This paper extends previous work on the behavior of the competitive firm under a single source of uncertainty to dual sources of uncertainty. The indirect expected utility function is used to derive symmetry restrictions for a general form of the direct utility function. These restrictions are then used to test for different forms of the utility function and the nature of risk aversion under price and exchange rate uncertainty. The empirical results are consistent with the expected utility maximization hypothesis, and support the existence of a separable utility function. © 1999 Elsevier Science Inc.

Keywords: Price uncertainty; Exchange rate uncertainty; Utility functions

JEL classification: D21, D81, C50

I. Introduction

In the theory of firm behavior under uncertainty, the most widely-analyzed case is that of a firm operating under a single source of risk [Sandmo (1971); Pope (1980); Chavas and Pope (1985); Dalal (1990)]. The basic Sandmo (1971) model was extended to multiple sources of risk by Chavas (1985), who incorporated initial wealth uncertainty, and by Park and Antonoviz (1992a, 1992b), who analyzed output and hedging decisions under spot and futures price uncertainty. This paper continues the above line of analysis by extending the Sandmo (1971) model of a firm operating under price uncertainty in the domestic market to an international firm operating under uncertainty in both the domestic and foreign markets. The source of price uncertainty in the domestic market is similar to the Sandmo (1971) model, and is attributable to demand not being known at the time when output decisions are made. The source of uncertainty in the foreign market is attributable to random exchange rates.1

1 See Clark (1973) for a discussion of why firms cannot avoid exchange rate risk even in the presence of forward currency markets.
A second objective of this paper is to contribute to the empirical literature on uncertainty. There exists a substantial body of theoretical literature on uncertainty which uses the expected utility maximization approach, but the corresponding empirical literature is sparse. This paper conducts an empirical analysis of the risk responsive behavior of international firms faced by domestic price and exchange rate uncertainty and uses the general expected utility maximization approach to determine the empirical form of the utility function [see Park and Antonovitz (1992a, 1992b); Dalal and Arshapalli (1993)]. The issue of empirically determining the form of the utility function cannot be overemphasized in any study of risk responsive behavior. Given that differing assumptions about risk preferences lead to differing economic outcomes, any study of risk responsive behavior must be presumed upon an empirical determination of the utility function.

Two recent papers, Goldberg and Kolstad (1995) and Chavas and Holt (1996), have made useful contributions in developing econometric models to test the economic implications of behavior under multiple sources of risk. Although both Goldberg and Kolstad (1995) and Chavas and Holt (1996) were concerned with economic behavior under multiple risks, the orientation of these papers and the problems they sought to solve are different from this paper. Goldberg and Kolstad (1995) investigated the determinants of production location between domestic and foreign plants under conditions of foreign demand and exchange rate risk, whereas Chavas and Holt (1996) were concerned with the effects of technology and risk preferences on producer behavior under price and production risk. Goldberg and Kolstad (1995) and Chavas and Holt (1996) were not concerned with the theoretical and empirical issues of how domestic price uncertainty and exchange rate uncertainty interact to affect domestic and foreign sales, a key focus of this paper. Whereas the Goldberg and Kolstad (1995) paper simply assumed an empirical form for the utility function, the Chavas and Holt (1996) paper did investigate the empirical form of the utility function, but proposed a specific form of an exponential utility function rather than the general form used here. The Chavas and Holt (1996) paper relied on numerical methods to determine the empirical form of the utility function, whereas the empirical method in this paper relies on using the general expected utility function, the envelope theorem and the properties of the indirect utility function. Chavas and Holt (1996), moreover, were not concerned with deriving, imposing or testing theoretical restrictions involving symmetry and separability conditions, an issue that is treated at length here.

This paper’s contribution lies in extending previous results in firm behavior under a single source of risk to dual risks. The empirical analysis here also provides some insight into certain unresolved issues on the macroeconomic effects of exchange rate risk on trade flows.

