Consumption vs. income taxes when private human capital investments are imperfectly observable

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Abstract

This paper considers optimal taxation in an endogenous growth model where private education investments are imperfectly observable. Consumption taxation is better than labor income taxation for public provision of goods unless educational investment is completely unobservable. If subsidies are feasible for observed education investment, the consumption tax rate is independent of the degree of observability but the subsidy rate is higher the lower is the observability. If subsidies are not feasible, the consumption tax rate is lower the more limited is the observability. Optimal tax rates for goods that provide consumption and education investment simultaneously are below normal rates for observed pure consumption. Growth and welfare are positively related to (independent of) the degree of observability without (with) subsidies. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

The conventional optimal taxation literature based on a representative agent focuses on efficiency arguments and draws conclusions in favor of consumption
taxes or taxes on initial capital stock over income or labor income taxes; see, e.g., Atkinson and Stiglitz (1981), Auerbach et al. (1983), Chamley (1986), Cooley and Hansen (1991), Devereux and Love (1994), Jones et al. (1993), Judd (1987), King and Rebelo (1990), Lucas (1990), Pecorino (1993, 1994), Perroni (1995), Rebelo (1991), Summers (1981), Trostel (1993). When the importance of human capital is recognized a key question arises: can governments distinguish private human capital investment from private consumption when a consumption tax is used? For many goods and services, the answer tends to be negative. A wide range of goods and services have both a pure consumption and human capital investment component. Major commodities like food, shelter, and clothing, for example, are all in this category — they are essential for bringing children up and also for maintaining the human capital of adults. Also, books, magazines, computers, radio, TV, and private lessons are recreational as well as educational. In fact, it is easier to make a list of things which have very little human capital investment aspect — e.g. tobacco — than it is to list all the goods and services which people use to build up or maintain their human capital. Thus, it is prohibitively costly to impose different taxes based strictly on actual uses, and it is realistic to take as a fact that governments have only a partial ability to distinguish private consumption from private educational investment.

Owing to this fact, the publicly identifiable portion of private investment in education is generally exempted from a consumption tax or subsidized while the remaining portion is taxed. Clearly, taxing private educational investment to some extent under a consumption tax distorts individuals’ decisions. Thus, governments’ partial ability to differentiate private consumption from private educational investment poses some challenging questions. Firstly, can a consumption tax do better than other taxes in terms of welfare when private educational investment is (at least partly) subject to this tax? Secondly, what are the implications of the degree of this partial ability for growth and for the level of government expenditure which should, and perhaps will, be chosen?

Much work on optimal taxes focuses on lump-sum transfers of tax revenue to individuals. In practice, however, there are substantial uses of tax revenue to provide goods and services, including public consumption and investment in education in many countries. Most of the OECD countries spend more government revenue on the provision of goods and services than transfers net of social security.\footnote{According to ILO (1987), many OECD countries in fact have earning-dependent social security which conditions an individual’s benefits on his contribution made from payroll taxes. Evidently, such a social security program differs from redistributive transfer programs assumed in the taxation literature. For the effects of social security on growth, see Zhang (1995) and others.} Using the neoclassical growth model, Krusell et al. (1996) have recently shown that if government outlays are used for redistribution through lump-sum transfers, then income taxes are not necessarily worse in welfare terms, and may even be better than consumption taxes. In the political equilibrium of their model,
income taxes are attractive precisely because they are more distortionary, since this implies that low equilibrium transfer levels will be chosen. In the case of public in-kind provision, however, they show that the best tax is the least distortionary one, which in their model as in many others is the consumption tax. In recent endogenous growth models that include human capital and the labor-leisure choice, consumption taxes are found to be better than taxes on physical capital income or labor income (e.g., Lucas, 1990; Pecorino, 1993; Perroni, 1995). It is interesting to ask what happens then if the consumption tax is distortionary owing to governments' partial ability to distinguish between private education investment and private consumption.

Our purpose in this paper is to consider optimal flat-rate taxation for public provision of goods and services when there is partial public knowledge about the uses of private goods for consumption vs. investment in education. The limited observability of private spending appears in different forms with different types of expenditures (pure consumption, pure investments, or mixed), and we will investigate these different cases. In doing so, we assume that public and private educational investments are not perfect substitutes. We also consider endogenous growth of per capita income through human capital accumulation to capture the dynamic growth effects of taxes and public expenditure as in some recent optimal taxation literature (e.g., Barro, 1990; Lucas, 1990; Pecorino, 1993; Perroni, 1995; Stokey and Rebelo, 1995). Our attention is confined to taxes on consumption vs. labor income since our model abstracts from non-human capital.

In our model, both income and consumption taxes exert opposing forces on welfare and growth. On the one hand, both cause an intertemporal distortion due to their handling of the direct costs of human capital investment. (Indirect costs, that is forgone earnings, are implicitly deductible from either tax and therefore do not cause a problem.) Under a pure labor income tax there is no deduction for direct costs, and with partial observability some direct costs will be taxed under a consumption tax. These elements raise the cost of investment and reduce after-tax rates of return, tending to reduce welfare and the growth rate. On the other hand, using the tax revenue in part to provide public investment in education tends to raise welfare and the growth rate. The net effects on growth and welfare are of central importance. In particular, the degree of governments' partial ability to identify the actual uses of a private good will be critical to size the distortion of the consumption tax, and hence may affect the optimal rate of the consumption tax, and the optimal mix of income taxes and consumption taxes.

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1As stated in Pecorino (1993), the net effect of income taxation is to reduce the growth rate below its efficient value, and the increase in both the growth rate and welfare caused by replacing an income tax with a consumption tax is larger than that in Lucas (1990).

2In a model with only indirect costs of schooling, in the form of forgone earnings, proportional labor income or consumption taxes are both non-distortionary, since they each reduce the costs and benefits of going to school by the same fraction. See, e.g., Davies and Whalley (1989).
The main results in this paper are as follows. The optimal tax solution depends on the government’s ability to distinguish private human capital investment from private consumption. If it has some ability in this regard, only a consumption tax should be used to finance public provision of goods and services. If subsidies are feasible for observed education investment, the consumption tax rate is independent of the degree of observability but the subsidy rate is higher the lower is the observability. If subsidies are not feasible, the consumption tax rate is lower the more limited is the observability. Optimal tax rates for goods that provide consumption and education investment simultaneously are below normal rates for observed pure consumption. Growth and welfare are positively related to (independent of) the degree of observability without (with) subsidies. The growth effect of a consumption tax for public goods is also discussed.

The remainder of this paper is organized as follows. The next section introduces the model where private investments in education are fully observed. Section 3 investigates the model with limited observability of education investment. Section 4 makes some extensions. The last section gives some concluding remarks.

2. The model with full observability

To facilitate the comparison of this model with previous ones, we start with the conventional assumption that governments can fully observe the uses of private goods/expenditures for education and consumption. This model has an infinite number of periods and a constant population with measure one. Agents are identical, infinitely lived, and each is endowed with one unit of time per period. They allocate their time among leisure $z_t$, production $l_t$, and education $h_t$. There is a private good that can be either consumed or invested in human capital, and two publicly provided goods: public consumption and public education investment.

The representative agent’s preferences are defined over private consumption $c_t$, leisure $z_t$, and public consumption $g_t$ as

$$U_t = \sum_{s=t}^{\infty} \rho^{s-t} (\ln c_s + \ln z_s + \beta \ln g_s), \quad \beta > 0, \; 0 < \rho < 1,$$

where $t$ and $s$ refer to periods in time, $\beta$ measures the taste for public consumption, and $\rho$ is the discount factor. Correspondingly, the recursive value function is

$$V_t = \ln c_t + \ln z_t + \beta \ln g_t + \rho V_{t+1}. \quad (1)$$

As in Glomm and Ravikumar (1992), the production of the private good is proportional to the use of effective labor units according to:

$$y_t = H_l l_t.$$  \quad (2)
where \( y \) is output, \( l \) the units of labor, and \( H \) human capital (skill).

In contrast to optimal taxation models or public goods models that use a neoclassical framework (e.g. Krusell et al., 1996), here human capital or skills accumulate through education:

\[
H_{t+1} = A(q_t^{\theta} e_t^{1-\theta})^\alpha (h_t H_t) ^{1-\alpha}, \quad A > 0, \quad 0 < \theta < 1, \quad 0 < \alpha < 1.
\]  

(3)

In Eq. (3), \( q_t \) and \( e_t \) are, respectively, private and public investments of goods in education, \( h_t \) the units of time used in learning, \( A \) a productivity parameter, \( \alpha \) the importance of physical inputs, and \( \theta \) the importance of private investment relative to public investment. Note that private and public investments in education are not perfect substitutes.

We assume that government expenditures on public consumption \( g_t \) and public education investment \( e_t \) are financed by flat-rate taxes. The available tax instruments include a tax on the purchase of the private good at a rate \( \tau_c \), and a labor income tax at a rate \( \tau_h \). With full observability of private investment in education, it is important to see if there is any deductibility for educational expenditures under the optimal tax structure. Denote the deduction rate as \( x \) and let \( d = 1 - x \). Individuals’ budget and time constraints are given by

\[
(1 + \tau_c)c_t = (1 - \tau_c)H_{t+1} - (1 + \tau_h)dq_t, \quad \text{(4)}
\]

\[
z_t = 1 - h_t - l_t. \quad \text{(5)}
\]

When \( d = 0 \) (or \( x = 1 \)), there is full deductibility for education expenditures, and \( \tau_c \) becomes a consumption tax. When \( d = 1 \) (or \( x = 0 \)), there is no deductibility and hence \( \tau_c \) is a commodity/indirect tax. A negative \( d \) (or \( x > 1 \)) stands for a subsidy.

