Asymmetries in Risk and in Risk Attitude: The Duopoly Case

Bruno Larue* and Vincent Yapo

A simple duopoly model is used to show that the profits of both firms can rise under uncertainty if the aversions of the firms toward risk are not too asymmetric nor too pronounced. Unlike the competitive case, the assumptions of strategic substitutes and asymmetric risk attitudes can support equilibria in which one of the firms produce more under uncertainty than under certainty. We also show that it is possible for one firm, but not both, to achieve a level of expected utility that exceeds its certainty benchmark. © 2000 Elsevier Science Inc.

Keywords: Uncertainty; Duopoly; Risk aversion; Asymmetry

JEL classification: D82; D43

I. Introduction

Given that governments are increasingly constrained by multilateral and bilateral agreements in the way and the extent with which they intervene to affect market outcomes, more and more attention is devoted to competition policy and the behavior of firms. The latter must often make decisions without the benefit of having timely and accurate information. The behavior of the firm under uncertainty has been extensively studied in a nonstrategic environment (e.g., Sandmo, 1971; Baron, 1970; Leland, 1972; Katz and Paroush, 1979; Fishelson, 1986). Fewer papers have analyzed the behavior of the firm under uncertainty and Cournot competition. Cyert and DeGroot (1970) used a Bayesian framework to analyze the behavior of expected utility maximizing duopolists when the output of the rival is known up to a stochastic term. Horowitz (1983) studied the effect of uncertainty on entry. Ofek and Paroush (1986), Fishelson (1989), and Tessitore (1994) compare the degree of competition in an industry under uncertainty to the Cournot Centre de Recherche en Économie Agroalimentaire (CRÉA), Université Laval, Ste-Foy, Quebec (BL and VY); Department of Economics, Iowa State University, Ames, IA (BL).

Address correspondence to: Bruno Larue, 4417 Pavillon Paul-Comtois, CRÉA, Université Laval, Ste-Foy, Qc, Canada, G1K 7P4.
certainty benchmark.¹ Moner-Colonques (1998) rely on subjective priors about costs to demonstrate that the profits of oligopolists can increase under asymmetric information. This paper builds on this line of research by focusing on the effects of asymmetries in risk attitudes and in the level of risk perceived by duopolists.

In contrast with Fishelson (1989), our results show that uncertainty does not cause the best-response functions to shift down in a parallel fashion. Uncertainty and risk aversion make the slopes of the best-response functions pivot toward the origin around the perfect competition level of output. Building on this result, we analyze how informational asymmetry (resulting from different risk perceptions) and asymmetric risk attitudes affect the Nash equilibrium. Unlike the case with competitive firms, we are able to derive conditions under which the quantity chosen by a firm can increase under uncertainty and risk aversion. The intuition behind this is straightforward when the effect of uncertainty is decomposed into a direct effect and an indirect one. The direct effect of risk is to induce a risk-averse firm to lower its output while the indirect effect stems from the facts that the rival firm is also affected by risk and that quantities are strategic substitutes. This implies that a firm responds to a reduction in its rival’s output by increasing its own. Thus, if the indirect effect dominates, a firm’s output increases under uncertainty. We also show that under some conditions, the ex post level of profits of both firms can rise. However, only one firm can gain from uncertainty when the criterion is the level of expected utility.

II. The Modeling of Risk and the Benchmark Certainty Equilibrium

To make the assessment of the effect of uncertainty on firm behavior as transparent as possible, we will compare our results derived under uncertainty to a certainty benchmark. We chose to anchor our comparisons to the static Cournot duopoly model. This choice is motivated by the simplicity and the ubiquity of the Cournot model.² We begin this section by describing our benchmark. We then discuss the introduction of uncertainty into the model.

Suppose two competing firms selling a homogenous product. Both firms face the same constant average cost such that the firms’ total cost function can be represented by:

\[ C(x_i) = cx_i,\ c > 0. \]  

(1)

The demand for the firms’ output can be expressed in its inverse form as³:

¹ Klemperer and Meyer (1989) used a different framework to analyze the effect of uncertainty on the behavior of oligopolists. They contend that under uncertainty, it is advantageous for firms to rely on a supply function as their strategy. The equilibria can resemble either Cournot or Bertrand depending on the steepness of the supply functions. Demand uncertainty contributes to make the supply functions steeper.

² As argued by Mas-Colell, Winston and Green (1995, p. 394), the Cournot model is simple and it has pleasing comparative statics (about the number of firms and the demand and technology parameters). However, its theoretical foundation has been criticized (see Martin, 1994 p. 19–28). The Cournot duopolist recognizes that his profit is affected by what his rival does but he somehow believes that his rival is not affected by what he does. Each firm acts as though its rival does not react to output changes, when the rival’s best-response function indicate that they do react (i.e., the inconsistent conjecture objection). Another problem that has irked many economists is that there is no good story about the way, if any, by which the market reaches equilibrium. Tirole’s (1988, p. 216) imaginary auctioneer determining the market price is certainly not an appealing story while the common defense of the Cournot game which appeals to an approximation of a two-stage capacity-price game has also been criticized (see Tirole, 1988 p. 217 and Maggi, 1996). Nevertheless, the static Cournot model with constant marginal costs has proven to be useful in modelling strategic substitutes and it remains widely used.

³ The results derived below are not qualitatively influenced by the use of a more general slope coefficient in the demand equation.
where $X = x_i + x_j$ is the industry’s level of output. The profit of firm $i$ can therefore be defined as:

$$\pi_i = (\delta - x_i - x_j - c)x_i$$

(3)

The equilibrium under certainty is derived by assuming that the information available to both firms is certain when they make their decision. This eliminates any perceived risk and reduces the maximization of the expected utility of profits to the maximization of expected profits. The best-response function for firm $i$ under certainty is defined as:

$$R_i(x_{jc}) = \text{Arg max}\{ \pi_i(x_{ic}, x_{jc}) \}.$$ 

Noting that $\pi_i' = \frac{\partial \pi_i}{\partial x_{ic}} = (\delta - c) - 2x_{ic} - x_{jc}$, we obtain a well-known result:

$$R_i(x_{jc}) = \frac{\delta - c}{2} - \frac{x_{jc}}{2}, \quad R_i = -\frac{1}{2}.$$

(4)

The firm’s optimal behavior is to set its output such that its marginal revenue equals its marginal cost. The above equation tells us that the firm sets its output by subtracting half of its rival’s output from its monopoly output. The Nash equilibrium is determined by solving the pair of best-response functions: $x_{ic}^* = R_i(x_{jc})$. The solution to this problem is:

$$x_{ic}^* = \frac{\delta - c}{3}.$$  

(5)

