A Tiebout/tax-competition model

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Abstract

This paper analyzes a Tiebout/tax-competition model, where heterogeneity of consumer preferences is introduced into a standard tax-competition framework. Following the modern Tiebout tradition, consumer sorting in the model is achieved through the activities of profit-maximizing community developers. Once sorting is achieved, the equilibrium is equivalent to that in a standard tax-competition model with immobile, but heterogeneous, consumers. A principal lesson of the analysis is that, under capital taxation, consumers with high public-good demands are worse off than under a head-tax regime. In pursuit of high levels of public spending, high-demand communities impose high tax rates, which drive away capital. The analysis also establishes a number of other results. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Analysis of tax competition among local governments has generated a large literature since the mid-1980s. In models of tax competition, public expenditure is financed by a tax on mobile capital, and the analysis investigates the distortions created by this reliance on a mobile tax base. When a jurisdiction raises its capital-tax rate to increase spending on the public good, the net-of-tax return falls below that available elsewhere in the economy, and capital relocates to other jurisdictions until net returns are equalized. In the competitive case, which was...
first analyzed by Zodrow and Mieszkowski (1986), Wilson (1986), and Beck (1983), each jurisdiction is so small that the resulting capital outflow has no effect on the economy-wide net return. In the strategic case, where jurisdictions are sizable, the capital outflow occurring in response to a higher tax rate is sufficiently large to depress the net return. Wildasin (1988a), Bucovetsky (1991), and Mintz and Tulkens (1986) provided the first analyses of this case. In both cases, the shrinkage in the jurisdiction’s tax base lowers the incentive to raise taxes, which may lead to underprovision of public goods.

Two externalities are at work in generating this outcome. Firstly, a higher tax rate causes a beneficial inflow of capital into other jurisdictions. Also, if the given jurisdiction is large, an increase in its tax rate depresses the earnings of capital owners throughout the economy by lowering capital’s net return. The effect of the second externality vanishes when jurisdictions are symmetric because, in this case, each is a zero net exporter of capital. However, since each jurisdiction ignores the external gains from the capital flows caused by an increase in its tax rate, equilibrium tax rates are too low. The resulting undertaxation of capital can also be understood by recognizing that even though a fear of tax-base flight motivates low tax rates, the capital stock ends up evenly divided among jurisdictions in any symmetric equilibrium. As a result, jurisdictions’ fears are ultimately misplaced, so that taxes are set too low in equilibrium.


Unlike capital, which is mobile across jurisdictions in all tax-competition models, consumers are viewed as immobile in much of the literature. Both consumers and capital are mobile, however, in a number of recent papers, including Hoyt (1991a, 1993), Burbidge and Myers (1994), Henderson (1994, 1995), and Wilson (1997). Consumer mobility is a desirable assumption because it strengthens the connection between the tax-competition literature and the Tiebout (1956) tradition. In Tiebout models, mobile consumers enjoy freedom of choice in public-good consumption, a freedom that is gained through choice of a community of residence.²

¹Other early contributors to the literature are Oates and Schwab (1988), who analyze a model where tax competition is accompanied by competitive setting of environmental standards, and Epple and Zelenitz (1981), who investigate tax competition when jurisdictions maximize profits. Also, for a simulation analysis of strategic competition in the taxation of natural resources, see Kolstad and Wolak (1983).

²For more discussion of the Tiebout tradition, see Oates (1972) and Wildasin (1986).
Although consumer mobility adds an important element to the tax-competition literature, the link to the Tiebout tradition is still incomplete. The reason is that diversity in the demand for public goods, which leads to consumer sorting across jurisdictions in Tiebout models, is absent in the tax-competition framework. This absence reflects a key feature of all the models, namely the assumption of identical consumer preferences and endowments. With no variety in public-good demands, the literature is thus silent on an important question: what does an economy with tax competition and consumer sorting across jurisdictions look like? Such an economy resembles the one we live in, where local governments rely heavily on property taxation, and where consumers select communities partly on the basis of the public goods they provide.

The purpose of the present paper is to fill this gap in the literature by analyzing a tax-competition model where consumer preferences are heterogeneous. Since it blends two traditions, the resulting framework is referred to as a ‘Tiebout/tax-competition’ model. The model relies on standard tax-competition assumptions, and it achieves consumer sorting across jurisdictions by using the standard approach in the modern Tiebout literature. In particular, jurisdictions are formed by profit-maximizing community developers, and ‘adopted’ by consumers according to their preferences.

The analysis focuses on a perfectly competitive model, where community developers levy capital taxes but have no market power. Following Scotchmer and Wooders (1986), developers are ‘price takers’, who form communities in response to a parametric price function. In the present context, this function gives the relationship between the wage earned by community residents and the public-good level they consume. Developers choose the public-good level and capital-tax rate to maximize profit subject to this wage function, taking capital mobility into account. Simultaneously, consumers choose the best wage/public-good combination along the wage function. In equilibrium, developers offer the wage/public-good combinations demanded by consumers, and profits equal zero. Because consumers are free to choose their community of residence, equilibrium is characterized by consumer sorting, which generates homogeneous communities.

The analysis shows that the equilibrium in this model is equivalent to the outcome in a standard, perfectly competitive tax-competition model with immobile consumers, modified to allow intercommunity differences in preferences. The key feature of the equilibrium is that high-demand communities have high tax rates and small capital stocks, while low-demand communities have low tax rates and large capital stocks. Thus, in pursuit of a high public-good level, high-demand communities end up driving out investment, with capital relocating to low-demand communities, where taxes are lower. The analysis compares this outcome to the one that would obtain if public goods were financed instead by a head tax, which does not distort the allocation of capital across communities. The conclusion is that high-demanders are worse off, and that low-demanders may be better off, under the capital-tax system than under a head-tax regime. The analysis also
presents some comparative-static results, which show how the equilibrium is affected by changes in the distribution of preferences and the strength of public-good demands.

