On optimal non-linear taxation and public good provision in an overlapping generations economy

Jukka Pirttilä, Matti Tuomala

*Bank of Finland, Helsinki, Finland
Department of Economics, University of Tampere, P.O. Box 607, 33101 Tampere, Finland

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Abstract

Using the self-selection approach to tax analysis within an OLG framework, the paper examines optimal non-linear labour and capital income taxation and the provision of a durable public good. Under endogenous wages, the marginal tax rules depend on the influence of the tax instruments on self-selection and on the income earning abilities of the households. In particular, we found that production inefficiency occurs in the model, justifying capital income taxation. For the public good, the paper derives a dynamic analogue of the second-best Samuelson rule, encompassing both inter- and intragenerational redistributive considerations. Furthermore, the usual conditions guaranteeing the efficiency of the first-best Samuelson rule are not sufficient in the present model. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The analysis of capital income taxation and the optimal provision of public goods are classical topics in public economics. It is well-known already from the
result by Atkinson and Stiglitz (1976) that if utility in each period is separable between labour and other commodities, income from capital should not be taxed. In the field of optimal provision of a public good when the tax revenues for its financing come from distorting taxation, there are several key references. These include Atkinson and Stern (1974), who characterise the optimality conditions for a public good when commodity taxes are present, and Christiansen (1981) and Boadway and Keen (1993), who examine the case with non-linear income tax.

It is clear that the basic nature of the taxation of savings requires some kind of dynamic analysis. There are also several examples of public goods that have dynamic characteristics that could, and should, be formulated as durable goods: environmental quality, the basic infrastructure of the society, as well as the material and immaterial heritage of a nation. Thus, actions taken by the government today affect the position and the well-being of future generations. Therefore, the objective in the present paper is a joint examination of optimal taxation of savings and the provision of public good in a genuinely dynamic framework. To further this aim, the paper combines two well-known economic models: the Mirrlees (1971) model of optimal taxation and the overlapping generations model as represented in Diamond (1965). The reason for this is that this combination, encompassing heterogeneity both within and between generations, provides an appropriate means to study savings taxation and public good provision in a dynamic model with distortionary taxation.

There are several earlier papers that have analysed a partly similar situation. Ordover and Phelps (1979) set up the research agenda by thoroughly discussing the formulation of the objectives of the government in such a framework. They also demonstrate that if consumer utility is separable between consumption and labour, capital income should not be taxed. Brett (1997), using the self-selection interpretation of optimal taxation, continues the research and describes capital income taxation rules if preferences are not separable. Stiglitz (1987) extends the analysis to include general equilibrium considerations arising from endogenously determined pre-tax wage rates, as in his 1982 paper and in Stern (1982). In a more recent OLG-model, Boadway et al. (1998) treat labour and capital income taxation separately, in the spirit of the so-called dual income tax system. In their model bequests have a key role. Finally, by means of a somewhat different methodology of dynamic programming, Atkinson and Sandmo (1980) examine optimal savings income taxation under the case where there is no heterogeneity within generations. With respect to the public good provision rule, our analysis is related to that of Batina (1990) and Myles (1997). Batina considers ways to affect production efficiency by modifying a (static) public good provision rule in a representative

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1 There is some earlier work done by the same authors; see the references cited in Ordover and Phelps (1979).

2 Using the same approach, Park (1991) relaxes the homogeneity assumption but does not derive many concrete results. For an excellent overview of various OLG tax models, see Renström (1998).
consumer economy with linear taxes, when government debt policy is restricted. Myles, in turn, derives a dynamic formulation of the Samuelson principle using a representative consumer model with no distortionary taxation and no OLG setting. Their approaches are, therefore, quite different to ours.\(^3\)

In this study we push the analysis somewhat further in two main ways. First, we characterise the capital income tax schedule similar to the way the labour income tax schedule has been analysed before in the literature. Here, we allow for a rich description of the production technology that enables the before-tax wage rate to depend on, e.g., the labour supply of different households. We find this to be a quite plausible and important assumption in an essentially long-term, dynamic, framework. Allowing for endogenously determined wages is also important because it gives rise to new forms of distorting taxation and production inefficiency, along the lines of Naito (1999). Second, in addition to the analysis of marginal tax rates for labour and capital income, we examine the conditions for the optimal provision of public good under distorting taxation. The public good is modelled as a stock that preserves from one period to another and provides a potentially intriguing link between the taxation of different households. Indeed, it is shown how our dynamic formulation of the Samuelson rule in a second-best world addresses efficiency and distributional considerations both within and between generations. Finally, worth noting is the fact that the modified Samuelson rule the paper develops is derived in a setting where the tax policy instruments (the possibility for non-linear income and savings taxation) are practically relevant.

