Structural Uncertainty and Breakpoint Tests: An Application to Equilibrium Velocity

John B. Carlson, Ben Craig, and Jeffrey C. Schwarz

The recent breakdown in the relationship between M2 velocity and its opportunity cost has raised serious questions about the reliability of money measures as policy targets or indicators. We apply newly developed breakpoint procedures to examine the stability of the velocity relation. The tests provide a rigorous basis for estimating and testing break dates. Our results lead us to conclude that there were two breaks: one in 1981 and one in 1991. These estimated breaks occur around events that one might expect would affect velocity. Another important finding is that the estimated opportunity cost elasticity increases across the three periods, but most substantially between the last two periods, when it doubles. © 2000 Elsevier Science Inc.

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I. Introduction

During the 1990s, the reliability of money measures as policy targets and indicators was called into question. It became clear that the simple relationship between M2 and nominal income had become permanently altered. Although the Humphrey–Hawkins Act requires that the Federal Reserve specify growth ranges for money and credit, their role has been greatly diminished. In July 1993, Alan Greenspan, Chairman of the Board of Governors of the Federal Reserve, reported that “at least for the time being, M2 has been downgraded...
as a reliable indicator of financial conditions in the economy, and no single variable has yet been identified to take its place.\textsuperscript{1}

\(M_2\)'s breakdown is evident in the pattern of its velocity. Figure 1 reveals that for more than 20 years, \(M_2\) velocity varied around a moderately increasing trend. Although the deviations from trend were typically persistent, they were systematically related to changes in the opportunity cost of \(M_2\)—measured here as the simple difference between the 3-month Treasury bill rate and the share-weighted rate paid on \(M_2\) components. In the early 1990s, however, \(M_2\) velocity increased substantially above its trend, while \(M_2\) opportunity cost essentially fell. Thus, the simple relationship between \(M_2\) velocity and its opportunity cost clearly appears to have been subject to structural change.

The role of money in policy is often justified on the basis of the quantity theory, which presumes that the velocity of money is stable and relatively predictable.\textsuperscript{2} If velocity is neither stable nor predictable, then the reliability of \(M_2\) as a policy target or indicator is seriously impaired. The breakdown in the relationship between \(M_2\) velocity and its opportunity cost casts doubt on both the stability and predictability of \(M_2\) velocity.

Below we apply some recently developed techniques to test for structural change in the relationship between \(M_2\) velocity and its opportunity cost. Our purpose is to examine in a rigorous manner whether the recent breakdown was unique or whether the velocity relation has been plagued by numerous structural changes. We also seek to identify dates at which any breaks occur. If structural change occurs near an event that is likely to have

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{M2 Velocity and Opportunity Cost.}
\end{figure}

\begin{footnotesize}
\footnote{This is illustrated clearly by the \(P^*\) indicator of inflation proposed by Hallman et al. (1991), which we discuss below. Early versions of the \(P^*\) approach assumed that velocity reverted to some constant mean level \(V^*\) in the long run.}
\end{footnotesize}
a predictable effect on velocity (such as deregulation), then policymakers may be able to anticipate such effects and adjust policy prescriptions in an appropriate manner.

Our choice of testing procedures is explained in Section II. In Section III, we present our results. We find evidence of at least two breaks. Because the testing procedures estimate breakpoint dates, we are able to associate them with events that provide an economic basis for explaining the structural change. Importantly, we find that the elasticity of opportunity cost has increased over time. In Section IV, we compare our results with the findings of Orphanides and Porter (1998). This comparison suggests that the structural change in the 1990s did not occur abruptly at a single date but evolved gradually over a period of several years. The implications for policy and the limitations of our analysis are discussed in the concluding remarks.