This constitutes the benchmark for the individual firms’ output. The benchmark for the other variables of interest, namely the output of the industry, the price level, the profits of the individual firms and the profits of the industry are given by:

$$x_c^* = \frac{2(\delta - c)}{3}; \quad p_c^* = \frac{\delta + 2c}{3}; \quad \pi_i^* = \left(\frac{\delta - c}{3}\right)^2; \quad \pi_c^* = 2\left(\frac{\delta - c}{3}\right)^2$$

(6)

Fig. 1 illustrates a symmetric benchmark equilibrium with $\delta = 5$ and $c = 2$. The
best-response functions of firms 1 and 2 are represented by the \( ab \) and \( cd \) schedules respectively. The equilibrium occurs at point \( e \) and each firm sells one unit of output. The industry output, \( x^*_c \), is 2 and the equilibrium price, \( p^*_c \), is 3. The profits of individual firms and of the industry are: \( \pi^*_i = 1 \), \( \pi^*_c = 2 \). The schedule \( hef \) is the iso-industry output curve corresponding to the benchmark level (i.e., \( x^*_c = 2 \)). The \( cb \) schedule is the combination of outputs that generates the monopoly/collusive output level of 1.5. The fact that \( cb \) lies under \( hef \) indicates that the firms produce more under Cournot competition than under collusion. However, the firms are less competitive than under perfect competition (i.e., \( h < a \)).

As suggested in our introduction, uncertainty can be introduced in different ways in oligopoly models (e.g., the intercept of the demand, costs, conjecture). One of the first papers to investigate this issue is Cyert and DeGroot (1970). They argued that since a firm cannot react to its rival’s output choice in a static simultaneous-move game, the output of the rival must be treated as an unknown for which the firm might hold a subjective expectation about. Asymmetries in the Cournot model have important welfare implications. Salant and Shaffer (1999) show that asymmetric equilibria are likely to be socially optimal even when asymmetries are induced by government actions (e.g., R&D or export subsidies to identical firms). In our case, we assume that firms are asymmetric to begin with. Following Paroush (1981), we allow for potential asymmetries by defining \( \bar{x}_j \) as the level of firm \( j \)’s output when firm \( i \) is choosing its output level such that:

\[
\bar{x}_j = x_j + \gamma_i e_j, \tag{7}
\]

where \( e_j \sim \mathcal{N}(0, 1) \).\(^4\) The variance about firm \( j \)’s reaction as perceived by firm \( i \) is given by \( \gamma_i^2 \) where \( \gamma_i \) is a parameter capturing the quality of information available to firm \( i \) regarding firm \( j \)’s response. As opposed to the standard static Cournot game, each firm knows that its rival will bring a quantity to market but the exact level of that output is unknown. Firm \( i \)’s normally distributed profits can then be represented by:

\[
\tilde{\pi}_i = \left[ \delta - x_i - \bar{x}_j - c \right] x_i, \tag{8}
\]

where \( x_i \) is firm \( i \)’s output choice given the unknown output of firm \( j \). The first two moments of Eq. (8) are the expected value of the profits: \( E(\pi_i) = (\delta - c)x_i - (x_i + x_j)x_i \) and the conditional variance: \( \text{var}(\pi_i) = \gamma_i^2 x_i^2 \) which can be interpreted as the level of risk perceived by firm \( i \). It is directly related to the quality of information it has about firm \( j \)’s reaction function at the time of making its output decision.

Suppose that firms are risk neutral and maximize expected profits. Then the best-response function of firms \( i \) and \( j \) can be represented by:

\[
x_i = \frac{\delta - c - E_i[x_j]}{2} \quad \text{and} \quad x_j = \frac{\delta - c - E_j[x_i]}{2}. \]

If demand and cost parameters are common knowledge and that firm \( i \) knows firm \( j \) best-response function, firm \( j \)’s best-response function can then be inserted into firm \( i \)’s best-response function to yield:

\[
x_i = \frac{\delta - c}{2} - \frac{\delta - c - E_i[E_j[x_i]]}{4} + \frac{E_i[E_j[x_i]]}{4}. \]

\(^4\) We assume that errors in the expectations about the rival’s output are not correlated between firms. It is common to restrict the correlation to be zero or to be bounded between zero and one. In that restricted range, zero correlation allows for greater heterogeneity and, as a result, many results found in the literature depend on a “low” correlation (e.g., Moner-Colonques, 1998). Therefore, we do not believe that this assumption is objectionable.
If we keep substituting for the expected output levels, it is easy to show that:

\[ x_i = \frac{\delta - c}{2} \left[ 1 + \left( -\frac{1}{2} \right)^2 + \left( -\frac{1}{2} \right)^4 + \cdots \right] = \frac{\delta - c}{3}, \]

which is the Cournot output level derived in Eq. (5). This made Cyert and De Groot (1970, p. 1173) claim that the Cournot outcome can be reached under reasonable assumptions. Daugherty (1985) reached the same conclusion by deriving a conjectural variation (instead of assuming it) through his infinite-regress model. Ofek and Paroush (1986) generalized Cyert and De Groot’s (1970) approach by having firms maximize expected utility and by allowing for risk aversion. They derived conditions under which an increase in the number of firms need not lead to an increase in industry output but they did not explore the implications of asymmetries in risk perceptions and in risk attitudes.

Another method of introducing uncertainty in the Cournot model is to add a stochastic component in the intercept of the inverse demand:

\[ \tilde{d}_i = d_1 + \gamma x_i, \quad (9) \]

Fishelson (1989) and Tessitore (1994) analyzed the effect of uncertainty using a specification similar to Eq. (9). Finally, uncertainty can be introduced through the cost structures of rivals. Shapiro (1986) and Moner-Colonques (1998) supposed that firm \( i \)'s subjective conditional expectation about firm \( j \)'s marginal cost can be represented by:

\[ E_i[c_j | c_i] = \mu_j + \rho \sigma_{ij} \sigma_j, \]

where \( \mu_j \) is the unconditional expectation of \( c_j \), \( \rho \in [0, 1] \) is the coefficient of correlation between \( c_i \) and \( c_j \) and \( \sigma_j^2 \) is the unconditional variance of \( c_j \). If costs are known to move together (i.e., \( \rho = 1 \)), then cost uncertainty is equivalent to demand uncertainty (Shapiro, 1986, p. 445). It is also easy to demonstrate that introducing uncertainty through Eqs. (7) or (9) will produce similar results because in both cases the variance of profits is given by the same expression (i.e., \( \text{var}(\pi_i) = \gamma^2 x_i^2 \)). In this paper, we follow Cyert and De Groot (1970) by concentrating on risk in the conjecture as defined by Eq. (7).