Section 2 of the paper develops the model and provides a diagrammatic illustration of the equilibrium. Relying partly on the diagrammatic approach, Section 3 analyzes the properties of the equilibrium. Section 4 offers conclusions.

2. The model

2.1. The community-developer framework

As in the standard tax-competition model, competitive firms in each community produce a numeraire private good with a common, constant-returns technology represented by the production function \( F(K, N) \), where \( K \) gives the capital input in community \( i \) and \( N \) is the labor input. Because each community resident inelastically supplies one unit of labor, \( N \) equals the community population. A tax per unit of capital is levied in each community, with \( t \) denoting the tax rate in community \( i \). Letting \( \rho \) denote capital’s net-of-tax return, which is taken as parametric by all agents in the economy, the after-tax cost of capital in community \( i \) is \( \rho + t \). Private-good producers equate capital’s marginal product to this after-tax cost, satisfying \( F(K, N) = \rho + t \). Letting \( w \) denote community \( i \)’s wage, the wage bill is then \( wN = F - KF_K \). Rewriting the first equation and dividing the second by \( N \) then yields the conditions

\[
\begin{align*}
  f'(k_i) &= \rho + t_i, \\
  w_i &= f(k_i) - k_i f'(k_i),
\end{align*}
\]  

where \( k_i = K_i/N \) equals capital per worker and \( f(k_i) = F(k_i, 1) \) gives private-good output per worker. Eqs. (1) and (2) determine \( k_i \) and \( w_i \) as functions of community \( i \)'s tax rate. Differentiating (1) yields \( \partial k_i / \partial t_i = 1/f''(k_i) < 0 \) and \( \partial w_i / \partial t_i = -k_i f''(k_i) \partial k_i / \partial t_i = -k_i < 0 \), indicating that capital relocates and the wage falls in response to a higher tax rate.

The public good \( z \) is a publicly-produced private good, an assumption that follows the tax-competition literature. The public good is produced from the private good at a constant cost of \( c \) per unit, which means that the cost of providing \( z \) units to each resident of community \( i \) is \( cNz \). Along with the constant-returns assumption on \( F \), the fact that \( z \) is private means that community populations in the model are indeterminate, simplifying the analysis. The main conclusions are unaffected, however, when \( z \) has nonprivate congestion properties, yielding a unique optimal community size.

The public good is provided by the community developer, who covers the cost
of provision with revenue from the capital tax, which he levies. Unlike a local
government, however, the developer seeks to earn a profit from providing the
public good. With tax revenue equal to $Nt_k$, the developer’s profit is given by
$N(t_k - cz_i)$. In maximizing profit, the developer takes into account the effect of
his capital-tax choice on the capital usage and wages paid by competitive
private-good producers.

On the consumer side of the model, consumer types are distinguished by
differences in preferences. Let the types be indexed by $j$, with $j = 1, 2, \ldots, J$, and
let $n^j$ denote the number of consumers of type $j$. The well-behaved utility function
of a type-$j$ consumer is $u^j(x^j, z^j)$, where $x^j$ and $z^j$ denote type-$j$ consumption of
the private and public goods.

Private-good consumption is equal to the consumer’s wage income plus income
from capital, as in the standard tax-competition model. As usual, it is assumed that
the ownership of the economy’s fixed stock of capital, denoted $K$, is equally shared
among the population. Therefore, individual capital endowments are given by
$k = K/N$, where $N$ is the economy’s total population. Income from capital
ownership is then $\rho k$ for each consumer type, and adding wage income, $x^j$ is given
by $w^j + \rho k$, where $w^j$ denotes the wage earned by a type-$j$ consumer.

A key assumption of the model is that both developers and consumers face a
parametric ‘wage function’, which gives allowable combinations of wages and
public-good levels in the economy. This function, which is written $w = P(z)$, is
analogous to the parametric price faced by buyers and sellers in a competitive
market for a private good. The difference is that, instead of being a scalar, $P(\cdot)$
gives an entire schedule of wages, with a different value for each public-good
level. As will be seen below, $P$ must be a decreasing function. Use of such
function follows Scotchmer and Wooders (1986), who argue that a proper
competitive model of local public-good provision should be based on price-taking
behavior. The analogous price function in their model specifies, for each level of
the public good, the allowable magnitude of a community entry fee, which is the
mirror image of the wage payment in the current model.

While the wage function is parametric, its properties must be consistent with
achievement of equilibrium. An equilibrium wage function must first satisfy a
‘market-clearing’ requirement. In particular, the communities created by develop-
ers in response to the wage function must match those demanded by consumers,
who also react to this function. In addition, an equilibrium wage function must
generate zero profits for developers. These requirements are discussed further
below.

Consider now the optimization problem faced by the developer of community $i$.
By suitable choice of $t_i$ and $z_i$, the developer maximizes profit subject to
$w_i = P(z_i)$, taking account of the fact that $k_i$ and $w_i$ are functions of $t_i$ via (1) and

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3Berglas and Pines (1980) offer a similar analysis in an earlier contribution that is less well known.
(2). As noted above, the developer thus recognizes the effect of his choices on the decisions of private-good producers. The Lagrangian expression for this problem is

\[ N(t_k, -c_{z_j}) + \lambda[w_i - P(z_j)], \]

where \( \lambda \) is the multiplier, and the first-order conditions are

\[ N(k + t_i \partial k_i / \partial t_i) + \lambda \partial w_i / \partial t_i = 0 \]

and

\[ -N(c - AP'(z_j)) = 0. \]

Eliminating \( \lambda \) and substituting for the derivative expressions yields

\[ \frac{c}{1 + t_i/k_j} = -P'(z_j). \] (3)

Although \( f'' < 0 \) holds, the denominator expression in (3) must be positive in order for tax revenue \( t_k \) to be increasing in \( t_i \) (the solution then lies on the uphill side of the ‘Laffer’ curve). Therefore, \( P'(z_j) \) must be negative, indicating that the wage function is decreasing in the public-good level. It should be noted that (3) is derived holding the community population \( N \) fixed. Choice of this variable by the developer is considered below.