II. Testing for Structural Change

The seminal work of Chow (1960) and Quandt (1992) and the CUSUM test of Brown et al. (1994) focused on testing for structural change at a single specified (hence known) break date. Recently, however, an extensive literature has led to the development of methods that allow for estimation and testing of structural change at unknown break dates. These include tests proposed by Andrews (1993), Andrews and Ploberger (1994), Andrews et al. (1996), Liu et al. (1997), and Bai and Perron (1998). The procedures proposed by Bai and Perron are most directly applicable for our purposes. We describe the procedures and their advantages more thoroughly in the appendix.

A key feature of the Bai and Perron test is that it allows us to test for multiple breaks at unknown dates. It is well known that the effects of financial innovation and deregulation on money are difficult to model. Thus, simple frameworks for velocity—like the velocity- 
opportunity cost relation—typically do not incorporate explanatory variables to control for such factors. The Bai–Perron test thus allows us to examine whether the omission of such relevant factors has frequent implications for the stability of velocity.

Another advantage to the Bai–Perron procedure is that it allows us to investigate breaks in subsets of parameters in the velocity relation. For example, it may be that financial innovation has affected the mean level of velocity but not the elasticity of opportunity cost. Our choice of procedures allows us to compare estimated break dates under alternative specifications about what parameters are affected by structural change.

We should note that Bai and Perron develop two procedures to test the hypotheses of \( l + 1 \) breaks given \( l \) breaks. The first, a purely sequential procedure, bases each null hypothesis on the previous significant break date generated. Therefore, under the null hypothesis of \( l \) breaks, breakpoint dates are not necessarily a global minimizer of the sum of squared residuals. The second procedure is based on \( l \) break dates that are global minimizers of the sum of squared residuals—we refer to this procedure as the sequential test under the global null. We follow the recommendation of Bai and Perron and give most weight to the sequential test under the global null.

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4 The tests proposed by Andrews (1993) and Andrews and Ploberger (1994) do not test for multiple breakpoints at unknown dates. Andrews et al. (1996) consider multiple structural changes but require a known variance. Liu et al. (1997) also test for multiple unknown change points but considers only the pure structural change case where all parameters are subject to shifts.
III. A Specification of Equilibrium Velocity

The role for $M_2$ as a useful anchor for the price level was demonstrated in the $P^*$ model developed by Hallman et al. (1991). Simply put, $P^*$ is the eventual price level implied by the current level of the $M_2$ monetary aggregate. It is defined as:

\[ P^* = \frac{MV^*}{Q^*}, \]

where $V^*$ is an estimate of the long-run value (equilibrium) of the GDP velocity of $M_2$—assumed to be constant and equal to its mean value from 1955Q1 to 1988Q4—and $Q^*$ is an estimate of potential output. The approach was also useful for understanding the monetary dynamics of inflation. When the current price level, $P$, deviates from its equilibrium level, $P^*$, inflation tends to adjust so as to close this gap—the price gap.

Time-Varying Velocity

It is clear that the original $P^*$ approach was vulnerable to shifts in equilibrium velocity. Indeed, when $M_2$ velocity jumped sharply in the early 1990s, the price-gap framework began to persistently under-predict inflation. In an effort to remedy the problem, Orphanides and Porter (1998) proposed a more general approach to specifying equilibrium velocity. They begin with the simple notion that in the short run, velocity varies with the opportunity cost of money:

\[ V = V^* + a_1 \hat{OC} + e, \]

where $\hat{OC}$ denotes deviations of the opportunity cost of money, $OC$, from its average norm, $a_1$ is the elasticity of velocity with respect to opportunity cost, and $e$ is a stationary mean zero error term. In the case where $V^*$ is constant, it can be estimated as the intercept of the population regression.

The experience of the early 1990s, however, suggests that the assumption of a constant velocity is untenable. In fact, the evidence suggests that equilibrium velocity increased in a permanent way. Moreover, a minor change in the definition of $M_2$ had an important implication for $M_2$ velocity. Specifically, when measured on the basis of the new definition, $M_2$ velocity exhibited a modest upward trend prior to 1990. To account for these elements, Orphanides and Porter posit

\[ V = a_0 + a_1 \hat{OC} + a_2 Time + a_3 D(\tau) + e, \]

where $V^* = a_0 + a_2 Time + a_3 D(\tau)$, $Time$ is a simple time trend, and $D$ is a dummy variable defined parametrically on an unknown quarter, $\tau$, such that it equals 0 before quarter $\tau$ and 1 thereafter. A time trend is often included as a proxy variable to account for the effects of financial innovation on the trend of velocity.