The government budget constraint is given by

\[
g_t + e_t = R_t = \tau_c \hat{c}_t + \tau_h \hat{H}_t + \tau_d \hat{q}_t, \quad \text{(6)}
\]

where \( R_t \) is the total tax revenue. Eq. (6) means that the tax revenue per household equals taxes from average consumption \( \hat{c}_t \), average labor income \( \hat{H}_t \), and average private education investment \( \hat{q}_t \). With identical agents, \( \hat{c}_t = c_t, \hat{H}_t = H_{t+1}, \) and \( \hat{q}_t = q_t \). We allow an optimal allocation of \( R_t \) between \( g_t \) and \( e_t \) with \( e_t = \psi R_t \), and \( g_t = (1 - \psi)R_t \), for \( 0 < \psi < 1 \).

Given the tax rates \( \{\tau_c, \tau_h, d\} \), the publicly provided goods \( \{g_t, e_t\} \) and initial human capital \( \{H_t, \hat{H}_t\} \), the representative agent solves the following concave programming problem:

\[\text{Deductibility in principle is feasible under an income tax, but in practice has been observed mainly under consumption taxes.}\]
\[ V(H_t, \tilde{H}_t, \tau, \tau_i, d) \]

\[
= \max_{h_t, l_t, H_{t+1}} \left\{ \ln \frac{h_t l_t (1 - \tau_t) - (1 + d \tau_t) H_{t+1} l_t^{\alpha \theta} (a \theta) l_t^{(\alpha - 1) (a \theta) - 1)}{1 + c_t} + \ln(1 - h_t - l_t) + \beta \ln g_t + \rho V(H_{t+1}, \tilde{H}_{t+1}, \tau, \tau_i, d) \right\}. \quad (7)
\]

Observe that average human capital \( \tilde{H} \) affects individuals' welfare via \( e_t \) and \( g_t \) by (6) and (7). The first-order conditions for the above optimization problem are

\[ h_t : 1/(1 - h_t - l_t) = q_t (1 + d \tau_t) (1 - \alpha) / [(1 + \tau_t) c_t h_t \alpha \theta], \quad (8) \]

\[ l_t : 1/(1 - h_t - l_t) = H_t (1 - \tau_t) / [(1 + \tau_t) c_t], \quad (9) \]

\[ H_{t+1} : (1 + d \tau_t) q_t / [(1 + \tau_t) c_t \alpha \theta H_{t+1}] = \rho \partial V_{t+1} / \partial H_{t+1}. \quad (10) \]

The envelope condition is \( \partial V_t / \partial H_t = [l_t (1 - \tau_t) + (1 - \alpha)(1 + d \tau_t) q_t / (\alpha \theta H_t)] / [(1 + \tau_t) c_t]. \)

Eqs. (8) and (10) and the envelope condition state that the loss in utility from investing in human capital now will be compensated by the gain in utility from higher productivity later. Eq. (9) equates the loss in utility from working for an additional unit of time (less leisure) to the gain in utility from earning more income for private consumption.

Solving the above first-order conditions, we have

\[ h_t = h = \frac{\rho (1 - \alpha)}{2 - \rho [1 - \alpha (1 - \theta)]}, \quad (11) \]

\[ l_t = l = \frac{1 - \rho (1 - \alpha)}{2 - \rho [1 - \alpha (1 - \theta)]}, \quad (12) \]

\[ c_t = \frac{(1 - \rho [1 - \alpha (1 - \theta)]) (1 - \tau_t)}{2 - \rho [1 - \alpha (1 - \theta)]} H_t = \gamma_t H_t, \quad (13) \]

\[ q_t = \frac{\alpha \theta \rho (1 - \tau_t)}{2 - \rho [1 - \alpha (1 - \theta)]} H_t = \gamma_t H_t. \quad (14) \]

From the above solution, the time allocation among \( h_t, l_t \) and \( z_t \) is the same in each period, and independent of all the tax rates due to our specification of the
preferences and production technology. However, private consumption \(c_t\) and private education investment \(q_t\) are proportional to the agent’s own human capital stock, changing over time, and responsive to these taxes. An increase in the labor income tax rate, or in the indirect tax rate for \(d > 0\) (partial deductibility), reduces both private consumption and private education investment.

Using the above results, our logarithmic preferences give the following value function:

\[
V_t = B + D \ln H_t + E \ln \tilde{H}_t,
\]

\[
B = \frac{1}{1 - \rho} \left\{ \ln \gamma_t(1 - \ell - l) + \beta \ln[(1 - \psi)\gamma_t] + \frac{\rho(1 + \beta)}{1 - \rho} \ln \gamma_t \right\},
\]

\[
D = \frac{1}{1 - \rho[1 - \alpha(1 - \theta)]},
\]

\[
E = \frac{\beta[1 - \rho[1 - \alpha(1 - \theta)]] + \alpha \rho(1 - \theta)}{(1 - \rho)[1 - \rho(1 - \alpha(1 - \theta))]} \tag{15}
\]

with \(\gamma_t = Ah^{-\alpha} \gamma_t(\psi \gamma_t)^{\alpha(1 - \theta)} = H_{t+1}/H_t\) and \(\gamma_t = \tau_c \gamma_t + \tau_l + \tau_d \gamma_t\). In (15), \(B\) depends on the taxes while \(D\) and \(E\) are independent of the taxes. The growth rate of the representative agent’s income is given by

\[
\mu = \gamma_t - 1. \tag{16}
\]

Here, \(\mu\) is positive if the productivity parameter \(A\) is large enough. We now determine the optimal allocation of tax revenue \(R_t\) between public investment and public consumption.

**Proposition 1.** The optimal share of public education investment is

\[
\psi^* = \alpha \rho(1 + \beta)(1 - \theta)/(\beta(1 - \rho) + \alpha \rho(1 + \beta)(1 - \theta)).
\]

**Proof.** Maximizing \(B\) (hence \(V_t\)) with respect to \(\psi\) requires \(B_\psi = [-\beta(1 - \rho)\psi + \alpha \rho(1 - \theta)(1 + \beta)(1 - \psi)]/(1 - \rho)^2 \phi(1 - \psi) = 0\). The value of \(\psi^*\) follows. \(\square\)

The division of tax revenue between public consumption and investment depends on their relative importance in preferences and technology. The more
important the public consumption (larger $\beta$), the larger the fraction of tax revenue
devoted to public consumption [larger $(1 - \psi^*)$]. Similarly, the more important the
public investment in education [larger $\alpha(1 - \theta)$], the larger the fraction of tax revenue
devoted to public investment (larger $\psi^*$). Obviously, if $\beta = 0$, then
$\psi^* = 1$; if $\theta = 1$, then $\psi^* = 0$.

Define $I = \alpha \rho \theta (1 + \beta)/(1 - \rho)$ and $J = \beta + \alpha \rho (1 - \theta)(1 + \beta)/(1 - \rho)$. Opti-
mal taxation is characterized by:

**Proposition 2.** With full observability of private investment in education, all
combinations of $\tau_c > 0$ and $\tau_i \geq 0$ such that $d^* = -\tau_i/\tau_c$ and $\gamma^*_R = J l/(1 + I + J)$
are optimal. In particular, $\tau^*_i = d^* = 0$ and $\tau^*_c = J$ is an optimal scheme.

**Proof.** See Appendix A.

The intuition for the optimal tax is to remove dynamic inefficiency and to
optimize the size of tax revenue for public goods. The factor $(1 - \tau_c)/(1 + \tau_i d)$ in
(14) represents the dynamic inefficiency of the flat-rate taxes. The inefficiency is
eliminated if $d = 0$ and $\tau_i = 0$ or if we allow a subsidy such that $d = -\tau_i/\tau_c$. With
this subsidy, it is easy to see from (4) the equivalence between proportional
consumption taxes and labor income taxes: $(1 + \tau_i) c = (1 - \tau_c)(H_i l_i - q_i)$. Conse-
quently, all combinations of taxes that correct the dynamic inefficiency and
generate the optimal tax revenue for public goods will be optimal. The optimal tax
revenue relative to income $\gamma^*_R$ depends positively on the taste for public
consumption $\beta$ and the role of public investment in education $\alpha(1 - \theta)$.

The full deductibility for educational expenditures under the optimal tax without
subsidies ($d^* = 0$) in Proposition 2 arises under the conventional assumption that
private investments in education are fully observable. Also, under full observability
the optimal tax without subsidies and without labor income taxes is found to be
equivalent to the ones with both subsidies and labor income taxes. However, what
happens if there is limited observability of private investment in education?

3. The model with limited observability

Now we assume that the government can only distinguish a fraction $(1 - \lambda)$ of
private investment in education from private consumption. The observed fraction
is exempted from the consumption tax with or without subsidies. The remaining
fraction $\lambda$ of private educational investment will unavoidably be subject to a
consumption tax, however. Since subsidized and unsubsidized cases with respect

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^In a static model Sandmo (1974) noted that this result requires some restrictive assumptions, but it
is much more general than the Cobb–Douglas case here. Our approach features problems of dynamic
inefficiency of taxation with the presence of purposive human capital investment.
to observed education expenditures are all relevant in practice, we will investigate both.

The budget constraints become

\[ g_i + e_i = R_i = \tau_e \hat{c}_i + \tau H \hat{l} + [\lambda \tau_e + (1 - \lambda)\tau_q]q_i, \]  

(17)

\[ (1 + \tau_e)c_i = H \hat{l}(1 - \tau_i) - [\lambda(1 + \tau_e) + (1 - \lambda)(1 + \tau_q)]q_i. \]  

(18)

When \( \tau_e = 0 \), there is no subsidy or no tax on observed private investment. When \( \tau_q < 0 \) (< >), there is a subsidy (tax) on observed private investment. Note that if either (i) \( \lambda = 1 \), or (ii) \( \tau_q = 0 \) (the optimal solution if either \( \theta = 0 \) or \( \alpha = 0 \)), then we obtain the equivalence between proportional consumption and labor income taxes which holds in the standard model, since the budget constraint (18) becomes \( (1 + \tau_e)(c_i + q_i) = H \hat{l}(1 - \tau_i) \) in case (i), and \( (1 + \tau_e)c_i = H \hat{l}(1 - \tau_i) \) in case (ii). Thus for consumption taxation to differ in its effects from labor income taxation we need that private goods as well as time are used in human capital production and that the government is able to prevent consumption taxes from falling fully on these private human capital investment goods.

