Spatial price discrimination and the merger paradox

R. Rothschild*a,*, John S. Heywoodb,c, Kristen Monaco*d

*aDepartment of Economics, Lancaster University, Lancaster LA1 4YX, UK
bDepartment of Economics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA
cDepartment of Commerce, University of Birmingham, Birmingham B15 2TT, UK
dDepartment of Economics, University of Wisconsin-Eau Claire, Eau Claire, WI 54702, USA

Received 20 July 1998; received in revised form 25 April 2000; accepted 28 April 2000

Abstract

A familiar result in the literature on mergers is that the principal beneficiaries from such activity are the firms which are excluded from participation. The possible existence of this ‘merger paradox’ contrasts strongly with the frequently expressed view that merger is anti-competitive. This paper examines the question within the context of a model of spatial competition in which firms choose their locations in anticipation of forming a merger, and practise price-discrimination. We allow for differences in firms’ shares in the benefits of merger, and for the possibility that the firms will attach probabilities to merger formation.

© 2000 Elsevier Science B.V. All rights reserved.

Keywords: Spatial competition; Price discrimination; Merger paradox

JEL classification: D43; L41

1. Introduction

One of the robust insights afforded by traditional models of oligopolistic behaviour is that the principal beneficiaries from a merger are the non-participants.

*Corresponding author. Tel.: +44-1524-594-217; fax: +44-1524-594-244.
E-mail address: r.rothschild@lancaster.ac.uk (R. Rothschild)
In the standard Cournot model, for example, the parties to a merger exploit their increased market power by restricting output and raising price. But the consequence is that excluded firms increase their output to raise their own profits and, as Salant et al. (1983) have observed, this increase is generally sufficient to offset any profit gain by the merged entity. The literature which has grown since the work of Salant et al. broadly reinforces their original conclusion (see Pepall et al., 1999, for an extensive treatment).

The theoretical findings which have emerged stand in marked contrast to the widely held perception that, in practice, mergers are anti-competitive (see Boyer, 1992, for a discussion). Indeed, as White (1988) has observed, the parties most likely to file suits intended to overturn mergers are those firms which are excluded from the process. Excluded rivals may legitimately fear that the merged firm will acquire the larger financial resources necessary for successful predation, in the form of ‘dumping’ or other types of underpricing, or for encroaching on established product lines. There may arise also the perception that competitors which are enlarged by merger may be in a position, through the use of potentially resource intensive pre-emptive tactics, to block the expansion of smaller rivals into other markets. In any case, given the conflict between theory and experience, the natural question is whether there can be found circumstances in which horizontal mergers can be simultaneously profitable for the participants (a requirement which must be satisfied if the merger is to take place), anti-competitive (the basis for the filing of a suit) and socially inefficient (a justification for government intervention). This paper addresses that question.

The approach we take is based upon a model of spatial competition in which firms practise price discrimination. The ‘spatial’ approach to the question of collusion and merger is not in itself new. Writers such as Jehiel (1992) and Friedman and Thisse (1993) have investigated differing degrees of collusion between spatially competitive firms, but on the assumption that customers pay transport costs (so-called f.o.b pricing). The work of Levy and Reitzes (1992) is similarly focused. Our justification for considering price discrimination is that, as several writers have noted (see, for instance, Thisse and Vives, 1988; Norman and Thisse, 1996), such pricing may be preferred by both customers and firms over f.o.b pricing. This difference between our approach and that of Jehiel and Friedman–Thisse, aside, the importance of their work is that it shows the extent to which location choices which are made in anticipation of the possibility of merger

---

1 Or, more generally, in any model in which the firms’ strategic variables are strategic substitutes.

2 Pepall et al. (1999) refer to this phenomenon as the ‘paradox of merger’. It should be noted, however, that this paradox does not invariably arise. In Gaudet and Salant (1992), for example, the authors observe that when the relevant variables are strategic complements the paradox is eliminated. See also Deneckere and Davidson (1985), whose results can be explained in this way. Other writers, such as Perry and Porter (1985), demonstrate that the paradox can be eliminated if the technological basis of Salant et al.’s model is changed.
may be different from those which are not. This idea is an entirely reasonable one, and we accord it a central position in the present paper.

Previous research allowing the possibility of price discrimination, for example Reitzes and Levy (1995), does not consider the impact of anticipated merger on location choice. As a consequence, merger raises the prices set by the merged firms while leaving unaffected the optimal discriminatory prices of excluded rivals. Moreover, since merger simply results in a transfer from customers to the merged entity, there are no welfare effects.