As discussed in Section II, the Bai–Perron procedures are particularly well suited to identify and test for such multiple breaks (allowing for an unknown number of Ds). Moreover, the procedures also allow us to test for breaks in any subset of the parameters. The specification above exhibits a high degree of first-order autocorrelation. Thus, we include lagged velocity as a regressor.

\[ ^5 \text{For a description of the newly defined } M_2 \text{ see Whitesell and Collins (1996)} \]
This specification accommodates a gradual impact of parameter shifts at each break date. For example, a one-time shift in the intercept at time $t_0$ has an initial impact of $D(t_0)$ but a long-run impact of $D(t_0)/(1 - a_4)$.

### IV. The Results

We test for three potential sets of breaks: intercept only; intercept and coefficient of opportunity cost; and intercept and coefficients of opportunity cost and time trend. Panel A in Table 1 presents test statistics and estimated coefficients for the case in which only the intercept is allowed to change. Three break dates are found—1970Q2, 1981Q1, and 1991Q4—using the Bai–Perron sequential test under the global null. The estimation package provided by Bai and Perron also includes test statistics based on an unconditional
null, a purely sequential test, Bayesian Information Criteria (BIC), and Liu, Wu, and Zidek (LWZ) tests. The BIC test also finds three breaks, but the purely sequential test and the LWZ test reveal only one break, in 1991Q4.

Following Bai and Perron, we feature the results of the sequential test under the global null. Thus, we conclude that there are three breaks for the case in which only the intercept is allowed to shift. Figure 2A illustrates the estimates of both the long- and short-run equilibrium trends for M2 velocity evaluated at the mean value of opportunity cost based on the Bai–Perron estimated breakpoint dates. The short-run paths are based on the estimated AR(1) process.

The results for the tests allowing for shifts in both the intercept and coefficient of opportunity cost are provided in panel B. Again, three breaks are found using the

<table>
<thead>
<tr>
<th>Parameter estimates using sequential test under the global null</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{M2 \text{ vel.(t-1)}}$</td>
<td>0.676750 (0.0395)</td>
</tr>
<tr>
<td>$\beta_{\text{time}}$</td>
<td>0.000855 (0.0002)</td>
</tr>
<tr>
<td>$\delta_{\text{M2 opp. cost.1}}$</td>
<td>0.007314 (0.0021)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.1}}$</td>
<td>0.510616 (0.0636)</td>
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<tr>
<td>$\delta_{\text{M2 opp. cost.2}}$</td>
<td>0.009900 (0.0016)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.2}}$</td>
<td>0.483295 (0.0617)</td>
</tr>
<tr>
<td>$\delta_{\text{M2 opp. cost.3}}$</td>
<td>0.010447 (0.0022)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.3}}$</td>
<td>0.459063 (0.0596)</td>
</tr>
<tr>
<td>$\delta_{\text{M2 opp. cost.4}}$</td>
<td>0.021528 (0.0049)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.4}}$</td>
<td>0.496657 (0.0624)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter estimates using purely sequential test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{M2 \text{ vel.(t-1)}}$</td>
<td>0.689653 (0.0380)</td>
</tr>
<tr>
<td>$\beta_{\text{time}}$</td>
<td>0.000348 (0.0001)</td>
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<tr>
<td>$\delta_{\text{M2 opp. cost.1}}$</td>
<td>0.009923 (0.0013)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.1}}$</td>
<td>0.485983 (0.0602)</td>
</tr>
<tr>
<td>$\delta_{\text{M2 opp. cost.2}}$</td>
<td>0.023837 (0.0047)</td>
</tr>
<tr>
<td>$\delta_{\text{constant.2}}$</td>
<td>0.530953 (0.0624)</td>
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</tbody>
</table>

* Time, opportunity cost of M2, and 1-quarter-lagged M2 velocity were used as regressors in all models. Those variables not mentioned explicitly as changing (δ coefficients) were included as constant-coefficient regressors (β coefficients). In all cases, a minimal break fraction of 0.15 was used, corresponding to a minimal regime length of 21 periods.

