Endogenous markups and the effects of income taxation: Theory and evidence from OECD countries

Yangru Wu\textsuperscript{a,*,} Junxi Zhang\textsuperscript{b}

\textsuperscript{a}Department of Finance and Economics, Faculty of Management, Rutgers University, Newark, NJ 07102-1820, USA
\textsuperscript{b}University of Hong Kong, Hong Kong, People's Republic of China

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Abstract

Existing studies on the effects of fiscal policy under imperfect competition typically treat each firm's price-cost markup as fixed. This paper examines the implications of endogenising the markup in a simple model of income taxation under monopolistic competition. It is demonstrated that an increase in income tax reduces the number of firms, lessens competition among surviving firms and raises the optimal markups in the new steady state. Through this channel, the effects of income taxation on the key macroeconomic variables as well as on consumer welfare are unambiguously enlarged. The simple model prediction is largely supported by cross-section regressions with data from OECD countries. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

The literature on 'the equilibrium approach to fiscal policy', summarised by Aschauer (1988) and Barro (1989), examines the macroeconomic effects of various policies within a restricted version of the basic neoclassical model. The
framework adopted in the equilibrium approach is one with perfect competition and constant returns to scale. In a general one-sector model, Baxter and King (1993) study these effects quantitatively. Among a number of policy experiments, one concerns the changes in balanced-budget tax rates. They find that permanent tax-financed shifts in government expenditure induce declines in output and capital accumulation. The policy implication of this result is appealing in that a balanced-budget tax cut is preferred by many economists and politicians.

In the last 20 years or so, there have been voluminous empirical studies suggesting that imperfect competition and increasing returns to scale more accurately describe the behaviour of firms (see the discussion in Farmer and Guo, 1994). Following the classic models of Spence (1976) and Dixit and Stiglitz (1977), many imperfectly competitive models were developed to study various economic issues; in particular, along this line there is an incipient literature of the so-called `new macroeconomics of imperfect competition.' It has been shown that the model with imperfect competition and increasing returns exhibits many characteristics distinct from the model without the two elements. However, these studies are based on a strong assumption that entry and exit of firms do not affect the intensity of competition and thus the markups of incumbent firms. In this sense it is an analysis with exogenous markups. It is widely agreed that policy which induces entry will tend to increase competition in the market, and changes in the number of firms will influence output and capital accumulation.

This paper explores the effects of income taxation in a model with imperfect competition and increasing returns to scale. Our model departs from the above conventional models by endogenising each incumbent firm’s price-cost markup. We assume that the intensity of competition among a continuum of monopolistically competitive intermediate goods producers increases with the level of economic development and the associated range of intermediate products available. Since markups vary with the number of firms, entry and exit of firms will affect each firm’s price-cost margins. The presence of market power then drives wedges between factor shares and their respective elasticities of production. To summarise the main results at the outset, we find that a higher tax rate, in the long run, lowers the equilibrium levels of capital stock and consumption, while leaving work effort unchanged. Moreover, because a higher tax rate leads to higher industry concentration and raises equilibrium markups, the effects on capital stock and consumption are notably higher than those in conventional models of imperfect competition with constant markups. Our analysis of the transitional

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1See Dixon and Rankin (1995) for a comprehensive survey of this literature.
2There exist a number of papers, e.g. Startz (1989) and Dixon and Lawler (1996), that discuss the implications of entry for the effectiveness of fiscal policy in a monopolistically competitive context. These models typically treat the markup as fixed, independent of the number of firms. Dixon and Lawler (1996) contain an informal discussion of the possible implications of entry for the markup and thus key macroeconomic variables.
dynamics after an increase in the tax rate identifies two offsetting effects on consumption in the short run: the income effect and intertemporal substitution effect; hence, consumption can either increase or decrease during the transition depending on the relative magnitudes of these two effects. We also conduct a simple welfare analysis and find that the welfare cost of a given increase in the tax rate under endogenous markups is unambiguously higher than that under constant markups. Based on these results, we conclude that existing studies on the effects of income taxation might have markedly underestimated the magnitudes of these effects.

Our results can be given an intuitive interpretation: income taxation lowers savings, which induce declines in capital accumulation, output, consumption and utility. We term this usual effect as the primary effect. Interestingly, in our model, since each incumbent firm’s price-cost markup is endogenous, it will generate a secondary effect. Higher returns to capital stock lead to lower price-cost markups and cause firms to exit the market. This triggers the following process: exit leads to less competition, higher markups and lower returns to labour, which reduce consumer’s income; lower income leads to lower demand and further declines in capital accumulation, output, consumption and welfare. Therefore, the effects in this model are significantly higher than those in existing models.

The remainder of the paper is organised as follows. In Section 2, we cast the basic model and define a perfect foresight competitive equilibrium. Section 3 establishes the unique saddle-path equilibrium of the dynamic economy. This section also conducts a steady-state analysis by solving for the optimal markups and other variables. Section 4 first studies the effect of taxation on the number of firms, and then investigates the effects on consumption, capital accumulation and work effort in both the short run and the long run. Section 5 examines the welfare consequences. In Section 6, we provide some empirical evidence regarding the main implications. Finally, Section 7 offers some concluding remarks.

2. The model

2.1. Producers

In the model economy, there exist a large number of monopolistically competitive intermediate goods producers, \(j = 1, \ldots, n\). Final output at time \(t\), produced in a competitive sector, is given by

\[
Y = \left[ n^{-1/\alpha} \sum_{j=1}^{n} x_j^{(\epsilon-1)/\epsilon} \right]^{1/(\alpha-1)},
\]  

(1)

\footnote{Superfluous time indices are suppressed when they are unnecessary.}
where $Y$ is output, $x_j$ is the quantity of the $j$th intermediate good used, and $\varepsilon > 1$ is the (intrasector) elasticity of substitution among intermediate goods. In (1), the final good is a bundle of $n$ varieties which are imperfect substitutes.

Denoting $P$ and $p_j$ as the prices of the final good and the $j$th intermediate good, respectively, we obtain the profit of a final good producer, $\Pi$:

$$\Pi = PY - \sum_{j=1}^{n} p_j x_j. \quad (2)$$

Throughout this paper, lower-case letters are used to represent individual-specific variables, while upper-case letters to represent economy-wide (or aggregate) variables. The solution to this problem yields the demand function for a typical intermediate good:

$$x_j = (\frac{p_j}{P})^{-\varepsilon} \left( \frac{Y}{n} \right). \quad (3)$$

The production technology available to an intermediate commodity producer is assumed to exhibit the Cobb–Douglas form (to ease the exposition, the firm index $j$ is omitted hereafter):

$$x = F(k,h) = A k^{\alpha} h^{1-\alpha}, \quad (4)$$

where $k$ and $h$ are capital and labour inputs, respectively, and $\alpha$ is the capital elasticity of production. The profit function for the $j$th intermediate good producer, $\pi$, is given by:

$$\pi = px - wh - rk - p\phi, \quad (5)$$

where $w$ is the wage rate, $r$ is the rental cost of capital, and $\phi$ is the fixed cost. In each instant, an amount $\phi$ of the intermediate good is immediately used up for administration purposes to keep production going, which is independent of how much output is produced. Because of this fixed cost, production in the intermediate goods sector displays increasing returns to scale, although capital and labour elasticities sum to one.