The present paper considers within a framework of spatial price discrimination the possibility that (a) in making their location choices, the parties to a prospective merger, as well as those which will be excluded, anticipate that such merger will take place, and (b) the profits from such merger may not be equally shared amongst the participants. In these two respects the paper is an extension, to a three-firm environment, of work by Gupta et al. (1997). Their paper involves a duopoly, but no ‘outside’ firms which might be excluded from a merger and, therefore, cannot examine the merger paradox. Moreover, in Gupta et al., the maximum price which firms can set post-merger is a constant. By contrast, in the present analysis this maximum depends, in a sense to be made precise below, upon the existence and location of the outside firm.

Many of the practical examples of the theoretical problem we consider here follow from interpreting the spatial dimension as an ordered product characteristic (see Schmalensee and Thisse, 1988; Tirole, 1988). For example, newspapers or magazines differ from one another in their editorial policies. For each publication, editorial policy could, in principle, be represented as a ‘location’ decision on a political spectrum running from ‘left’ to ‘right’. The worldwide wave of mergers and consolidations which has taken place among newspapers and magazines should then help to determine the editorial policy (location) to be adopted by a new entrant. In turn, the editorial policy of the entrant will influence the profits associated with a potential merger amongst existing publications.

A second example involves departure ‘slots’ at airports. In this case, the spatial dimension can be thought of as representing departure times over an interval running from early morning to late evening. Here, a new entrant’s choice of departure time (location on the interval) is influenced by the departure times offered by existing airlines. The profitability (and proximity to one another) of the departure times offered by the latter is determined in part by opportunities for merger, as well as by the departure slot chosen by the entrant.

Recent work in other branches of industrial organization theory also touches

---

1In similar vein, the potential influence of anticipated merger opportunities on investment decisions in a non-spatial context has been studied by Gatsios and Karp (1992).

2See also McAfee et al. (1992) for a model involving spatial price discrimination.

3However, it is assumed that the actual sharing rule, whatever this may be, is known with certainty before merger occurs.
upon some of the issues to be considered here. For example, Eaton and Schmitt (1994) address the question of flexible manufacturing in a spatial framework. In their discussion the delivered price schedules of the model of spatial price discrimination are replaced by marginal cost schedules which reflect the (increasing) costs of producing progressively larger variations on a basic product. The two schedules are in this sense analogous. The authors investigate the relationship between, on the one hand, the propensity to merge and, on the other, the profitability of market entry by firms which remain outside of the merger. They argue, inter alia, that, given the products produced by the established firms, the profitability of entry is independent of whether or not merger occurs. It will become clear that the approach which we adopt here bears also upon this question.

The paper proceeds as follows. In the next section we set out the model in detail; Section 3 analyses the implications of merger activity involving two firms and one excluded rival; Section 4 offers some concluding comments.

2. The model

We let the market segment be one-dimensional with support [0,1], and suppose that consumers’ locations are distributed uniformly (with unit density) over the interval. A consumer i located at \(x_i\in[0,1]\) buys one unit of a product which maximises utility

\[U^i = V - p(x_i).\]

Here, \(V\) is i’s reservation price and \(p(x_i)\) is the delivered price to i of a unit of the product produced by a firm at location \(L_0\in[0,1]\). We assume that \(V\) is identical for all consumers.

Firms incur no fixed costs of production, and marginal production costs are normalised at zero. The firm transports the product from the point of production to each consumer in its market segment, and pays the associated costs. The total costs involved in supplying \(k\) customers, each at a unique location \(x_i\), is

\[\sum_{i=1}^{k}[r|x_i - L_0|].\]

where \(r\) is unit transport cost and \(r|x_i - L_0|\) is the cost of moving one unit of the product from point of production \(L_0\) to point of consumption \(x_i\).

Let \(r_x\) denote the smallest delivered cost of the product at given location \(x\), and

\[\text{In order to economise on notation, but without loss of generality, we shall identify firms in terms of their location. Thus, a firm ‘}F_a\text{’ is deemed to be synonymous with location }L_a\text{ and will hereafter be referred to as such.}\]
let \( r_x \) denote the second smallest delivered cost at \( x \). Then the following price schedule applies:

\[
p(x) = \begin{cases} 
  r_1 x, & \text{if } V < r_1 x; \\
  V, & \text{if } r_1 x \leq V \leq r_2 x; \\
  r_2 x, & \text{if } r_2 x < V.
\end{cases}
\]

Suppose that \( V \) is arbitrarily large, so that all consumers in \([0,1]\) are served. Then it follows from the foregoing price schedule that each product is sold by the firm with the lowest delivered costs at \( x \) at a price equal to the second most `efficient' firm's delivered cost at \( x \). Here, and at the risk of some sacrifice of precision, we use the words `most efficient' to characterise the firm offering the lowest delivered cost of the product to a consumer located at \( x \).