** Indicates significance at the 5% significance level.

** Indicates significance at the 1% significance level.
sequential test under the global null. The estimated coefficient of opportunity cost increases over subsequent periods between breaks. The purely sequential test and BIC tests indicate one break, while the LWZ test reveals no break. Estimated dates are the same as for the corresponding breakpoints in which only the intercept shifts. Figure 2B reveals that the implications for the three-break result in equilibrium velocity are nearly identical to those for the intercept shift only.

When we allow for shifts in the intercept and coefficients of opportunity cost and time trend, we find only two shifts: 1981Q1 and 1991Q4 (see panel C). Again estimated opportunity cost elasticity increases in subsequent periods. The implications of the two-break result for trend velocity are presented in Figure 2C. The estimated time trend increases sharply between the two estimated breakpoints. It is notable, however, that after 1991Q4, velocity exhibits no statistically significant time trend.

To summarize, all tests find a sizable upward shift in 1991Q4. This large break is generally consistent with other estimates of the shift in velocity [see Orphanides and Porter (1998), Whitesell (1997), and Carlson et al. (1999)]. The preferred test (sequential

**Table 1.** (Continued)

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>C. Intercept and coefficients of opportunity cost and time vary.*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Global</strong></td>
<td></td>
</tr>
<tr>
<td>SupFₐ(1)</td>
<td>83.0316**</td>
</tr>
<tr>
<td><strong>Sequential under the global null</strong></td>
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<tr>
<td>SupFₐ(2</td>
<td>1)</td>
</tr>
<tr>
<td>SupFₐ(3</td>
<td>2)</td>
</tr>
<tr>
<td>SupFₐ(4</td>
<td>3)</td>
</tr>
<tr>
<td><strong>Purely sequential</strong></td>
<td></td>
</tr>
<tr>
<td>SupFₐ(2</td>
<td>1)</td>
</tr>
<tr>
<td>SupFₐ(2</td>
<td>1)</td>
</tr>
</tbody>
</table>

Number of breaks selected by:

- BIC 3
- LWZ 1

Parameter estimates using sequential test under the global null

\[
\begin{align*}
\hat{\beta}_{M2 \text{ vel.(t-1)}} &= 0.714851 (0.0406) \\
\delta_{\text{time,1}} &= 0.000296 (0.0001) \\
\delta_{\text{M2 opp. cost,1}} &= 0.010862 (0.0015) \\
\delta_{\text{constant,1}} &= 0.443949 (0.0649) \\
\delta_{\text{time,2}} &= 0.001140 (0.0002) \\
\delta_{\text{M2 opp. cost,2}} &= 0.011093 (0.0024) \\
\delta_{\text{constant,2}} &= 0.364976 (0.0664) \\
\delta_{\text{time,3}} &= -0.000178 (0.0004) \\
\delta_{\text{M2 opp. cost,3}} &= 0.024946 (0.0045) \\
\delta_{\text{constant,3}} &= 0.545612 (0.0592)
\end{align*}
\]

Parameter estimates using purely sequential test

Same as sequential under the global null

* Time, opportunity cost of M2, and 1-quarter-lagged M2 velocity were used as regressors in all models. Those variables not mentioned explicitly as changing (δ coefficients) were included as constant-coefficient regressors (β coefficients). In all cases, a minimal break fraction of 0.15 was used, corresponding to a minimal regime length of 21 periods.

* Indicates significance at the 5% significance level.