We suppose that firms maximize the expected utility of profits and that their utility functions are characterized by constant absolute risk aversion (i.e., \( U(\pi_i) = -e^{-\alpha_i \pi_i} \)). This implies that firm \( i \) maximizes:

\[ W_i(\pi_i) = E(\pi_i) - (\alpha_i/2) \text{var}(\pi_i), \quad (10) \]

where \( \alpha_i = -U''/U' \) is the Arrow–Pratt measure of absolute risk aversion.

The Nash equilibrium quantities brought to market by the two firms, \( x_{iu}^* \) and \( x_{ju}^* \), are determined by supposing that each firm behaves according to Eq. (10). Thus, firm \( i \) chooses its level of output such that:

\[ x_{iu}^* = R_{iu}(x_{ju}^*) = \arg \max W_i(x_{iu}^*; x_{ju}), \quad (11) \]

\( R_{iu}(\cdot) \) is commonly known as firm \( i \)'s best-response function even though firm \( j \)'s output is determined simultaneously. This schedule tells the firm the quantity to bring to market conditional on the level of output of its rival. The Nash equilibrium is determined at the point where beliefs are consistent with one another (i.e., where the best-response functions cross).
III. Uncertainty and Asymmetric Risk Attitudes

Our benchmark equilibrium fully characterized, we can now assess the effect of uncertainty on the firms’ behavior. This section investigates the effects of asymmetries in perceived risk and in risk aversion between the firms. It is shown that uncertainty induces risk averse firms to compete less aggressively. This results in a lower industry output than under certainty. We also show that asymmetries in risk and in risk attitude could confer sufficient “information” and “attitude” advantages to allow one of the firms to produce more under uncertainty than under certainty.

We begin our investigation with the effect of uncertainty on the best-response functions.

Proposition 1. Let \( R_{iu}(x_j; \cdot) \) be firm \( i \)'s best-response function given \((\alpha_i, \gamma_i^2)\). (a) Assuming \( \alpha_i = 0 \): \( R_{iu}(x_j; \cdot) = R_{ic}(x_j) \); Assuming \( \alpha_i > 0 \): (b) \( R_{iu}(x_j; \cdot) < R_{ic}(x_j) \) for \( 0 \leq x_j \leq (\delta - c) \); (c) \( R'_{iu} > R'_{ic} \).

Proof. According to Eq. (10), the firm maximizes:

\[
W_i(x_{iu}; x_{ju}) = [(\delta - c) - x_{iu} - x_{ju}]x_{iu} - (\alpha_i/2)\gamma_i^2 x_{iu}^2
\]  

The first order condition associated with this optimization problem is:

\[
(\delta - c) - x_{ju} - (2 + \alpha_i \gamma_i^2) x_{iu} = 0.
\]

Rearranging, we obtain the best-response function:

\[
R_{iu}(x_{ju}; \alpha_i, \gamma_i) = \frac{(\delta - c) - x_{ju}}{2 + \alpha_i \gamma_i^2} x_{iu} - \frac{x_{iu}}{2 + \alpha_i \gamma_i^2}
\]  

From Eq. (14), it becomes clear that: (a) \( R_{iu}(x_j; \cdot) = R_{ic}(x_j) \) if \( \alpha_i = 0 \); (b) \( R_{iu}(x_j; \cdot) < R_{ic}(x_j) \); and (c) \( R'_{iu} = -1/(2 + \alpha_i \gamma_i^2) > R'_{ic} = -1/2 \) if \( \alpha_i > 0 \).

Q.E.D.

The first part of the proposition simply states that the best-response function of a risk neutral firm is not affected by the uncertainty. The last two parts of the proposition imply that the best-response function of the risk averse firm shifts inward. By Eq. (14), we can deduce that the best-response function of firm \( i \) and firm \( j \) actually rotate inward around the coordinates \((0, \delta - c)\) and \((\delta - c, 0)\) as shown in Fig. 1 where \( aeb \) and \( ced \) are the best-response functions under certainty. Thus uncertainty induces the risk averse firm to bring less to market. Another implication of the proposition is that the effect of uncertainty depends on the rival’s expected level of output: the greater the expected output of the rival, the smaller the effect of uncertainty on the firm’s level of output. This reflects Leland’s (1972) principle of increasing uncertainty that links the level of potential losses to the level of profits. In this case, if the rival’s expected output is small, the residual demand faced by the firm will allow for higher profits. Because the firm has more to lose, it will be more affected by uncertainty. The effect of uncertainty on the strategy of the firm is directly related to the coefficient of risk aversion of the firm. As in Baron (1970), risk aversion renders the firm less aggressive in its output decision.

The first two elements of the above proposition are consistent with results derived in Fishelson (1989). However, Fishelson’s best response functions shift downward in a
parallel fashion. In light of this, we assessed the robustness of our results by using a general Von Neumann-Morgenstern utility function.

**Proposition 2.** Assuming firm $i$ has a general Von Neumann-Morgenstern utility function and $R_{iu}^u(x_i)$ and $R_{iu}^c(x_i)$ are the slopes of the best-response functions of firm $i$ under uncertainty and certainty respectively. a) $R_{iu}^u(x_i) \neq R_{iu}^c(x_i)$; b) If $U'' = 0$, $R_{iu}^u < 0$, $R_{iu}^c = 0$.

**Proof.** The first order condition to the utility maximization problem can be represented by:

$$E[\pi'(x_i)U'(\pi_i)] = E[\pi'(x_i)] + \frac{\text{cov}[U'(\pi_i), \pi_i]}{E[U'(\pi_i)]} = 0$$  \hspace{1cm} (15)

Using the definition of covariance, the above condition can be rearranged in a more intuitive manner:\footnote{By definition, \(\text{cov}(U', \pi'_i) = E[U' - E[U']][\pi'_i - E[\pi'_i]]\). Since \(\pi'_i = (\delta - c) - 2x_i - x_i - \gamma e_i\) and \(\pi'_i - E[\pi'_i] = -\gamma e_i\), \(\text{cov}(U', \pi'_i) = -\gamma e_i\). Since \(\pi'_i - E[\pi'_i] = -\gamma e_i\), \(\text{cov}(U', \pi'_i) = -\gamma E[U'] + \gamma E[U'_e] = -\gamma E[U'_e] = -\gamma E[U'_e].\)

\[E[\pi'(x_i)] - \frac{E[U'(\pi_i)e_i]}{E[U'(\pi_i)]} = 0.\]  \hspace{1cm} (16)

Now, unless utility is linear in profits, the second term on the left-hand side will be a function of $x_i$. Fishelson (1989) obtains his result (see his equations 3 and 4) that the slope of the best-response function remains the same as under certainty by assuming that the second term is independent of $x_i$.\footnote{A parallel shift of $R_i(\cdot)$ is possible if the objective function of the firm had been $W_i = E[\pi_i] - (\alpha/2)\frac{\gamma^2}{\alpha i}$. Uncertainty would have had the same effect as a rise in the unit cost of production since $(\alpha E[\pi_i]/\alpha x_i) - \alpha i = 0$.}