II. The Model

The basic model is of a competitive firm which sells a single product in the domestic and foreign markets. Output prices in both the domestic and foreign markets are not known ex ante and are, therefore, random variables. Domestic price uncertainty arises from uncertain demand at the time when production decisions are made, and foreign price uncertainty stems from random exchange rates. The objective of the firm is to maximize the expected utility of profits $E[U(\Pi)]$ given by:

$$\text{Max}_{Q_d, Q_f} E[U(\Pi)] = E[U(P_dQ_d + P_fQ_f - C(w, Q) + H)],$$

(1)
where profits are denominated in domestic currency; \( P_d \) is the random domestic output price; \( P_f \) is the random foreign output price (denominated in domestic currency); \( Q_d \) and \( Q_f \) are domestic and foreign output; total output \( Q = Q_d + Q_f \); \( C \) is a cost function; \( w \) is an input price vector, and \( H \) is a shift parameter with an initial value of zero.\(^2\) The random variables are given by: \( P_i = \bar{P}_i + \sigma_i \epsilon_i \), where \( i = f, d \); \( \bar{P}_i = E(P_i) \); \( \sigma_i \) are shift parameters with initial values of unity, and \( \epsilon_i \) are random variables with \( E(\epsilon_i) = 0 \), \( E(\epsilon_i^2) = 1 \) and \( E(\epsilon_f, \epsilon_d) = 0.\(^3\) The first order conditions for equation (1) are given by:

\[
Z_1 = \left. \frac{\partial E[U(\Pi)]}{\partial Q_f} \right|_{d\alpha = 0} = E[U'(\Pi)(P_f - C_q)] = E[U'(\Pi) F] = 0; \tag{2}
\]

\[
Z_2 = \left. \frac{\partial E[U(\Pi)]}{\partial Q_d} \right|_{d\alpha = 0} = E[U'(\Pi)(P_d - C_q)] = E[U'(\Pi) D] = 0, \tag{3}
\]

where \( C_q = \partial C/\partial Q \). The second order condition, \( \Delta = [Z_{11}Z_{22} - (Z_{12})^2] > 0 \), will be satisfied with \( U'' < 0 \) (risk aversion) and \( C_{qq} \geq 0 \) (non-decreasing marginal cost). Although the second partial derivatives \( (Z_{11}, Z_{22}) \) are unambiguously negative, the sign of \( Z_{12} \) is, in general, indeterminate. However, it is possible to determine the sign of \( Z_{12} \) under certain assumptions. This is related to the notion of the compensated function, which is explained in the next section.

III. Compensated Functions

Chavas and Pope (1985) and Dalal (1990) pointed out that a number of implications of expected utility theory can be stated using compensated functions, and they showed that a number of symmetry restrictions can be expressed in terms of compensated functions. Following Dalal (1990),\(^4\) the compensated or substitution effect is defined as the effect on optimal output when a parameter changes and the shift parameter, \( H \), is adjusted to hold maximum expected utility \( (V) \) constant. Thus,

\[
\left. \frac{dQ}{d\alpha} \right|_{d\alpha = 0} = S_{Q,\alpha} = \left. \frac{\partial Q}{\partial \alpha} \right|_{d\alpha = 0} + \left. \frac{\partial Q}{\partial H} \right|_{d\alpha = 0} \left. \frac{dH}{d\alpha} \right|_{d\alpha = 0}, \tag{4}
\]

where \( Q_i = Q_{i,d}; Q_{i,f} \); \( \alpha = \bar{P}_d, \bar{P}_f, \bar{P}_d, \bar{P}_f, \bar{w}_i, \bar{w}_f \); and \( S_{Q,\alpha} \) represents the substitution effect of a change in \( \alpha \) on \( Q_i \). The cross-price substitution effects are given by \( S_{Q,\bar{P}_d} \) and \( S_{Q,\bar{P}_f} \); the own price substitution effects by \( S_{Q,\bar{P}_d} \) and \( S_{Q,\bar{P}_f} \); and the compensated input effects by \( S_{Q,\bar{w}_d} \) and \( S_{Q,\bar{w}_f} \).