The first-order condition with respect to leisure is the same as that in (9), while the other first-order conditions are

\[ q_i:1/(1 - h_i - l_i) = q_i[\lambda(1 + \tau_e) + (1 - \lambda)(1 + \tau_q)]/(1 + \tau_e)c_i, \]  

(19)

\[ H_{l+1}:\lambda(1 + \tau_e) + (1 - \lambda)(1 + \tau_q)]q_i/[((1 + \tau_e)c_i, \alpha \theta H_{l+1}) = \rho \partial V_{l+1}/\partial H_{l+1}. \]  

(20)

These first-order conditions lead to the same solution for \( c_i, h, l, \) and \( V_i \) as in Section 2, and

\[ q_i = \frac{\alpha \theta \rho(1 - \tau_i)}{2 - \rho[1 - \alpha(1 - \theta)]}[\lambda(1 + \tau_e) + (1 - \lambda)(1 + \tau_q)]H_i = \gamma_i H_i. \]  

(21)

By (21), the strength of the negative effect of the consumption tax on goods invested in education increases with \( \lambda \), that is as the government’s knowledge about the distinction between private consumption and private education investment worsens. The optimal \( \psi^* \) does not depend on \( \lambda \) and the optimal taxes are:

**Proposition 3.** With limited observability of private investment in education, the optimal tax features are: (i) for \( \lambda < 1 \) without subsidies, \( \tau^*_q = \tau^*_\psi = 0 \) and \( \tau^*_h > 0 \) with \( \partial \tau^*_\psi / \partial \lambda < 0 \). (ii) For \( \lambda < 1 \) with subsidies, \( \tau^*_h = 0 \), \( \tau^*_\psi = J, \) and \( \tau^*_q = - \lambda J / (1 - \lambda). \) (iii) For \( \lambda = 1 \), \( \tau^*_q = 0 \) and all combinations of \( \tau_e \geq 0 \) and \( \tau_i \geq 0 \) such that
\( \gamma^N = JI(1 + I + J) \) are equivalent to the case with \( \tau^N = 0 \) and \( \tau^L = J(1 + I + J) \).

**Proof.** See Appendix A.

Both the consumption tax and the labor income tax have their benefits and costs in terms of their effects on the welfare of the representative agent. A rise in the consumption tax rate or the labor income tax rate increases both public human capital investment and public consumption at the cost of reducing private consumption and private human capital investment. However, there is an important difference between these two taxes. An increase in the labor income tax rate reduces private investment in education by the same percentage as it reduces private consumption while an increase in the consumption tax rate decreases private investment in education less than it decreases private consumption in percentage terms for \( 0 < \lambda < 1 \). Balancing the costs and benefits of these two taxes leads to the optimal tax rates. It turns out that it is always optimal to use only the consumption tax unless the government is entirely unable to distinguish between private educational investment and private consumption, in contrast to the results with full observability and subsidies. (Taxing observed education investment is non-optimal because it generates the tax revenue only through reducing private education investment.)

The relation between the optimal consumption tax rate and the degree of observability is very intuitive. If observability worsens (larger \( \lambda \)), the proportion of private investment that is taxed becomes larger, and hence the cost of raising the tax rate increases. Thus, without subsidies the optimal tax rate depends negatively on the fraction of private human capital investment that is taxed (\( \lambda \)). With optimal subsidies for observed education investment, the subsidy rate rises in response to worsening observability (larger \( \lambda \)) to offset the increased distortion of the consumption tax on unobserved education investment (\( \lambda \tau^c \)) such that the net tax on education investment \( \lambda \tau^c (1 - \lambda) \) is zero. As a result, the optimal consumption tax rate with subsidies is independent of the degree of observability.

In the special case where \( \lambda = 1 \), the government has no distinction at all between \( c \) and \( q \), so all private human capital investment is taxed. Then all combinations of \( \tau^c \geq 0 \) and \( \tau^q \geq 0 \) that optimize the public provision of goods are

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Note that part (ii) of Proposition 3 implies that when subsidies are allowed, if on average \( x\% \) of expenditures on some class of goods are for educational purposes, then \( x\% \) of the expenditures on this class of goods should be deductible. This result may be special to the log utility function and the Cobb–Douglas human capital production function. With other functional forms, the results could be more complicated. However, we expect some deduction would still feature optimal taxation with alternative functional forms in order to minimize tax distortions on education investment that lead to dynamic inefficiency.

Subsidies are not possible when education investment is fully unobserved.
equivalent to the case with an optimal labor income tax and a zero consumption tax to finance public expenditure. As we noted earlier, according to (17) and (18), if a commodity (or indirect) tax is imposed on both private consumption and private investment in human capital at the same rate, then this tax is equivalent to a labor income tax. Therefore, the well-known equivalence between a proportional labor income tax and a uniform commodity tax holds in our dynamic setting where the commodity can be consumed or invested. It would also hold, of course, if private goods were not used in human capital formation ($\alpha = 0$ or $\theta = 0$), as noted earlier. Overall, the difference between the consumption tax on one side and a proportional labor income tax or a uniform commodity tax on the other depends on the fraction of private education investment subject to a consumption tax and the importance of goods in the human capital production function.

With $\lambda = 1$, the optimal labor income tax rate without the consumption tax depends positively on the taste for public consumption $\beta$ and the importance of public investment in education $\alpha(1 - \theta)$. This is because more tax revenue is needed if tastes for public goods become stronger, and because without consumption taxes tax revenue comes only from the labor income tax.

Now we turn to the effects of taxation on growth. Since the optimal labor income tax rate is zero in our general case with limited observability, we focus on the effect of a consumption tax. Also, since the growth effect of subsidizing education is obviously positive, and since taxing observed education investment is non-optimal, we concentrate on the case with zero subsidies (zero taxes) on observed education investment. Let $m = (1 + \lambda \tau_{g})/(1 + \tau_{c}) \leq 1$, $I_{c}^{*} = \gamma_{c}(1 + \tau_{c})$ and $I_{g}^{*} = \gamma_{g}(1 + \lambda \tau_{g})$ where $I_{c}^{*}$ and $I_{g}^{*}$ are constant at $\tau_{g}^{0} = 0$, and independent of $\lambda$.

**Proposition 4.** A rise in the consumption tax rate with zero subsidies increases (does not affect, decreases) the growth rate of the representative agent’s income if $(1 - \theta)(m^{2}I_{c}^{*} + \lambda I_{g}^{*})/[(\theta \lambda \tau_{c}(mI_{c}^{*} + \lambda I_{g}^{*}))] > 1$ ($= 1, < 1$).

**Proof.** Eqs. (11)–(13), (16), and (21) under $\tau_{g} = 0$ imply

$$\frac{\partial \mu}{\partial \tau_{c}} = \frac{\partial \gamma_{c}}{\partial \tau_{c}} = \frac{\alpha \gamma_{c}[(1 - \theta)(m^{2}I_{c}^{*} + \lambda I_{g}^{*}) - \theta \lambda \tau_{c}(mI_{c}^{*} + \lambda I_{g}^{*})]}{m \tau_{c}(1 + \tau_{c})(1 + \lambda \tau_{g})(\gamma_{c} + \lambda \gamma_{g})}. \tag{22}$$

Obviously, if $(1 - \theta)(m^{2}I_{c}^{*} + \lambda I_{g}^{*})/[(\theta \lambda \tau_{c}(mI_{c}^{*} + \lambda I_{g}^{*}))] > 1$ ($= 1, < 1$), then $\partial \mu/\partial \tau_{c} > 0$ ($= 0, < 0$). $\square$

This proposition has an intuitive interpretation. An increase in the consumption tax rate with zero subsidies affects both private investment and public investment in education. On the one hand, an increase in the consumption tax rate increases
public investment in education. The increase in public investment in education speeds up human capital accumulation. On the other hand, since the consumption tax is also imposed on unobserved private investment in education for \( \lambda > 0 \), an increase in the consumption tax rate will decrease private investment in education with zero subsidies. The decrease in private investment in education slows down human capital accumulation. The net effect on human capital accumulation of an increase in the consumption tax rate with zero subsidies depends on which of the above two effects dominates. It turns out that if \((1 - \theta)(m^2I_c + \lambda I_c) / [\theta \alpha \tau_c (mI_c + \lambda I_p)] > 1 \), then the first (second) effect dominates; as a result, human capital accumulation speeds up (slows down). If \((1 - \theta)(m^2I_c + \lambda I_c) / [\theta \alpha \tau_c (mI_c + \lambda I_p)] = 1 \), then these two effects exactly offset each other, so an increase in the consumption tax rate with zero subsidies does not affect human capital accumulation. Since the representative agent’s income is proportional to his human capital stock, the growth rate of the representative agent’s income is the same as the growth rate of human capital. By (13), (17), and (21), an income measure that includes public consumption will also grow at the same pace as that of human capital.

A closer look at two special cases of Proposition 4 with \( \lambda = 0 \) and \( \lambda = 1 \) may be helpful. From (22), we can see that if \( \lambda = 0 \) then \( \partial \mu / \partial \tau > 0 \). That is, an increase in the consumption tax rate will always increase the growth rate of per capita income when the government has full ability to differentiate private educational investment from private consumption. This is because if \( \lambda = 0 \), then an increase in the consumption tax rate will have only the first effect mentioned above. If \( \lambda = 1 \), then like private consumption, all private human capital investment is taxed, and (22) implies that the net effect of an increase in the consumption tax rate depends on the initial tax rate. If \( \tau_c < (1 - \theta) / \theta \) (\( =, > \)), then the first effect dominates (exactly offsets, is dominated by) the second effect. As a result, an increase in the consumption tax rate increases (does not affect, decreases) the growth rate.

It is also interesting to know how the degree of observability affects growth and welfare under a consumption tax with or without subsidies. In Propositions 3 and 4, we have seen that \( \lambda \) affects the optimal rate as well as the growth effect of the consumption tax without subsidies when the tax revenue is used for public goods. Owing to the fact that the optimal tax rate without subsidies is only implicitly determined for \( 0 < \lambda < 1 \), we have computed numerical solutions to investigate these effects. We summarize the simulation results as follows:

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9As government expenditures on public consumption and public investment in education are proportional to the tax revenue, an increase in the tax revenue will increase public investment in education proportionally.