A Taylor expansion of $U'(\pi_i)$ around $E[\pi_i]$ will help us show that, generally speaking, the slope of the best-response function is affected by uncertainty.

$$U'(\pi_i) = U'(E[\pi_i]) + U''(E[\pi_i]) [\pi_i - E[\pi_i]] + \frac{U'''(E[\pi_i])}{2!} [\pi_i - E[\pi_i]]^2 + R.$$  \hspace{1cm} (17)

Limiting the approximation to the terms appearing in Eq. (17) and multiplying by $e_j$, we obtain:

$$U'(\pi_i)e_j = U'(E[\pi_i])e_j + U''(E[\pi_i]) [\pi_i - E[\pi_i]]e_j + \frac{U'''(E[\pi_i])}{2!} [\pi_i - E[\pi_i]]^2 e_j.$$  \hspace{1cm} (18)

Noting that $\pi_i - E[\pi_i] = -\gamma x_i e_i$, and that $E[(\pi_i - E[\pi_i])^2, e_j] = \text{cov}(\pi_i, E[e_i])^2, e_j$, which is in turn equals to $\gamma^2 x_i^2 \text{cov}(e_i^2, e_j)$ and taking the expectation of Eqs. (17) and (18), we get:

$$E[U'(\pi_i)] = U'(E[\pi_i]) + \frac{U'''(E[\pi_i])}{2!} \gamma^2 x_i^2.$$  \hspace{1cm} (19)
Inserting Eqs. (19) and (20) into Eq. (16) confirms that its second left-hand side term is indeed a function of $x_i$. It can be shown that $U''(\cdot) = 0$ is a sufficient condition to have linear best-response functions and hence avoid potential multiple equilibria. Thus, under rather general conditions, the introduction of uncertainty will change the slopes of the best-response functions but it need not transform best-response functions that are linear under certainty into nonlinear ones.

**Q.E.D.**

It is well known that the perfectly competitive firm reduces its output under uncertainty if it is risk averse. We now want to verify whether this result extends to the oligopolistic firm with constant absolute risk aversion.

**Proposition 3.** (1) Let $x_{iu}^*(\alpha_i, \alpha_j; \gamma_i, \gamma_j)$ be the sales of the individual firms. (a) $x_{iu}^* < x_{ij}^*$ if $\alpha_i \geq 1/\gamma_i^2$; (b) $x_{iu}^* < x_{ij}^*$ if $\alpha_i > 0$ and $\alpha_i = 0$. (2) Assuming symmetry in risk perceptions ($\gamma_i = \gamma_j = \gamma$) and that $0 < \alpha_i < 1/\gamma^2$, then $x_{iu}^* > x_{ij}^*$ if $\alpha_i > 2\alpha_i/(1 - \alpha_i^2)$.

**Proof.** The Nash equilibrium quantities under uncertainty are given by:

$$x_{iu}^* = a_{iu}(\alpha_i, \alpha_j, \gamma_i, \gamma_j) \cdot (\delta - c)$$

where $a_{iu} = (1 + \alpha_i \gamma_i^2)[(2 + \alpha_i \gamma_i^2)(2 + \alpha_j \gamma_j^2) - 1]$ and $i \neq j$. Note that the coefficients of risk aversion and the level of risk for both firms enter explicitly in the equilibrium quantities. This contrasts with the expression derived for the best-response functions. From Eqs. (5) and (21), we can assert that $a_{ic} = 1/3$. After some rearranging, it can be shown that:

$$a_{iu} = \frac{1}{3} + \frac{\alpha_i \gamma_i^2 \cdot (1 - \alpha_i \gamma_i^2) - 2\alpha_i \gamma_i^2}{9 + 6(\alpha_i \gamma_i^2 + \alpha_j \gamma_j^2) + 3\alpha_i \gamma_i \alpha_j \gamma_j}$$

From Eq. (22), it is clear that $a_{iu} < 1/3$ if $\alpha_i \geq 1/\gamma_i^2$ for all $\alpha_i \geq 0$. It is also evident that $a_{iu}(\alpha_i, 0, \gamma_i, \gamma_j) = 1/(3 + 2\alpha_i \gamma_j^2) < 1/3$ for all $\alpha_i > 0$. This proves part (1) of the proposition. If we impose symmetry on the level of risk, Eq. (22) simplifies to:

$$a_{iu} = \frac{1}{3} + \frac{\alpha_i \gamma(1 - \alpha_i \gamma^2) - 2\alpha_i \gamma^2}{9 + 6(\alpha_i + \alpha_j) \gamma^2 + 3\alpha_i \alpha_j \gamma^4}$$

A careful inspection of Eq. (23) reveals that:

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7 The expression

$$\frac{\text{cov}(U'_i, \epsilon_i)}{E[U'_i]} = -\gamma_i \cdot \left[ \frac{U'(E[\pi_i]) x_i - \frac{U''(E[\pi_i])}{2!} \gamma_i x_i^2 \text{cov}(\epsilon_i', \epsilon_i)}{U'(E[\pi_i]) + \frac{U''(E[\pi_i])}{2!} \gamma_i x_i^2} \right]$$

will have two roots unless the third derivative of the utility function vanishes.
This condition is respected if and only if: 

\[ \alpha_i \gamma^2 \leq 1 \] and 

\[ \alpha_j \geq \psi(\alpha_i) = \frac{2\alpha_i}{1 - \alpha_i \gamma^2} \]. 

Q.E.D.

The above proposition states that Sandmo-like behavior on the part of the firm is observed if the rival firm is risk neutral. Of particular interest is the fact that the level of output of the risk averse oligopolistic firm is not necessarily smaller under uncertainty. The condition under which the firm’s output is higher under uncertainty amounts to a large risk aversion differential between the “slightly” risk averse firm and its rival. The size of that risk aversion differential is in turn function of the level of risk. More specifically, the output of the firm rises under uncertainty if it is “not too risk averse,” \( \alpha_i \leq 1/\gamma^2 \), and if the rival firm is “sufficiently risk averse,” that is: \( \alpha_j \geq \psi(\alpha_i) = \frac{2\alpha_i}{1 - \alpha_i \gamma^2} \). Note that \( \psi(0) = 0, \psi(1/\gamma^2) = \infty, \psi' = 2(1 - \alpha_i \gamma^2)^2 > 0 \). This means that the minimum level of risk aversion on the part of firm \( j \) that will allow firm \( i \) to increase its output under uncertainty is a positive and increasing function of firm \( i \)'s level of risk aversion.