Like the developer, the consumer’s wage/public-good combination is constrained by the wage function, with \( w = P(z) \). The consumer’s optimization problem is then to choose \( x \) and \( z \) to maximize utility subject to the budget constraint \( x = P(z) + \rho k \). The first-order condition for this problem is

\[ \frac{u'(w + \rho k, z)}{u'(w + \rho k, z)} = -P'(z), \] (4)

which says that the marginal rate of substitution is equated to the trade-off between \( w \) and \( z \) along the wage function. From (4), it is clear that all \( n \) consumers of type \( j \) select the same wage/public-good combination, with an analogous conclusion applying to consumers of other types.

With developer and consumer behavior characterized by (3) and (4), the requirements of equilibrium can now be discussed. As noted above, the market-clearing requirement says that the communities created by developers are the same as the ones demanded by consumers. This means that the developer’s first-order condition must hold at each of the wage/public-good combinations chosen by consumers. In other words, the developer’s condition (3) must hold at the \( z \)'s that satisfy (4), for \( j = 1, 2, \ldots, J \). This requirement means that wage function derivatives on the right-hand sides of (3) and (4) are the same, being evaluated at the same \( z \)'s. This coincidence allows the equations to be combined, yielding

\[ \frac{u'(w_j + \rho k, z_j)}{u'(w_j + \rho k, z_j)} = \frac{c}{1 + t_i/k_j} \] (5)

It is important to realize that this condition characterizes the consumption bundle...
in a community that has been selected or ‘adopted’ by type-$j$ consumers after its creation by developers. Because preferences differ, no other consumer types favor this particular community. As a result, the market-clearing process leads to consumer sorting, as individuals of different types adopt different communities.

This outcome is reflected by the notation in (5), where the community index $i$ on the RHS has been changed to the consumer-type index $j$, reflecting the convention that type-$j$ consumers live in community $j$. In addition, (5) reflects the substitutions $w' = w_j$ and $z' = z_j$, which indicate that the wage and public-good level for type-$j$ consumers are those prevailing in community $j$.

An additional requirement of equilibrium is that each developer earns zero profit, so that the condition

$$t_j k_j - cz_j = 0$$

holds for all $j$. Once the zero-profit condition is imposed, profit in any community $j$ (given by $N_j$ times the LHS of (6)) is invariant to the choice of population, being equal to zero regardless of the value of $N_j$. Therefore, the developer is indifferent to community size, and it can be assumed without loss of generality that all members of each consumer type are housed in a single community, as is done above.

The final requirement of equilibrium is that the total use of capital in the economy adds up to the available stock, implying $\sum_{j=1}^{J} N_j k_j = K$. Because community $j$ contains all the type-$j$ consumers, $N_j = n_j$ holds, as does $\sum_{j=1}^{J} n_j = N$. The above condition then reduces to

$$\sum_{j=1}^{J} \theta_j k_j = \bar{k}$$

where $\theta_j = n_j/N$ gives the type-$j$ population share. Equilibrium values for the variables $k_j, t_j, w_j, z_j, j = 1, 2, \ldots, J$, and $\rho$ are determined by the $4J + 1$ equations consisting of (1), (2), (5), (6), for $j = 1, 2, \ldots, J$ and (7) (note that $j$ replaces the previous $i$ index in (1) and (2)). It should be noted that the wage function does not appear in any of these conditions. Its only purpose is to guide the economy toward equilibrium, where markets clear and profits equal zero. While development of the above equilibrium conditions has been largely informal, the requirements of equilibrium are easily stated in a more-formal fashion.

The preceding equilibrium conditions are the same as those emerging from the

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Footnote: Following Scotchmer and Wooders (1986), a competitive equilibrium in the model consists of a wage function $P(\cdot)$ and values $\rho^*, z^*_j, t_j^*, k_j^*, w_j^*, j = 1, \ldots, J$, for the endogenous variables such that (i) for each $j$, $u'(w_j^* + \rho^* k_j^*) = u'(w_j + \rho k_j, z_j)$ holds for all $w_j$ and $z_j$ satisfying $w_j = P(z_j)$; (ii) for each $j$, $t_j^* k_j^* - c z_j^* = t_j k_j - c z_j$ holds for all $z_j, t_j, k_j$ satisfying $t_j' (k_j) = \rho^* + t_j, w_j = P(z_j) - k_j f'(k_j)$, and $w_j = P(z_j)$; (iii) $t_j^* k_j^* - c z_j^* = 0$ holds for each $j$, and (iv) $\sum_j \theta_j k_j = \bar{k}$. Requirement (i) reflects utility maximization, (ii) reflects profit maximization and market clearing, (iii) says that profits are zero, and (iv) indicates that the supply and demand for capital are equal.
standard tax-competition model, modified to include preference differences across communities. Most importantly, the key condition (5) is the same as the first-order condition from the standard tax-competition model, which is written in various forms in the literature. In a survey article, for example, Wilson (1999) writes the RHS of (5) as \( c/(1 - \tau \epsilon) \), where \( \epsilon \) is the absolute elasticity of the demand for capital with respect to its gross return \( \rho + t_i \), and \( \tau = t_i/((\rho + t_i)) \).

Given this equivalence, it follows that the equilibrium is identical to the one that would result if consumers were immobilized in homogeneous communities, with the capital tax in each community chosen in standard fashion to maximize the utility of its residents. This shows the usual coincidence between competitive equilibrium with community developers and the planning solution, where utility is maximized subject to a resource constraint (see Scotchmer and Wooders, 1986). Note, however, that since the economy is distorted by capital taxation in the present setting, the relevant planning solution is a second-best solution.

Given that the equilibrium could be generated from the standard model by simply introducing preference diversity across communities, it is natural to ask why the community-developer model is even needed. The answer is that a key element of Tiebout models is endogenous consumer sorting across jurisdictions, which is achieved in the modern literature via the community-developer model. Without use of that model, sorting would have to be imposed by assumption in the present analysis rather than derived as an equilibrium outcome, an unattractive prospect.