Profit maximisation for the intermediate good producer suggests that the wage rate of labour and the rental rate of capital are paid according to their respective marginal revenue productivities:

$$w = \frac{e-1}{e} \frac{(1-\alpha)px}{h}, \quad r = \frac{e-1}{e} \frac{\alpha px}{k}. \quad (6)$$

\(*\)Since in equilibrium, $p = P = 1$, the formulation of the fixed cost in Eq. (5) is equivalent to that if it is introduced at the level of the production function (4). Thus, the results of this paper do not depend on how the fixed cost is introduced. Moreover, it can be shown that our qualitative results will hold in an alternative formulation where the fixed cost and a scale parameter are modelled at the level of the production function (4). The result is contained in a technical appendix available upon request.
where \( e = - \frac{\partial x}{\partial p} \frac{p}{x} \) is the price elasticity of demand. In a symmetric Nash equilibrium, to calculate \( e \), we follow Yang and Heijdra (1993) by taking into account both the direct and indirect effects, i.e. both the intrasector and intersector elasticities of substitution:

\[
e(n) = \varepsilon - (\varepsilon - 1) \frac{1}{n}, \quad n \in (1, \infty).
\]

Note that the second term represents the indirect effect through \( p \) on \( Y \), which is neglected in conventional models with monopolistic competition.

Let \( \mu(n) = e(n)/[e(n) - 1] > 1 \), then, \( \mu(n) \) corresponds to the firm’s price markup over its marginal cost. Since the perceived elasticity varies with the number of firms, so does the markup. Thus, in this sense, the markup is endogenised. Moreover, it can be easily shown that \( \mu'(n) < 0 \), implying that the smaller the number of firms in the industry, the less intense the competition, and then the higher the degree of market power and the monopoly profits for incumbent firms.  

It should also be noted that in Eq. (6), \( (1 - \alpha)/\mu(n) \) and \( \alpha/\mu(n) \) correspond to the labour and capital shares, respectively. By the property that \( \mu(n) > 1 \) for \( n \in (1, \infty) \), factor shares do not necessarily equal their respective elasticities of production and may sum to less than one; moreover, these shares vary with the number of firms in the industry or the firms’ markups. By contrast, in standard models with perfect competition, \( \mu(n) = 1 \); and, in conventional models with monopolistic competition, such as Spence (1976) and Dixit and Stiglitz (1977), \( \mu(n) \) is a constant parameter larger than one.

In the symmetric equilibrium, all firms employ the same amounts of inputs, produce the same quantities and set the same prices: \( k = K/n \), \( h = H/n \), \( x = Y/n \), and \( p = P = 1 \) (by normalisation). Finally, free entry into the intermediate sector forces profits to be zero,

\[
\frac{1}{\mu(n)} = 1 - \frac{\phi n}{AK^\alpha H^{1-\alpha}}.
\]

\footnote{It should be pointed out that for a finite number of firms to play a static Nash equilibrium at each period, such as the one in this paper, some technical justification is needed. It is known that with finite numbers firms may play a collusive game and use punishments to support many subgame perfect equilibria, including the static Nash equilibria. This collusive behaviour has been ruled out throughout the paper. Alternatively, we could adopt a continuum measure of the firm number, which shall avoid such a problem, and replace the constant elasticity of substitution \( \epsilon \) by a function in \( n \), \( \epsilon(n) \), where \( \epsilon'(n) > 0 \). Then Eq. (7) would become \( e(n) = \epsilon(n) \) (notice that in Dixit and Stiglitz (1977) the price elasticity of demand \( \epsilon \) is simply equal to the elasticity of substitution); that is, each firm’s price will have no effect on the output of the final good. But, since \( \epsilon \) is assumed to be a function of \( n \), all the results of this paper would remain valid. We wish to thank an anonymous referee for clarifying this point.}

\footnote{We also assume that \( \lim_{n \to 1} \mu(n) = \bar{\mu} \in (1, \infty) \).}

\footnote{Alternatively, one can express the net output, \( Y_{net} \), as: \( Y_{net} = n(x - \phi) \).}
2.2. Consumers and the government

A representative consumer’s utility is assumed to be a function of consumption, $C$, and work effort, $H$. The individual seeks to maximise the objective function:

$$U = \int_{0}^{t} e^{-\rho t} u(C, H) dt,$$

(9)

where $\rho$ is a constant, positive rate of time preference. Function $u$ is assumed to be strictly concave and twice continuously differentiable, satisfying $u_C > 0$, $u_H < 0$, $u_{CC} < 0$, $u_{CH} \leq 0$, $u_{HH} < 0$, and the Inada conditions.

At each instant, the individual is endowed with one unit of time and supplies $H$ units of labour input. She is also engaged in accumulating capital, $K$, which she rents to firms. Total income comes from wage and rental income, which are liable to a tax rate of $\tau_w$ and $\tau_K$, respectively. Then, she fully allocates her disposable income to purchasing of the final good, leading to the dynamic budget constraint

$$C + \dot{K} = (1 - \tau_w)wH + (1 - \tau_K)rK,$$

(10)

where $\dot{K}$, the time derivative of $K$, is the change in the capital stock. For simplicity, the depreciation rate of capital is assumed to be zero.

The government is assumed to purchase an amount of $G$ of the final good in each time and finance the spending with the income taxes. We assume that the government balances its budget every instant,

$$G = \tau_H wH + \tau_K rK.$$

Accordingly, the tax rates are referred to as the balanced-budget tax rates. To simplify our analysis and focus on the issues of interest, we abstract from all issues associated with alternative means of government financing in order to isolate the impact of changes in the income taxes.

2.3. Perfect foresight equilibrium

The representative consumer is assumed to behave competitively, taking all prices as given. Her problem is to choose a path $\{C, H, K\}^\infty$ to maximise (8) subject to (9). Let $\lambda$ be the shadow price used to value increments to capital in the budget constraint (9), then the first-order conditions for this problem are:

$$u_C(C, H) = \lambda,$$

(11a)

$$- u_H(C, H) = \lambda(1 - \tau_H)w,$$

(11b)

$$\dot{\lambda} = \rho \lambda - (1 - \tau_K) r \lambda,$$

(11c)
where the standard transversality condition is assumed to hold. On the margin, Eq. (11a) states that goods must be equally valuable in their two uses — consumption and capital accumulation, while Eq. (11b) states that time must be equally valuable in its two uses — leisure and production. Eq. (11c) gives the rate of changes of the shadow price of investment $\lambda$.