10Note that here \( \theta < 1 \) is assumed. If \( \theta = 1 \), then human capital accumulation will be independent of public investment, and all tax revenue will be used to finance public consumption. As a result, an increase in the consumption tax rate will reduce the growth rate of the representative agent’s income (\( \partial \mu / \partial \tau < 0 \)).

11We have performed many numerical experiments. Some of them are reported in Tables 2 and 3 in the earlier version of this paper; see Davies et al. (1998).
Simulation result. The growth rate and welfare will tend to be higher under the optimal consumption tax without subsidies the better the government is able to distinguish private educational investment from private consumption.

For the case with subsidies, we have

**Proposition 5.** For $\lambda < 1$ under the optimal tax with subsidies, a rise in $\lambda$ has no effect on the growth rate, $\mu$, and welfare, $V$.

**Proof.** Under $(\tau^*_q, \tau^*_q, \tau^*_q) = (0, J, -\lambda J/(1 - \lambda))$ for $\lambda < 1$ in Proposition 3, the net tax on $q_i$ is $\lambda \tau^*_q + (1 - \lambda)\tau^*_q = 0$. (Subsidies are not feasible at $\lambda = 1$.) Also $\tau^*_q = J$ is independent of $\lambda$. Thus, $\gamma_q$, $\gamma_q$, and $B$ and hence $\mu$ and $V$ are independent of $\lambda$. □

For $\lambda = 0$ or 1, we have the following analytical results in regard to differences in growth rates and welfare in these two special cases under the optimal consumption tax without subsidies.

**Proposition 6.** Under the optimal tax without subsidies, the growth rate and welfare are higher with $\lambda = 0$ than those with $\lambda = 1$.

**Proof.** See Appendix A.

The results regarding the growth and welfare implications of different degrees of observability under optimal taxes with or without subsidies for the whole range of $\lambda \in [0, 1]$ can be interpreted as follows. An increase in the government’s identification ability increases private educational investment (by reducing its taxed portion), and thus it raises the growth rate provided subsidies are not feasible. From our simulation results, the magnitudes of the changes in growth rates and the optimal tax rates without subsidies are significant as $\lambda$ changes. An increase in the government’s identification ability without subsidies also reduces the distortionary effect on welfare of the consumption tax, so welfare improves. These improvements in current welfare due to more public knowledge about actual uses of a private good (smaller $\lambda$) are found to be small in our numerical experiments. However, the significant improvements in growth imply substantial future gains in welfare as $\lambda$ falls.

When the subsidy rate is optimally chosen in Proposition 5, it varies with the degree of observability to exactly offset any impact of a change in the observability on the unobserved, and hence taxed, proportion of private investment.

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12We found that the effect on welfare of the government’s identification ability is sensitive to the discount factor $\rho$ under the optimal tax without subsidies. The effect is stronger when the discount factor is larger.
Consequently, under the optimal tax with subsidies, the growth rate and welfare are independent of the degree of observability.

When private investment is fully observable (\( \lambda = 0 \)), the optimal tax is the same with or without subsidies. When private investment is fully unobservable (\( \lambda = 1 \)), subsidies are not implementable. According to Proposition 6, growth rates and welfare are higher when the government has full knowledge about actual uses of private goods than those when the government has no such knowledge at all under the optimal consumption tax. These special results accord in essence with the numerical results under the optimal tax without subsidies when the government has partial knowledge about uses of private goods.

Given the two optimal tax structures with and without subsidies, we need to know which one leads to a higher welfare level when observability is limited.

**Proposition 7.** For \( 0 < \lambda < 1 \), individuals are better off under the optimal tax with subsidies than without subsidies.

**Proof.** The optimal tax without subsidies requires \( \tau_q = 0 \), while the optimal tax with subsidies is obtained from the same model without this restriction. Thus the former cannot lead to higher welfare than the latter. When \( 0 < \lambda < 1 \), the net tax on education investment is positive under the optimal tax without subsidies but zero under the one with subsidies, implying that individuals’ welfare must be different between the two cases. The claim follows. \( \square \)

Without subsidies, some taxes on education investment are unavoidable with limited observability. Such a distortion must imply a loss in welfare. An optimal subsidy yields zero net taxes on education investment, and eliminates such a welfare loss. While this is true in our simple model, the comparison of welfare between the two cases could be different or even opposite if some realistic features are considered. For example, subsidizing observed private educational expenditures requires additional administrative costs, which generates some welfare loss. Consequently, if the welfare gain of the subsidy is lower than its welfare loss due to the additional cost, then the optimal tax without subsidies can do better.

According to Propositions 5 and 7, the welfare levels are the same for all \( \lambda \in [0,1) \) under the optimal tax with subsidies, and higher under the optimal tax with subsidies than without subsidies for \( \lambda \in (0,1) \). Also, when \( \lambda = 0 \) there is full observability and welfare is the same with or without subsidies as in Proposition 2. It then follows:

**Corollary.** Under the optimal tax without subsidies, welfare levels for \( 0 < \lambda < 1 \) are lower than that for \( \lambda = 0 \).

This corollary complements the welfare comparison in Proposition 6. Thus welfare is lower for \( \lambda > 0 \) than for \( \lambda = 0 \) under the optimal tax without subsidies.
4. Some extensions

In Sections 2 and 3, we focused on one pure private consumption and one pure private investment. In this section, we extend the model to consider more types of private expenditures for consumption as well as education. With multiple types of educational expenditures, we also consider whether different functional forms of the education technology alter our results.

4.1. A good entering utility and education simultaneously

So far we have separated private consumption and education investment. As in the examples mentioned earlier, in the real world many expenditures simultaneously provide elements of both consumption and human capital investment. What is the optimal tax structure in this case? To answer this question, we allow another form of private consumption \( c_{2t} \), which enters the process of human capital creation:

\[
H_{t+1} = A(q^0_t) c^0_{1t} e^{1-\theta_1-\theta_2} (h_t H_t)^{1-\alpha}
\]

(23)
in addition to pure private consumption \( c_{1t} \). Here \( \theta_2 \) is the relevance of private consumption \( c_{2t} \) in education. When \( \theta_2 = 0 \) the model is the same as in the previous set-up. An agent’s budget constraint with full observability and two types of private consumption expenditures is

\[
(1 + \tau_{c_1}) c_{1t} + (1 + \tau_{c_2}) c_{2t} = H_t l_t (1 - \tau_t) - (1 + \tau_q) q_t.
\]

(24)

Accordingly, the government budget constraint is

\[
g_t + e_t = R_t = \tau_{c_1} \tilde{c}_{1t} + \tau_{c_2} \tilde{c}_{2t} + \tau_l \tilde{l} + \tau_q \tilde{q}_t.
\]

(25)

Also, we replace \( \ln c \) in (1) with \( \omega \ln c_{1t} + (1 - \omega) \ln c_{2t} \). Let \( \theta = \theta_1 + \theta_2 \).

**Proposition 8.** With full observability of pure consumption \( c_{1t} \), pure education investment \( q_t \), and a mixed expenditure \( c_{2t} \) for consumption and education, the optimal tax features are \( \tau^*_c = \tau^*_q = 0 \), \( \tau^*_l = J \), and \( \tau^*_c = (1 - \omega) J/(1 - \omega + \theta_2 J) \). For \( 0 < \omega < 1 \), \( \tau^*_c > \tau^*_q > 0 \).

**Proof.** See Appendix A.

According to Proposition 8, the inclusion of private consumption in the formation of human capital leads to a nonzero optimal tax rate on this class of expenditures. The tax rate on such a mixed expenditure is lower the greater is its contribution to education. In general, its optimal tax rate is lower than the one associated with pure private consumption. This justifies nonzero but lower tax rates on some expenditures that are essential for human capital accumulation and
consumption than on other consumption expenditures that have little impact on human capital.

4.2. Education investment distinguishable from some types of private consumption but indistinguishable from other types of private consumption

When there are pure education investment and multiple types of pure consumption expenditures, some types of consumption may be easily distinguished from education investment while others cannot. What will be the optimal tax rates for such different types of expenditures? To answer this question, we assume two pure private consumption expenditures and one pure private education investment: \( c_{1t} \) has full observability and \( c_{2t} \) is fully indistinguishable from education investment \( q_t \). A tax on \( c_{2t} \) applies also to \( q_t \) under this assumption. The private budget constraint is

\[
(1 + \tau_{c_1})c_{1t} + (1 + \tau_{c_2})c_{2t} = H_1(1 - \tau_t) - (1 + \tau_{c_2})q_t. \tag{26}
\]

The government budget constraint is

\[
g_t + e_t = R_t = \tau_{c_1} \tilde{c}_{1t} + \tau_{c_2} (\tilde{c}_{2t} + \tilde{q}_t) + \tau_t \bar{l}. \tag{27}
\]

Preferences are the same as in Section 4.1. We have:

**Proposition 9.** When \( q_t \) is distinguishable from \( c_{1t} \) but not from \( c_{2t} \), the optimal tax features are \( \tau^*_1 = 0 \), \( \tau^*_2 = J \), and \( \tau^*_3 = (1 - \omega)J(1 - \omega + 1) \). For \( 0 < \omega < 1 \), \( \tau^*_1 > \tau^*_2 > 0 \).

**Proof.** See Appendix A.

The reason here is intuitive. When education investment is indistinguishable from some private consumption but distinguishable from other private consumption, an intermediate tax rate (nonzero but lower than normal rates for observable consumption) is optimal on this class of expenditures. This case is similar to that with partial observability examined in Section 3 where a parameter \( \lambda \) determines the degree of observability. Here, the parameter \( \omega \) in the utility function measures the degree of observability. When \( \omega \) is smaller, the larger is the fraction of income spent on private consumption that is unidentifiable from private investment in education, and the lower is the associated tax rate.