To solve equation (4) proceed as follows. Differentiate equations (2) and (3) with respect to any parameter, \( \tau = \bar{P}_d, \bar{P}_f, \bar{w}_d, \bar{w}_f, \bar{H} \). Thus:

\(^2\) The theoretical analysis would be unaltered with \( H \neq 0 \). \( H \) can be interpreted variously as \( H < 0 \) (fixed costs or a lump sum tax) and \( H > 0 \) (initial wealth or lump sum subsidy). If \( H < 0 \) is interpreted as fixed costs, \( C(w, Q) \) can be interpreted as a variable cost function. See, also, Dalal (1990).

\(^3\) Contrast equation (1) in this paper with the objective function in Chavas and Holt (1996). Chavas and Holt (1996) specified an objective function in which the arguments are not firm profits as in equation (1) but inputs to the production function (corn and soybeans acreage), technology parameters and risk preference parameters.

\(^4\) The definitions of the substitution effect used by Dalal (1990) and Chavas and Pope (1985) are essentially similar.
\[
E \left[ U''(\Pi) F \frac{\partial \Pi}{\partial \tau} \right] + \left( \frac{\partial P_f}{\partial \tau} - C_{qi} \frac{\partial w_i}{\partial \tau} - C_{qf} \frac{\partial Q_f}{\partial \tau} \right) E[U'(\Pi)] = 0; \tag{5}
\]
\[
E \left[ U''(\Pi) D \frac{\partial \Pi}{\partial \tau} \right] + \left( \frac{\partial P_d}{\partial \tau} - C_{qi} \frac{\partial w_i}{\partial \tau} - C_{qf} \frac{\partial Q_f}{\partial \tau} \right) E[U'(\Pi)] = 0, \tag{6}
\]
where \( \partial \Pi/\partial \tau = (Q_f \partial P_f/\partial \tau + P_f \partial Q_f/\partial \tau + Q_d \partial P_d/\partial \tau + P_d \partial Q_d/\partial \tau - C_i \partial w_i/\partial \tau - C_d \partial Q_d/\partial \tau + \partial H/\partial \tau); C_i = \partial C_i/\partial w_i; \) and \( C_{qi} = \partial C_i/\partial w_i. \)

Solving equations (5) and (6) simultaneously gives the expressions for \( \partial Q_f/\partial \alpha \) and \( \partial Q_d/\partial H. \) To determine \( dH/d\alpha \bigg|_{\alpha=0}, \)

note that when \( dv = 0, \) equation (1) implies:
\[
E \left[ U'(\Pi) \left( Q_f \frac{\partial P_f}{\partial \alpha} + P_f S_{Q_f} + Q_d \frac{\partial P_d}{\partial \alpha} + P_d S_{Q_d} - C_i \frac{\partial w_i}{\partial \alpha} + C_{qf} S_{Q_f} + \frac{dH}{d\alpha} \bigg|_{\alpha=0} \right) \right] = 0,
\]
which along with equations (2) and (3) leads to:
\[
\frac{dH}{dP_f} \bigg|_{\alpha=0} = -Q_f \frac{dH}{dP_d} \bigg|_{\alpha=0} = -Q_d \frac{dH}{d\alpha} \bigg|_{\alpha=0} = C_i, \tag{7}
\]

Using equations (5)–(7), the cross-price substitution effect, \( S_{\alpha,\hat{P}_f}, \) is given by:
\[
\frac{dQ_d}{dP_d} \bigg|_{\alpha=0} = S_{\alpha,\hat{P}_d} = \frac{1}{\Delta} \left[ Z_{12} E U'(\Pi) \right] = S_{\alpha,\hat{P}_d}, \tag{8}
\]
where \( S_{\alpha,\hat{P}_d} = S_{\alpha,\hat{P}_d} \) is implied by the symmetry of the cross-price substitution effects. Notice that the sign of \( S_{\alpha,\hat{P}_d} \) and \( S_{\alpha,\hat{P}_d} \) depends on the sign of \( Z_{12}. \) Similarly, the own-price substitution effects are given by:
\[
S_{\alpha,\hat{P}_d} = -\frac{1}{\Delta} \left[ Z_{22} E U'(\Pi) \right] > 0; \ S_{\alpha,\hat{P}_d} = -\frac{1}{\Delta} \left[ Z_{11} E U'(\Pi) \right] > 0, \tag{9}
\]