The profit of the most efficient firm (denoted, say, \( L_0 \)) in its market segment \( M_0 \) is given in this case by

\[
\pi_0 = \int_{x \in M_0} [p(x) - r_0 x] dx.
\]

We take the purpose of horizontal merger to be the maximisation of \( p(x) - r_x \) at any given \( x \) which is served by a party to the merger. Thus, whenever a member of a merger would deliver to \( x \) at a cost higher than that of another firm in the merger, then the former should in effect cease to deliver to \( x \). In this case, \( p(x) \) is set equal to the lower of \( V \) and the delivered cost of the most efficient `outsider' (that is, non-member of the merger), and all sales at \( x \) are made by the most efficient member.

We assume that firms’ locations are chosen simultaneously rather than sequentially, and the focus of our attention is on the determination of the equilibrium locations which emerge in this case. In determining the sequence of events we base our approach on the intuitively reasonable assumption that, in general, firms will make their location decisions before engaging in merger activity. The entry decision is thus a three-stage process, involving in the first stage the simultaneous choice of (irreversible) location and in the second stage the decision as to whether or not to merge. In the third stage, firms either engage in spatial price discrimination (if no merger has taken place) or adopt the post-merger price schedule. At the end of stage three the market clears.

3. Analysis

Suppose that the industry consists of three firms, \( L_1, L_2 \) and \( L_3 \), where \( L_1, L_2, L_3 \in [0,1] \) and \( L_1 < L_2 < L_3 \).\(^7\) We assume that the first two are potential

\(^7\)In other words, we assume that even though firms may ‘relocate’ according to their perceptions of the benefits of merger, they do not ‘leap’ over one another.
participants in a merger, while \( L_2 \) is the ‘outsider’. Figs. 1 and 2 illustrate the position. In these figures, the dashed lines represent the delivered cost schedules and the solid lines represent the price schedules.

---

\(^{8}\)We note that merger is only profitable for the participants if the firms involved are adjacent to one another.
The profits of the three firms are as follows:

\[ \pi_1 = \int_0^b r(L_2 - x_i)dx - [0.5rL_1^2 + 0.5r(b - L_1)^2] + \rho \alpha z, \quad (1) \]

\[ \pi_2 = \int_0^c r(x_i - L_1)dx + \int_c^d r(L_3 - x_i)dx - [0.5r(d - L_2)^2 + 0.5r(L_2 - b)^2] + \rho \beta z \]

and

\[ \pi_3 = \int_0^1 r(x_i - L_2)dx - [0.5r(1 - L_1)^2 + 0.5r(L_1 - d)^2], \quad (3) \]

where \( b = (L_1 + L_2)/2 \), \( c = (L_1 + L_3)/2 \) and \( d = (L_2 + L_3)/2 \). The magnitudes \( \alpha \) and \( \beta \), \( \alpha, \beta \in [0,1] \), represent the shares of \( L_i \) and \( L_2 \), respectively, in post-merger incremental profits, \( z \). In what follows we shall assume that \( \beta = 1 - \alpha \). The parameter \( \rho \), \( \rho \in [0,1] \), represents the probability — which we take to be common knowledge — that merger will take place.\(^9\)

The value of \( z \), the increment in the participants’ combined revenues due to merger, is found by considering the following. The merged entity is able to charge a price at each point in its combined market interval which corresponds to the delivered cost of \( L_3 \)'s product to that point. The total revenue obtained by the merged firms is thus given by

\[ 0.5rL_3^2 - 0.5r(L_3 - d)^2, \]

and therefore the increment in their revenues due to merger is the difference between the foregoing amount and the original revenues of both firms, or

\[ z = r[L_3^2 + 2L_1L_3 - L_2(L_2 + 2L_1)]/4. \]

The equilibrium locations are given by differentiating expressions (1), (2) and (3) with respect to \( L_1 \), \( L_2 \), and \( L_3 \), respectively, setting the derivatives equal to zero to obtain the implicit reaction functions of the three firms, and solving. This yields

\[ L_1^* = \frac{1 - \rho^2 \alpha^2 + \rho \alpha(1 + \rho)}{\rho^2 \alpha^2 - \rho \alpha(5 + \rho) + 6(1 + \rho)}, \quad (4) \]

\(^9\)It seems reasonable to consider the possibility that shareholders, anti-trust officials or others might fail to approve even a profitable merger.
\[ L_2^* = \frac{\rho^2 \alpha^2 + \rho \alpha (1 - \rho) + 3}{\rho^2 \alpha^2 - \rho \alpha (5 + \rho) + 6(1 + \rho)} \]  
(5)

and

\[ L_3^* = \frac{\rho^2 \alpha^2 - \rho \alpha (3 + \rho) + 4 \rho + 5}{\rho^2 \alpha^2 - \rho \alpha (5 + \rho) + 6(1 + \rho)} \]  
(6)

We turn now to consider the implications of the foregoing results.