Fig. 2 illustrates the combinations of risk aversion coefficients required for firm \( I \) to maintain its certainty level of output under uncertainty. The closest schedules to the origin are associated with higher levels of risk. The space to the left of each schedule maps the region over which the level of output of firm \( I \) is greater under uncertainty than under certainty. Thus, the higher the level of risk \( \gamma^2 \), the smaller the region in the \( (\alpha_1, \alpha_2) \) space that will support an equilibrium in which the output of firm \( I \) is higher under uncertainty than under certainty.

The following proposition investigates the effect of uncertainty on the industry’s level of output.

**Proposition 4.** Let \( x^*_u(\alpha_i, \alpha_j, \gamma^2) \) and \( p^*_u(\alpha_i, \alpha_j, \gamma^2) \) be the industry sales and market price when the firms face a symmetric level of risk. If \( \alpha_i > 0 \), then \( x^*_u < x^*_c \) and \( p^*_u > p^*_c \), \( \forall \alpha_j \geq 0 \).

**Corollary 4.1.** \( \alpha_i > 0, \alpha_j = 0 \Rightarrow \pi^*_u = \pi^*_i + \pi^*_u > \pi^*_c \).
Proof. We must show that \( a_u = a_{iu} + a_{ju} < 2/3 \) for \( \alpha_i > 0 \) and \( \alpha_j \geq 0 \). For the case least favorable to the proposition (i.e., when the rival firm is risk neutral (\( \alpha_j = 0 \)), we have:

\[
a_u = \frac{2}{3} - \frac{\alpha_i \gamma^2}{9 + 6\alpha_i \gamma^2}.
\]

From Eq. (25), it is apparent that for \( \alpha_i > 0, a_u(\alpha_i, \alpha_j, \gamma^2) < 2/3 \). The corollary follows from the fact that the industry output will lie between the Cournot and monopoly level. Thus when one firm is risk averse and the other is risk neutral, industry profits are bounded by the profits associated with the Cournot and monopoly certainty equilibria.

Q.E.D.

Proposition 5. Assuming symmetry in risk (\( \gamma^2_i = \gamma^2_j = \gamma^2 \)), \( \delta = 5, c = 2 \), and \( 0 < \alpha_i < (5 + 3\sqrt{3})/2 \gamma^2 \), then \( \pi^*_u > \pi^*_u, \pi^*_j > \pi^*_j \) if \( 0 < \lambda < \alpha_i < \alpha_j \) where \( \lambda = g(\alpha_i; \gamma^2) \) and \( \tilde{\alpha} = h(\alpha_i; \gamma^2) \).

Proof. The certainty benchmark profits and the profits under uncertainty are:

\[
\pi^*_u = 1 \quad \text{and} \quad \pi^*_i = \beta_i(\alpha_i, \alpha_j, \gamma) = \frac{9(1 + \alpha_i \gamma^2)(1 + \alpha_j \gamma^2)^2}{[3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4]^2}.
\]
Asymmetries in Risk Attitude: Duopoly

For the profits of both firms to be higher under uncertainty, it must be shown that the following conditions are satisfied: $\beta_i(\alpha_i, \alpha_j, \gamma) > 1$ and $\beta_i(\alpha_i, \alpha_j, \gamma) > 1$. Solving for $\beta_i = 1$ and $\beta_j = 1$, we obtain the boundaries on firm $j$'s risk aversion in terms of firm $i$'s risk aversion:

$$g(\alpha_i; \gamma) = \frac{3 + 2\alpha_i \gamma^2 - 2\alpha_i^2 \gamma^4 - 3\sqrt{(1 + \alpha_i \gamma^2)^3}}{-5 + \alpha_i^2 \gamma^4 - 5\alpha_i \gamma^2},$$

$$h(\alpha_i; \gamma) = \frac{-3 + 4\alpha_i \gamma^2 + 5\alpha_i^2 \gamma^4 + 3\sqrt{(1 + \alpha_i \gamma^2)^3(1 + 5\alpha_i \gamma^2)}}{2(2 + \alpha_i \gamma)^2}.$$

The $g(\cdot)$ schedule is a mapping of the levels of firm $j$'s risk aversion for which the profits of firm $i$ under uncertainty are equal to the certainty benchmark. The $h(\cdot)$ schedule has a similar interpretation for firm $j$. From the $g(\cdot)$ schedule, a region $A_i$ can be defined in the $(\alpha_i, \alpha_j)$ space in which $\pi_{iu}^* > \pi_{iu}^c$:

$$A_i = \{(\alpha_i, \alpha_j): 0 < \alpha_i < \frac{5 + 3\sqrt{5}}{2\gamma^2} \text{ and } \alpha_j > g(\alpha_i)\}.$$  

From $h(\cdot)$, we can also define a region $A_j$ in which $\pi_{ju}^* > \pi_{ju}^c$:

$$A_j = \{(\alpha_i, \alpha_j): \alpha_i > 0 \text{ and } \alpha_j < h(\alpha_i) < \frac{5 + 3\sqrt{5}}{2\gamma^2}\}.$$  

For the ex post profit of both firms to be higher, there must exist a region $A$ such that $A_i \cap A_j = A$ is non-empty. The existence of $(\alpha_i, \alpha_j)$ combinations defining $A$ is proven by noting that: 1) $g(\cdot)$ and $h(\cdot)$ are continuous functions over the interval $[0, (5 + 3\sqrt{5})/2\gamma^2]$, 2) $g'(0) < h'(0)$, 3) $g(\cdot)$ and $h(\cdot)$ are respectively convex and concave in the $(\alpha_i, \alpha_j)$ space (see Fig. 3). Thus, there exist many combinations of risk aversion levels bounded by: $0 < \alpha_i < (5 + 3\sqrt{5})/2\gamma^2$ and $g(\alpha_i, \gamma^2) < \alpha_j < h(\alpha_i, \gamma^2) < (5 + 3\sqrt{5})/2\gamma^2$ that support an equilibrium characterized by higher profits for both firms under uncertainty.