The own-price substitution effects can be interpreted as the slopes of the compensated supply functions. These are positive and imply that an increase (decrease) in the expected price increases (decreases) compensated output supply. Chavas and Pope (1985) showed that the compensated supply curve is upward sloping in the presence of a single risk, and the results here show that this is preserved even under dual risks.\(^5\)

Now, note that if \( (S_{\alpha,\hat{P}_d} = S_{\alpha,\hat{P}_d}) < 0, \) an expected price increase (decrease) in one market leads to an increase (decrease) in compensated output supply in that market, but leads to a decrease (increase) in compensated output supply in the other market. Thus, domestic and foreign outputs are net substitutes. If \( (S_{\alpha,\hat{P}_d} = S_{\alpha,\hat{P}_d}) > 0, \) then an expected price increase (decrease) in one market leads to an increase (decrease) in compensated

\(^5\) Chavas and Pope (1985) showed the symmetry between compensated input and output functions for a single source of risk. Under dual risks this is given by the following: \( S_{\alpha,\hat{P}_d} = S_{\alpha,\hat{P}_d} + S_{\alpha,\hat{P}_d} = [EU'/(\Pi)\Delta] \) \([Z_{12} - Z_{11}]. \) Using equations (5)–(7), it can be shown that \( S_{\alpha,\hat{P}_d} = -C_{pi} / [EU'/(\Pi)\Delta] \) \([Z_{12} - Z_{11}, \) which implies the symmetry result \( S_{\alpha,\hat{P}_d} = -C_{pi} \). Now, \( C_{pi} = \partial C_{pi}/\partial w_i = X_i \) (input demand) by Shepard’s lemma, and \( C_{qi} \) is the symmetry result \( S_{\alpha,\hat{P}_d} = -C_{pi} \). Thus, \( S_{\alpha,\hat{P}_d} = -X_i / Q_i \) \((S_{\alpha,\hat{P}_d}) \) and \( S_{\alpha,\hat{P}_d} = -X_i / Q_i \) \((S_{\alpha,\hat{P}_d}) \).
output supply in both markets, implying that domestic and foreign output are net complements. These results imply the following:

**Lemma 1:** $Z_{12} < 0$ if and only if $(S_Q, P_f) = (S_d, k)$ implies that $Q_f$ and $Q_d$ are net substitutes (complements).

The notion of output substitutability/complementarity is useful in signing the comparative statics results. These are summarized in Table 1 for a constant absolute risk aversion (CARA) utility function and a separable utility function. The comparative statics results and the symmetry (reciprocity) results in Table 1 constitute refutable hypotheses which are tested in the empirical section.

### IV. Estimating Equations

The indirect expected utility function corresponding to equation (1) is given by $V(\bar{P}_p, \sigma_p, \bar{P}_d, \sigma_d, w, H)$, and the envelope theorem applied to it implies that:

---

6 A negative exponential utility function of the form $U(\Pi) = -e^{-\lambda \Pi}$ exhibits CARA, where $\lambda$ is the coefficient of CARA. The separable utility function is given by $U(\Pi) = \Pi - b(\Pi - E(\Pi))^2$, where $b > 0$. This function has been especially useful in the theoretical and empirical analysis of uncertainty [Pope (1980); Antonovitz and Roe (1986); Park and Antonovitz (1992a)]. The derivation of the comparative statics results reported in Table 1 are available from the author on request.
The parameter restriction for CARA and separability is given by:

\[
(\partial V / \partial \tilde{P} / \partial V / \partial H) = (V_{\tilde{p}}) / (V_{\tilde{n}}) = Q_f, \tag{10}
\]

\[
(\partial V / \partial \tilde{P}_d / \partial V / \partial H) = (V_{\tilde{p}_d}) / (V_{\tilde{n}_d}) = Q_d. \tag{11}
\]