3.1. The no-merger case (\( \rho = 0 \))

It is easily shown that when no merger takes place the equilibrium locations are \( L_1^* = 0.1666, \ L_2^* = (3L_1^* + 2)/5 = 0.5 \) and \( L_3^* = (L_1^* + 4)/5 = 0.833 \). In this case, each firm emerges with one-third of the total available market.

Substitution, as appropriate, of the foregoing equilibrium locations in the square bracketed cost components of expressions (1), (2) and (3) yields individual profits of \( \pi_1^* = 0.0833r, \pi_2^* = 0.0555r \) and \( \pi_3^* = 0.0833r \). The sum of the firms’ delivered costs is 0.0833r.

3.2. The case of certain merger (\( \rho = 1, \alpha \geq 0 \))

When merger takes place with certainty, expressions (4), (5) and (6) can be rewritten as

\[ L_1^* = \frac{1 - \alpha^2 + 2\alpha}{\alpha^2 - 6\alpha + 12}, \]  
(4a)

\[ L_2^* = \frac{\alpha^2 + 3}{\alpha^2 - 6\alpha + 12} \]  
(5a)

and

\[ L_3^* = \frac{\alpha^2 - 4\alpha + 9}{\alpha^2 - 6\alpha + 12}. \]  
(6a)

The derivatives of the foregoing expressions with respect to \( \alpha \) are all positive. Moreover, since the derivatives of the ratios \( L_2^*/L_1^* \) and \( L_3^*/L_2^* \) with respect to \( \alpha \) are

\[ \frac{2(\alpha^2 + 4\alpha - 3)}{(\alpha^2 - 2\alpha - 1)^2} \]

and
respectively, it is the case that while the distance between $L_1$ and $L_2$ is monotonically decreasing in $\alpha$, that between $L_3$ and $L_4$ is decreasing only if $\alpha \leq 0.64$, approximately. The two ratios can easily be shown to be equal to each other when $\alpha = 0.75$, approximately, and both are equal to 3 when $\alpha = 0$. The precise locations of the three firms for given $\alpha$, together with the associated total transport costs and individual profits, are set out in Table 1.

It is clear from the table that the rightward shift of the firms’ locations gives rise to monotonically increasing profits for $L_1$ and monotonically decreasing profits for both the other party to the merger, $L_2$, and the excluded firm, $L_3$. The combined profits of the parties to the merger always exceed their combined profits in the no-merger situation. We note, however, that if $\alpha \leq 0.34$, the profits of $L_1$ are lower than they would be in the absence of merger. By the same reasoning, if $\alpha \geq 0.894$, the profits of $L_2$ are less than 0.055, the no-merger level for that firm. Similarly, if $\alpha \geq 0.873$, the profits of $L_3$ are lower than they would be in the absence of merger. Thus, it is possible for the excluded rival to be hurt by an individually rational merger if $\alpha \in [0.87, 0.89]$, approximately.