4.3. One fully observed investment and one partially observed investment: the Cobb–Douglas case

In previous sections, we assumed only one private investment which is either fully or partially observable. Assume now there are two private investments in
education: a fully observed \( q_{1t} \) (e.g. tuition) and a partially observed \( q_{2t} \) (e.g. books). We also assume that the human capital production function is Cobb–Douglas in these two private investments:

\[
H_{t+1} = A(q_{1t}^{\theta_1}q_{2t}^{\theta_2}e_t^{1-\theta_1-\theta_2})(h_tH_t)^{1-\alpha}.
\] (28)

The representative agent and the government’s budget constraints are, respectively

\[
(1+\tau_c)\bar{c}_t = H_t(1-\tau_t) - (1+\tau_{q_1})q_{1t} - [\lambda(1+\tau_c)+(1-\lambda)(1+\tau_{q_2})]q_{2t},
\] (29)

and

\[
g_t + e_t = R_t = \tau_c\bar{c}_t + \tau_t\bar{H}_t + \tau_{q_1}\bar{q}_{1t} + [\lambda\tau_c+(1-\lambda)\tau_{q_2}]\bar{q}_{2t}.
\] (30)

Solving the representative agent’s optimization problem gives the same solution for \( h_t, l_t \), and \( c_t \) as in Sections 2 and 3 and

\[
q_{1t} = \frac{\alpha\theta_1\rho(1-\tau)}{[2-\rho(1-\alpha(1-\theta))](1+\tau_{q_1})H_t} = \gamma_{q_1}H_t,
\] (31)

\[
q_{2t} = \frac{\alpha\theta_1\rho(1-\tau)}{[2-\rho(1-\alpha(1-\theta))](1+\lambda\tau_c+(1-\lambda)\tau_{q_2})H_t} = \gamma_{q_2}H_t.
\] (32)

The value function is the same as before with exactly the same coefficients \( D \) and \( E \). The coefficient \( B \) takes the same form as in (15) with \( \gamma_\alpha = Ah^{1-\alpha}\gamma_{q_1}^{\alpha\theta_1}\gamma_{q_2}^{\alpha\theta_2}(\psi\gamma_{q_2})^{(1-\alpha)} \), and

\[
\gamma_\beta = \tau_c\gamma_c + \tau_l + \tau_{q_1}\gamma_{q_1} + [\lambda\tau_c+(1-\lambda)\tau_{q_2}]\gamma_{q_2},
\]

where \( \theta = \theta_1 + \theta_2 \). We examine optimal taxation with and without subsidies.

**Proposition 10.** If the education technology is Cobb–Douglas with respect to fully observed \( q_{1t} \) and partially observed \( q_{2t} \), then the optimal tax features are: (i) for \( \lambda < 1 \) without subsidies, \( \tau_c^* = \tau_{q_1}^* = \tau_{q_2}^* = 0 \) and \( \tau_c^* > 0 \) with \( \partial\tau_c^*/\partial\lambda < 0 \); (ii) for \( \lambda < 1 \) with subsidies, \( \tau_c^* = \tau_{q_1}^* = \tau_{q_2}^* = J \) and \( \tau_c^* = -AJ/(1-\lambda) \); (iii) for \( \lambda = 1 \), \( \tau_c^* = \tau_{q_1}^* = \tau_{q_2}^* = 0 \) and \( \tau_c^* = J/(1+I\theta_2/\theta) \).

**Proof.** See Appendix A.

In this case with one fully observable investment and one partially observable investment, our previous conclusions remain. The optimal tax rate on the fully observable investment is zero (or the expenditure is fully deductible) as in Proposition 2, while the optimal tax rates associated with private consumption and the partially observable investment are the same as those in Proposition 3. The results here may change if we use a different education technology with respect to the two investments \( q_{1t} \) and \( q_{2t} \).
4.4. One fully observed investment and one partially observed investment: the Leontief case

Now we examine optimal taxation when the human capital production function is Leontief in a fully observed investment \( q_{1t} \) and a partially observed investment \( q_{2t}; q_t = \min(q_{1t}, q_{2t}) \), which leads to \( q_t = q_{1t} = q_{2t} \) for optimality.\(^{13}\) The education technology takes the same form as (3). The private and government budget constraints are the same as (29) and (30). To solve the representative agent’s optimization problem, we use the optimization condition \( q_t = q_{1t} = q_{2t} \) to rewrite the private budget constraint as

\[
(1 + \tau_t)c_t = H_t, (1 - \tau_t) - \{(1 + \tau_{q_1}) + [1 + \lambda\tau_t + (1 - \lambda)\tau_{q_2}]\}q_t. \tag{33}
\]

Following the same procedure as in previous sections, we get the same solution for \( h_t, l_t \), and \( c_t \) as in Sections 2 and 3 and

\[
q_t = q_{1t} = q_{2t} = \frac{\alpha\theta\rho(1 - \tau_t)}{2 - \rho[1 - \alpha(1 - \theta)][2 + \tau_{q_1} + \lambda\tau_t + (1 - \lambda)\tau_{q_2}]\}H_t, \tag{34}
\]

We also have the same form of the value function as in Sections 2 and 3 with exactly the same coefficients \( D \) and \( E \). The coefficient \( B \) takes the same form as in (15) with \( \gamma_0 = A^{1 - \sigma}q^{\sigma}(\psi\gamma_b)^{\alpha(1 - \theta)} \) and \( \gamma_b = \tau_t\gamma_t + \tau_l + [\tau_{q_1} + \lambda\tau_t + (1 - \lambda)\tau_{q_2}]\). Similar to the Cobb–Douglas case, we consider optimal taxation with and without subsidies.

**Proposition 11.** If the education technology is Leontief with respect to fully observed \( q_{1t} \) and partially observed \( q_{2t} \), then the optimal tax features are: (i) for \( \lambda = 1 \) without subsidies, \( \tau_t^* = \tau_{q_1}^* = 0 \) and \( \tau_{q_2}^* > 0 \) with \( \frac{\partial\tau_{q_2}^*}{\partial\lambda} < 0 \); (ii) for \( \lambda < 1 \) with subsidies, \( \tau_t^* = 0, \tau_{q_1}^* = J, \) and all combinations of \( \tau_{q_1}^* \) and \( \tau_{q_2}^* \) such that \( \tau_{q_1}^* + \lambda J + (1 - \lambda)\tau_{q_2}^* = 0 \); (iii) for \( \lambda = 1 \) with subsidies, \( \tau_t^* = 0, \tau_{q_1}^* = \tau_{q_2}^* = J, \) and \( \tau_{q_2}^* = -J \).

**Proof.** See Appendix A.

The intuition behind the optimal tax without subsidies here is the same as explained in Section 3. However, note that even if \( \lambda = 1 \), the optimal labor income tax rate is zero now. As a result, there is no equivalence between the optimal labor income tax and a combination of a labor income tax and a consumption tax.\(^{14}\) This

\(^{13}\)More generally, the production function should be written as \( q_t = \min(q_{1t}, q_{2t}) \) where \( \nu > 0 \). However, this simplification does not qualitatively affect our results.

\(^{14}\)This result is not unique to the Leontief case. It also applies to the Cobb–Douglas case.
is because investment $q_1$ is fully observed while $q_2$ is not. Generally speaking, whenever the government has some observability of certain private investments, the optimal labor income tax rate is zero.

In contrast to the Cobb–Douglas case in the preceding subsection, the Leontief function leads to multiple solutions for the optimal tax with subsidies in the sense that $\tau_{q_1}$ and $\tau_{q_2}$ can now have different values as long as they jointly lead to a zero net tax on education investment. The solution $\{\tau_{q_1}^*, \tau_{q_2}^*, \tau_{q_1}^z, \tau_{q_2}^z\}$ (for $\lambda < 1$) in the Cobb–Douglas case is also optimal in the Leontief case. Another special solution is $\{\tau_{q_1}^*, \tau_{q_2}^*, \tau_{q_1}^z, \tau_{q_2}^z\} = \{0, J, -J, J\}$ for both $\lambda < 1$ and $\lambda = 1$, which says that a government can treat partially observed private investment like books as ordinary consumption with the optimal tax rate $\tau_{q_2}^z = \tau_{q_2}^* = J$ and then allow a deduction at a rate 200% on observed education investment like tuition (if we view $\tau_{q_1} = \tau_1(1 - x)$ then the deduction rate $x$ equals 2 given $\tau_{q_1}^z = -J$ and $\tau_{q_2}^z = J$). Therefore, the government can compensate for its partial observability of some educational expenditures by allowing a larger deduction for other expenditures it does observe.

The difference in results between the Cobb–Douglas and Leontief cases comes from the relations of investments in the education technology. The two investments $q_1$ and $q_2$ are net substitutes in the Cobb–Douglas case, and hence they each respond to their own net taxes. However, they are perfect complements in the Leontief case, and therefore their combined, rather than own, net tax matters. The optimal solutions with subsidies in the Leontief case all lead to a zero combined net tax, $\tau_{q_1} + \lambda \tau_{q_2}^* + (1 - \lambda)\tau_{q_2}^z = 0$.

5. Conclusions

This paper has investigated optimal taxation in an endogenous growth model where governments can only distinguish partially between private human capital investment and private consumption. We have shown that optimal taxation for public provision of goods and services depends on the government’s ability to distinguish private human capital investment from private consumption. The government should use only the consumption tax instrument as long as some fraction of private investment in education can be identified. The latter should ideally be subsidized.

If subsidies are not feasible, the lower is the government’s identifying ability, the lower is the optimal consumption tax rate. With subsidies, the optimal consumption tax rate is independent of the degree of observability but the subsidy rate is higher the lower is the government’s observability. If the government is totally unable to identify the uses of private goods, then it can use either the consumption tax, or the labor income tax, or any mix of them. Optimal tax rates for goods with mixed benefits of consumption and human capital investment, and
for fully indistinguishable pure consumption and investment are positive but below normal rates for observed pure consumption. It can also be shown that without subsidies, the growth rate and welfare are positively related to the government’s identifying ability, through numerical solutions with general values for this ability as well as analytical solutions with special values for this ability. With subsidies, the growth rate and welfare are independent of the degree of the observability if there is no additional cost associated with the provision of subsidies.