Equations (10)–(11) are uncertainty analogues of Hotelling’s lemma and represent the foreign (export) and domestic supply functions. These supply functions hold for any general unrestricted utility function. The exact functional form for the indirect expected utility function is unknown, but can be approximated by a second-order Taylor series expansion around an expansion point (Z) thus:

\[
V(\tilde{P}, \sigma, \tilde{P}_d, \sigma_d, w, H) = V(Z) + \sum_{i=1}^{6} V_i(Z) D_i + \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} V_{ij}(Z) D_i D_j, \tag{12}
\]

where \( V_1 = V_{\tilde{P}}, \ V_2 = V_{\sigma}, \ V_3 = V_{\tilde{P}_d}, \ V_4 = V_{\sigma_d}, \ V_5 = V_w, \ V_6 = V_H \) and \( V_0 = V_f \) are second partial derivatives. All derivatives are evaluated at the expansion point \( Z \), and \( D_i \) represent deviations from \( Z \). Differentiating equation (12) with respect to \( P, P_d, \) and \( H \), and substituting into equations (10) and (11) yields the estimating equations:

\[
Q_f = \left[ V_1 + \sum_{i=1}^{6} V_{1i}(D_i) \right] \left[ V_6 + \sum_{i=1}^{6} V_{6i}(D_i) \right] ; \tag{13}
\]

\[
Q_d = \left[ V_3 + \sum_{i=1}^{6} V_{3i}(D_i) \right] \left[ V_6 + \sum_{i=1}^{6} V_{6i}(D_i) \right] , \tag{14}
\]

where all partial derivatives are evaluated at the expansion point, and the cross-equation restriction, \( V_{31} = V_{13} \) is imposed in equation (14). For estimation purposes, an input price index \( (w) \), rather than individual input prices, is used and the shift parameter, \( H \), is set equal to its initial value of zero. The estimating equations above are homogeneous of degree zero in the parameters. To identify these parameters, \( V_H = V_6 \) is normalized to 1.

V. Hypothesis Testing

The first hypothesis to be tested is the symmetry restriction \( (\partial Q / \partial \tilde{P}_d) = (\partial Q_d / \partial \tilde{P}_d) \) (see Table 1), which is implied by both CARA and separability. The empirical restriction corresponding to this is given by differentiating equation (10) with respect to \( \tilde{P}_d \) and equation (11) with respect to \( \tilde{P}_d \) and evaluating at the expansion point of the indirect expected utility function. Noting that at the expansion point \( V_H = V_6 = 1 \), the first parameter restriction for CARA and separability is given by:

\[
V_{61} = (V_{o1} V_f) / (V_3), \tag{15}
\]
which is imposable in equations (13) and (14). Next, note that \( Q = (V_P + V_D)(V_p) \). The parameter restriction implied by the second symmetry condition (see Table 1), evaluated at the approximation point, is given by:

\[
V_{11} = \frac{[(V_{13} + V_{53}) - V_{65}(V_{13} - V_{53})]}{[(V_{53} - V_{65}V_{3})]} + \frac{V_{65}V_{1} (V_{1} + V_{3}) - V_{13}}{V_{3}}.
\]

(16)

where the prior restriction (equation (15)) is imposed in equation (16). If the restrictions given by equations (15) and (16) are rejected, then both CARA and separability must be rejected.

The comparative statics of the model provide one source of empirically refutable symmetry restrictions. Additional symmetry restrictions can be easily derived by using the derivatives of the indirect expected utility function [Dalal (1994)]. The indirect function \((V)\) corresponding to the separable utility function is given by:

\[
V(P, \sigma_P, \sigma_D, \sigma_{w}, H) = [\bar{P}Q_{P} + \bar{D}Q_{D} - C(w, Q) + H] - b(Q^{2}_{P}\sigma_{P}^{2} + Q^{2}_{D}\sigma_{D}^{2}),
\]