A perfect foresight equilibrium of this model can be characterised by the set of the first-order conditions (11) and the pricing equation (6); moreover, the labour market, the capital market, and the final good market all clear, where the latter leads to

$$C + K + G = wH + rK.$$  

3. Steady state analysis

This section characterises the steady state of the model. To this end, we begin by deriving the steady-state equations and showing that there exists a unique stationary state which satisfies $\dot{C} = \dot{K} = \dot{H} = 0$. Let an asterisk denote the steady state value of a variable, then the stationary state is defined by the following equations:

$$\begin{align*}
(1 - \tau_K)r &= \rho, \\
-\frac{\mu K}{\mu C} &= (1 - \tau_H)w. 
\end{align*}$$

To yield an analytical solution, it remains to specify the utility function. As a standard practice in the literature, we employ the functional form of constant elasticity of substitution (CES) so as to investigate the steady state properties and thereby the effects of income taxation:

$$u(C,H) = \frac{[C^\omega (1 - H)^{1-\omega}]^{1-\sigma} - 1}{1-\sigma},$$

where $\omega$ and $\sigma$ are constant parameters. Hence, the stationary state becomes:

$$\begin{align*}
\frac{1 - \omega}{\omega} \frac{C^*}{1 - H^*} &= \frac{(1 - \tau_H)(1 - \alpha)}{\mu(n^*)} AK^{*\alpha} H^{* - \alpha},
\end{align*}$$

A sufficient condition for the first-order conditions to maximise (9) is that the Hamiltonian is concave in $K$ after the controls $C$ and $H$ are substituted out with their maximising values. It can be easily shown that this condition is satisfied.

When $\sigma = 1$, $u$ becomes additively logarithmic. It can also be verified that our results in this paper hold for a simpler form of utility reflecting indivisible labour, $u(C,H) = \ln(C - BH)$, where $B$ is a constant parameter; see, e.g., Hansen (1985), Rogerson (1988), and Cooley and Hansen (1989).
where \( \tau_y = (1 - \alpha)\tau_n + \alpha \tau_K \) is an average of the individual’s tax on labour and capital income, weighted by the capital elasticity of production \( \alpha \). Eqs. (14)–(16) together with (8) constitute a system of four equations in four variables: \( C^* \), \( K^* \), \( H^* \) and \( n^* \). This system can be further reduced to a non-linear equation in \( n^* \) or \( \mu(n^*) \):

\[
\frac{n^*\mu(n^*)^{1/(1-a)}}{\mu(n^*) - 1} \left[ \frac{(1 - \tau_K)^{a/(1-a)}}{\alpha} + \frac{\alpha(1 - \omega)}{\omega(1 - \alpha)} \right] = D,
\]

(17)

where \( D = A^{1/(1-a)(\alpha/\rho)^{a/(1-a)}/\phi} \) is a positive constant. To determine the number of solutions in (17), we provide an important condition in the following proposition.

**Proposition 1.** Suppose (17) has solutions. Then, when \( \bar{\mu} \leq 1/\alpha \), there exists a unique steady state in this economy.

**Proof.** Let \( I(n) \) be the expression in the left-hand side of (17). Direct differentiation of \( I(n) \) yields:

\[
\frac{I''(n)}{I(n)} = \frac{1}{n} - \frac{\mu'(n)}{\mu(n)} \left[ \frac{\mu(n)}{\mu(n) - 1} - \frac{1}{1 - \alpha} \right].
\]

Using the property of function \( \mu(n) \), \( \mu'(n) < 0 \), it is clear that a sufficient condition for the second term in the right-hand side of the above expression and hence \( I''(n) \) to be positive is \( \mu(n) \leq \bar{\mu} \leq 1/\alpha \). Moreover, by the assumptions, we have \( \lim_{n \to 1} \mu(n) = \bar{\mu} \in (1, \infty) \) and \( \lim_{n \to \infty} \mu(n) = \epsilon/(\epsilon - 1) > 1 \), because \( \epsilon > 1 \). So, when \( n \) approaches 1, \( I(n) \) approaches a constant which is assumed to be smaller than \( D \); whereas, when \( n \) approaches \( \infty \), \( I(n) \) also approaches \( \infty \). Given that the \( I' \) curve is monotonically upward-sloping, there exists a unique equilibrium \( n^* \) that solves (17) (see Fig. 1). □

To investigate the empirical plausibility of the above condition, we turn our attention to some of the empirical evidence and the parameter values generally agreed by economists. Estimates of the capital elasticity of production, \( \alpha \), range from 0.25 (e.g. Lucas, 1988) to 0.42 (e.g. Rotemberg and Woodford, 1994), whilst those for markups, based on either the gross output measure or the value added measure, lie within the range 1.05 to 2.3 (e.g. Morrison, 1990; Norrbin, 1993;
Roeger, 1995). On the basis of this evidence, we view the condition identified in Proposition 1 to be empirically plausible.

Given the unique steady-state value of \( n^* \), it is straightforward to derive the steady-state values for other key model variables:

\[
H^* = \frac{1}{1+\varepsilon}, \text{ where } \varepsilon = \frac{1-\omega}{\omega(1-\alpha)} \left[(1-\alpha) + \alpha \frac{1-\tau_k}{1-\tau_H}\right], \tag{18}
\]

\[
K^* = H^* \left[\frac{\alpha A(1-\tau_k)}{\rho \mu(n^*)}\right]^{1/(1-\alpha)}, \tag{19}
\]

\[
C^* = K^* \left(\frac{1-\tau_f}{1-\tau_k\alpha}\right). \tag{20}
\]

Next, we examine the local stability of the stationary state. To do so, we reduce the dimensionality of the system by eliminating the variable \( H \) so as to concentrate entirely on the transitions of \( C \) and \( K \). For simplicity, we will work on a case of a flat income tax, \( \tau_k = \tau_H \), in the rest of this section. From (11a), (11b), the utility

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10 For a comprehensive review of some of the empirical evidence, see a recent paper by Schmitt-Grohe (1997, pp. 130–132).
function and the factor price equation (6), we can express $H$ as a function of $C$ and $K$: $H = H(C,K)$. It can be easily shown that $H_C(C,K) < 0$ and $H_K(C,K) > 0$. Substituting this into (6), we find the wage rate, $w$, and the rental rate of capital, $r$, are functions of both $C$ and $K$.