** Indicates significance at the 1% significance level.
Figure 2. Estimates of Breakpoint and Trends in GDP Velocity of M2* (Bai-Perron Sequential Test under a Global Null Hypothesis) A) Intercept only varies. B) Intercept and coefficient of opportunity cost vary. C) Intercept and coefficients of opportunity cost and time vary. Time, opportunity cost of M2, and 1-quarter-lagged M2 velocity were used as regressors in all models. Those variables not mentioned explicitly as changing (β coefficients) were included as constant-coefficient regressors (β coefficients). In all cases, a minimal break fraction of 0.15 was used, corresponding to a minimal regime length of 21 periods. Dotted lines represent short-run adjustments to equilibrium, solid lines are long-run equilibrium.
under the global null) finds a smaller downward shift in the intercept in 1981Q1 for tests of parameter changes in all three specifications. In the two cases where the coefficient of opportunity cost is allowed to change, we estimate that the elasticity of opportunity cost increases after each breakpoint, regardless of the number of estimated breaks.

Discussion

Taken together, we conclude that our results provide strong evidence of two breaks, one in early 1981 and one in early 1991. Although we find an additional break when we do not allow for a change in the coefficient of the time trend, we believe that this result is spurious.

One interpretation of the coefficient of the time trend is that it is a measure of the degree to which financial innovation affects velocity. Because estimates of this coefficient vary substantially in the three periods between the two breakpoints in the most flexible specification, we conclude that it is inappropriate to restrict the coefficient to one value across all periods.

Another important finding is that the estimated opportunity cost elasticity increases across the three periods, but most substantially between the last two periods, when it doubles. Such a result is consistent with the proliferation of financial instruments, which have likely increased the sensitivity of $M_2$ demand (and hence of $M_2$ velocity) to fluctuations in its opportunity cost. This has important implications for policymakers. Given that the own rates paid on $M_2$ components respond sluggishly to changes in market rates, $M_2$ velocity has become much more sensitive to changes in interest rates induced by policy actions that change the federal funds rate.

Estimated breakpoint dates in the early 1980s and 1990s correspond to well-documented events that are consistent with structural changes in the equilibrium velocity relation. The downward break in the intercept of the equilibrium velocity relation in 1981Q1 occurs around the implementation of banking deregulation that ultimately eliminated interest rate ceilings. It was expected at that time that such deregulation would enhance banks’ abilities to compete with nonbank intermediaries for funds and thereby induce a portfolio reallocation from nondepository instruments (not included in $M_2$) to bank instruments, particularly checking, savings, and time deposits. Indeed, many models of $M_2$ demand include an intercept dummy to account for a one-time shift in $M_2$ velocity around this time. Our test result provides some statistical evidence in favor of such a practice.

The upward break in velocity in the early 1990s is now well understood to be related to a break in $M_2$ demand. Numerous studies have found an association between the $M_2$ demand shortfall and the meteoric growth in stock and bond mutual funds, which are not included in $M_2$. The consensus of this literature is that mutual funds are relatively close substitutes for time deposits, and what occurred in the early 1990s was a massive reallocation in household portfolios from time deposits to bond funds and, to a lesser extent, stock funds. The question remains, however, why such a reallocation occurred at the time it did. Carlson et al. (1999) conclude that the timing largely reflected the confluence of two factors. First, innovations reduced transactions costs of mutual funds and increased their accessibility to households. These instruments allowed individuals to buy into a diversified portfolio of long-term bonds, while maintaining some liquidity with check-writing privileges. These features made them increasingly attractive as substitutes
for time deposits. Second, beginning in the late 1980s, many depositories, especially thrift institutions, found themselves in poor financial condition. A large number of such institutions failed to meet minimum capital standards and hence were constrained from acquiring assets and thus from competing aggressively for funds (see Carlson and Parrott 1991).