(17)

where \(\sigma_{P}^{2}\) and \(\sigma_{D}^{2}\) are the variance of the domestic and foreign price distributions. The envelope theorem applied to equation (17) immediately implies that \(V_{P_{1}} = Q_{P}, V_{P_{3}} = Q_{D}, V_{\sigma_{P}} = -2bQ^{2}_{P}\sigma_{P}, V_{\sigma_{D}} = -2bQ^{2}_{D}\sigma_{D}, V_{w} = -\partial C/\partial w = -C_{w}, V_{H} = 1,\) and Young’s theorem implies that:

\[
(V_{P_{1}}, V_{P_{3}}, V_{\sigma_{P}}, V_{\sigma_{D}}) \Rightarrow \frac{\partial Q_{P}}{\partial P_{1}} = \frac{\partial Q_{D}}{\partial P_{3}};
\]

(18)

\[
(V_{\sigma_{P}}, V_{\sigma_{D}}) \Rightarrow \frac{\partial Q_{P}}{\partial \sigma_{P}} = \frac{\partial Q_{D}}{\partial \sigma_{D}};
\]

(19)

\[
V_{hh} = V_{\theta h} = 0 \text{ where } \theta = (\bar{P}, \bar{D}, \sigma_{P}, \sigma_{D}, w, H).
\]

(20)

Now, equations (15) and (16) are valid restrictions for both CARA and the separable utility function. If these restrictions are rejected, then both CARA and separability must be rejected. However, if these restrictions cannot be rejected, then CARA or separability are possible forms for the utility function. To distinguish separability from CARA, impose the additional restrictions for separability given by equations (18), (19) and (20). If these restrictions are rejected, the utility function is consistent with CARA but is not separable. However, if these restrictions cannot be rejected, then the utility function is separable and does not display CARA.

VI. Data Description and Stationarity Properties of the Data

All required data for this study were drawn from the Monthly Abstract of Statistics and British Business. The data pertain to one of the United Kingdom’s most important
industries—the Chemicals industry—and is available from 1974Q1–1987Q4. Data on total sales ($Q$) and domestic sales ($Q_d$) is available from various issues of British Business; foreign sales ($Q_f$) was calculated as the difference between the two. The expected price series, ($\hat{P}_d, \hat{P}_f$) was calculated as a four-quarterly moving average of the chemical industry’s domestic and foreign price indexes, and a moving standard deviation of $\hat{P}_d, \hat{P}_f$ was used to generate $\sigma_d$ and $\sigma_f$. A price index of materials and fuel purchased by the chemicals industry was used as the input price index ($w$).

The notion that British firms in the chemicals industry face domestic and foreign price uncertainty needs to be elaborated. Domestic price risk arises from random domestic demand, whereas foreign price risk arises from random exchange rates. Available data indicate that over the period 1979–1987, at least 47% (by value) of the chemical industry’s export contracts were denominated in foreign currency and, thus, exposed to currency risk. Of these, only 11% were covered by forward contracts. Thus, by and large, British firms were exposed to foreign price uncertainty.

Before conducting empirical tests, it was necessary to determine the stationarity properties of the data. The results of a Dickey-Fuller unit root test indicated the following (Dickey-Fuller) test statistic values: $\hat{P}_f = -2.52; \hat{P}_d = -3.60; \sigma_f = -3.74; \sigma_d = -2.05; w = -2.91$. The applicable MacKinnon critical value (5% level) is $-1.95$. Clearly, the hypothesis of a unit root was rejected for all variables; these are therefore stationary variables.

VII. Empirical Results

Equations (13) and (14) were estimated for the Chemicals industry of the United Kingdom for the period 1974Q1–1987Q4, using a Seemingly Unrelated Regression Estimation (SURE) procedure. The results of this estimation are reported in Table 2, and constitute the parameter estimates of the Unrestricted Model. The log of the likelihood function of the Unrestricted Model is $-629.79$.

A useful check on the appropriateness of the model specification is to test if the empirical form of the indirect utility function satisfies concavity or convexity conditions. The indirect utility function, $V$, is quasi-convex in prices. This convexity requirement was satisfied at the point of approximation using the estimates of the Unrestricted Model in Table 2. The satisfaction of this requirement provides reassuring evidence that the utility function is well-behaved and there are no model specification errors.