To aid the exposition, we define the after-tax rental cost of capital as $r(K,C,t) = (1 - \tau)r$ and the after-tax income as $g(K,C,t) = (1 - \tau)(wH + rK)$. Then, the dynamics of this economy are described by

$$\dot{K} = g(K,C,t) - C, \quad (21)$$

$$\dot{C} = \eta C[r(K,C,t) - \rho], \quad (22)$$

where $\eta = 1/[1 - \omega(1 - \sigma)] > 0$ is constant. Linearising (21) and (22) around the steady state yields:

$$\begin{bmatrix} \dot{K} \\ \dot{C} \end{bmatrix} = \begin{bmatrix} g_K(K^*,C^*,\tau) & 1 + g_c(K^*,C^*,\tau) \\ \eta C^* r_K(K^*,C^*,\tau) & \eta C^* r_c(K^*,C^*,\tau) \end{bmatrix} \begin{bmatrix} K - K^* \\ C - C^* \end{bmatrix}.$$  

The determinant of this Jacobian matrix can now be calculated as

$$\text{Det} = \eta C^* [r_K(K^*,C^*,\tau)[1 - g_c(K^*,C^*,\tau)] + r_c(K^*,C^*,\tau)g_K(K^*,C^*,\tau)].$$

Note that the uniqueness result of Proposition 1 implies that $r_K < 0$. It can be shown that standard differentiations yield $g_c < 0$, $g_K > 0$, and $r_c < 0$. Thus, Det<0. This result immediately yields:

**Proposition 2.** When there is a unique steady-state equilibrium, there exists a unique stable path converging to this equilibrium.

This proposition implies that the steady-state equilibrium is a saddle point and there is a one-dimensional stable manifold. Since $C$ is a ‘jumping’ variable, we can always find a unique initial value of $C$ on this stable manifold for a given initial value of $K$. Fig. 2 depicts the phase diagram of the dynamical system (21) and (22).

4. Effects of income taxation

In this section, the model is used to study the effects of the balanced budget fiscal policy introduced in Section 2, in which increases in the rates of taxes applied to income from labour and capital are accompanied by a rise in government spending equal in value to the resulting expansion in tax revenues. We consider both the long-run and short-run effects.
4.1. The long-run effects

First, we look at the effects on industry concentration $n^*$ and thus the markups for the surviving firms $\mu(n^*)$. It is straightforward to note that an increase in the rate of tax on labour income tax, $\tau_h$, shifts the $I'$ curve upward, resulting in a decrease in $n^*$. This in turn yields a higher markup, $\mu(n^*)$. Unfortunately, the effects of a rise in the rate of tax on capital income, $\tau_k$, are generally ambiguous, because there exist two opposite forces (see (17)): on one hand, a rise in $\tau_k$ raises the first term in the square bracket; on the other hand, it reduces the second term for $1 - \alpha/(1 - \alpha) > 0$. Interestingly, if a flat rate of income tax is levied, i.e. $\tau_k = \tau_h = \tau$, a rise in this rate will lower $n^*$ and raise $\mu(n^*)$. These results are conveniently summarised as follows.

**Proposition 3.** In the long run, an increase in the rate of tax on labour income leads to higher industry concentration (lower $n^*$) and markups ($\mu$) for the surviving firms, while the effects of an increase in the rate of tax on capital income are generally ambiguous. In the case of a flat income tax, higher industry concentration and markups are also obtained for a higher tax rate.
These results accord reasonably well with intuition and we take the effects of the flat rate as an example. As shown below (also see Eq. (24) and footnote 8), the number of firms is increasing in the aggregate stock of capital. An increase in \( \tau \) reduces the capital stock, which suggests that the steady-state number of firms will also be reduced. That is, some firms are forced to exit from the industry. As a result, the variety of intermediate goods available is reduced and competition becomes less intense, which leads to higher equilibrium markups.

Next, the effects on other key endogenous variables are examined. Given the fact that the effects of a change in \( \tau_k \) are ambiguous, we will focus exclusively the analysis in the rest of this section on the effects of changes in the flat rate \( \tau \). The following proposition presents one of the main results of this paper.

**Proposition 4.** Higher income taxes reduce the long-run equilibrium values of consumption and capital stock, but do not affect that of the work effort. Moreover, the magnitudes of these effects, if any, are notably higher than those in conventional models of imperfect competition with constant markups.

**Proof.** Under the presumption of \( \tau_k = \tau_h = \tau \), the steady-state values of consumption and capital stock are given by

\[
C^* = \frac{(1 - \tau)^{1/(1-\alpha)} D_1}{\mu(n^*)^{1/(1-\alpha)}}, \quad K^* = \frac{(1 - \tau)^{1/(1-\alpha)} D_2}{\mu(n^*)^{1/(1-\alpha)}},
\]

where \( D_1 \) and \( D_2 \) are positive functions of model parameters. Take the effect on \( K^* \) as an example. Straightforward differentiation with respect to \( \tau \) yields:

\[
\frac{K^*_\tau}{K^*} = -\frac{1}{1-\alpha} \frac{1}{1-\tau} - \frac{1}{1-\alpha} \frac{\mu_r(n^*)}{\mu(n^*)} < 0,
\]

where hereafter a variable with the subscript \( \tau \) represents its partial derivative with respect to changes in \( \tau \). Note that the second term, warranted by Proposition 3, is a new effect — the secondary effect, which is absent in models with constant markups.\(^1\) Therefore, the results follow. \( \square \)

To understand the result for \( H^* \), we refer to (18). It is evident that a rise in \( \tau_h \) reduces \( H^* \), while a rise in \( \tau_k \) raises \( H^* \). Intuitively, a rise in \( \tau_h \) reduces an individual’s disposable income from labour, invoking a familiar disincentive

\(^1\) Alternatively, if we let \( n = n(K) \), \( n' > 0 \), (24) can be rewritten as

\[
\frac{K^*_\tau}{K^*} = -\left( \frac{1}{1-\alpha} \frac{1}{1-\tau} \right) \left[ 1 + \frac{\mu'(n^*) n'(K^*) K^*}{(1-\alpha) \mu(n^*)} \right].
\]

In general, the denominator is positive despite the second term is negative (this can be deduced from the expression for \( \tau_k \)).
effect. However, a rise in $\tau$ reduces the after-tax return on capital, so there will be a tendency for firms to substitute labour for capital. When the flat rate is imposed, the resulting effect simply suggests that the two changes offset one another.

It should be made especially clear that our result is obtained in an imperfectly competitive model with entry and endogenous markups,\textsuperscript{12} which is in contrast to conventional models of monopolistic competition with constant markups. Proposition 4 reveals that the conventional analysis has underestimated the effects of income taxation, in that only the primary but not the secondary effects are documented. This also yields an important policy implication: the benefit of a balanced-budget tax cut appears to be notably higher than what we currently understand.