Section 2 introduces the basic framework of the study. In the model, individual households are assumed to differ in two respects: by their birth date and income-earning ability. Applying the self-selection approach to tax analysis, there are assumed to be two types of households in each generation. The government tax policy is, hence, restricted by the self-selection constraint of high-income households. The wage rates of the households are endogenously determined at the labour market and depend on the labour supply decisions, the capital stock and the level of the public good. Thus, we examine a very general class of public goods, which, in addition to their impact on consumer utility, also affect the production possibilities of the economy. Section 2 also sets up the key feature in our model, the formalisation of the public good as a stock variable. Section 3 presents optimal marginal tax rules for labour and capital income, for a given level of the public good. The marginal labour income tax rate is shown to depend on the endogenous wage impact of labour supply in a way similar to Stern (1982), while the marginal savings tax depends on the relative valuation between present and future

\(^3\)In addition, our paper is related to the work on optimal environmental policy in an OLG economy, where the environment is modelled as a stock variable (see, for instance John and Pecchenino, 1994). These papers, with the first-best world assumptions, also have a notably different emphasis to the current study.
consumption by true low-ability households and mimickers, and on the impact of capital stock on the income earning abilities of future generations. The latter impact, arising from the presence of endogenous wages, also justifies distorting capital income taxation even in the case where household preferences are separable between goods and leisure. The dynamic analogue of the Samuelson rule for public good provision is discussed in Section 4. Section 4 also demonstrates that the usual separability conditions guaranteeing the efficiency of the first-best Samuelson rule are not sufficient in the present set-up with endogenous wages. Finally, Section 5 contains some concluding remarks.

2. The framework

We consider an overlapping generations economy where individual households live two periods, supplying labour at the first and consuming a composite good on both periods. We assume for simplicity that there is no population growth. Each generation consists of two households that have different productivity at the labour market. As usual in the optimal taxation literature, type 2 households’ wage rate, \( w_2 \), is higher than the wage rate of type 1 households, \( w_1 \). We apply the following notation: \( c_i^t \) denotes the consumption of a household of type \( i \) born at time \( t \) when young, \( x_i^t \) the consumption of the same household when old, \( l_i^t \) household \( i \)'s labour supply, and \( G_t \) the level of the public good at period \( t \). The utility function, which is assumed to be identical between the households, may then be written as \( U = U(c_i^t, x_i^t, y_i^t, G_t) \). Individual optimisation, given a government’s labour income tax schedule \( T(w_i^t) \), enables the marginal income tax rate to be written as

\[
\text{MTR}(w_i^t) = \frac{1}{w_i^t} \left( \frac{U_{y_i^t}}{U_{l_i^t}} \right) + 1
\]

where \( U_{y_i^t} \), for instance, depicts the partial derivative of the utility of household \( i \) at period \( t \) with respect to its labour supply.

The production side of the economy utilises four factors: capital denoted by \( k \), the two kinds of labour, and the public good. Hence, we write the production function as \( F = F(k^t, l_1^t, l_2^t, G^t) \). The production function is assumed to exhibit constant returns to scale with respect to labour and capital under perfect competition. Factor prices are then determined by the marginal productivities as follows: \( F_{k_i}^t = r_i^t \), \( F_{l_1}^t = w_1^t \) and \( F_{l_2}^t = w_2^t \). As mentioned above, the public good contributes positively to the output, i.e., \( F_{G}^t \) is positive. Because \( G \) appears both in the utility and the production functions, it is a good like roads used both by

\*Note that households enjoy utility from the public good both when young and when old. For brevity, we omit from the notation below.
consumers and for commercial traffic, or environmental quality enjoyed by the consumers but which is at the same time beneficial for production. Note that the way the production function is specified here involves endogenous determination of the wage rates of the two household types. The relative wage between the two types may, therefore, be written as a function $\omega^t = w_1^t / w_2^t = \omega(k_1^t, l_1^t, l_2^t, G^t)$.