**Proposition 1.** When merger is certain, the locations of all firms are increasing in

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$L_1^*$</th>
<th>$L_2^*$</th>
<th>$L_3^*$</th>
<th>Cost</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.083</td>
<td>0.250</td>
<td>0.750</td>
<td>0.1042</td>
<td>0.0208r</td>
<td>0.1875r</td>
<td>0.1875r</td>
</tr>
<tr>
<td>0.1</td>
<td>0.104</td>
<td>0.264</td>
<td>0.754</td>
<td>0.1021</td>
<td>0.0380r</td>
<td>0.1746r</td>
<td>0.1806r</td>
</tr>
<tr>
<td>0.2</td>
<td>0.125</td>
<td>0.280</td>
<td>0.760</td>
<td>0.1002</td>
<td>0.0564r</td>
<td>0.1611r</td>
<td>0.1726r</td>
</tr>
<tr>
<td>0.3</td>
<td>0.147</td>
<td>0.300</td>
<td>0.766</td>
<td>0.0982</td>
<td>0.0760r</td>
<td>0.1469r</td>
<td>0.1632r</td>
</tr>
<tr>
<td>0.4</td>
<td>0.168</td>
<td>0.324</td>
<td>0.775</td>
<td>0.0963</td>
<td>0.0969r</td>
<td>0.1321r</td>
<td>0.1524r</td>
</tr>
<tr>
<td>0.5</td>
<td>0.189</td>
<td>0.351</td>
<td>0.783</td>
<td>0.0945</td>
<td>0.1191r</td>
<td>0.1169r</td>
<td>0.1402r</td>
</tr>
<tr>
<td>0.6</td>
<td>0.210</td>
<td>0.384</td>
<td>0.795</td>
<td>0.0929</td>
<td>0.1425r</td>
<td>0.1013r</td>
<td>0.1267r</td>
</tr>
<tr>
<td>0.7</td>
<td>0.230</td>
<td>0.421</td>
<td>0.807</td>
<td>0.0915</td>
<td>0.1671r</td>
<td>0.0857r</td>
<td>0.1118r</td>
</tr>
<tr>
<td>0.8</td>
<td>0.250</td>
<td>0.464</td>
<td>0.821</td>
<td>0.0906</td>
<td>0.1926r</td>
<td>0.0702r</td>
<td>0.0957r</td>
</tr>
<tr>
<td>0.9</td>
<td>0.269</td>
<td>0.514</td>
<td>0.838</td>
<td>0.0905</td>
<td>0.2187r</td>
<td>0.0551r</td>
<td>0.0787r</td>
</tr>
<tr>
<td>1.0</td>
<td>0.285</td>
<td>0.571</td>
<td>0.857</td>
<td>0.0918</td>
<td>0.2449r</td>
<td>0.0408r</td>
<td>0.0612r</td>
</tr>
</tbody>
</table>

---

10 This fact — suggested by inspection of Table 1 — is easily confirmed by setting the profits expression in (1) equal to 0.0833$r$ and solving for $\alpha$.

11 The intuition here is clear. Merger results in a change in firms’ locations from those selected in the no-merger state. Such relocation may cause a reduction in the participants’ profits, and this has to be more than offset by the gains from merger if the process is to be profitable. For some values of $\alpha$ (that is $\alpha$ either ‘too small’ or ‘too large’) this offsetting increase in profits does not occur for one or the other of the participants.

12 This value of $\alpha$ is obtained by setting the profits expression in (3) equal to 0.0833$r$. 

---
$\alpha$, $L_1$’s share of the gains from merger. The profits of $L_1$ increase monotonically in $\alpha$, while those of $L_2$ and $L_3$ decrease. Merger is individually rational for the participants (and should therefore be expected to occur) only if $0.34 < \alpha < 0.894$; it is profit-increasing for all firms only if $0.34 < \alpha < 0.873$. For values of $\alpha$ in this interval, the participants’ gain from merger is greater than that of the excluded firm. Even so, when $0 < \alpha \leq 0.5$, approximately, the excluded firm obtains larger profits than does either party to the merger on its own.

There are two observations in particular which arise from the foregoing analysis. First, while locations of all three firms increase with $\alpha$, the distance between them does not necessarily do so. Second, when $L_2$ locates to the left of 0.5, the excluded firm always gains from the merger. This outcome occurs for all $\alpha < 0.873$. In other words, only when $\alpha$ is ‘large’, so that $L_2$ lies to the right of its no-merger location at 0.5, does $L_3$ obtain profits smaller than it would in the absence of merger.

The intuition upon which this logic rests can be compared with that which underlies the ‘non-spatial’ analysis of horizontal merger found in Salant et al. (1983). In that case, when products are homogeneous, the Cournot equilibrium response of the merged firm implies lower profits for it and higher profits to the excluded rivals. This follows because the merging firms are unable to commit to a level of output to be produced in the post-merger environment (see Pepall et al., 1999, for a detailed discussion). In ‘spatial’ or differentiated product markets, by contrast, this problem does not arise in general. By committing to their locations in advance, and thus retaining these locations post-merger, firms are able to ensure an output level which renders the merger profitable. Our model represents an intermediate position in which the locations of firms (the analogue of production in the non-spatial framework) depend upon the post-merger allocation of profits. In most cases this means that the post-merger equilibrium also results in greater profit for the excluded rival. Yet, in cases of $\alpha$ very large, the second firm locates far enough right prior to merger, so that the profit of the excluded rival is harmed.

It is clear that, when $\alpha \leq 0.34$, the inability of the participants to commit to locations is insufficiently offset by any gain in profits from the merger by $L_1$, so that for this firm merger is not individually rational; by the same reasoning, when $L_2$ obtains a very small share of the profits from merger (as when $\alpha \geq 0.894$) the implication is that $L_2$ is worse off. It is only when $L_2$ locates to the right of its location in the no-merger state (that is, when $\alpha \geq 0.873$) that the excluded rival is ‘squeezed’ sufficiently to experience a fall in its profits below those obtainable pre-merger.