2.2. Graphical representation

The above conditions can be illustrated graphically. In Fig. 1, the dotted curve shows the budget constraint \( x = P(z) + \rho k \) faced by consumers. The goal of each consumer is to reach the highest indifference curve along this constraint, which requires a tangency between a curve and the constraint, as shown in Fig. 1 for two consumer types. These tangencies are characterized by the previous condition (4), which requires that the indifference-curve and budget-constraint slopes are equal.

Developer choices are illustrated using iso-profit loci, which consist of collections of \((w, z)\) pairs consistent with a given level of profit. An iso-profit locus is defined by the equations

\[
tk - cz = \pi \tag{8}
\]

\[
f'(k) - \rho - t = 0 \tag{9}
\]

\[
w - f(k) + kf'(k) = 0, \tag{10}
\]
where \( \pi \) is the constant level of profit. This equation system gives solutions for \( w, k, \) and \( t \) as functions of \( z, \rho, \) and \( \pi \). The \( w \) solution is written \( w = \tilde{w}(z, \rho, \pi) \), and its graph yields an iso-profit locus.

The developer’s goal is to choose the \((w, z)\) pair along the wage function \( w = P(z) \) that lies on the lowest iso-profit locus, which corresponds to the highest profit level.\(^7\) Such a solution is achieved at a point of tangency between an iso-profit locus and the wage function. To characterize a tangency, the system (8)–(10) is differentiated to find the slope of an iso-profit locus, which yields

\[
\frac{\partial w}{\partial z} = -\frac{c}{1 + t/kf''(k)}. \tag{11}
\]

Given (11), it is evident that the previous condition (3) equates the iso-profit and wage-function slopes for community \( i \), thus characterizing a tangency solution.

To illustrate this solution in Fig. 1, observe that since the dotted curve in Fig. 1 is the graph of \( P(z) + \rho k \), it represents an upward translation of the wage function.

\(^6\)Community population \( N \) is held fixed, so that it can be suppressed in writing the profit condition (8).

\(^7\)Differentiation of (8)–(10) shows that lower iso-profit loci have higher profit levels. In particular, \( \partial w/\partial \pi = -kf''/(kf'' + t) < 0 \).
by the amount $\rho k$. Therefore, in Fig. 1, the developer’s goal is to find the point on the dotted curve lying on the lowest translated iso-profit locus, represented by the graph of $\tilde{w}(z, \rho, \pi) + \rho k$. In Fig. 1, the solid concave curve represents the lowest translated locus, which is tangent to the dotted curve at several points.⁸

As explained above, achievement of equilibrium requires that consumer and developer tangencies on the dotted curve coincide, so that the communities created by developers are the same as those demanded by consumers. But this coincidence of tangencies means that indifference curves and a translated iso-profit locus must be tangent to each other at the chosen $(x, z)$ points. Furthermore, the zero-profit condition means that the relevant locus is the translated zero-profit locus. Thus, the equilibrium $(x, z)$ values in the different communities correspond to points of tangency between indifference curves of the various consumer types and the translated zero-profit locus. Letting the zero-profit locus be written $w(z, \rho) = \tilde{w}(z, \rho, 0)$, the translated locus is given by $x = w(z, \rho) + \rho k$.

Given this requirement, the properties of an equilibrium wage function are clear. An equilibrium function $P(z)$, when translated by $\rho k$, must generate a dotted curve that ‘threads’ the points of tangency between the indifference curves and the translated zero-profit locus, in the manner shown in Fig. 1. A wage function that fails to do so will violate either the market-clearing or zero-profit requirements, or both. Inspection of Fig. 1 shows that many wage functions within this equilibrium class can be constructed. For example, the zero-profit locus itself represents an equilibrium wage function.⁹

Finally, to see in Fig. 1 the equivalence between the planning solution and community-developer equilibrium, note that the translated zero-profit locus represents the planner’s resource constraint. He maximizes consumer utility subject to this constraint, which leads to the tangency solutions shown in Fig. 1.

### 3. Analysis of the equilibrium

#### 3.1. The implications of consumer sorting

To explore the implications of consumer sorting in the tax-competition model, the first step is to restate Fig. 1’s tangency conditions in a slightly different form. To do so, set $\pi = 0$ in (8) and let the $k$ and $t$ values that satisfy (8)–(10) for given $\rho$ be denoted $k(z, \rho)$ and $t(z, \rho)$. As noted above, the corresponding $w$ solution is

⁸Concavity of the translated iso-profit locus requires $\tilde{a}^3 w(z) \tilde{a}^2 z < 0$. While this inequality need not be satisfied in general, it is guaranteed to hold if $f'' < 0$.

⁹When the wage function coincides with the zero-profit locus, the dotted curve in Fig. 1 is identical to the translated zero-profit locus, so that developers are indifferent among their possible choices. Community formation is then guided solely by consumer preferences, so that market clearing occurs vacuously. In addition, the zero-profit condition holds identically.
given by \( w(z, \rho) \), the expression for the zero-profit locus. For simplicity, let the translated zero-profit locus, \( x = w(z, \rho) + \rho k \), be denoted the ‘capital-tax constraint’. Rewriting (5), the condition for a tangency between the capital-tax constraint and a type-\( j \) indifference curve is written

\[
\frac{u'_j[w(z_j, \rho) + \rho k, z_j]}{u'_i[w(z_i, \rho) + \rho k, z_i]} = c \left( 1 + \frac{t(z_j, \rho)}{k(z_j, \rho)f''(k(z_j, \rho))} \right)^{-1}.
\]  

(12)

Note that (12) captures the variation in \( k \) and \( t \) that occurs as \( z \) changes moving along the capital-tax constraint. Indeed, since \( z_j \) is the only unknown in (12), the equation directly determines the equilibrium \( z_j \) as a function of \( \rho \). This solution, which gives the public-good level in community \( j \), is denoted \( z^*_j = z^*_j(\rho) \). The equilibrium level of capital per worker in community \( j \), denoted \( k^*_j \), is then given by \( k^*_j = k(z^*_j(\rho), \rho) \). Substituting in (7), this condition can be rewritten as

\[
\sum_{j=1}^{J} \theta_j k(z^*_j(\rho), \rho) = \bar{k}.
\]  

(13)

which directly determines the equilibrium value of \( \rho \). Finally, let \( t^*_j = t(z^*_j(\rho), \rho) \) denote the equilibrium value of \( t_j \), and let \( w^*_j \) and \( x^*_j \) be defined similarly.