The government’s problem is to maximise intertemporal social welfare that includes welfare comparison both within and between generations. We apply here the interpretation by Brett (1997) of the social welfare function, and assume that the government is utilitarian within the generation, but the utilities of different cohorts are taken into account via a general social welfare function $W$. Alternatively, we could choose a formulation with a general social welfare function within generations as well. Because the assumption of within-generation utilitarianism is not crucial for the results but may be expositionally briefer, we apply it below. In both regimes, the government chooses optimal tax rates for both income and savings taxation, subject to a resource constraint for each period. It may be of some interest to note that our model is some version of the so-called dual income tax system. As in our model, under the dual income tax system capital income is taxed separately from labour income.5

As mentioned in the introduction, the public good is a stock variable that evolves from period to another according to the relationship

$$G^t = \beta G^{t-1} + g^t$$

(2)

where $\beta$ is a depreciation parameter that takes values from zero to one, and $g^t$ is the decision-based increase of the public good at period $t$. Supposing that the producer price of the incremental public good in terms of the numeraire, private consumption, is $p$, the resource constraint of the economy for each period may be written as

$$k^{t+1} = F(k_1^t, l_1^t, l_2^t, G^t) + k^t - c_t - x_{t-1} - pg^t$$

(3)

where $c_t$ depicts the aggregate consumption by the two young individuals born at time $t$, and $x_{t-1}$ the aggregate consumption of the old at the same period.

The informational assumptions we apply are similar to the standard Mirrlees-type income tax problem. The government can observe the income of the two household types, but not their type, i.e., their wage rate. This implies that the tax schedule must be planned to fulfil the self-selection constraints of the households. Concentrating on the ‘normal’ case where the redistribution occurs from the high-ability type to the low ones, the self-selection constraint that may bind is

$$U(c_1^t, x_1^t, \omega(k_1^t, l_1^t, l_2^t, G^t)) \geq U(c_2^t, x_2^t, \omega(k_1^t, l_1^t, l_2^t, G^t))$$

(4)

5See Nielsen and Sørensen (1997) for a recent contribution on the dual income tax system.
where \( \omega'(k^l_1, l^l_2, G^l)l^l_1 \) gives the required labour supply of a high-ability type when mimicking the choice of the low-ability type to earn the income level of the true low-ability type. Note that as the individuals retire in the second-period of their life, at that period their types are public information because of their decision at the first period. Hence, no self-selection constraints for the old need to be considered.

3. Optimal tax rules

If we ignore the generation that is old at period 1, we are now in a position to write the Lagrangean for the government optimisation problem as follows:

\[
L = \mathbb{W}[U(c^1_1, x^1_1, l^1_1, G^1) + U(c^1_2, x^1_2, l^1_2, G^1), \ldots]
+ \sum_{i=1}^{\infty} \lambda^i [U(c^i_1, x^i_1, l^i_1, G^i) - U(c^i_2, x^i_2, \omega^i l^i_1, G^i)]
+ \sum_{i=1}^{\infty} \rho^i \left[ F(k^l_1, l^l_2, G^l) + k^l - c^l - x^{l-1} - pg^l - k^{l+1} \right]
+ \sum_{i=1}^{\infty} \mu^i \left[ \beta G^l + g^{l+1} - G^{l+1} \right]
\]