4.2. The short-run effects

We now investigate the character of the transitional path after a change in the rate of income tax, $\tau$. The economy is assumed to be in the steady-state equilibrium initially, and then an unanticipated increase in $\tau$ occurs at time 0. To slightly abuse the notation, we make dependence of a variable on time and $\tau$ explicit by the following: e.g. $C(t;\tau)$ and $K(t;\tau)$. Since the initial value of capital is predetermined, we seek how the initial value of consumption responds to a change in $\tau$. Before presenting the result, it is helpful to understand qualitatively the possible effects on $C(0;\tau)$. The response may consist of two effects: the income effect and the intertemporal substitution effect. It is understood that the income effect is always negative. However, the intertemporal substitution effect works to increase present consumption and to decrease future consumption, which works through altering the interest rate. Thus, the overall effect will be clouded by these opposite effects, depending on the relative magnitude of each one.

Formally, applying Judd’s (1982, 1985) method of comparative dynamics, we obtain:

**Proposition 5.** If the income effect dominates (is dominated by) the intertemporal substitution effect, a rise in the tax rate has a negative (positive) effect on consumption; that is, $C_C(0;\tau) < (>) 0$. Also, both the income and substitution effects are found to be notably larger than when constant markups are assumed.

**Proof.** See Appendix A. \hfill $\square$

Fig. 3 describes the transitional dynamics associated with an increase in $\tau$ for case (a) only. In both cases, the $C = 0$ locus shifts to the left, while the $K = 0$ locus shifts downward. In case (a), $C$ initially jumps downward and then decreases, and

\textsuperscript{12}While we focus on $C^*$ and $K^*$ in Proposition 4, the result on other model variables, such as net output and wage rate, is essentially the same, which are omitted to conserve space.
$K$ gradually decreases. In case (b), on the other hand, $C$ initially jumps upward and then decreases, and $K$ gradually decreases. The transitional path of $C$ is delineated in Fig. 4, where that of $K$ is not drawn.

A remark should be made against case (b). This case, albeit documented in theoretical studies elsewhere, e.g. Futagami et al. (1993), is inconsistent with empirical findings; among others, Hall (1988) and Campbell and Mankiw (1989) found that consumption depends positively on disposable income. Due to this reason, in what follows, we will only consider case (a).

Next, we explore the speed of convergence from the original equilibrium $E$ to the new one $E'$. From the long-run analysis, we have learned that for a given rise in $\tau$ the long-run equilibrium value of $C^*$ is lower in our model than that in the conventional models with constant markups; in addition, from the above short-run analysis, it is found that the initial decline in $C^*$ is also bigger. Thus, it is interesting to compare the speed of convergence during the transitions.

We observe from Proposition 2 that one of the eigenvalues of the Jacobian matrix in the dynamical system of (21) and (22) is positive and the other is negative. By definition (see, e.g., Barro and Sala-i-Martin (1995)), the convergence coefficient, $\beta$, corresponds to the absolute value of the negative eigenvalue, which is given by
Fig. 4. The transitional path of consumption in case (a).

\[ \beta = \frac{\sqrt{\text{Tr}^2 - 4 \cdot \text{Det} \cdot \text{Tr}}}{2}, \tag{25} \]

where \( \text{Tr} = g(K^*, C^*, \tau) + \eta C^* r_c(K^*, C^*, \tau) \) is the trace of the Jacobian matrix. Differentiating functions \( g \) and \( r \) with respect to \( K \), we can now state:

**Proposition 6.** The speed of convergence in this model is slower than that in models with constant markups.

Obviously, this result points out the importance of considering the short-run effects in our model. In summary, the above analyses demonstrate that this imperfectly competitive model with endogenous markups exhibits characteristics

---

13More precisely, the two eigenvalues can be shown as

\[ \theta_+, \theta_- = \frac{\text{Tr} \pm \sqrt{\text{Tr}^2 - 4 \cdot \text{Det}}}{2}. \]

Making use of the notations introduced in Appendix A, we find \( a_{12}, a_{21} > 0 \). Thus, one can obtain that \( \theta_+ > a_{11} \). For the convergence coefficient, we have \( \beta = -\theta_- \).
distinct from that of a conventional model with constant markups. As a further confirmation, attention now briefly turns to the welfare comparison.

5. The welfare effects of changes in income taxes

For simplicity, we suppose that the utility function takes the logarithmic form, or \( \sigma = 1: u(C,H) = \omega \ln C + (1 - \omega) \ln (1 - H) \). Since a change in the flat rate of income does not affect \( H \) (Proposition 4), we ignore this term in the following analysis. Notice also that by definition, \( \eta = 1 \).

From Eq. (22), we have

\[
C(t;\tau) = C(0;\tau) \exp \left\{ \int_0^t \left[ r(K(s;\tau),C(s;\tau),\tau) - \rho \right] ds \right\}. \tag{26}
\]

Substituting (26) into the objective function (9) yields:

\[
U = \int_0^\infty \omega \ln [C(0;\tau)] e^{-\rho t} dt + \int_0^\infty \omega \int_0^t [r(K(s;\tau),C(s;\tau),\tau) - \rho] ds \cdot e^{-\rho t} dt. \tag{27}
\]

Evidently, the attained lifetime utility depends on both the initial consumption caused by a change in the tax rate, \( C(0,\tau) \), and the time path of \( K, K(s,\tau) \). Direct differentiation of (27) with respect to \( \tau \) leads to:

\[
\frac{dU}{d\tau} = \frac{\omega C(0;\tau)}{C(0;\tau)} + \int_0^t \int_0^t \omega [r_s(K^*,C^*,\tau)K(s;\tau) + r_c(K^*,C^*,\tau)C(s;\tau) + r_f(K^*,C^*,\tau)] ds \cdot e^{-\rho t} dt. \tag{28}
\]

As shown in Appendix B, under the assumption that the economy is initially in equilibrium, (28) can be rewritten as

\[
\frac{dU}{d\tau} = \frac{\omega C(0;\tau)}{C(0;\tau)} + \frac{\omega r_s(K^*,C^*,\tau)}{\rho(\rho + \beta)} < 0. \tag{29}
\]

To compare the magnitude with a standard model with constant markups, we find that for the same initial equilibrium \( K^* \) and \( C(0;\tau) = C^* \), \( \beta \) in our model is smaller (Proposition 6), while the absolute values of \( C_s(0;\tau) \) and \( r_f(K^*,\tau) \), both of which are negative, are larger (Proposition 5 and Eq. (A.5)). These result in:

**Proposition 7.** When markups are endogenous, a change in the rate of income tax
induces a larger change (in the opposite direction) in consumers’ lifetime utility than otherwise.