To develop the implications of Fig. 1, suppose that the consumer types can be ranked unambiguously according to the strength of their public-good demands, as reflected in the marginal rate of substitution between \( z \) and \( x \). In other words, suppose that for any fixed values of \( x \) and \( z \), \( u'_i(x, z)/u'_i(x, z) \) is increasing in \( j \), being highest for type-\( J \) consumers and lowest for type-1 consumers. This assumption allows unambiguous identification of consumers who are high and low demanders of the public good.

Under this assumption, it is clear that the tangencies in Fig. 1 are arrayed along the capital-tax constraint according to the strength of public-good demands. High-demand consumers, whose indifference curves are steep, live in communities with high public-good levels and low wages, implying low consumption of the private good. Conversely, communities inhabited by low-demand consumers have low public-good levels and high wages, implying high private-good consumption.

The communities inhabited by the different consumer types also have different values of \( t \) and \( k \). To deduce these differences, the effect on \( t \) and \( k \) of movements along the capital-tax constraint can be derived. This is done by computing \( \partial t/\partial z \) and \( \partial k/\partial z \) from (8)–(10) (with \( \pi = 0 \)). Differentiating these equations yields

\[
\frac{\partial t}{\partial z} = \frac{c/k}{1 + t/kf''(k)} > 0,
\]  

(14)

Note that \( j \) subscripts (denoting communities) are used instead of superscripts to denote type-\( j \) variables, recognizing that each type forms a different community.
Therefore, high $z$s are accompanied by high values of the tax rate $t$ and low values of capital per worker, $k$. Combining these results with the previous conclusions yields

**Proposition 1.** High-demand consumers live in communities with high public-good levels, low wages and private-good consumption levels, high capital taxes, and low amounts of capital per worker. The reverse conclusions apply in low-demand communities. In other words, $z^*_j$ and $t^*_j$ are increasing in the demand index $j$, while $w^*_j$, $x^*_j$ and $k^*_j$ are decreasing in $j$.

Thus, high-demand communities impose high tax rates in their pursuit of high public-good levels, leading to capital flight. The beneficiaries of this capital outflow are low-demand communities, whose low tax rates make them attractive havens. This conclusion seems to be confirmed by casual empiricism, which suggests that real estate developers tend to avoid siting shopping malls and office complexes in communities with high property-tax rates. The projects are located instead in communities with lower tax rates, where public-good demands may be more moderate.

### 3.2. Comparison to a head-tax regime

The next step is to compare the tax-competition equilibrium with heterogeneous preferences to the one that emerges under a head-tax regime. Capital taxes are absent under such a regime, which implies that capital’s marginal product is equalized across communities. Capital per worker must then be the same in all communities, and from condition (7), this common value of $k$ must equal $\bar{k}$. Capital’s net return is then fixed at $\rho = f'(\bar{k})$, and wages are the same in each community and equal to $w = f(\bar{k}) - kf'(k)$. Disposable income $w + \rho \bar{k}$ is also the same for all consumers, being equal to $f(\bar{k}) - kf'(\bar{k}) + kf'(\bar{k}) = f(\bar{k})$.

Community developers levy head taxes on consumers in the form of a community entry fee, being constrained by a price function that gives allowable combinations of the head tax and the public-good level. As before, this head-tax function must be consistent with equilibrium in that markets must clear and profits must be zero. The equilibrium is once again characterized by consumer sorting, and as before, the equilibrium $(x, z)$ pairs can be found using the developer’s zero-profit locus. That locus is given simply by $T = cz$, where $T$ is the head tax. Substituting into the consumer budget constraint, which is written $x = f(\bar{k}) - T$, then yields the ‘head-tax constraint’, $x = f(\bar{k}) - cz$. The indifference-curve tangencies along this constraint indicate the equilibrium $(x, z)$ values of the various consumer types.
Comparing the outcomes under the two tax regimes requires a comparison of the constraints in the two cases. The key to the comparison is to use (8)–(10) to rewrite the capital-tax constraint in terms of $k$, as follows:

\[
x = f(k) - kf'(k) + \rho \tilde{k} = f(k) - k(\rho + t) + \rho \tilde{k} = f(k) + (\tilde{k} - \tilde{k})\rho - cz.
\]

Note that in (16), $k$ is implicitly a function of $z$, with $k = k(z, \rho)$ from (8)–(10). Next, the mean value theorem is used to rewrite $f(k)$ in the last line of (16) as $f(\tilde{k}) + f'(\tilde{k})(k - \tilde{k})$, where $\tilde{k}$ lies between $k$ and $\tilde{k}$. Then, (16) can be rewritten as

\[
x = f(\tilde{k}) + (f'(\tilde{k}) - \rho)(k(z, \rho) - \tilde{k}) - cz,
\]

where the dependence of $k$ on $z$ is made explicit. The term $f'(\tilde{k}) - \rho$ equals the tax rate in a community whose level of capital per worker, $\tilde{k}$, lies between $k(z, \rho)$ and $k$. Since tax rates must be positive, the middle term in (17) is then positive (negative) when $k(z, \rho) - \tilde{k} > (\leq) 0$. Recalling that $k$ and $z$ are inversely related from (15), it follows that the middle term is positive (negative) when $z < (>) \tilde{z}$, where $\tilde{z}$ is the $z$ value associated with $k = \tilde{k}$ ($\tilde{z}$ satisfies $k = k(\tilde{z}, \rho)$). Summarizing yields

**Lemma 1.** The capital-tax constraint lies above the head-tax constraint at $z$ values satisfying $z < \tilde{z}$, and lies below it at values satisfying $z > \tilde{z}$.