The development of the stock of the public good is captured by the last constraint, with the multiplier referring to the social marginal value of the stock at period \( t \). An important assumption to note is that the government possesses perfect control over the capital stock. As shown by e.g. Atkinson and Sandmo (1980), this implies that only one from the resource constraint and government budget constraint need to be taken into account. Because of this, together with the observation that the resource constraint may be derived from combining the government’s and the households’ budget constraints in the Mirrleesian fashion, the multiplier associated with the resource constraint, \( \rho^l \), may be interpreted as the shadow price of government’s revenue.\footnote{It is a usual feature of dynamic optimal taxation exercises that it is optimal for the government to collect the bulk of the revenue by confiscating the capital stock at the outset. We want to abstract from these complications by leaving out the generation 0. For a good overview on these issues, see Domeij and Klein (1998).} For a given level of the public good, the first-order conditions at an exemplary date \( t \) revealing the optimal tax structure are the following:

\[
k^l: \rho^l (F^l_k + 1) - \rho^{l-1} - \lambda^l \tilde{O}^l_{z,t} \frac{\partial \omega^l}{\partial k^l} l^l_1 = 0
\]
where the hat-terms refer to the type-2 when mimicking the choice of the type-1.

Although our main purpose is to focus on capital income taxation and public goods, we briefly discuss the properties of labour income tax schedule:

**Proposition 1.** Within each cohort, the marginal labour income tax rate is negative for the high-ability type, and positive for the low-ability type.

**Proof.** The marginal income tax rate for the high-ability type may be deduced by dividing (12) by (8) and combining the result with the property in (1):

\[
\text{MTR}(w^l_i) = \frac{\lambda^i \hat{U}^i_{2,1} \partial \omega^i}{\rho^i w^l_i \partial l^i_2} l^i_1, \quad (13)
\]

which is negative, since \( \hat{U}^i_{2,1} \) is negative and the increase in the labour supply of the high-ability type increases the relative wage of the low-ability workers (i.e., \( \partial \omega^i / \partial l^i_2 > 0 \)). The corresponding tax rate for the low-ability type follows from dividing (11) by (7) and some rearrangements:

\[
\text{MTR}(w^l_i) = \frac{\lambda^i \hat{U}^i_{2,1} \partial \omega^i}{\rho^i w^l_i \partial l^i_2} l^i_1 + \frac{\lambda^i \hat{U}^i_{2,1} \partial \omega^i}{\rho^i w^l_i \partial l^i_2} l^i_1 = 0 \quad (14)
\]

To sign this expression, note first that the latter term, referring to the endogenous wage structure, is positive, since increasing labour supply of the low-ability type decreases their relative wage rate (\( \partial \omega^i / \partial l^i_2 < 0 \)). The term in the brackets is positive, because of the standard single-crossing property: the indifference curve for the high-ability type at a given point in (income, consumption)-space is assumed to be flatter than that of the low-ability type. This results from the fact
that the type-2 households find it easier to transform leisure to consumption because of their higher ability. It may be shown that under the single-crossing assumption, the marginal rate of substitution for the mimicker is determined by $(\hat{U}_{2,t}/\hat{U}_{2,c})\omega^t$, and that the term in the brackets is positive. Combining the effect of the two forces in (14), the marginal income tax rate for type-1 households is positive. □

Proposition 1 reveals that the characteristics of an optimal income tax schedule under endogenous wages derived in a static framework by Stern (1982) and Stiglitz (1982) also apply in the dynamic case of the present paper. Our result is a modest extension to the finding by Ordover and Phelps (1979), which showed that the standard properties of optimal non-linear income taxation in the static case under no general-equilibrium impacts of the wage rates also carry over to the overlapping generations model. Worth noting is the fact that unlike in the standard model, the marginal tax rate faced by the high-ability households is negative instead of zero. The intuition, as noted by Stern (1982), is clear: on top of the income tax, it is optimal for the government to rely on the redistributive capacity of the economy by pushing the labour supply of the high-ability type further, and hence to achieve an increase in the relative wage of the low-ability type.

We now turn to the properties of the optimal non-linear taxation of capital income (savings), which may be deduced by comparing the marginal rate of substitution between the present and the future consumption imposed on the households. By performing these calculations, we obtain the following result:

**Proposition 2.** Within each generation, the optimal marginal capital income tax rate (MTR for savings) is positive (negative) for the high-ability household, if an increase in capital reduces (raises) the relative wage rate of the low-ability type, $\omega^t$. The MTR for savings of the low-ability household is positive (negative), if an increase in capital reduces (raises) the relative wage rate and the mimickers value future consumption more (less) than the true low-ability types. Otherwise the MTR for savings of the low-ability type households is ambiguous.