Carlson et al. argue further that the restructuring of depositories acted as a catalyst in the development of mutual funds, especially bond mutual funds. Bond funds are subject to capital losses in the short run, but in the long run they yield relatively higher rates than deposit instruments. When short-term interest rates began falling in 1989, the mutual fund industry intensified marketing strategies that informed households about bond funds, which were yielding significantly higher returns. Capital-constrained thrifts were effectively limited in their pricing responses. As a result, many households, apparently for the first time, diversified their portfolios out of $M_2$ deposits into bond mutual funds. It appears now that, for many of these households, bond funds have become a permanent and significant part of their portfolios, thus supplanting bank CDs.6

V. Comparison with Orphanides and Porter

Whereas we seek to identify structural break dates based on rigorous estimation and testing procedures, Orphanides and Porter seek to estimate the level of equilibrium $M_2$ velocity recursively in order to forecast inflation. More precisely, they apply a regression tree procedure to estimate changes in equilibrium velocity. Their recursive approach yields a series of real-time estimates of $V^*$, which in turn can be applied to forecast inflation within the $P^*$ framework. Their forecasting exercise begins in 1989.

Although our approach is not designed to accommodate such forecasts, we view our results as complementary and supportive of the regression tree results. Our estimates of equilibrium velocity are adjusted to accommodate structural changes and hence are based on rigorous testing procedures. As such, they essentially provide a benchmark against which to compare regression tree estimates.

Estimated breakpoint dates suggest that a permanent upward shift in velocity occurred around 1991, early in the forecast horizon examined by Orphanides and Porter. Among the alternative velocity estimates, their full regression tree approach generates the most comparable path for velocity. As with theirs and other studies, our results indicate that equilibrium velocity has stabilized in recent years to a level about 20 percent higher than it was in the 1980s.

As noted above, a key advantage to the regression tree approach is that it allows for real-time estimates of equilibrium velocity that can be applied to out-of-sample forecasts of inflation in the $P^*$ framework. Orphanides and Porter compare such forecasts based on regression tree estimates with forecasts based on alternative estimates of equilibrium velocity, including the original approach, which assumes $V^*$ is constant. The error statistics reveal that the full regression tree estimates of $V^*$ provide superior forecasts over the alternative models. Table 2 reveals that inflation forecasts based on the assumption of

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6 In principle, one would expect that substitution from time deposits to bond funds should be associated with a corresponding change in the relative returns. Feinman and Porter (1992), Mehra (1997), and Lown and Rich (1997) examine evidence on this issue. Mehra and Feinman and Porter develop more comprehensive measures of opportunity cost that include capital market yields as alternative rates. These approaches, however, also fall short as a complete explanation.
constant velocity substantially underpredict inflation, while the forecasts based on the full regression tree are unbiased since 1990. Our estimates of $V^*$ are based on the complete history of velocity. They are in this sense backward looking. Nevertheless, we believe that a comparison of inflation projections based on our estimates with those of Orphanides and Porter yields insight into the nature of the structural change. Table 2 illustrates that projections based on our estimates tend to overpredict inflation. Indeed, our approach appears to overcompensate, yielding, on average, an overprediction of inflation over the same horizon. One might expect such an outcome if the estimated break date was too early, or if a break occurred at the estimated date but evolved as a smooth process over some interval. In both cases $V^*$ would be overstated initially, implying more inflationary pressure than otherwise predicted.

Discussion

We stress that our estimates of equilibrium velocity are not designed for forecasting. Nevertheless, we conjecture that the poor forecasting performance probably reflects that structural change in M2 velocity was one that occurred over a period of several years, reflecting, in part, the diffusion of the knowledge and accessibility of newly developed financial instruments. Under the Bai–Perron test hypothesis, multiple breaks can occur but only at dates separated in time. Thus, by design the procedure cannot test for structural change that occurs over an interval of two or more contiguous periods, i.e., a smooth change. The regression tree results suggest that the change in the trend of equilibrium velocity evolved over a period of at least two years before it stabilized. Although the inclusion of lagged velocity in our specification allows us to specify a short-run path for the adjustment of velocity to its estimated equilibrium trend, the attenuation is not adequate.