This result is illustrated in Fig. 2. The intersection point of the two constraints is denoted $B$, which has a $z$ coordinate equal to $\tilde{z}$.

Lemma 1 can be used to compare the welfare of different consumer types between the capital-tax and head-tax regimes. Inspection of Fig. 2 shows that a switch to the head-tax regime raises utility for consumers whose tangency on the capital-tax constraint lies to the right of $B$. Consumers with tangencies to the left of $B$ may be hurt by the switch, an outcome that is illustrated in Fig. 2. However, when the capital-tax tangency lies very close to $B$, it is possible that the tangent indifference curve dips below the head-tax constraint to the right of $B$. In this situation, which is not shown in Fig. 2, a switch to the head-tax regime would raise the consumer’s utility. The welfare effect of the switch is thus ambiguous for consumers with tangencies to the left of $B$, suggesting the following conclusion:

**Proposition 2.** Type-J consumers (the highest public-good demanders) are worse off under capital taxation than under a head-tax regime. For other consumer types, utility comparisons between the regimes are ambiguous.

Note that Proposition 2 relies on the fact that at least one tangency, namely that of
the highest-demand type, must lie to the right of $B$. If this were not true, $k$ values would all lie below $k$, an impossibility.

Proposition 2 follows naturally from Proposition 1. Since high tax rates lead to a flight of capital from communities with high public-good demands, it is natural that the residents of these communities are worse off than under a head-tax regime, where the allocation of capital is undistorted. While one might expect that the residents of low-demand communities, who receive the fleeing capital, would be worse off in the absence of this distortion, Proposition 2 shows that this is not necessarily true. A welfare loss requires that the tangent indifference curve under capital-taxation lies entirely above the head-tax constraint in Fig. 2, an outcome that is more likely the farther the tangency lies to the left of $B$. Assuming there are just two demand types, high and low, such a location for the tangency would appear to require a substantial difference in public-good demands as well as a distribution of types that does not place too much weight on the low demanders. If the share of low demanders is very high, or if their preferences are close to those of the high demanders, then the $k$ value in their community must necessarily be close to $k$, putting the tangency close to $B$ and leading to a potential welfare gain from head taxation.

In the standard tax-competition model, where consumer preferences are identical, switching to a head-tax regime raises the common level of utility. The switch thus generates a Pareto improvement, in contrast to Proposition 2. To see
this, note that with identical consumers, \( k = \bar{k} \) holds in each community, which implies that the common equilibrium consumption bundle lies at the intersection of the head-tax and capital-tax constraints.\(^{11}\) Since the equilibrium indifference curve must be tangent to the capital-tax constraint at this point, it follows that the curve cuts the head-tax constraint. This implies that each of the identical consumers is better off under the head-tax regime, in possible contrast to the case with heterogeneous preferences.\(^{12}\)

In addition to raising utility, this movement to a new tangency on the head-tax constraint increases the common level of consumption of the public good. With heterogeneous preferences, however, the outcome is not as simple. Assuming that the public good is normal, inspection of Fig. 2 shows that switching to the head-tax regime raises \( z \) for any consumer whose tangency on the capital-tax constraint lies to the right of \( B \). However, for consumers with tangencies to the left of \( B \), the switch to the head-tax regime has an ambiguous effect on \( z \). Fig. 2 shows a situation where \( z \) is higher under the head-tax regime for type-1 consumers, but the opposite outcome is also possible. This yields

**Proposition 3.** Public-good consumption for type-J consumers (the highest demanders) is higher under a head-tax regime than under capital taxation. Consumption levels for the other consumer types may be higher or lower under a head-tax regime.

Note that this result again reflects the fact that the highest-demand consumers are the only type whose tangency is guaranteed to lie to the right of \( B \).

Proposition 3 shows that the public good need not be underprovided for each consumer type relative to the level that would be chosen under a head-tax regime. The explanation is that, while a head tax reduces the effective cost of the public good by eliminating capital flight, it also generates income effects when consumers are heterogeneous. These effects arise because the uneven distribution of capital across communities under capital taxation is levelled under the head-tax regime. Since the income effect is positive for high-demand consumers, their public-good consumption necessarily rises. But the negative income effect for low-demand consumers may offset the reduction in effective public-good cost, leading to the possibility of a decline in \( z \).

Despite these conclusions, a result more reminiscent of the standard model may

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\(^{11}\)Note that the capital-tax constraint will lie in a different position than in Fig. 2, a consequence of a different equilibrium value of \( \rho \) in the identical-consumer case.

\(^{12}\)It is important to note that the welfare conclusions from the two models would coincide if adoption of a head tax were accompanied by redistributive transfers among consumers. If appropriate transfers were made, switching from the distortionary capital tax to a nondistortionary head tax would necessarily generate a Pareto improvement. The transfers in this case would flow from high-demand consumers, who benefit from the switch in their absence, to low-demand consumers.
be stated. In particular, it is easy to see that the public good is underprovided in each community conditional on the equilibrium distribution of capital. To see this, first observe that, holding capital in a given community fixed, the trade-off between $x$ and $z$ is given by the last line of (16). With $k$ fixed, the highest utility along this constraint is achieved at the point where the marginal rate of substitution equals $c$, the marginal cost of the public good. But since the RHS of (12) is greater than $c$, it follows that the MRS exceeds $c$ at the chosen consumption bundle. As a result, the chosen bundle contains too little of the public good and too much of the private good, a conclusion that applies within each community. Note that since the distribution of capital changes in switching to the head-tax regime, this result is not relevant to a comparison between regimes.