Note that this is also true even if there are no transitional dynamics involved; that is, given an unanticipated change in the tax rate, the economy instantly switches from one equilibrium to another. Then, $C_s(0;\tau)$ corresponds to the change in consumption and $\beta$, the speed of convergence, is equal to infinity. As a result, the second term in (29) disappears. Thus,

**Corollary.** The welfare cost (gain) of a balanced-budget tax rise (cut) with the consideration of transitional effects is unequivocally higher than without such a consideration.

This result implies that the consideration of transitional dynamics is important as it causes an additional effect in the welfare aspect of public policy. To conclude, from a policy perspective, our welfare comparison in this section suggests that the benefit, in terms of welfare, of a balanced-budget tax rate also appears to be higher than we thought.

6. Some empirical evidence

In this section, we employ data to test one important implication generated from our model on the relationship between income taxation and equilibrium markups. Proposition 3 in Section 4 has demonstrated that an increase in the income tax rate forces some firms in the intermediate goods sector to be out of the market and in turn lessens competition among surviving firms. As a result, their optimal markups in the new steady state will go up. It has been shown that this result is crucial in driving the wedge between our model and the orthodox model not only on the effects of income taxation (in both the long run and the short run) but also on the welfare effect. Hence, our empirical strategy is to perform some simple and robust checking on whether this implication is supported by international data.

To conduct a comprehensive test of the theory, sufficient observations on direct measures of markups and taxes are required. Unfortunately, to the best of our knowledge, existing data of this sort are quite limited and therefore our empirical analysis is confined to the information we have available. We managed to collect a data set for a group of 14 OECD countries where direct measures of markups and income tax rates are available. Oliveira Martins et al. (1996) use micro data from 1980 to 1992 and regression techniques to estimate markup measures for 14 OECD countries and five industry sectors. These countries include: United States, Japan, Germany, France, Italy, United Kingdom, Canada, Australia, Belgium, Denmark, Finland, The Netherlands, Norway and Sweden. The industry sectors are: (1) manufacturing; (2) electricity, gas and water; (3) construction; (4)
wholesale, retail trade, restaurants and hotels; and (5) transport, storage and
communication. We take the observations on markup measures from their Table 3,
and obtain individual marginal income tax rate data for the year 1985 from Price
Waterhouse (1985). We first run a simple regression of markup level on individual income tax rate as follows:

\[
\text{Markup}_i = b_0 + b_1 \text{Tax}_i + \epsilon_i, \tag{30}
\]

where the country index \(i\) goes from 1 to 14; \(\epsilon_i\) is a regression error term; and \(b_0\) and \(b_1\) are regression parameters. Our model implies that \(b_1\) should be positive.

Prior to presenting the empirical results, we make two remarks. Firstly, the linear functional form in Eq. (30) is adopted mainly for empirical tractability, since the purpose of our regression is not to identify the exact relationship between equilibrium markups and income taxation but rather to show that our model prediction is consistent with the data.

Secondly, it should be noted that because Eq. (30) is a cross-section regression, it is reasonable to conjecture that the error term \(\epsilon_i\) may be heteroscedastic. To this end, for each regression we formally test for heteroscedasticity of the error term using three tests. These are White’s (1980) general test; Breusch and Pagan’s (1979) Lagrangian multiplier test under the assumption that the variance of \(\epsilon_i\) depends on the regressor \(\text{Tax}_i\); and Goldfeld and Quandt’s (1965) test, where the whole sample is divided into two equal subsamples, according to low and high tax rates, and observations from the low and high tax groups are employed to calculate the test statistic.

Panel A of Table 1 documents the least squares regression and test results. The three tests do not show significant evidence of heteroscedasticity in general, although admittedly the power of these tests is low given that the sample contains only 14 observations. In two cases, we do find significant evidence of heteroscedasticity. Namely, in the regression for industry sector 2 (electricity, gas and water) with the Goldfeld-Quandt test, and for sector 3 with the Breusch-Pagan test. Therefore, as a check for robustness, we also construct White’s heteroscedasticity-consistent \(t\)-ratio. Overall, we find that the parameter \(b_1\) has the expected positive sign for all regressions except the one for sector 2, which is significantly negative using the White \(t\)-ratio, but not significant using the OLS \(t\)-ratio. Based on the OLS \(t\)-ratio and using a two-sided test, we find that \(b_1\) is significantly positive at the 5% level for sector 3 and at the 10% level for sector 4. Based on White’s

\(^1\)Price Waterhouse provides a complete marginal tax rate schedule for each country, which typically includes some 5 to 10 distinct marginal tax rates depending on various income measures. The results reported in this paper are based on the highest marginal tax rates. We have also employed the median marginal tax rates to conduct the same analysis and obtained similar results. These results are not reported here but are available from the authors upon request.
Table 1
Regression of markup on tax rate for 14 OECD countries\textsuperscript{a,b}