**Proof.** Individual optimisation between the present and future consumption, for a given after-income-tax income, implies that the marginal savings tax rate (with $s_i^t$ denoting the savings of household $i$) may be expressed by

$$\text{MTR}(s_i^t) = \frac{U_i^t}{U_i^t} + 1 + r^{t+1}. \quad (15)$$

Note next that leading and rearranging (6) yields

$$\rho^{t+1} = \frac{\left(\rho^t + \lambda^{t+1} \hat{U}_{i,t} \frac{\partial \omega^{t+1}}{\partial \kappa^{t+1} \hat{U}_{i+1}} \right)}{(1 + r^{t+1})}.$$
By means of this property and the one in (15), and dividing (8) by (10), we obtain

$$\text{MTR}(s^t) = \frac{\lambda^{t+1} U^{t+1}_{2,s} \frac{\partial \omega^{t+1}}{\partial k^{t+1}} t^{t+1}_{1} U^{t}_{2,s}}{\rho^{t} U^{t}_{2,s}}$$

This expression reveals that if an increase in savings (and hence in the level of capital) reduces the relative wage rate of the low-ability households, it is optimal to impose a positive marginal savings (=capital income) tax on type-2 households.

The MTR for savings of the low-ability type may be derived in a similar manner. Divide first (7) by (9) and use the properties above to get

$$\frac{U^{t}_{1,c}}{U^{t}_{1,s}} \left[ 1 + \frac{\lambda^{t+1} U^{t+1}_{2,l} \frac{\partial \omega^{t+1}}{\partial k^{t+1}} t^{t+1}_{1}}{1 + r^{t+1}} \right] = \frac{\lambda^{t} U^{t}_{2,s} \left( \frac{U^{t}_{2,c}}{U^{t}_{2,s}} - \frac{U^{t}_{1,c}}{U^{t}_{1,s}} \right) + 1}{r^{t}}$$

This enables us to write the tax rate as follows:

$$\text{MTR}(s^t) = \frac{\lambda^{t+1} U^{t+1}_{2,s} \frac{\partial \omega^{t+1}}{\partial k^{t+1}} t^{t+1}_{1} U^{t}_{1,c} - \lambda^{t} U^{t}_{2,s} \left( \frac{U^{t}_{2,c}}{U^{t}_{2,s}} - \frac{U^{t}_{1,c}}{U^{t}_{1,s}} \right)}{\rho^{t} U^{t}_{2,s}} (1 + r^{t+1})$$

This expression encompasses two distinct influences: the first term on the right is analogous to the expression in the MTR for savings of the high-income type, and captures the effect of extra capital on the relative wage rate. If \( \frac{\partial \omega^{t+1}}{\partial k^{t+1}} \) is negative, the endogenous wage term in (18) implies a positive marginal tax rate on savings, and vice versa for a positive \( \frac{\partial \omega^{t+1}}{\partial k^{t+1}} \). The latter term in (18), arising from the self-selection considerations, becomes positive (negative), if a mimicker has a higher (lower) valuation of future consumption than a true type-1 representative. Clearly, in the case where the two impacts have different signs, the MTR for savings of type-2 remains ambiguous.

It is useful to compare our result to those derived in earlier contributions. To our knowledge, the part of the capital income tax referring to the impact on endogenous wages has not been presented in a marginal tax format before.\(^8\) Its intuition is straightforward: if an increase in savings and, hence, in the capital stock, leads to a decrease in the relative wage of the low-income households, it becomes optimal for the government to distort the savings level downwards because of redistributive impacts that arise. The self-selection concerns expressed in the marginal tax rule for the low-ability type have been, however,

\(^8\)Stiglitz (1987) provides similar results but does not discuss their implications on marginal tax rules.
discussed earlier in Brett (1997). As he points out, if mimickers have a lower valuation of future consumption, taxing savings is a device to relax the self-selection constraint and obtain a Pareto-improvement through easier redistribution. This part also makes it clear why, with weakly separable preferences between goods and leisure, capital income taxation cannot be used to deter mimicking. The reason is that in that case, a true type-1 and a mimicker has the same valuation for savings, and the self-selection term in (18) vanishes.