The profile of equilibrium velocity based on the full regression tree estimates supports this view. It reveals that though the most recent estimate for equilibrium velocity is comparable to ours, the change is more attenuated. Although the Bai–Perron approach

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Table 2. Summary of Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Orphanides-Portera</th>
<th>Carlson-Craig-Schwarzb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant $V^*$</td>
<td>Full Regression Tree $V^*$</td>
</tr>
<tr>
<td>Mean Error</td>
<td>1.65</td>
<td>0.03</td>
</tr>
<tr>
<td>Standard Deviation of Error</td>
<td>1.14</td>
<td>0.59</td>
</tr>
<tr>
<td>Mean Absolute Error</td>
<td>1.74</td>
<td>0.46</td>
</tr>
</tbody>
</table>

a Estimates taken from Orphanides and Porter (1998), Table 1. These are based on out-of-sample forecasts of $V^*$ (1991Q1 to 1997Q4). Constant $V^*$ is estimated as the mean of the velocity of M2 from 1960Q1 to 1988Q4, and the full regression tree estimate of $V^*$ is evaluated at the average opportunity cost of M2.

b Estimates based on within-sample errors from 1991Q1 to 1997Q4. The methodology for calculating the errors is identical to that of Orphanides and Porter (1998); however the in- versus out-of-sample difference makes strict comparison impossible.

c Model C as presented in Table 1 and Figure 1, including as varying-coefficient regressors a constant, opportunity cost, and a time trend, and including a one-quarter-lagged velocity of the M2 variable as a fixed-coefficient regressor. The short-run adjustment of M2 velocity (as depicted by the dashed lines in Figure 2C) were used.

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7 Indeed, the procedure requires that breakpoint dates be separated by a minimal interval (see Bai and Perron [1998]). A nonparametric test proposed by Lombard (1987) does allow for smooth changes. However, the Lombard test applies under the assumption that errors are independently and identically distributed.
appropriately finds a break, its limitations prevent it from identifying a range of time over which a structural change may occur.

VI. Concluding Remarks
The breakdown in \( M_2 \) velocity in the 1990s has raised serious questions about the reliability of money measures as policy targets or indicators. We have applied newly developed breakpoint procedures to examine the stability of the velocity relation. The tests provide a rigorous basis for estimating and testing break dates.

Our results lead us to conclude that there were two breaks: one in 1981 and one in 1991. These estimated breaks occur around events that one might expect would affect velocity. Deregulation in the early 1980s, for example, enabled banks and thrifts to compete more effectively for funds, thereby increasing depository share of financial holdings. The second break date is associated with the proliferation of stock and bond mutual funds. This increased the accessibility of capital markets to ordinary investors, who previously had few alternatives to holding time deposits. Accessibility of these funds, in turn, reduced the quantity demanded of \( M_2 \) relative to its measured opportunity cost, hence raising its velocity.

Allowing for structural changes in parameters reveals important findings for policymakers going forward. Recent experience indicates that opportunity cost elasticity has increased substantially. Moreover, our tests suggest that since 1991 equilibrium velocity has been trendless, suggesting that at least for the time being, \( M_2 \) velocity appears to be unaffected by financial innovation.

Appendix:
Bai–Perron sequential procedure for estimating breakpoints
The Bai–Perron (1998) test is based upon an information criterion in the context of a sequential procedure, and allows one to find the number of breaks implied by the data, as well as estimating the timing of the breaks and the parameters of the processes between breaks. Among the many important advantages offered by this test is that it is not computationally excessive, in spite of what, on the surface, would appear to be an extremely computer intensive and difficult numerical problem. To illustrate, consider the following \( m \)-break model:

\[
\begin{align*}
y_t &= x_t^\prime \beta + z_t^\prime \delta_1 + u_t, t = 1, 2, \ldots, T_1 \\
y_t &= x_t^\prime \beta + z_t^\prime \delta_2 + u_t, t = T_1 + 1, \ldots, T_2 \\
& \vdots \\
y_t &= x_t^\prime \beta + z_t^\prime \delta_{m+1} + u_t, t = T_m + 1, \ldots, T,
\end{align*}
\]