<table>
<thead>
<tr>
<th>Industry markup measures</th>
<th>(b_0)</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>White test (P)-value</th>
<th>BP test (P)-value</th>
<th>GQ test (P)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Simple bivariate regression</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Manufacturing</td>
<td>1.1394</td>
<td>0.0868</td>
<td>–0.0597</td>
<td>–0.0095</td>
<td>1.7034</td>
<td>0.5295</td>
<td>0.7569</td>
</tr>
<tr>
<td></td>
<td>(0.5213)</td>
<td>(0.3693)</td>
<td>(0.4266)</td>
<td>(0.4668)</td>
<td>(0.4664)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Electricity, gas and water</td>
<td>2.0465</td>
<td>–0.9185</td>
<td>0.9359</td>
<td>–0.0333</td>
<td>1.4534</td>
<td>0.7580</td>
<td>11.3546</td>
</tr>
<tr>
<td></td>
<td>(5.5314)</td>
<td>(–1.4062)</td>
<td>(1.1830)</td>
<td>(–0.1463)</td>
<td>(0.4879)</td>
<td>(0.3840)</td>
<td>(0.0093)</td>
</tr>
<tr>
<td>3. Construction</td>
<td>1.0260</td>
<td>0.3082</td>
<td>3.1086</td>
<td>3.3124</td>
<td>0.1722</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.5974)</td>
<td>(1.9795**)</td>
<td>(0.2113)</td>
<td>(0.0688)</td>
<td>(0.9619)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Wholesale, retail trade, restaurants and hotels</td>
<td>1.0033</td>
<td>0.7386</td>
<td>0.6993</td>
<td>0.7124</td>
<td>0.3800</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(4.1944)</td>
<td>(1.7546*)</td>
<td>(0.7049)</td>
<td>(0.3987)</td>
<td>(0.8441)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Transport, storage and communication</td>
<td>1.0403</td>
<td>0.7116</td>
<td>0.4115</td>
<td>1.1256</td>
<td>0.0026</td>
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<tr>
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<td>(4.0400)</td>
<td>(1.1815)</td>
<td>(0.8140)</td>
<td>(0.2887)</td>
<td>(1.0000)</td>
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<td></td>
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<tr>
<td>B. Regression with other control variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Manufacturing</td>
<td>1.7014</td>
<td>0.0678</td>
<td>–0.0597</td>
<td>–0.0095</td>
<td>1.7034</td>
<td>0.5295</td>
<td>0.7569</td>
</tr>
<tr>
<td></td>
<td>(2.2116)</td>
<td>(0.9340)</td>
<td>(–0.7250)</td>
<td>(–0.4110)</td>
<td>(0.4266)</td>
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<td>(0.4664)</td>
</tr>
<tr>
<td>2. Electricity, gas and water</td>
<td>–6.7646</td>
<td>–0.6371</td>
<td>0.9359</td>
<td>–0.0333</td>
<td>1.4534</td>
<td>0.7580</td>
<td>11.3546</td>
</tr>
<tr>
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<td>(–0.8987)</td>
<td>(–0.8971)</td>
<td>(1.1830)</td>
<td>(–0.1463)</td>
<td>(0.4879)</td>
<td>(0.3840)</td>
<td>(0.0093)</td>
</tr>
<tr>
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<td>–0.1070</td>
<td>8.1588</td>
<td>3.9634</td>
<td>0.1722</td>
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<tr>
<td></td>
<td>(2.1885)</td>
<td>(1.5931)</td>
<td>(–1.4534)</td>
<td>(–2.3582)</td>
<td>(0.0169)</td>
<td>(0.0465)</td>
<td>(0.9619)</td>
</tr>
<tr>
<td>4. Wholesale, retail trade, restaurants and hotels</td>
<td>4.6056</td>
<td>0.6311</td>
<td>–0.3980</td>
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<td>0.8238</td>
<td>0.9212</td>
<td>0.3800</td>
</tr>
<tr>
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<td>(0.9377)</td>
<td>(1.3619)</td>
<td>(–0.7565)</td>
<td>(0.6628)</td>
<td>(0.6624)</td>
<td>(0.3372)</td>
<td>(0.8441)</td>
</tr>
<tr>
<td>5. Transport, storage and communication</td>
<td>–3.4320</td>
<td>0.8523</td>
<td>0.4865</td>
<td>–0.0412</td>
<td>0.4181</td>
<td>0.8252</td>
<td>0.0026</td>
</tr>
<tr>
<td></td>
<td>(–0.4711)</td>
<td>(1.2400)</td>
<td>(0.6253)</td>
<td>(–0.1871)</td>
<td>(0.8114)</td>
<td>(0.3637)</td>
<td>(1.0000)</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Numbers inside () and [ ] underneath the parameter estimates are OLS and White’s heteroskedasticity-consistent t-ratios; *, ** and *** denote the parameter estimate \(b_1\) is significantly different from zero at the 10, 5 and 1% levels, respectively, using a two-sided test.

\textsuperscript{b} The White test statistic is asymptotically distributed as \(\chi^2(2)\). The Breusch and Pagan’s (BP) test statistic is asymptotically distributed as \(\chi^2(1)\). As for the Goldfeld and Quandt’s (GQ) test, we divide the whole sample into two equal subsamples, according to low and high tax rates, and use observations from the low and high tax groups to conduct the test. The GQ test statistic follows the \(F(n_1 – 1, n_2 – 1)\) distribution, where \(n_1\) is the number of observations in the group with low tax rates, while \(n_2\) is the number of observations in the group with high tax rates. In each regression, \(n_1 + n_2 = 7\).
t-ratio, we find stronger results, namely, $b_1$ is larger than zero at the 5% level for sectors 1, 3 and 4, and at the 1% level for sector 5.

The above results should be interpreted with caution, however, because there can be omitted country characteristics on the right hand side of Eq. (30). Omitted variables can create an endogeneity problem and cause the parameter estimates to be unreliable. One possible way to alleviate the problem due to omitted country characteristics is to include country-specific effects in the regression. Unfortunately, the limited data available only permit us to run a cross-section regression rather than a panel regression. Within the cross-section regression, including country-specific dummy variables will make the number of intercept terms equal to the number of data points, thereby rendering the model unidentified. As a possible remedy for this problem, we add two variables, initial levels of real per capita GDP and trade/GDP ratio, in an attempt to control for individual-specific effects. In particular, the first measure allows for differences in the output elasticity of labour across countries, while the second captures possible effects of the degrees of openness or barriers to trade on equilibrium markup levels. We obtain the measures of real per capita GDP and trade/GDP ratio from the Summers and Heston (1991) data set. Our regression with the added control variables is specified as follows:

$$\text{Markup}^i = b_0 + b_1 \text{Tax}^i + b_2 \text{GDP}^i + b_3 (\text{Trade}^i / \text{GDP}^i) + \nu^i,$$

Panel B of Table 1 reports the estimation results. There is significant evidence of heteroscedasticity for sector 2 (with the Goldfeld-Quandt test) and sector 3 (with the White and Breusch-Pagan tests). Broadly speaking, these results are qualitatively similar to those obtained from the simple bivariate regression in Panel A. All estimates of $b_1$ have the expected positive sign except for sector 2, in which case it is insignificant using either $t$-ratio. Furthermore, based on the White $t$-ratio with a two-sided test, we find $b_1$ to be significantly positive at the 10% level for sectors 1 and 3, at the 5% level for sector 4 and at the 1% level for sector 5. Therefore, we conclude that our results are reasonably robust, despite the fact that the sample contains only 14 observations.

In summary, the analysis in this section leads us to conclude that the OECD data seem to be supportive of our theoretical prediction that higher income tax reduces the number of firms and raises the equilibrium markups.

7. Conclusion

This paper has developed a simple model of income taxation under monopolistic competition with endogenous markups. Standard models in the literature assumed a constant price-cost markup for each firm. This modelling strategy is rather restrictive because it has ignored some potentially important long-run effects. The
objective of this paper has been to examine the implications of relaxing such an assumption. We have demonstrated that an increase in the income tax forces some firms in the intermediate goods sector to exit the market, thereby lessening the competition among surviving firms. Consequently, incumbent firms raise their optimal markups in the new steady state. This qualitative prediction is largely supported by simple cross-section regressions with data from OECD countries.

Moreover, it has been shown that through this channel, the effects of income taxation on some key endogenous variables, including capital stock, consumption, output, wage rate and utility, are unambiguously enlarged. Therefore, our results suggest that the effects of income taxation on the key macroeconomic aggregates and consumer welfare might have markedly been underestimated by existing studies. Or, put it differently, these studies appear to have understated the benefit of a balanced-budget tax cut.