We finally note that even if preferences are separable between goods and leisure, the endogenous wage term in the tax rules provides justification for distorting capital income taxation:

**Corollary 1.** If consumer preferences are weakly separable between goods and leisure, it is optimal to impose a tax (subsidy) on capital income, if an increase in the savings — i.e., investment — by generation t leads to an decrease (increase) of the relative wage of the low-income households (w_t^{t+1}) of the next generation, t + 1.

In the standard taxation model in an OLG economy (in the model examined by Ordover and Phelps (1979), for instance), there is no reason to rely on taxation between the generations. This is because the income earning abilities of the generations are independent from each other, and there is no device to transfer income from one generation to another. It seems, therefore, to be unnoticed in the existing literature that the possibility that changes in capital stock may influence income earning abilities and wage differences in the future provides a rationale to resort to distorting taxation of savings of the present generation in favour of future cohorts. This result can also be interpreted as a reservation to the old issue of the optimality of the so-called expenditure taxation (which does not involve taxation of savings).

Perhaps the most interesting way to discuss the implications of Corollary 1 is in terms of production efficiency. The presence of the endogenous wage term (the last term) in expression (6) implies production inefficiency at the optimum of our model. In other words, the across-time ratio of marginal social values of commodities is not equal to the marginal rate of transformation of commodities across time, i.e., \(1 + r' \neq \rho t^{-1}/\rho t\). Thus, this kind of production inefficiency justifies capital income taxation in our model. This is in contrast with the famous efficiency theorem by Diamond and Mirrlees (1971). The discussion here bears an obvious similarity with the important recent work by Naito (1999) who shows that, in the presence of endogenous wages, the introduction of distortions in the public sector can Pareto-improve welfare.

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9The result that there should be no capital income taxation if preferences are weakly separable between goods and leisure has been shown originally by Ordover and Phelps (1979).

10We are grateful to a referee for this suggestion.
An important further question to discuss concerns the usual time inconsistency problem in capital income taxation (see, for instance, the analysis in Renstrom, 1998). While we fully agree that the tax schedules presented here suffer from the time inconsistency syndrome, we regard analysing the issue in depth as another, albeit important, topic. In a model utilising the socially optimal taxation approach, such as ours, the analysis of the case where the government can commit to the tax schedules for further generations serves as a useful benchmark case.

4. The optimal provision of a public good

This section proceeds to examine the optimal provision of a public good as a stock variable. The optimality conditions are the following:

\[
G^t: \frac{\partial W}{\partial U_t} (U^t_{1,t} + U^t_{2,t}) + \frac{\partial W}{\partial U_{t-1}} (U^t_{1,t-1} + U^t_{2,t-1}) + \lambda (U^t_{2,t} - \hat{U}^t_{2,t}) + \lambda^{t-1} (U^t_{2,t-1} - \hat{U}^t_{2,t-1}) - \lambda^t \hat{U}^t_{2,t} \frac{\partial \omega^t}{\partial G^t} f^t_1 + \rho^t \hat{F}^t + \mu^t \beta - \mu^{t-1} = 0
\]  

(19)

\[
g^t: = \rho^t p + \mu^{t-1} = 0
\]  

(20)

Note that the level of the public good affects at the same time both the young and the old living at that period and it also influences the production possibilities and the relative wage in the economy. Eq. (19) may be reformulated by substituting from (7)–(10), and by defining the marginal rate of substitution between the public good and the consumption when young and the consumption when old as \(\text{MRS}_{i,GC} = U_{i,t}/U_{i,x} \) and \(\text{MRS}_{i,GX} = U_{i,t}/U_{i,x} \), respectively. Conducting these operations in (19) yields

\[
\text{MRS}_{i,GC} \left( \frac{\lambda^t \hat{U}^t_{2,c}}{\rho^t} + 1 \right) + \text{MRS}_{i,GX} \left( \frac{\lambda^{t-1} \hat{U}^{t-1}_{2,c}}{\rho^t} + 1 \right) + \text{MRS}_{2,GC} \\
+ \text{MRS}_{2,GX} \left( \frac{\lambda^t \hat{U}^t_{2,c}}{\rho^t} - \frac{\lambda^{t-1} \hat{U}^{t-1}_{2,c}}{\rho^t} - \frac{\lambda^t \hat{U}^t_{2,\beta}}{\rho^t} \frac{\partial \omega^t}{\partial G^t} f^t_1 + \rho^t \hat{F}^t + \frac{\mu^t}{\rho^t} \beta - \frac{\mu^{t-1}}{\rho^t} \right) \\
= 0
\]  