where \( y_t \) = observed value of the dependent variable at time \( t \); \( x_t = p \times 1 \) vector of covariates with corresponding vector of coefficients, \( \beta \); \( z_t = q \times 1 \) vector of covariates with corresponding vector of coefficients, \( \delta_j \) \( (j = 1, 2, \ldots, m+1) \); \( u_t \) = disturbance (error) at time \( t \); and \( T_1, \ldots, T_m \) = breakpoint dates, treated as unknowns. Note that in this general framework, not all coefficients need to be subject to structural change. A proposed
set of breakpoint dates implies a partition of the data into segments of time, each of which has a separate model associated with it.

The Bai–Perron test looks at all the possible partitions for a given number of such breakpoints, \( m \). The partition that minimizes the sum of the squared residuals when one iteratively estimates the parameters \( \beta \) and \( \delta_j \) associated with it is chosen as the most likely partition. The computational burden rapidly becomes intolerable for \( m > 2 \) (there are \( T!(T - m)! \) different possible combinations of breakpoints, each of which would imply a different estimate). An important innovation of Bai and Perron is an efficient computational method based on dynamic programming that reduces the number of computations to the order of \( T^2 \) for any number of breaks greater than one.

The Bai–Perron procedure applies the least squares principle to estimate \( \beta \), \( \delta_j \) \((j = 1, \ldots, m + 1)\), and \( T_i \) \((i = 1, \ldots, m)\) using \( T \) observations on \((y_t, x_t, z_t)\). That is, for each \( m \)-partition \( \beta \) and \( \delta_j \) \((j = 1, \ldots, m + 1)\) are estimated by...

\[
\text{Min } (y - x\beta - z\delta)'(y - x\beta - z\delta) = \sum_{i=1}^{m+1} \sum_{t=T_i-1}^{T_i} [y_t - x'_t\beta - z'_t\delta]
\] (1)

In practice, however, the number of breaks is unknown. Bai and Perron propose the following sequential approach:

First, estimate the model with a small number of breaks that are thought to be necessary (or start with no break). Second, perform parameter constancy tests for every subsample, adding a break to a subsample associated with a rejection of the null hypothesis of no break using the test \( F_T(l + 1/l) \) given by

\[
f_T(l + 1/l) = \{ S_T(T_1, \ldots, T_l) - \min_{1 \leq i \leq l \leq T} S_T(T_1, \ldots, T_{i-1}, \tau, T_i, \ldots, T_l) \}/\sigma^2.
\] (2)

where

\[
\Lambda_{i, \eta} = \{ \tau: T_{i-1} + (T_i - T_{i-1}) \eta \leq \tau \leq T_i + (T_i - T_{i-1}) \eta \}
\] (3)

and \( \hat{OC} \) is a consistent estimate of \( \hat{OC} \) under the null hypothesis. Note that for \( i = 1, S_T(T_1, \ldots, T_{i-1}, \tau, T_i, \ldots, T_l) \) is understood as \( S_T(\tau, T_1, \ldots, T_l) \) and for \( i = 1, \) as \( S_T(T_1, \ldots, T_l, \tau) \).

This process is repeated by increasing \( l \) sequentially until the test fails to reject the null hypothesis.

There are two variants of this sequential procedure proposed by Bai and Perron. One uses a null hypothesis based on a global minimization of the sum of squared residuals to estimate a given number of breakpoints, \( m \). The other approach is more strictly sequential, using a null hypothesis of \( m \) breaks determined sequentially, not globally. Bai and Perron show that an estimation strategy need not simultaneously estimate the location of breaks in order to consistently determine the number of breaks. They extend their procedure to accommodate serial correlation in the disturbance term (giving their method an advantage over alternative approaches suggested by Liu et al. 1997) or, alternatively, to accommodate the inclusion of a lagged dependent variable in the list of regressors.

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References