We have kept our theoretical framework simple so as to address the important points. Needless to say, our work is preliminary and is subject to several limitations. Firstly, we have ignored the possible utility generated from government consumption, nor have we considered the productivity of government spending. Secondly, our modelling has abstracted from all issues associated with alternative means of government financing. Finally our model is abstracted from long-run growth. These as well as other interesting issues will be left for future research.

Acknowledgements

We thank Thomas Piketty, a co-editor of this journal, and two anonymous referees for helpful comments and suggestions. Constructive conversations and suggestions from Huw Dixon and Mike Wickens are also acknowledged. We are grateful to Chia-Jane Wang for data assistance. The usual caveat applies.

Appendix A

Proof of Proposition 5

This proof follows from Futagami et al. (1993), who use Judd’s (1982, 1985) method. Differentiating the dynamical system of (21) and (22) with respect to $\tau$ produces:

$$
\begin{bmatrix}
\dot{K}_r \\
\dot{C}_r
\end{bmatrix} =
\begin{bmatrix}
g_K(K^*,C^*,\tau) & -1 + g_c(K^*,C^*,\tau) \\
\eta C^* r_K(K^*,C^*,\tau) & \eta C^* r_c(K^*,C^*,\tau)
\end{bmatrix}
\begin{bmatrix}
K_r \\
C_r
\end{bmatrix}
+ \begin{bmatrix}
g_c(K^*,C^*,\tau) \\
\eta C^* r_c(K^*,C^*,\tau)
\end{bmatrix}.
$$

(A.1)
Denote the Jacobian matrix as:

\[
J = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix},
\]

and the positive and negative eigenvalues as \(\theta_+\) and \(\theta_-\), respectively. Also, denote \(\tilde{K}(s)\) and \(\tilde{C}(s)\) as the Laplace transformation of \(K(t)\) and \(C(t)\), respectively; e.g. \(\tilde{K}(s) = \int_0^\infty K(t)e^{-st}dt\). Thus, (A.1) becomes

\[
\begin{bmatrix}
s\tilde{K}_r \\
\tilde{C}_r - C_r(0,\tau)
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
\tilde{K}_r \\
\tilde{C}_r
\end{bmatrix} + \begin{bmatrix}
g_s(K^*,C^*,\tau)/s \\
\eta C^* r_s(K^*,C^*,\tau)/s
\end{bmatrix},
\]

where use has been made of \(K(0;\tau) = 0\) (predetermined). It can be reduced to

\[
\begin{bmatrix}
\tilde{K}_r \\
\tilde{C}_r
\end{bmatrix} = (I - J)\begin{bmatrix}
g_s(K^*,\tau)/s \\
C_r(0,\tau) + \eta C^* r_s(K^*,\tau)/s
\end{bmatrix},
\]

where \(I\) is the identity matrix. Note that this system has a unique bounded solution, even if \(s = \theta_+\), which implies that \((I - J)\) in (A.2) becomes singular. Therefore, the following condition must be satisfied:

\[
a_{21} g_s(K^*,C^*,\tau)/\theta_+ + (\theta_+ - a_{11})[C_r(0,\tau) + \eta C^* r_s(K^*,\tau)/\theta_+] = 0. \quad (A.3)
\]

Rearranging, we have

\[
C_r(0,\tau) = -\frac{\eta C^* r_s(K^*,\tau)}{\theta_+} - \frac{a_{21} g_s(K^*,C^*,\tau)}{\theta_+ - a_{11}}.
\]

To determine the sign of each term in the right-hand side of (A.4), we differentiate the \(r\) and \(g\) functions with respect to \(\tau\), evaluated at the steady state, to obtain:

\[
r_s(K^*,C^*,\tau) = -\frac{\alpha AK^{a-1}H^{*1-a}}{\mu(n*)} - \frac{(1 - \tau)\alpha AK^{a-1}H^{*1-a}}{[\mu(n*)]^2} \mu_s(n*) < 0,
\]

\[
g_s(K^*,C^*,\tau) = -\frac{AK^{a}H^{*1-a}}{\mu(n*)} - \frac{(1 - \tau)AK^{a}H^{*1-a}}{[\mu(n*)]^2} \mu_s(n*) < 0,
\]

where use has been made of \(\mu_s(n*) > 0\) from Proposition 3. Thus, the first term in the right-hand side of (A.4) is positive, which corresponds to the intertemporal substitution effect; whereas, the second term is negative given that \(a_{21} < 0, g_s < 0\) (from (A.6)) and \(\theta_+ > a_{11}\) (also see footnote 10), which corresponds to the income effect. This completes the proof for the first part.

As for the second part, we observe that the second terms in the right-hand sides of both (A.5) and (A.6), which will vanish in the case of constant markups, work to reinforce the first terms. Then, the second part immediately follows.
Appendix B

Derivation of Eq. (29)

To simplify Eq. (27), we note that the response of $K(s;\tau)$ to a change in $\tau$ is given by:

$$K_s(s;\tau) = K^*_s(1 - e^{-\beta s}), \quad (B.1)$$

where again $K_s(0,\tau) = 0$ and $\beta$ is the speed of convergence. Substituting (B.1) into (28) produces:

$$\frac{dU}{d\tau} = \frac{\omega C_s(0;\tau)}{\rho C(0;\tau)}$$

$$+ \int_0^t \int_0^t \omega \left[ r_s(K^*,C^*,\tau)K^*_s + r_c(K^*,C^*,\tau)C^*_s + r_r(K^*,\tau) \right]$$

$$- r_s(K^*,C^*,\tau)K^*_s e^{-\beta s} ds \cdot e^{-\rho \tau} dt. \quad (B.2)$$

Given the assumption that the economy is initially in equilibrium, we get $r_s(K^*,C^*,\tau)K^*_s + r_c(K^*,C^*,\tau)C^*_s + r_r(K^*,\tau) = 0$. Then, (B.2) becomes:

$$\frac{dU}{d\tau} = \frac{\omega C_s(0;\tau)}{\rho C(0;\tau)} + \int_0^t \int_0^t \omega r_s(K^*,C^*,\tau) e^{-\beta s} ds \cdot e^{-\rho \tau} dt$$

$$= \frac{\omega C_s(0;\tau)}{\rho C(0;\tau)} + \frac{\omega r_r(K^*,C^*,\tau)}{\rho \beta} (1 - e^{-\beta t}) e^{-\rho \tau} dt$$

$$= \frac{\omega C_s(0;\tau)}{\rho C(0;\tau)} + \frac{\omega r_r(K^*,C^*,\tau)}{\rho (\rho + \beta)},$$

which is Eq. (29) in the text.

References