The cost minimising (or consumer welfare maximising) equilibrium location configuration is found by summing all firms’ costs, differentiating with respect to $\alpha$ and solving as appropriate.

Proposition 2 sets out more fully the cost implications of the merger:

**Proposition 2.** Where merger is certain, it results, given any $\alpha$, in an increase in
transport costs over the no-merger level. When $\alpha = 0.859$ transport costs are 0.0904, the minimum level attainable under merger. Thus, transport costs are minimised when almost the entire gain from merger is appropriated by $L_1$, and approximately in the region of $\alpha = 0.873$, for which value the profits of the excluded firm are reduced to a level below those obtainable by that firm in the no-merger case.

3.3. The case of uncertain merger ($\rho \in [0,1]$, $\alpha \approx 0$)

We consider now issues which arise when the firms contemplate the probability, rather than the certainty, that merger will take place. In this case, we can show that the locations of all firms, and consequently their profits as well, are influenced by the value of the probability, $\rho$.

The derivatives of the ratios $L_2^\# / L_1^\#$ and $L_3^\# / L_2^\#$ with respect to $\rho$ are, respectively,

$$
\frac{2\alpha [\rho^2 \rho^2 - \rho \alpha (4 - \rho) - 4\rho - 1]}{[\rho^2 \rho^2 - \rho \alpha (1 + \rho) - 1]^2}
$$

and

$$
\frac{2[2\rho^2 \alpha (\alpha^2 - 2\alpha + 1) - 2\rho \alpha (\alpha - 1) - 7\alpha + 6]}{[\rho^2 \alpha (\alpha - 1) + \rho \alpha + 3]^2}.
$$

The first of these derivatives has negative sign if

$$
\alpha < \sqrt{\rho^2 + 8 \rho + 20 + \rho - 4},
$$

which is true for all $\alpha, \rho \in [0,1]$. The distance between the participants in the merger is thus always decreasing in $\rho$. The second of the derivatives does not admit of a simple analytic solution, but can be shown by numerical methods to be positive for all $\alpha < 0.89$, approximately. Consequently, the distance between $L_2$ and $L_3$ is increasing in $\rho$ for $\alpha < 0.89$, and decreasing in $\rho$ for larger values of $\alpha$.

Tables 2–4 show in some detail the relationships among $\alpha$, $\rho$ and the optimal location for each firm.

**Proposition 3.** Given uncertain merger ($0 < \rho < 1$), the locations of all firms shift rightwards with $\alpha$. Given $\alpha$, the location of $L_1$ typically moves rightwards as $\rho$ increases, except for $\alpha$ in the region within which merger is not individually rational for that firm. In this region, $L_1$ moves leftwards. The locations of $L_2$ and $L_3$, given $\alpha$, typically move leftwards as $\rho$ increases, except for values of $\alpha$ in the region within which merger is not individually rational for $L_2$. In this region, $L_2$ and $L_3$ move rightwards.

The foregoing results enable us to determine how the impact on the profits of
Table 2
Location of \( L \) as a function of \( a \in [0,1] \) and \( \rho \in [0,1] \)

\[
\begin{array}{cccccccccc}
\alpha & \rho \\
1.0 & 0.285 & 0.275 & 0.265 & 0.254 & 0.242 & 0.231 & 0.219 & 0.206 & 0.194 & 0.180 & 0.166 \\
0.9 & 0.269 & 0.259 & 0.249 & 0.239 & 0.229 & 0.219 & 0.209 & 0.198 & 0.188 & 0.177 & 0.166 \\
0.8 & 0.250 & 0.241 & 0.232 & 0.224 & 0.215 & 0.207 & 0.199 & 0.190 & 0.182 & 0.174 & 0.166 \\
0.7 & 0.230 & 0.223 & 0.215 & 0.208 & 0.201 & 0.195 & 0.189 & 0.183 & 0.177 & 0.172 & 0.166 \\
0.6 & 0.210 & 0.204 & 0.198 & 0.193 & 0.188 & 0.183 & 0.179 & 0.175 & 0.171 & 0.169 & 0.166 \\
0.5 & 0.189 & 0.185 & 0.181 & 0.177 & 0.174 & 0.171 & 0.168 & 0.167 & 0.166 & 0.166 & 0.166 \\
0.4 & 0.168 & 0.165 & 0.163 & 0.161 & 0.160 & 0.159 & 0.158 & 0.159 & 0.160 & 0.163 & 0.166 \\
0.3 & 0.147 & 0.146 & 0.145 & 0.145 & 0.146 & 0.147 & 0.149 & 0.151 & 0.155 & 0.160 & 0.166 \\
0.2 & 0.125 & 0.126 & 0.128 & 0.129 & 0.132 & 0.135 & 0.139 & 0.144 & 0.150 & 0.157 & 0.166 \\
0.1 & 0.104 & 0.107 & 0.110 & 0.114 & 0.118 & 0.123 & 0.129 & 0.136 & 0.144 & 0.154 & 0.166 \\
0.0 & 0.083 & 0.088 & 0.093 & 0.098 & 0.104 & 0.111 & 0.119 & 0.128 & 0.139 & 0.152 & 0.166 \\
\end{array}
\]