These results may have some policy implications. Due to governments’ partial knowledge about actual uses of private goods, the advantages of consumption taxes over labor income taxes are smaller than those suggested in the literature. In practice, the provision of subsidies to observed educational investment involves some additional administrative costs. If these costs are small, subsidizing observed educational investment may be the best option. When such costs are substantial, exemption of observed educational investment from tax may do better although its gain tends to be smaller the more limited is the observability. In either case with or without subsidies, replacing income taxes with consumption taxes will yield smaller gains than the proposed ones in previous work. Finally, in the real world there may be good reasons for the exemptions or lower consumption tax rates often observed on goods and services which may contribute to human capital investment as well as consumption.

The gains of switching from income to consumption taxation may fall further in a model that includes physical capital. As pointed out by a referee, replacing income taxes with consumption taxes will increase the tax rates required to raise a given amount of tax revenue because the tax base becomes smaller, especially when physical capital income taxes are considered as well. If the government is sufficiently poor at identifying investment in human capital and if administrative costs of subsidies are too great, net taxation of inputs into human capital may therefore rise when income taxation is replaced by consumption taxation in models with observed investment in physical capital. This suggests the possibility that a move from a comprehensive income tax to consumption taxes may generate very small welfare gains or even welfare losses. Investigation of this issue would be a good area for future research.

In a model like ours, without non-human capital, the traditional contrast between an income tax which distorts saving, and a consumption tax which is neutral in that regard, does not apply. This naturally tends to produce more favorable growth and welfare effects under an income tax than when non-human capital is included. But while we abstract from this traditional contrast, we also bring in that between the distortionary impact of an income tax on investment in human capital vs. the less distortionary impact of a consumption tax. Recent literature has stressed the general importance of human capital and its specific role in the growth process. Our results should therefore be of interest even though they abstract from the form of income tax distortion which was formerly considered to be of central interest.
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Appendix A

Proof of Proposition 2

The first-order conditions for maximizing $B(t, d, t)$ (hence $V$) with respect to $(t, d, t)$ are given by

\[ B_t = \frac{1}{1 + \tau} \left( \frac{J \gamma + d \gamma}{\gamma} - d \right) \geq 0, \quad \tau \geq 0, \quad B_{\tau} \tau = 0, \quad (A.1) \]

\[ B_d = \frac{\tau}{(1 + \rho)(1 + dt)} \left[ \frac{J \gamma}{\gamma} - 1 \right] \geq 0, \quad d \geq 0, \quad B_{\delta} d = 0, \quad (A.2) \]

\[ B_{\tau} = \frac{1}{1 + \tau} \left[ \frac{J \gamma}{\gamma} - (1 + I) \right] \geq 0, \quad \tau \geq 0, \quad B_{\tau} \tau = 0. \quad (A.3) \]

Eq. (A.2) leads to

\[ \gamma \geq \frac{(1 - \tau)J I}{(1 + d \tau)(1 + I + J)}. \quad (A.4) \]

Eq. (A.3) yields

\[ \gamma \geq \frac{J I}{1 + I + J}. \quad (A.5) \]

Strict equality holds in (A.4) and (A.5) simultaneously if (i) $\tau = d = 0$ (without subsidy) or (ii) $d = -\tau / \tau$ (with a subsidy). It is not possible to have $d > 0$ and $\tau > 0$ at the same time for optimality.

Under condition (i), if we start with $\tau = 0$ then (A.1), (A.2), the budget constraints, and the solution for $h, l, \gamma$ and $\gamma$ imply $d = 0$ and $\tau = I$. Similarly, if we start with $d = 0$ then (A.1), (A.5), the budget constraints, and the solution for $h, l, \gamma$ and $\gamma$ lead to $\tau = 0$ and $\tau = I$. Therefore, $\tau = d = 0$ and $\tau = I$ without subsidies. Correspondingly, $\gamma = J I / (1 + I + J)$.

In the case with a subsidy, $d = -\tau / \tau$ implies: $\gamma = \gamma$ by (A.4); and $(1 + \tau)c_i = (1 - \tau)(I h_l - q_l)$ by (4). Thus, all combinations of $\tau > 0$ and $\tau \geq 0$ that satisfy $\gamma = \gamma$ and $d = -\tau / \tau$ are optimal and equivalent to the solution $\tau = d = 0$ and $\tau = I$. \qed

Before we prove Proposition 3, we provide the following lemma.
Lemma 1. Let \( \tilde{\tau}_e \) be the solution to \( \tilde{\tau}_e = \frac{J_y}{\gamma_e} - 1 = 0 \). If \( \tau_e = \tau_q = 0 \) then \( J/(1 + \lambda I) = \tilde{\tau}_e \leq J \), \( f(\tau_e) > 0 \) for \( \tau_e \in (0, \tilde{\tau}_e) \), and \( f(\tau_e) < 0 \) for \( \tau_e \in (\tilde{\tau}_e, \infty) \).

Proof. Set \( \tau_e = \tau_q = 0 \). From \( f(\tilde{\tau}_e) = 0 \), we have \( J_y/\gamma_e = \tilde{\tau}_e (\gamma_e + \lambda \gamma_q) \). Obviously, \( \tilde{\tau} \leq J \). Then we show that \( \tilde{\tau} \geq J/(1 + \lambda I) \). Define \( I_q = \gamma_q (1 + \tau_q) = [1 - \rho(1 - \alpha) - \alpha \rho \theta] / [2 - \rho(1 - \alpha) - \alpha \rho \theta] \) and \( I_q^* = \gamma_q (1 + \lambda \tau_q) = \alpha \rho \theta / [2 - \rho(1 - \alpha) - \alpha \rho \theta] \). By definition, we have \( I_q^* / I_q = (1 + \beta (1 - \alpha(1 - \theta))) / (1 - \rho) = 1 + J \), or equivalently \( I_q^* = I_q (1 + J) \) and \( I_q^* (1 + J) = (I_q + I_q^*) (1 + I + J) \). Let \( m = (1 + \lambda \tau_q) / (1 + \tau_q) \) where \( 0 < m \leq 1 \). If \( \tilde{\tau}_e = J/(1 + \lambda I) = \tilde{\tau}_e^* \), then

\[
m = \frac{1 + \lambda \tilde{\tau}_e^*}{1 + \tilde{\tau}_e^*} = \frac{1 + \lambda I + \lambda J}{1 + \lambda I + J},
\]

and thus

\[
f(\tilde{\tau}_e) = \frac{J_y}{\gamma_e} - 1 = \frac{J m I_q / I_q^*}{\tilde{\tau}_e^* (\lambda + m I_q / I_q^*)} - 1
\]

\[
= \frac{\lambda J (1 + I + J)}{(I_q^* (1 + \lambda I + \lambda J) + \lambda (1 + \lambda I + J))} \geq 0.
\]

Also, \( f'(\tau_e) < 0 \) for all \( \tau_e \). It follows that \( f(\tau_e) > 0 \) for \( \tau_e \in (0, J/(1 + \lambda I)) \). Therefore, \( \tilde{\tau}_e \geq J/(1 + \lambda I) \). The remaining part of this lemma holds due to \( f(\tilde{\tau}_e) = 0 \) and \( f'(\tau_e) < 0 \). \( \square \)

Proof of Proposition 3. Use \( B \) in (15) where \( \gamma_q, \gamma_e \) and \( \gamma_q \) are defined in Section 3. Then \( B_u = 0 \), \( B_{u_1} = 0 \), and \( B_{u_2} = 0 \) for \( (\tau_e, \tau_q) \geq 0 \) are respectively equivalent to

\[
m \frac{J y_e}{\gamma_e} - 1 \geq 0, \quad \tau_e \geq 0, \quad (A.6)
\]

\[
\frac{J I}{\gamma_q} - (1 + I + J) \geq 0, \quad \tau_q \geq 0, \quad (A.7)
\]

\[
J y_q/\gamma_q - I \geq 0, \quad \tau_q \geq 0, \quad (A.8)
\]

Also, \( B_{u_1} = 0 \), \( B_{u_1} = 0 \), and \( B_{u_2} = 0 \). Eq. (A.8) holds in strict equality when subsidies are allowed (i.e. no restriction on \( \tau_q \)).

Part (i). Rewrite (A.7) and (A.8) as \( \gamma_q, \tilde{\tau}_q \geq 0 \geq J I/(1 + I + J) \) and \( \gamma_q, \tilde{\tau}_q \geq 0 \geq J y_q / I \). Since the right-hand sides of the two equations have the relation \( (J I/(1 + I + J))/ (J y_q/I) = \lambda (1 + \tau_q) + (1 - \lambda) (1 + \tau_q) (1 - \tau_q) > 1 \) for \( (\tau_e, \tau_q) > 0, \tau_q > 0 \) and \( \tau_q > 0 \) cannot hold at the same time. Therefore there are two possibilities: \( \tau_q = 0 \) and \( \tau_q = 0 \), and \( \tau_q = 0 \).