The first hypothesis to be tested was the symmetry restrictions implied by both CARA and separability. The empirical restrictions implied by this, equations (15) and (16), were imposed on equations (13) and (14), and the resulting parameter estimates are reported as Restricted Model 1 in Table 2. The log likelihood function of this constrained model was $-632.30$. The calculated test statistic was $5.02$, and as this is less than the critical $\chi^2$ value of $5.99$, the hypothesis that CARA and separability hold could not be rejected. Consequently, Model 1 was a maintained hypothesis for all subsequent tests.

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13 Data is not available beyond this point because of rebased indexes. Later data is therefore not comparable with earlier data.
14 The data indicate that the coefficient of variation is 33% for the domestic price index and 31% for the foreign price index over the period covered by the study. This is, by any measure, a high level of volatility.
15 The test statistic is given by $-2 \ln \lambda$, where $\lambda$ is the ratio of the likelihood functions for the restricted and unrestricted models. Under the null, this is distributed as $\chi^2$ with degrees of freedom equal to the number of independent restrictions.
The next hypothesis to be tested was the hypothesis that the direct utility function is separable but does not display CARA. Imposition of the additional restrictions required by separability (equations (18), (19) and (20)), led to Restricted Model 2 (see Table 2). The log likelihood was $-2634.50$, and the calculated test statistic was $4.40$ (calculated by treating Model 1 as the maintained hypothesis). As the critical $x^2$ value with four degrees of freedom at the 5% level is $9.49$, the hypothesis that the direct utility function is separable could not be rejected.

The empirical results imply that the data is best described by a direct utility function that is separable. We, therefore, turn to an analysis of this form and the implications for comparative statics results. The estimated form for the separable function (Restricted Model 2 in Table 2) implies that the signs of the parameters are as follows: $V_{\hat{P},\hat{P}} > 0$.

### Table 2. Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unrestricted Model</th>
<th>Restricted Model 1</th>
<th>Restricted Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\hat{P}} [V_1]$</td>
<td>1389.77 (29.25)</td>
<td>1395.52 (32.21)</td>
<td>1369.64 (27.86)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{1,1}]$</td>
<td>4.60 *</td>
<td>18.67 (4.63)</td>
<td></td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{1,2}]$</td>
<td>18.87 (6.24)</td>
<td>-24.70 (9.43)</td>
<td>-14.28 (4.72)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{1,3}]$</td>
<td>-22.33 (7.64)</td>
<td>-14.23 (9.48)</td>
<td>-3.04 (4.72)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{1,4}]$</td>
<td>-98.70 (66.13)</td>
<td>-17.10 (87.34)</td>
<td>22.14 (15.38)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{1,5}]$</td>
<td>20.24 (10.94)</td>
<td>11.38 (13.71)</td>
<td>-5.53 (1.81)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{6,1}]$</td>
<td>-0.01 (0.039)</td>
<td>*</td>
<td>0</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{6,2}]$</td>
<td>0.02 (0.0333)</td>
<td>-0.0075 (0.0395)</td>
<td>0</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{6,3}]$</td>
<td>-0.064 (0.0040)</td>
<td>-0.0085 (0.0042)</td>
<td>0</td>
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<tr>
<td>$V_{\hat{P},\hat{P}} [V_{6,4}]$</td>
<td>-0.0680 (0.0419)</td>
<td>-0.0184 (0.0548)</td>
<td>0</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{6,5}]$</td>
<td>0.0135 (0.0063)</td>
<td>0.0074 (0.0080)</td>
<td>0</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{3,2}]$</td>
<td>2131.09 (30.03)</td>
<td>2179.84 (22.94)</td>
<td>2143.87 (27.90)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{3,3}]$</td>
<td>50.53 (72.21)</td>
<td>-5.75 (85.66)</td>
<td>7.20 (7.93)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{3,4}]$</td>
<td>-15.92 (6.72)</td>
<td>-19.16 (7.80)</td>
<td>1.78 (4.92)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{3,5}]$</td>
<td>-163.01 (88.75)</td>
<td>-72.49 (115.18)</td>
<td>22.51 (13.91)</td>
</tr>
<tr>
<td>$V_{\hat{P},\hat{P}} [V_{3,6}]$</td>
<td>26.71 (13.40)</td>
<td>15.90 (16.63)</td>
<td>*</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-629.79</td>
<td>-632.30</td>
<td>-634.50</td>
</tr>
<tr>
<td>Restrictions</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Calculated $\chi^2$</td>
<td>—</td>
<td>5.02</td>
<td>4.40</td>
</tr>
<tr>
<td>Critical $\chi^2$ (5%)</td>
<td>—</td>
<td>5.99</td>
<td>9.49</td>
</tr>
</tbody>
</table>