The excluded firm (given \( a \) in advance of the merger) is affected by the probability that the merger will occur. The derivative of \( L^*/L_2^* \) with respect to \( a \) when \( \rho > 0 \) is given by

\[
\frac{2\rho[2\rho^2a^2 - 2\rho a(1 + 2\rho) + 2\rho^2 - \rho - 7]}{[\rho^2a^2 + \rho a(1 - \rho) + 3]^2},
\]

which can be shown to be negative for all \( 0 < \rho \leq 1 \). It follows, provided that \( \rho > 0 \), that the profits of the excluded rival are always smaller as the share of \( L \) in the incremental profits from merger is larger.

**Proposition 4.** Given uncertain merger \((0 < \rho < 1)\), the profits of \( L_1 \) increase

Table 3
Location of \( L_2 \) as a function of \( a \in [0,1] \) and \( \rho \in [0,1] \)

\[
\begin{array}{cccccccccc}
\alpha & \rho \\
1.0 & 0.571 & 0.565 & 0.559 & 0.552 & 0.545 & 0.538 & 0.531 & 0.524 & 0.516 & 0.508 & 0.500 \\
0.9 & 0.514 & 0.514 & 0.513 & 0.512 & 0.511 & 0.510 & 0.509 & 0.508 & 0.506 & 0.504 & 0.502 & 0.500 \\
0.8 & 0.464 & 0.468 & 0.472 & 0.476 & 0.479 & 0.483 & 0.486 & 0.490 & 0.493 & 0.497 & 0.500 \\
0.7 & 0.421 & 0.428 & 0.436 & 0.443 & 0.450 & 0.458 & 0.466 & 0.474 & 0.482 & 0.491 & 0.500 \\
0.6 & 0.384 & 0.393 & 0.403 & 0.414 & 0.424 & 0.435 & 0.447 & 0.459 & 0.472 & 0.486 & 0.500 \\
0.5 & 0.351 & 0.363 & 0.375 & 0.388 & 0.401 & 0.415 & 0.429 & 0.445 & 0.462 & 0.480 & 0.500 \\
0.4 & 0.324 & 0.337 & 0.350 & 0.364 & 0.379 & 0.395 & 0.413 & 0.432 & 0.452 & 0.475 & 0.500 \\
0.3 & 0.300 & 0.314 & 0.328 & 0.343 & 0.360 & 0.378 & 0.397 & 0.419 & 0.443 & 0.470 & 0.500 \\
0.2 & 0.280 & 0.294 & 0.309 & 0.325 & 0.342 & 0.362 & 0.383 & 0.407 & 0.434 & 0.464 & 0.500 \\
0.1 & 0.264 & 0.277 & 0.292 & 0.309 & 0.327 & 0.347 & 0.370 & 0.395 & 0.425 & 0.459 & 0.500 \\
0.0 & 0.250 & 0.263 & 0.278 & 0.294 & 0.313 & 0.333 & 0.357 & 0.385 & 0.417 & 0.455 & 0.500 \\
\end{array}
\]
Table 4
Location of $L_1$ as a function of $a \in [0,1]$ and $\rho \in [0,1]$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.857 0.855 0.853 0.851 0.848 0.846 0.844 0.841 0.839 0.836 0.833</td>
</tr>
<tr>
<td>0.9</td>
<td>0.838 0.837 0.837 0.836 0.836 0.836 0.835 0.835 0.834 0.833</td>
</tr>
<tr>
<td>0.8</td>
<td>0.821 0.823 0.824 0.825 0.826 0.828 0.829 0.830 0.831 0.832 0.833</td>
</tr>
<tr>
<td>0.7</td>
<td>0.795 0.798 0.801 0.805 0.808 0.812 0.816 0.820 0.824 0.829 0.833</td>
</tr>
<tr>
<td>0.6</td>
<td>0.783 0.788 0.792 0.800 0.805 0.810 0.815 0.821 0.827 0.833</td>
</tr>
<tr>
<td>0.5</td>
<td>0.775 0.779 0.783 0.788 0.793 0.798 0.804 0.811 0.817 0.825 0.833</td>
</tr>
<tr>
<td>0.4</td>
<td>0.766 0.771 0.776 0.781 0.787 0.793 0.799 0.806 0.814 0.823 0.833</td>
</tr>
<tr>
<td>0.3</td>
<td>0.760 0.765 0.770 0.775 0.781 0.787 0.794 0.802 0.811 0.821 0.833</td>
</tr>
<tr>
<td>0.2</td>
<td>0.754 0.759 0.764 0.770 0.776 0.782 0.790 0.798 0.808 0.820 0.833</td>
</tr>
<tr>
<td>0.1</td>
<td>0.750 0.754 0.759 0.765 0.771 0.777 0.786 0.795 0.806 0.818 0.833</td>
</tr>
</tbody>
</table>

