\beta(1 - b)(\phi b)^{1-a}Y_t^a}{k} \sum_{s=0}^{\infty} \beta^s E[Y_{t+s}^{1-a}]$$

$$= \frac{\beta(1 - b)(\phi b)^{1-a}Y_t}{k} \sum_{s=0}^{\infty} \beta^s (\phi b)^{(1-a)s} E[Y_{t+s}^{1-a}]^s.$$  by equation (1)

Hence, if $\beta(\phi b)^{1-a} E[Y_{t+s}^{1-a}] < 1$, $\sum_{s=0}^{\infty} \beta^s (\phi b)^{(1-a)s} E[Y_{t+s}^{1-a}]^s$ is a convergent geometric series, and:

$$q_t = \frac{\beta(1 - b)(\phi b)^{1-a}Y_t E[1/\theta] E[Y_{t+s}^{1-a}]}{k(1 - \beta(\phi b)^{1-a} E[Y_{t+s}^{1-a}])}.$$  (7)