Suppose first \( \tau_q = 0 \). For convenience, we first investigate the optimal consumption tax without subsidies by assuming that the optimal labor income tax is zero.
Then we verify that the optimal labor income tax is indeed zero. The optimal consumption tax $\tau^*_c$ with $\tau^*_l = 0$ is determined by (A.6). Define the left-hand side of (A.6) as

$$F(\tau_c) = m\left(\frac{Jy_c}{y_R} - 1\right) + \frac{J\lambda y_q}{y_R} - \lambda I.$$  (A.9)

To establish a unique optimal tax rate $\tau^*_c$, it is sufficient to show that (a) $F(\tau_c) > 0$ if $\tau_c = 0$; (b) $F(\tau_c) < 0$ for $\tau_c \in (\tilde{\tau}_c, \infty)$; and (c) $F'(\tau_c) < 0$ for all $\tau_c \in (0, \tilde{\tau}_c]$. (a) If $\tau_c = 0$, then obviously $F(0) = +\infty$. (b) If $\tau_c = \tilde{\tau}_c$, then $F'(\tilde{\tau}_c) = \lambda Jy_q/y_R - \lambda I = \lambda y_q/y_R - \lambda I = J/y_R - (1 + \lambda I) \leq 0$ since $\tilde{\tau}_c$ is the solution to $Jy_q/y_R - 1 = 0$ and $\tilde{\tau}_c = J/(1 + \lambda I)$. If $\tau_c > \tilde{\tau}_c$, $(Jy_q/y_R - 1)m < 0$ by Lemma 1. Also $\tau_c > \tilde{\tau}_c$ leads to $\lambda Jy_q/y_R - \lambda I < 0$ since this function is decreasing in $\tau_c$ and is either zero or negative at $\tilde{\tau}_c$. So $F(\tau_c) < 0$ for $\tau_c \in (\tilde{\tau}_c, \infty)$. (c) Differentiating $F(\tau_c)$ with respect to $\tau_c$ gives

$$F'(\tau_c) = -\frac{mJy_c}{y_R^2} \left(\frac{y_q}{1 + \tau_c} + \frac{\lambda y_q}{1 + \lambda \tau_c}\right) - \frac{J\lambda y_q}{y_R^2} \left[\frac{\lambda(y_q + y_R)}{1 + \lambda \tau_c} + \frac{y_c}{1 + \lambda \tau_c}\right]$$

$$- \frac{(1 - \lambda)}{(1 + \tau_c)^2} \left(\frac{Jy_c}{y_R} - 1\right)$$

where the first two terms are always negative, and the last term is either zero or negative for $\tau_c \in (0, \tilde{\tau}_c]$ by Lemma 1. Thus, $F'(\tau_c) < 0$ for $\tau_c \in (0, \tilde{\tau}_c]$. This is equivalent to $B_{\tau_c} < 0$, the second-order condition for a maximum at $\tau^*_c = 0$. Therefore there exists a unique optimal tax rate $\tau^*_c \in (0, \tilde{\tau}_c]$.

We now verify $\tau^*_c = 0$ for $\tau^*_c \in (0, \tilde{\tau}_c]$. Eq. (A.7) with $\tau^*_l = 0$ and $\tau^*_c > 0$ leads to

$$B_{\gamma l}\big|_{\tau^*_l = 0} = 0 \Leftrightarrow \gamma_l\big|_{\tau^*_l = 0} = \frac{\gamma_c^*(mI_c^* + \lambda I_q^*)}{1 + \lambda \gamma_c^*} \leq \frac{J(I_c^* + I_q^*)}{1 + I + J},$$  (A.10)

Eq. (A.6) with $\tau^*_l = 0$ and $\tau^*_c > 0$ implies

$$B_{\gamma l}\big|_{\tau^*_l > 0} = 0 \Leftrightarrow \gamma_l\big|_{\tau^*_l > 0} = \frac{\gamma_c^*(mI_c^* + \lambda I_q^*)}{1 + \lambda \gamma_c^*} \leq \frac{J(mI_c^* + \lambda I_q^*)}{(1 + \lambda \gamma_c^*)m + \lambda I(1 + \lambda \gamma_c^*)},$$  (A.11)

Eq. (A.10) and $I_q^* = II_q^*/(1 + J)$ yield

$$B_{\gamma l}\big|_{\tau^*_l = 0} \leq 0 \Leftrightarrow \tau^*_c(m + \lambda I) \leq Jm.$$  (A.12)

Similarly, (A.11) and $I_q^* = II_q^*/(1 + J)$ produce
\[ B_{\bar{c}} \big|_{\tau_{\bar{c}} > 0} = 0 \iff \tau_{\bar{c}}^* (m + \lambda I) = \frac{Jm(m(1 + J) + (\lambda I/m))}{m(1 + J) + \lambda I} . \] (A.13)

Since \( m \leq 1, \ [m(1 + J) + (\lambda I/m)]/\{m(1 + J) + \lambda I\} \geq 1 \). Then, (A.13) implies \( \tau_{\bar{c}}^* (m + \lambda I) > Jm \) for \( m < 1 \). Therefore, if \( 1/(1 + \tau_{\bar{c}}^*) < m < 1 \) (i.e. \( 0 < \lambda < 1 \)) then (A.12) holds in strictly inequality and, equivalently, \( B_{\bar{c}} \big|_{\tau_{\bar{c}} = 0} < 0 \). By Kuhn-Tucker theorem, \( \tau_{\bar{c}}^* \) must be zero. For \( \lambda = 0 \), it is easy to verify that \( B_{\bar{c}} = 0, \ \tau_{\bar{c}}^* = 0, \) and \( \tau_{\bar{c}}^* = J \).

Denote \( F \) at \( \tau_{\bar{c}}^* \) as \( F(\tau_{\bar{c}}^*, \lambda) \). Differentiating (A.9) with respect to \( \lambda \) at \( \tau_{\bar{c}}^* \), we have

\[
\frac{\partial \tau_{\bar{c}}^*}{\partial \lambda} = - \frac{\partial F(\tau_{\bar{c}}^*, \lambda)}{\partial \lambda} \frac{\partial F(\tau_{\bar{c}}^*, \lambda) / \partial \tau_{\bar{c}}^*}{\partial F(\tau_{\bar{c}}^*, \lambda) / \partial \tau_{\bar{c}}^*}
\]

where

\[
\frac{\partial F(\tau_{\bar{c}}^*, \lambda)}{\partial \lambda} = \frac{(J\gamma_0 - I)}{1 + \lambda \tau_{\bar{c}}^*} \left( \frac{J\tau_{\bar{c}}^* \gamma_0}{\gamma_0 (1 + \lambda \tau_{\bar{c}}^*)} (m\gamma_0 + \lambda \gamma_0 + \lambda \gamma_0) \right).
\]

Note that \( F(\tau_{\bar{c}}^*, \lambda) = 0 \) and \( J\gamma_0 / \gamma_0 - 1 \geq 0 \) at \( \tau_{\bar{c}}^* \in (0, \bar{\tau}) \) imply that \( J\lambda \gamma_0 / \gamma_0 - \lambda I \leq 0 \) at \( \tau_{\bar{c}}^* \). It becomes that \( \partial F(\tau_{\bar{c}}^*, \lambda) / \partial \lambda < 0 \). Also, \( \partial F(\tau_{\bar{c}}^*, \lambda) / \partial \tau_{\bar{c}}^* < 0 \) as shown above. Therefore, \( \partial \tau_{\bar{c}}^*/\partial \lambda < 0 \).

Now let \( \tau_{\bar{c}} = 0 \). Suppose \( \tau_{\bar{c}} > 0 \). Then Eqs. (A.6) and (A.8) lead to \( \tau_{\bar{c}} = J \) and \( \lambda \gamma_0 + (1 - \lambda)\tau_{\bar{c}} = 0 \) that imply \( \tau_{\bar{c}} < 0 \) for \( 0 < \lambda < 1 \). By this contradiction, \( \tau_{\bar{c}}^* = 0 \), and again \( \tau_{\bar{c}}^* \in (0, J] \), along with \( \tau_{\bar{c}}^* = \tau_{\bar{c}}^* = 0 \), is determined by (A.6) as shown above.

Part (ii). Under no restriction on \( \tau_{\bar{c}} \), solving (A.6) and (A.7) provides \( \tau_{\bar{c}}^* = 0, \ \tau_{\bar{c}}^* = J, \) and \( \tau_{\bar{c}}^* = - \lambda J/(1 - \lambda) \) for \( \lambda < 1 \).

Part (iii). When \( \lambda = 1 \), equality holds for both (A.6) and (A.7), and there is equivalence between labor income taxes and consumption taxes as mentioned earlier. (For \( \lambda = 1 \), all education investment is unobserved and hence \( \tau_{\bar{c}} = 0 \).) These two equations and the solution for \( I, \gamma_0, \gamma_0 \) and \( \gamma_0 \) imply \( \gamma_0 = J I/(1 + I + J) \) and equivalently \( (1 - \tau_{\bar{c}})/(1 + \tau_{\bar{c}}) = (1 + I)/(1 + I + J) \) for any \( \tau_{\bar{c}} \geq 0 \) and \( \tau_{\bar{c}} \geq 0 \). When \( \tau_{\bar{c}} = 0, \ \tau_{\bar{c}}^* = J/(1 + I + J) \). Thus, with \( \lambda = 1 \) all combinations of \( \tau_{\bar{c}} \geq 0 \) and \( \tau_{\bar{c}} \geq 0 \) such that \( \gamma_0 = \gamma_0^* \) are equivalent to the optimal tax solution \( \tau_{\bar{c}}^* = 0 \) and \( \tau_{\bar{c}}^* = J \).

**Proof of Proposition 6.** In this proof, we show the difference in growth rates in part (1) and the difference in welfare in part (2) in the two cases corresponding to \( \lambda = 0 \) or 1 under optimal consumption taxes without subsidies. According to Proposition 3, \( \tau_{\bar{c}}^* = \tau_{\bar{c}}^* = 0 \) at \( \lambda = 0 \) without subsidies; \( \tau_{\bar{c}}^* = \tau_{\bar{c}}^* = 0 \) and \( \tau_{\bar{c}}^* > 0 \) such that \( \gamma_{\bar{c}}^* = J I/(1 + I + J) \) is an optimal tax scheme at \( \lambda = 1 \). By (A.9) for \( \tau_{\bar{c}}^* = \tau_{\bar{c}}^* = 0, \ \tau_{\bar{c}}^* = J \) with \( \lambda = 0, \) and \( \tau_{\bar{c}}^* = J I/(1 + I) \) with \( \lambda = 1 \).

Part (1). From (16), it is sufficient to show the difference in growth rates in the
two cases by focusing on \(\gamma_l\). Denote \(\gamma_l(\lambda = 0)\) and \(\gamma_l(\lambda = 1)\) as the values of \(\gamma_l\) at \(\lambda = 0\) and \(\lambda = 1\), respectively. Then, from (11)–(15), (21), and the definitions of the optimal tax rates, we have

\[
\ln \gamma_l(\lambda = 0) - \ln \gamma_l(\lambda = 1) = \alpha \theta \ln \left(1 + \frac{J}{1 + I}\right) > 0,
\]

since \(I > 0\) and \(J > 0\).