\textit{Q.E.D.}\textsuperscript{9}

To complement the above proof, an example is constructed to demonstrate that the profits of both firms can rise under uncertainty. This is the case for the pair $(g(\cdot), h(\cdot)) = (0.8\alpha_i, 1.25\alpha_j)$. To see this, rewrite Eq. (26) as:

$$\beta_i = 1 + \frac{3\gamma^2(2\alpha_j - \alpha_i) + D_i(\alpha_i, \alpha_j, \gamma)}{[3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^4 + \alpha_i \alpha_j \gamma^2]^2},$$

where $D_i(\alpha_i, \alpha_j, \gamma) = 5\alpha_i^2 \gamma^4 - 4\alpha_i \gamma^2 + 5\alpha_i \gamma^2 - 4\alpha_i \gamma^2 - 4\alpha_i \gamma^2 - 4\alpha_i \gamma^2 + 4\alpha_i \gamma^2 - 4\alpha_i \gamma^2$ and the level of profit under certainty is 1. From this representation of the profits of firm $i$ under uncertainty, it can be shown that for a given level of $\alpha_i$: 1) $\beta_i > 1$ if $\alpha_i \alpha_j < 4/\gamma^2$ and $\alpha_j > 4\alpha_i/5$, and 2) $\beta_j > 1$ if $\alpha_i \alpha_j < 4/\gamma^2$ and $\alpha_j < 5\alpha_i/4$. According to these conditions, a small level of risk can generate a more profitable equilibrium for both firms if there is not too

\textsuperscript{9} By evaluating the derivatives of $g(\cdot)$ and $h(\cdot)$ at $\alpha_i = 0$, we get $g'(0) = 0.5 < h'(0) = 2$.
\textsuperscript{9} We replaced our original assumption on demand and cost parameters (i.e., $\delta > c > 0$) by $\delta = 5$ and $c = 2$ because that the gains in tractability and intuition far outweigh the loss of generality. Clearly, equilibria characterized by higher profits for both firms can be supported by different values of $\delta$ and $c$. 

much asymmetry in the levels of risk aversion between the firms. In the case of symmetrical and unitary risk attitudes and risk levels, we have $\beta_i = \beta_j = 1.125 > \pi^*_i = \pi^*_j = 1, \beta = 2.25 > \pi^*_k = 2$.

Fig. 3 illustrates the last proposition. The convex schedules represent the combinations of risk aversion levels that maintain the level of firm 1’s profit to the certainty benchmark for different levels of risk. The combinations to the left of each schedule provide higher profits for firm 1. The higher profit region shrinks with the level of risk as shown by the respective position of the three schedules. The same interpretation can be given to the concave iso-certainty profit schedules of firm 2. The ellipses made by both sets of iso-certainty profit schedules define the combinations of risk aversions that support higher profits for both firms for different levels of risk. The size of the ellipse clearly shrinks with the level of risk.

Instead of representing the region of increased profits for both firms in risk aversion space, we can define it in terms of quantities. Fig. 1 displays the best-response functions under certainty and under uncertainty as well as the iso-profit curves associated with the Nash equilibrium under certainty. The ellipse formed by these iso-profit curves defines the region within which the profits of both firms can rise. The particular Nash equilibrium under uncertainty ($\alpha_i = \alpha_j = \gamma = 1$) shown in Fig. 1 falls in that region.
IV. Uncertainty and Asymmetry in Risk

In the previous section, we have investigated asymmetries in risk attitudes by holding risk perceptions symmetric. In this section, we will focus on asymmetries in risk. The following proposition captures the implication of asymmetric risk on the level of output of the firms.

Proposition 6. Let $x^*_i(\alpha_i, \gamma_i^2, \gamma_j^2)$ be the sales of firm $i$. Assuming symmetric risk attitudes ($\alpha_i = \alpha_j = \alpha$), $x^*_i > x^*_j$ if $\alpha < 1/\gamma_i^2$ and $\gamma_j^2 > 2\gamma_i^2/(1 - \alpha \gamma_i^2)$.

Proof. For simplicity, let $\alpha = 1$, $\gamma_i^2 = \gamma_j^2 > 0$ and $\gamma_j^2 = \mu \gamma_i^2$, ($\mu = \gamma_j^2/\gamma_i^2$). Then:

$$a_{iu} = \frac{1 + \mu \gamma^2}{(2 + \mu \gamma^2)(2 + \gamma^2) - 1} \text{ and } a_{ju} = \frac{1 + \gamma^2}{(2 + \mu \gamma^2)(2 + \gamma^2) - 1}. \quad (27)$$

After some manipulations, these output coefficients can be represented by:

$$a_{iu} = \frac{1 + \mu(1 - \gamma_j^2) - 2 \gamma_j^2}{9 + 6(1 + \mu) \gamma_j^2 + 3 \mu \gamma_j^2} \text{ and } a_{ju} = \frac{1 + \gamma_j^2}{9 + 6(1 + \mu) \gamma_j^2 + 3 \mu \gamma_j^2}. \quad (28)$$

The above expressions reveal that $a_{iu}^* > 1/3$ if $\mu(1 - \gamma_j^2) - 2 > 0$, i.e. $\gamma_j^2 < 1$, $\mu > 2/(1 - \gamma_j^2)$.

Q.E.D.

The above proposition indicates that a sufficiently large asymmetry in risk may allow one of the firms to sell more under uncertainty than under certainty. This is true even if both firms are risk averse. Thus, the above proposition complements proposition 3 in defining conditions under which the output of one of the firms can be higher under uncertainty.

V. Comparative Statics

As opposed to comparing the behavior of the firms under uncertainty to a certainty benchmark, we now investigate the marginal effects of uncertainty. The effects of marginal increases in risk are sensitive to the level of risk prevailing prior to the increase. Thus, we define $\gamma_0^2$ as the initial level risk. The next two propositions summarize the effects of increases in risk aversion and in the level of risk on the output and profits of the firms and the industry.

Proposition 7. (1) Effects of a change in risk aversion $\alpha_i$: (a) $\partial x^*_i/\partial \alpha_i < 0$; (b) $\partial x^*_j/\partial \alpha_i > 0$; (c) $\partial x^*_i/\partial \alpha_j < 0$. (2) Effects of a change in risk $\gamma_j^2$: (a) $\partial x^*_j/\partial \gamma_j^2 < 0$; (b) Assuming $\gamma_0^2 = 0$: (i) $\partial x^*_i/\partial \gamma_j^2 \equiv 0$ if $\alpha_j \equiv 2\alpha_i$; (c) Assuming $\gamma_0^2 > 0$: Let $\alpha_j > 2\alpha_i$, then $\partial x^*_j/\partial \gamma_j^2 \equiv 0$ if $\gamma_0^2 \equiv \gamma_j^2(\alpha_j, \alpha_i)$.