The model so far assumes that consumers have identical endowments of capital. However, this assumption is easily changed to allow endowments to differ, with $k^j$ denoting the type-$j$ endowment. In this case, the capital-tax constraint becomes type specific, being given by $x = w(z, \rho) + \rho \hat{k}^j$ for type $j$, and a different version of Fig. 1 applies for each type. Each type also has its own version of Fig. 2 showing only its own indifference curves, with the location of the tangencies relative to $B$ depending on the shape of the curves and the magnitude of capital usage relative to $\hat{k}^j$.

Although the analysis assumes equal endowments, capital taxation leads to an implicit form of redistribution, which effectively transfers resources away from the highest-demand consumer type. It is interesting to compare this implicit redistribution to a program of lump-sum redistribution carried out under a head-tax regime, which explicitly alters endowments. The nondistorting character of lump-sum transfers means that, if the intention is to redistribute away from high-demand consumers, this goal could be achieved more efficiently under a head-tax regime than via capital taxation’s implicit redistribution. However, one may question whether this direction of redistribution would ever be in society’s interest. This follows when it is recognized that high public-good demanders may represent households with school-age children or elderly citizens who value public safety, who are not generally viewed as sources of funds for redistribution.

3.3. Comparative statics

It is interesting to explore how parametric changes affect the capital-tax equilibrium. Suppose, for example, that the distribution of the population shifts toward low-demand consumer types. How would this change affect consumption and consumer welfare under the capital-tax regime? Before proceeding to answer this question, it is important to note that such a population shift has no consumption or welfare effects under a head-tax regime. Changing the population distribution would simply shrink the high-demand communities and enlarge the low-demand communities, with no effect on the consumption bundle or welfare of any consumer. By contrast, such a parametric change does generate welfare
impacts under capital taxation, as shown below. The reason is that parameters such as population shares of the various types affect the extent of the capital-tax distortion, and thus consumer welfare.

The analysis focuses on the impact of a given parametric change on $r$, capital’s net return. As $\rho$ varies in response to parameter changes, the capital-tax constraint in Fig. 2 shifts, and the welfare impact on different consumer types can be appraised. To analyze how changes in $\rho$ shift the constraint, the relationship $x = w(z, \rho) + \rho\bar{k}$ is differentiated, yielding

$$\frac{\partial x}{\partial \rho} = \frac{\partial w}{\partial \rho} + \bar{k}$$

$$= \bar{k} - k + \frac{tlf''(k)}{1 + tlf''(k)}.$$  

(18)

Eq. (18) uses $\partial w/\partial \rho = k/[1 + tlf''(k)]$, which is computed from (8)–(10). An understanding of (18) comes from referring to the last line of (16), which shows that $x$ is equal to community output $f(k)$ plus earnings from capital exports, $(k - k)\rho$, minus public good costs. For low-demand communities, which import capital, export earnings are negative and become more negative as $\rho$ rises, which means that the capital-tax constraint shifts downward to the left of $B$ as $\rho$ rises. The opposite effect occurs in high-demand communities, which are capital exporters, and the result is a tendency for the constraint to rotate around $B$ in a counterclockwise fashion. These effects are captured by the $\bar{k} - k$ term in (18), which is negative to the left and positive to the right of $B$. But since $\partial k/\partial \rho = [1/f''(k)]/[1 + tlf''(k)] < 0$, a higher $\rho$ also reduces capital usage, which lowers $x$ in each community. The effect is given by $[f'(k) - \rho]\partial k/\partial \rho = t\bar{k}/\partial \rho < 0$, which equals the last term in (18). Therefore, the rotation of the capital-tax constraint is accompanied by a downward shift, so that the constraint effectively rotates around a point to the right of $B$. This rotation is shown in Fig. 3.

To appraise the welfare effects of an increase in $\rho$, observe that low-demand consumers, whose tangencies lie to the left of $B$, are made worse off by rotation of the capital-tax constraint. The welfare impact on high-demand consumers is ambiguous, however. The reason is that the location of the high-demand tangencies relative to the rotation point in Fig. 3 cannot be determined. While some consumer types may have tangencies to the right of that point, in which case they enjoy a welfare gain, no tangencies at all may lie in this range.

With the welfare effects of a change in $\rho$ understood, the next step is to show how $\rho$ varies in response to parametric changes, using comparative-static calculations. These calculations require differentiation of (13), which is the sole equation determining $\rho$. To sign the resulting comparative-static derivatives, the sign of

$$\Omega = \frac{\partial}{\partial \rho}\left[ \sum_{j=1}^{J} \theta' k(z_j^*(\rho), \rho) \right]$$

(19)
is needed. This expression gives the derivative of the total demand for capital with respect to the net return $r$, and a standard stability argument would require that its sign is negative, so that $\Omega < 0$. Appendix A shows that this stability condition is satisfied under the following assumption:

**Lemma 2.** When preferences are homothetic, $\Omega < 0$.

As a first parametric change, consider a shift in the population distribution toward low-demand consumer types. In particular, suppose $\theta''$ increases by $d\alpha$ for some $m < J$, with $\theta'$ decreasing by $d\alpha$ for some $s > m$. Since low-demand communities have higher values of $k$, these changes have the effect of raising the demand for capital in the economy. Demand rises by $k(z(r), r)\rho > \rho'$, where the inequality follows because $z_m(\rho) < z_s(\rho)$ by Proposition 1 and $\partial k/\partial z < 0$. When capital demand is decreasing in $\rho$, as under Lemma 2, a higher value of $\rho$ is then required to offset the effects of this population shift, restoring equality between capital demand and supply. Thus,

\[1\text{In the underlying adjustment process, } \rho \text{ would increase (decrease) over time as the excess demand for capital is positive (negative). For this process to converge, the excess demand must be positive (negative) when } \rho \text{ is below (above) the equilibrium value. This requires that the excess demand for capital is a decreasing function of } \rho, \text{ implying that (19) is negative.}\]
An increase in $\rho$ also occurs in response to other parametric changes that raise the demand for capital. For example, suppose that the type-$j$ utility function depends on a parameter $\beta'$, with a higher value raising the MRS. Then $z_\beta^j$ is written $z_\beta^j(\rho, \beta')$, with $\partial z_\beta^j / \partial \beta' > 0$. Recalling the inverse relationship between $k$ and $z$, it follows that a decrease in $\beta'$ raises $k(z_\beta^j(\rho, \beta'), \rho)$. This in turn increases capital demand in the economy, raising $\rho$.