Part (2). We know that \(l\) and the tax rates affect welfare only through \(B\) in (15), given initial human capital. Denote \(B(l = 0)\) and \(B(l = 1)\) as the values of \(B\) at \(l = 0\) and \(l = 1\), respectively. From (11)–(15), (21), Part (1), and the optimal tax rates given above, we have

\[
B(\lambda = 0) - B(\lambda = 1) = \frac{1}{(1 - \rho)} \left[ (1 + I) \ln \left(1 + \frac{J}{1 + I}\right) - \ln (1 + J) \right].
\]

Define \(\mathcal{B}(I) = B(\lambda = 0) - B(\lambda = 1)\). Then, (i) \(\mathcal{B}(I = 0) = 0\); and (ii)

\[
\mathcal{B}'(I) = \frac{1}{1 - \rho} \left[ \ln(1 + \frac{J}{1 + I}) - \frac{J}{(1 + I + J)} \right] > 0
\]

for \(I > 0\) and \(J > 0\). Thus, \(\mathcal{B}(I) > 0\). □

**Proof of Proposition 8.** The solution in this case for \(h\) and \(l\) is the same as in (11) and (12). The solution for the remaining set of variables is given by

\[
\begin{align*}
\gamma_1 &= \frac{(1 - \tau_l)\alpha \theta \theta_1}{(1 + \tau_l)(2 - \rho[1 - \alpha(1 - \theta_1 - \theta_2)])}, \\
\gamma_2 &= \frac{(1 - \tau_l)[\omega \alpha \theta_2 + (1 - \omega)[1 - \rho(1 - \alpha) - \alpha \rho \theta_1]]}{(1 + \tau_l)(2 - \rho[1 - \alpha(1 - \theta_1 - \theta_2)])}, \\
\end{align*}
\]

Also, \(\gamma_R = \tau_l \gamma_1 + \tau_h \gamma_2 + \tau_l I + \tau_h \gamma_h\). The parameters \(D\) and \(E\) of the value function \(V\) are constant and \(B\) is given by

\[
B = \text{constant} + \frac{\omega}{1 - \rho} \ln \gamma_1 + \frac{(1 - \omega)}{1 - \rho} \ln \gamma_2 + \frac{\beta}{1 - \rho} \ln (1 - \psi)
\]

\[
+ \frac{\beta}{1 - \rho} \ln \gamma_R \left[ \frac{\rho(1 + \beta)}{(1 - \rho)^2} (1 - \theta_1) \ln \gamma_4 + \alpha \theta_1 \ln \gamma_4 + \alpha(1 - \theta_1 - \theta_2)(\ln \psi + \ln \gamma_R) \right].
\]

The solution, \(\tau_l = \tau_h = 0\), \(\tau_l = J\), and \(\tau_h = (1 - \omega)J/(1 - \omega + I\theta_2/I\theta)\), arises from the first-order conditions \(B_{\tau_l} = B_{\tau_h} = B_{\gamma_1} = B_{\gamma_2} = 0\). Correspondingly, \(\gamma_R = J\theta(1 + I + J)\). Also, \(B_{\tau_l} < 0\) and \(B_{\tau_h} < 0\) at this tax solution. Thus, \(\tau_l > 0\) leads
to $B_q < 0$; and $\tau_q > 0$ leads to $B_q < 0$. As a result, $\tau_l^* = \tau_q^* = 0$ is necessary for the optimal taxation. It is obvious that $\tau_{c_1}^* > \tau_{c_2}^* > 0$ for $0 < \omega < 1$. □

**Proof of Proposition 9.** The solution for $h$ and $l$ is the same as in (11) and (12). The solution for $g$ is given by

$$
\gamma_h = \frac{\omega(1 - \tau_l)[1 - \rho(1 - \alpha) - \alpha\rho\theta]}{(1 + \tau_l)(2 - \rho(1 - \alpha) - \alpha\rho\theta)},
$$

$$
\gamma_c = \frac{(1 - \omega)(1 - \tau_l)[1 - \rho(1 - \alpha) - \alpha\rho\theta]}{(1 + \tau_l)(2 - \rho(1 - \alpha) - \alpha\rho\theta)},
$$

$$
\gamma_q = \frac{(1 - \tau_l)\alpha\rho\theta}{(1 + \tau_l)[2 - \rho(1 - \alpha) - \alpha\rho\theta]},
$$

and $\gamma_0 = \gamma_h + \gamma_c + \gamma_q + l\tau_l$. The remaining proof is similar to that for Proposition 8. □

**Proof of Proposition 10.** The first-order conditions are given by

$$
\left[ \frac{Jl}{\gamma_l} - (1 + I + J) \right] \frac{1}{1 - \tau_l} \leq 0, \quad \tau_l \geq 0, \quad (A.14)
$$

$$
\frac{J}{\gamma_c} \left[ \frac{\gamma_h}{1 + \tau_l} + \frac{\lambda\gamma_q}{1 + \mu\tau_l + (1 - \mu)\tau_q} \right] - \frac{1}{1 + \tau_l} - \frac{\lambda(1 + \mu\tau_l + (1 - \mu)\tau_q)}{1 + \mu\tau_l + (1 - \mu)\tau_q} \leq 0, \quad \tau_c \geq 0, \quad (A.15)
$$

$$
\left[ \frac{J\gamma_{c_1}}{\gamma_c} - \frac{I\theta_l}{\theta} \right] \frac{1}{1 + \tau_{c_1}} \leq 0, \quad \tau_{c_1} \geq 0, \quad (A.16)
$$

$$
\left[ \frac{J\gamma_{c_2}}{\gamma_c} - \frac{I\theta_l}{\theta} \right] \frac{(1 - \lambda)}{1 + \mu\tau_l + (1 - \mu)\tau_q} \leq 0, \quad \tau_{c_2} \geq 0. \quad (A.17)
$$

The first inequality in each line with respect to $\tau_l$ is equivalent to $B_l \leq 0$, and $B_q \neq 0$ must hold for all $i = l, c, q_1,$ or $q_2$. With subsidies on $a_2$, (i.e. no sign restriction on $\tau_q$), Eq. (A.17) always holds with strict equality.

Part (i). Rewrite (A.14), (A.16), and (A.17) as $\gamma_h|_{\tau_q = 0} \geq Jl/(1 + I + J)$, $\gamma_c|_{\tau_q = 0} \geq J\gamma_{c_1}\theta/(I\theta_l)$, and $\gamma_{c_2}|_{\tau_q = 0} \geq J\gamma_{c_2}\theta/(I\theta_l)$. As in Part (i) of the proof of Proposition 3, the pair-wise relations of the right-hand sides of the three equations imply: $\tau_q > 0$ and $\tau_{c_1} > 0$ cannot be true at the same time; $\tau_l > 0$ and $\tau_{c_2} > 0$ cannot be true at the same time; and for $\tau_{c_1} > 0$ and $\tau_{c_2} > 0$, $\lambda\tau_l + (1 - \lambda)\tau_{c_2} = \tau_{c_1}$. There are two possibilities: $\tau_l \geq 0$ and $\tau_{c_1} = \tau_{c_2} = 0$; and $\tau_l = 0$, $\tau_{c_1} \geq 0$, and $\tau_{c_2} \geq 0$. The rest of the proof of Proposition 10 is parallel to that of Proposition 3. □
Proof of Proposition 11. The first-order condition with respect to \( \tau \) is the same as (A.14), and the other first-order conditions are

\[
\frac{J}{\gamma_t} \left[ \frac{\chi}{1 + \tau_e} + \frac{2\lambda \gamma_t}{2 + \tau_{q_1} + \lambda \tau_e + (1 - \lambda)\tau_{q_2}} \right] - \frac{1}{1 + \tau_e} - \frac{\lambda J}{2 + \tau_{q_1} + \lambda \tau_e + (1 - \lambda)\tau_{q_2}} \leq 0, \quad \tau \geq 0,
\]

\[
\left[ \frac{2J\gamma_t}{\gamma_t} - I \right] \left[ \frac{1}{2 + \tau_{q_1} + \lambda \tau_e + (1 - \lambda)\tau_{q_2}} \right] \leq 0, \quad \tau_{q_1} \geq 0,
\]

\[
\left[ \frac{2J\gamma_t}{\gamma_t} - I \right] \left[ \frac{(1 - \lambda)}{2 + \tau_{q_1} + \lambda \tau_e + (1 - \lambda)\tau_{q_2}} \right] \leq 0, \quad \tau_{q_2} \geq 0.
\]

The proof for Part (i) is then similar to those for Propositions 3 and 10 with the restriction \( q = q_1 = q_2 \). For Part (ii), it can be easily verified that \( B_{\gamma_1} = B_{\gamma_2} = B_{\tau_1} = B_{\tau_2} = 0 \) if \( \tau_1 = 0, \tau_2 = J \) and \( \tau_{q_1} + \lambda J + (1 - \lambda)\tau_{q_2} = 0 \) for \( \lambda < 1 \) without sign restrictions on \( \tau_{q_1} \) and \( \tau_{q_2} \). This optimal taxation in Part (ii) is obviously equivalent to \( \gamma_1 = 0, \gamma_2 = J \), and all combinations of \( \tau_{q_1} \) and \( \tau_{q_2} \) such that \( \gamma_2 = J/(1 + I + J) \). Part (iii) is a special case of Part (ii) with \( \tau_{q_2} = J \) since \( q_2 = 0 \) is now fully unobserved at \( \lambda = 1 \). □

References

Economy 98 (5), S103–S125.
Econometrica 54, 607–622.
Reserve Bank of Minneapolis, Institute for Empirical Macroeconomics, Discussion paper 38.
Davies, J.B., Whalley, J., 1989. Taxes and capital formation: how important is human capital, NBER,
investments are imperfectly observable, University of Western Ontario, Working Paper.
of Political Economy 101, 485–517.
Economy 95, 695–709.