Proof. To prove the above results, we need to compute the partial derivatives of $a_{iu}^*$ and $a_{ju}^*$.
that depends on \( \gamma(1 + \alpha_i \gamma^2)(2 + \alpha_i \gamma^2) \) \[\frac{\partial a_{i \alpha}}{\partial \alpha_i} = \frac{\gamma(1 + \alpha_i \gamma^2)(2 + \alpha_i \gamma^2)}{[3 + 2\alpha_i \gamma^2 + 2\alpha_i \gamma^2 + \alpha_i \gamma^4]^2}, \] \( \frac{\partial a_{i \gamma}}{\partial \gamma} = \frac{\alpha_i - 2\alpha_i}{[3 + 2\alpha_i \gamma^2 + 2\alpha_i \gamma^2 + \alpha_i \gamma^4]^2}, \] \( \frac{\partial a_{\gamma \gamma}}{\partial \gamma} = \frac{\alpha_i + 2\alpha_i + 4\alpha_i \gamma^2 + \alpha_i \gamma^4 + \alpha_i \gamma^6}{[3 + 2\alpha_i \gamma^2 + 2\alpha_i \gamma^2 + \alpha_i \gamma^4]^2}. \]

Eqs. (29), (30), and (32) prove the results in (7.1). Furthermore, Eq. (33) confirms that \( (\partial \pi_i^u / \partial \gamma^2) < 0 \) from Eq. (31), \( (\partial \pi_i^u / \partial \gamma^2) = (\alpha_i - 2\alpha_i)/9 \) when \( \gamma^2 = 0 \). This proves part (7.2.b). To prove (7.2.c), suppose \( \Delta = \alpha_i - 2\alpha_i > 0 \). Then there exist values of \( \gamma^2 \) consistent with \( \Delta - 2\gamma^2 \alpha_i \alpha_j - \gamma^2 \alpha_i \alpha_j^2 > 0 \) which implies a positive numerator for Eq. (31). The possible values of \( \gamma^2 \) that satisfy this condition must be inferior to a critical level \( \gamma^2 \) that depends on \( (\alpha_i, \alpha_j) \).

Q.E.D.

Tessitore (1994) obtained some of the results summarized by proposition 7.\(^{11}\) Our contribution lies in the identification of an explicit link between the maximum level of risk that allows the output of one of the firms to rise with a reference level of risk, \( \gamma^2 \) which is a function of the firms’ risk aversion \( (\alpha_i, \alpha_j) \).

The elements of the first part of proposition 7 are intuitive. The firm’s output is a decreasing function of its own risk aversion but it is increasing with its rival’s level of risk aversion. Both (7.1.c) and (7.2.a) reflect the fact that the output relinquished by a firm is never completely recovered by its rival. Finally, we showed that if one firm is sufficiently less risk averse than its rival and the level of risk is sufficiently small, the output of the less risk averse firm is increasing with the level of risk. Fig. 4 illustrates this phenomenon for three different pairs of risk aversion coefficients.

The marginal effects of uncertainty on the profits of the firms and the industry are described in proposition 8.

**Proposition 8.** (1) Effects of a change in risk aversion \( \alpha_i \): (a) \((\partial \pi_i^u / \partial \alpha_i) < 0 \); (b) \((\partial \pi_i^u / \partial \alpha_i) > 0 \); (c) \((\partial \pi_i^u / \partial \alpha_i) \equiv 0 \) as \( \gamma^2 \equiv 1/\sqrt{\alpha_i \alpha_j} \). (2) Effects of a change in risk \( \gamma^2 \): (a) Assuming \( \gamma_0 > 0 \): (i) \((\partial \pi_i^u / \partial \gamma^2) > 0 \); (ii) \((\partial \pi_i^u / \partial \gamma^2) > 0 \) if \( 0.5\alpha_i < \alpha_j < 2\alpha_i \); (b) Assuming \( \gamma_0 < 0 \): (i) \((\partial \pi_i^u / \partial \gamma^2) \equiv 0 \) if \( \gamma_0 \equiv 1/\sqrt{\alpha_i \alpha_j} \); (ii) When \( \alpha_i = \alpha_j \): \((\partial \pi_i^u / \partial \gamma^2) > 0 \); (ii) \((\partial \pi_i^u / \partial \gamma^2) > 0 \) if \( \gamma_0 < 1/\sqrt{\alpha_i \alpha_j} \); (iii) When \( \alpha_i \not= \alpha_j \): \((\partial \pi_i^u / \partial \gamma^2) > 0 \), \((\partial \pi_i^u / \partial \gamma^2) > 0 \) if \( \gamma_0 < 1/\sqrt{\alpha_i \alpha_j} \) and \( g(\alpha_i) \gamma^2 \not= \alpha_j < h(\alpha_i \gamma^2) \).

\(^{10}\)As an example, let \( \alpha_i = 0.45 \), \( \alpha_j = 1 \), \( \Delta = 0.1 \), then the numerator of \( \partial \pi_i^u / \partial \gamma^2 \) becomes \( 0.1 - 0.9 \gamma^2 - 0.45 \gamma^4 \) which has a positive root \( \gamma^2 = 0.1055 \).

\(^{11}\)The uncertainty was introduced through the intercept of the demand curve in Tessitore (1994). Such specification, depicted by (9), differs from (7). Similarities between Tessitore’s results and ours can be explained by the fact that in both cases, the variance of profits can be written as: \( \text{var}(\pi_i) = \gamma_i^2 \).
Proof. The proof is obtained by taking the partial derivatives of $\beta_i$ and $\beta = \beta_i + \beta_j$.

\[
\frac{\partial \beta_i}{\partial \alpha_i} = -\frac{9\gamma^2(1 + \alpha_i\gamma ^2)(1 + 2\alpha_i\gamma ^2 + \alpha_i\alpha_j\gamma ^4)}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3}
\]

\[
\frac{\partial \beta_i}{\partial \gamma ^2} = 9(1 + \alpha_i) \left[ \frac{2\alpha_i - \alpha_i}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3} \right] - \frac{2\alpha_i\gamma ^2 + \alpha_i\alpha_j\gamma ^4 + \alpha_i\alpha_j\gamma ^6 - 3\alpha_i\alpha_j\gamma ^2}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3}
\]

\[
\frac{\partial \beta}{\partial \alpha_i} = \frac{9(1 + \alpha_i\gamma ^2)^2(1 - \alpha_i\alpha_j\gamma ^4)}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3}
\]

\[
\frac{\partial \beta}{\partial \gamma ^2} = \frac{9[\alpha_i + \alpha_j + 4\alpha_i\alpha_j\gamma ^2 - 4\alpha_i^2\alpha_j^2\gamma ^4 - \alpha_i^2\alpha_j^2\gamma ^4 - \alpha_i^2\alpha_j^2\gamma ^4]}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3}
\]