Combining these conclusions with the previous results on welfare impacts, the following statement can be made:

**Proposition 4.** When the stability condition is satisfied (an outcome ensured by homothetic preferences), any parametric change that increases the economy’s demand for capital leads to a higher value for the net return $\rho$. The increase in $\rho$ in turn reduces the welfare of type-1 consumers (those with the lowest public-good demand). Utility for the other consumer types may rise or fall. Parametric changes that lead to such an effect include a population shift toward low-demand consumer types, and a decline in public-good demand for one or more types.

As before, the welfare effect is determinate only for the lowest-demand consumers, since this is the only type whose tangency is assured to lie to the left of $B$.

4. Conclusion

This paper has analyzed a Tiebout/tax-competition model, where heterogeneity of consumer preferences is introduced into a standard tax-competition framework. Following the modern Tiebout tradition, consumer sorting in the model is achieved through the activities of profit-maximizing community developers. Once sorting is achieved, however, the equilibrium is equivalent to that in a standard tax-competition model with immobile, but heterogeneous, consumers.

A principal lesson of the analysis is that, under capital taxation, consumers with high public-good demands are worse off than under a head-tax regime. The reason is that, in their pursuit of high levels of public spending, high-demand communities impose high tax rates, leading to an outflow of capital. The fleeing capital ends up in low-demand communities, an outcome that may make their residents better off than under a head-tax regime.

Although the analysis generates a number of other results of interest, the simple conclusion above seems especially relevant to the real-world economy. It is easy to point to examples where high-demand communities lose business investment through high property-tax rates. An example is Urbana, Illinois, whose average household appears to be a higher demander of elementary and secondary education.
than the average household in the adjacent city of Champaign, which has a separate school district. Urbana’s property-tax rate is notably higher than Champaign’s, and commercial and industrial investment is far more extensive in Champaign than in Urbana, as predicted by the model.

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Appendix A

This appendix proves Lemma 2. To sign (19), it is necessary to compute
\[ \frac{\partial}{\partial \rho} k(z^*_j(\rho), \rho) = \frac{\partial k}{\partial z} \frac{\partial z^*_j}{\partial \rho} + \frac{\partial k}{\partial \rho} \] (A.1)

for each \( j \). To compute \( \partial z^*_j / \partial \rho \), observe that the homotheticity assumption means that the MRS in (12) depends only on the ratio of \( x_j \) to \( z_j \). Letting \( \Phi_j \) denote the MRS for the type-\( j \) consumers, and rewriting (12), \( z^*_j \) satisfies
\[ \Phi_j ( \frac{w(z^*_j, \rho) + \bar{\rho} k}{z^*_j} ) + \frac{\partial w}{\partial z} = 0. \] (A.2)

Differentiating (A.2) yields
\[ \left[ \Phi_j ( \frac{\partial w}{\partial z} z^*_j - w^*_j - \bar{\rho} k) \frac{\partial z^*_j}{\partial z} + \frac{\partial^2 w}{\partial z^2} \right] \frac{\partial z^*_j}{\partial \rho} + \left[ \Phi_j ( \frac{\partial w}{\partial \rho} + \bar{\rho} k \frac{\partial z^*_j}{\partial \rho} + \frac{\partial^2 w}{\partial \rho \partial z} \right] d\rho = 0, \] (A.3)

where \( \Phi_j > 0 \). Rearranging (A.3) yields
\[ \frac{\partial z^*_j}{\partial \rho} = \left[ \frac{\partial w}{\partial \rho} + \frac{1}{\Phi_j} \frac{\partial^2 w}{\partial z^2} \right] + \left[ \frac{\partial z^*_j}{z^*_j} - \frac{\partial w}{\partial z} - \frac{1}{\Phi_j} \frac{\partial^2 w}{\partial z^2} \right]. \] (A.4)

Substituting (15) and using \( \partial k / \partial \rho = k / (kf''(k) + t) \), (A.1) can be written
\[ \frac{1}{k^*_j f''(k^*_j) + i^*_j} \left[ \frac{\partial z^*_j}{\partial \rho} + k^*_j \right]. \] (A.5)

Letting \( \eta \) denote the first term in brackets in (A.4) and \( \gamma \) denote the second term, (A.5) can be written as
\[
\frac{1}{\gamma(k_j^* f''(k_j^*) + \tau_j^*)} (c \eta + k_j^* \gamma). \tag{A.6}
\]

Since the denominator of (A.5) is negative, and since \( \gamma \) must be positive by the second-order condition for choice of \( z_j \), it follows that the denominator of (A.6) is negative. Therefore, the sign of (A.1) is the opposite of the sign of the numerator of (A.6). After substitution, the numerator of (A.6) can be written

\[
c \left( \frac{\partial w}{\partial \rho} + k \right) + k_j^* \left( \frac{x_j^*}{z_j^*} \frac{\partial w}{\partial z} \right) + \frac{1}{\Phi_j} \left( c \frac{\partial^2 w}{\partial \rho \partial \rho} - k_j^* \frac{\partial^2 w}{\partial z^2} \right). \tag{A.7}
\]

After substituting for the derivative expressions, (A.7) reduces to

\[
ck + \frac{k_j^* x_j^*}{z_j^*} + \frac{1}{\Phi_j} \frac{\partial w}{\partial z} \frac{\partial k}{\partial z} > 0, \tag{A.8}
\]

where the sign follows because the last two derivatives are both negative. Since (A.8) is positive, it follows that (A.1) is negative for all \( j \). As a result, \( \Omega \) in (19) is negative, establishing the lemma.

References