monotonically as $a$ rises, while those of $L_2$ and $L_3$ decrease. Conversely, given $a$, as the probability of merger rises, the profits of all firms typically increase. The exceptions arise when (a) $a$ is ‘small’, so that merger is not individually rational for $L_1$, and therefore $\pi_1$ decreases as $\rho$ rises; (b) when $a$ is so large that participation in merger is not individually rational for $L_2$, and therefore $\pi_2$ decreases as $\rho$ rises; and (c) when $a$ is so large that $\pi_3$ are reduced below the no-merger level as $\rho$ rises.

Fig. 3 summarises these critical relationships, for each firm, among $a$, $\rho$ and profits. The regions within which merger is not individually rational for one of the participants are shaded. For all combinations of $a$ and $\rho$ in region 1, merger results in a decrease in $\pi_1$ from the pre-merger level, while $\pi_1$ and $\pi_3$ increase; conversely, for combinations of $a$ and $\rho$ in region 4, merger reduces $\pi_1$ and increases $\pi_2$ and $\pi_3$. The profits of all firms increase above their no-merger levels for combinations of $a$ and $\rho$ in region 3. In region 2, merger decreases the profits of the excluded rival whilst simultaneously increasing those of $L_1$ and $L_2$.

4. Concluding comments

The ‘merger paradox’ arises when the principal beneficiaries to a merger are those firms which are excluded from the process. As noted in the Introduction, the theoretical possibility that this phenomenon might exist sits somewhat uncomfortably alongside the popular perception that, in general, merger harms outsiders.

The purpose of this paper has been to attempt a reconciliation of theory and practical experience. Our interest has therefore focused on the circumstances in
which merger is simultaneously (a) individually rational, (b) anti-competitive and (c) welfare reducing.

Our results show that, first, on one criterion of welfare (viz transport costs), all merger, irrespective of the distribution of benefits between the firms involved, reduces social welfare. We have also been able to show that the magnitude of the effect on welfare does depend upon the precise distribution of these benefits.

Second, our results show that when merger is individually rational, the gains to the participants exceed that of the excluded firm.

Third, with or without merger, the ‘middle’ firm ($L_2$) obtains lower profits than does the potentially excluded rival. Moreover, as the share of $L_1$ in the gain from merger increases, all firms move rightwards, and consequently the profits of both $L_2$ and the excluded rival fall.

Fourth, there exists a wide range of $\alpha$ for which merger is individually rational (that is, $[0.34,0.89]$, approximately). Yet, only a small part of this range ($[0.87,0.89]$, approximately) is associated with harm to the excluded rival. Thus, individually rational participation is generally not anti-competitive. With the exception of the narrow range identified above, anti-competitive mergers occur only when $L_2$ is ‘pressured’ into unprofitable participation. The most damaging instance of the phenomenon occurs when $\alpha = 1$, when the negative deviation (for both $L_2$ and $L_3$) from the no-merger level of profits is maximised.

Fifth, when merger does not occur with certainty (that is, when $1 > \rho \geq 0$) the

---

13The exception arises when $\alpha = 0$, and the profits of the two firms are equal (refer to Table 1).
profits of the three firms can be shown to reflect the impact of changes in $\alpha$, given $\rho$, and changes in $\rho$, given $\alpha$. In all of these cases, it is clear that, irrespective of $\rho$, the party with the larger expected share in incremental profits gains at the expense of the other firms. Conversely, given any expected allocation of incremental profits between the parties to the merger, and subject to both the individual rationality requirement and the effect of large $\alpha$ on the excluded rival, the profits of all firms will in general be lower as the probability of merger is smaller.

Acknowledgements

The authors would like to thank two anonymous referees for numerous very helpful comments on earlier drafts, and participants in the Graduate Economics Forum at UWM and in a seminar in the Economics Department at the University of Birmingham.

References

Salant, S.W., Switzer, S., Reynolds, R., 1983. Losses from horizontal merger: the effects of an