It is clear that $(\partial \beta/\partial \alpha_i) < 0$, $(\partial \beta/\partial \alpha_i) > 0$, $(\partial \beta/\partial \gamma ^2) = (\alpha_i + \alpha_j)/3 > 0$ when $\gamma ^2 = 0$ and that $(\partial \beta/\partial \alpha_i) \equiv 0$ if $\gamma ^2 \leq 1/\sqrt{\alpha_i\alpha_j}$. It is also clear that when the initial level of risk is zero, $(\partial \beta/\partial \gamma ^2) = (2\alpha_i - \alpha_i)/3 \equiv 0$ as $\alpha_i \equiv 0.5\alpha_i$. Regarding 8.2.b.i, we can prove it by rewriting Eq. (38) as follows:

\[
\frac{\partial \beta}{\partial \gamma ^2} = \frac{9(1 - \alpha_i\alpha_j\gamma ^3)[(\alpha_i + \alpha_j)(1 + \alpha_i\alpha_j\gamma ^4) + 4\alpha_i\alpha_j\gamma ^4]}{[3 + 2\alpha_i\gamma ^2 + 2\alpha_i\gamma ^4 + \alpha_i\alpha_j\gamma ^4]^3}
\]

For 8.2.b.ii, we impose symmetric risk attitudes and rewrite Eq. (36) as:
That the profits of a firm decrease with its level of risk aversion but increase with the risk aversion of its rival is an intuitive result. Similarly, it is not hard to rationalize that industry profit increases when one of the firms becomes more risk averse if the level of risk is sufficiently small. The positive marginal effects of risk on profits can be explained by recalling that risk deflates the output of the industry and that this moves the Nash equilibrium closer to the certainty collusion equilibrium. Both firms can benefit from higher profits after a rise in risk as long as their coefficients of risk aversion do not differ too much and as long as the initial level of risk is not too high. Fig. 5 captures the essence of our argument. Each schedule shows how the marginal profit of firm \( i \)'s varies with respect to risk changes when evaluated at different levels of initial risk. Each schedule is conditioned on specific coefficients of risk aversion. For each schedule, there is a critical level of risk around which the derivative of profits experiences a sign reversal: i.e.,

\[
\frac{\partial \beta_i}{\partial \gamma^2} = \frac{9(1 - \alpha^2 \gamma^4)(\alpha + 2\alpha^2 \gamma^2 + \alpha^3 \gamma^4)}{[3 + 4\alpha \gamma^2 + \alpha^2 \gamma^4]^3}.
\]

\( Q.E.D. \)

Because the profits of both firms are potentially higher under uncertainty, one might be tempted to infer that firms may like uncertainty (ex ante). To address this issue, we performed the comparative static for the expected utility of firms, as defined in Eq. (10), with respect to the risk aversion coefficients.

**Proposition 9.** Let \( W_i \) be the expected utility of firm \( i \). (1) Effects of a change in risk aversion \( \alpha_i \): (a) \((\partial W_i/\partial \alpha_i) < 0\); (b) \((\partial W_i/\partial \alpha_i) > 0\). (2) Effects of a change in risk \( \gamma^2 \): (a) \((\partial W_i/\partial \gamma^2) \equiv 0\); (b) Assuming \( \gamma_0 = 0 \); (c) \((\partial W_i/\partial \gamma^2) \equiv 0 \) as \( \alpha_i/\alpha_j \equiv 5/4 \).
The proof is obtained by taking the first derivatives of the expected utility of firm \( i \) defined as:

\[
W_i(\alpha_i, \alpha_j, \gamma^2) = \frac{9(2 + \alpha_i \gamma^2)(1 + \alpha_j \gamma^2)^2}{(3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4)^2}.
\]

Part 9.1.a is proven by:

\[
\frac{\partial W_i}{\partial \alpha_i} = \frac{-9\gamma^2(1 + \alpha_i \gamma^2)(5 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4)}{2(3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4)^3} < 0;
\]

Part 9.1.b is proven by:

\[
\frac{\partial W_i}{\partial \alpha_j} = \frac{9\gamma^2(1 + \alpha_i \gamma^2)(2 + 3\alpha_i \gamma^2 + \alpha_j \gamma^4)}{(3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4)^3} > 0;
\]

while 9.2.a is established by

\[
\frac{\partial W_i}{\partial \gamma^2} = \frac{9(1 + \alpha_i \gamma^2)^2(-5\alpha_i + 4\alpha_j - 2\alpha_i^2 \gamma^2 - \alpha_i \alpha_j \gamma^2 - \alpha_i^2 \alpha_j \gamma^4 - 2\alpha_i \alpha_j^2 \gamma^2 - \alpha_i^2 \alpha_j^2 \gamma^4)}{2(3 + 2\alpha_i \gamma^2 + 2\alpha_j \gamma^2 + \alpha_i \alpha_j \gamma^4)^3} \geq 0.
\]

Finally, 9.2.b is verified by noting that \( \frac{\partial W_i}{\partial \gamma^2} \big|_{\gamma^2=0} = 3(-5\alpha_i + 4\alpha_j)/2 \).

\[ Q.E.D. \]

Fig. 6 displays iso-expected utility schedules that map the levels of risk aversion coefficients that allow the firms to maintain their certainty level of utility. The convex and concave schedules apply to firm 1 and firm 2, respectively. For firm 1, the area above the schedule defines a region where the expected utility exceeds the certainty benchmark. This
region gets smaller as the level of risk increases. It should also be noted that the trade off between the coefficient of risk aversion (i.e., the slope of the schedule) is increasing very rapidly with own risk aversion. This means that firm 1 can only be better off (ex ante) as long as it is not too risk averse. This holds even when its rival is very risk averse. The fact that firm 1 and firm 2’s schedules do not cross means that only one firm can be better off under uncertainty. Because the space separating the schedules gets larger with the level of risk, we can infer that the likelihood that both firms be worse off (ex ante) increases with risk.

VI. Conclusion

This paper has analyzed the effect of uncertainty on the behavior of duopolists using the level of output as their strategies. We build on the framework pioneered by Cyert and De Groot (1970) to analyze asymmetries in risk and in risk attitudes. The results differ markedly from the competitive firm since one of the two firms may respond to uncertainty by increasing its output. The intuition behind this result is that uncertainty tends to reduce the output of all the firms but given that outputs are strategic substitutes, it is possible in equilibrium for the output of one of the firms to exceed its certainty benchmark. We also show that both firms can experience higher ex post profits under uncertainty if they face a low enough level of risk and if their attitudes toward risk are not too asymmetric. This does not mean that firms prefer uncertainty to certainty ex ante. We show that in terms of expected utility, both firms are likely to be worse off under uncertainty. However, we demonstrate that it is possible for one firm (but not both) to experience a level of expected utility in excess of the certainty benchmark.

Senior authorship is not assigned. The authors wish to thank Philippe Barla, Jean-Paul Chavas, Greg Leblanc, Christos Constantatos, Jean-Philippe Gervais and two anonymous referees for their insightful comments on an earlier version of this paper. The usual caveat about remaining errors applies.

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