Notes: Asymptotic standard errors are in parentheses. * See text for appropriate restrictions.
\( V_{P,t} < 0, V_{P,t} = V_{P,t} < 0, V_{P,t} > 0, V_{P,t} < 0, V_{P,t} > 0, V_{P,t} > 0, V_{P,t} < 0, \) and
\( V_{P,t} < 0. \) All these signs are unambiguously implied by theory (see Table 1, column 4), and all estimated parameters are in agreement with theoretical predictions for a separable utility function. The result that the utility function under uncertainty is best described by a separable function is in broad conformity with previous results by Antonovitz and Roe (1986) and Park and Antonovitz (1992a).

The empirical results imply that an increase in the expected price in any market leads to a supply increase in that market \( [V_{P,t} > 0, V_{P,t} > 0] \). Thus, supply curves are upward sloping even under dual sources of risk. The sign of the cross-price derivative is negative \( [V_{P,t} = V_{P,t} < 0] \), implying that an increase in the expected price in one market leads to a supply decrease in the other market. Thus, domestic and foreign output are net substitutes. An increase in risk in any market leads to a supply decrease in that market \( [V_{P,t} < 0, V_{P,t} < 0] \), but as domestic and foreign output are substitutes, supply increases in the other market \( [V_{P,t} > 0, V_{P,t} > 0] \). An increase in input prices leads to a supply decrease in both domestic and foreign markets \( [V_{P,t} < 0, V_{P,t} < 0] \).

These results have important implications for the empirical literature on exchange rate uncertainty. The focus in this literature is on the macro effects of exchange rate risk, but there is no consensus on the effects of exchange rate risk on the volume of foreign trade [Pozo (1992)]. This paper has examined the effect of uncertainty at the microeconomic level and concludes that an increase in uncertainty in the foreign market does reduce exports. However, it is possible that for a different risk preference function an increase in foreign market uncertainty need not necessarily reduce exports. If the utility function is consistent with CARA, for instance, an unambiguous conclusion regarding the effect of an increase in risk on output cannot be inferred (i.e., \( \partial Q_f/\partial s_f \equiv 0; \partial Q_d/\partial s_d \equiv 0 \)). The effect of risk preferences on output decisions under uncertainty, which are typically ignored in studies on the macro effects of exchange rate risk, may explain why these studies are unable to make a clear finding regarding the effect of uncertainty on the volume of trade.

VIII. Concluding Remarks

This paper has extended the Sandmo (1971) model of a competitive firm operating under a single source of uncertainty to an international firm operating under price uncertainty in the domestic market and exchange rate uncertainty in the foreign market. Symmetry restrictions were derived for a general form of the utility function, and these were then used to test different hypotheses regarding risk-averse firm behavior. The empirical results are consistent with the expected utility maximization hypotheses of firm behavior, and support the existence of a separable utility function. This paper shows that ad hoc assumptions about risk-averse behavior are unwarranted, as it is relatively straightforward to determine the empirical form of the utility function.

I would like to express my deep appreciation and thanks to Professor Ardeshir Dalal. I am grateful to Sherrill Shaffer (editor) and Mike Tansey for many constructive comments on earlier drafts of the paper. This paper was

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16 The estimated value for \( V_{P,t} \) is \(-2.24\), with a standard error of 1.92.
partly written while I was a consultant to the International Trade Division of the World Bank. I would like to thank its members (especially Anil Lal) for many helpful discussions.

References