(21)

Using \(\lambda^t \) as shorthand for \(\lambda^t \hat{U}^t_{2,c}/\rho^t \) and \(\lambda^{t-1} \hat{U}^{t-1}_{2,\beta}/\rho^t \), (21) may be rewritten as
The notion of SMB measures the social marginal benefit of an increase in the level of the public good at time \( t \); its interpretation is discussed shortly below. According to Eq. (22), the shadow value of the public good at one period should be equal to the value of an incremental unit of the public good (SMB) plus the shadow value of the public good next period. To arrive at the final formulation of the dynamic public good provision rule, lead (22) first by one, then two and so on periods, and substitute these values again in (22).\(^{11}\) Combining the result with (20) enables us to state the following proposition:

**Proposition 3.** In the presence of the optimal income and savings taxation scheme, the dynamic formulation of the Samuelson rule for a public good is given by

\[
p = \sum_{\tau=1}^{\infty} \frac{\rho^\tau}{\rho} \beta^{t-1} SMB_G^\tau.
\]

Let us begin the interpretation of this proposition by noting that the dynamic formulation of the Samuelson rule is written in a similar way to the static one: the left-hand side depicts the marginal rate of transformation between producing an incremental unit of public good in terms of private consumption. The right-hand side, in turn, measures the sum over time of the marginal social benefits of an incremental public good. The key aspect to note is that the expression at the right now measures not only the sum of the marginal rate of substitution within one generation, but a sum of all these intratemporal utilities together.\(^{12}\)

The marginal social benefit of the public good, as given in Eq. (23), takes into account all the influences of an increase in the public good stock at one period. The first term in this expression captures the sum of the marginal rate of

\(^{11}\)In other words, we simply add the first-order conditions for the public good evaluated at different periods together to obtain the present value of an increase in the public good.

\(^{12}\)This discussion could be related to the literature on cost–benefit analysis of public projects. An important early contribution in this field within the OLG-models is Pestieau (1974).
substitution for the public good of the two types of households that are working at that period. This term is hence equal to the corresponding term in the standard Samuelson rule of the static case. Within the overlapping generations model, however, the presence of the public good affects two cohorts at the same time. Thus, the optimality condition also takes into account the sum of the marginal rate of substitution for the public good of the households that are old at the examined period. This influence is encapsulated in the second term in (23). The third and the fourth terms in the public good provision rule address the impact of the public good on the self-selection constraint. Recall that our framework belongs to the optimal non-linear income taxation literature, and it is well-known that in the second-best circumstances where the government cannot observe households’ abilities, its redistributive efforts are limited by the self-selection constraint of the rich households. For instance, as demonstrated by Boadway and Keen (1993), if the individual valuation of the public good decreases with leisure, the government may ease the self-selection constraint and assist redistribution by distorting the public good provision upwards. This follows from the observation that, in that case, the true type-1 households have a higher valuation of the public good than the mimickers, and revenue-neutral increases in the public good provision hence deter mimicking. These self-selection impacts are taken into account in the present model as well, but for the both generations living at the period considered.

The remaining terms in (23) refer to the presence of the public good in the production function. The first from these, \( F_G^t \), simply measures the direct influence of the public good on the production possibilities. If this impact is positive, it increases the marginal social benefit of public good provision. The second one arises from the fact that the public good may also affect the relative wage in the economy. If, for instance, the public good increases the relative wage of type-1 workers, this impact raises the social benefit of public good provision. To conclude, the divergence of the MSB at a given point of time from the fully efficient Samuelsonian principles stems from the imperfect redistributive capacity of the government. In these circumstances, under- or over-provision of the public good may be used for intragenerational redistributive purposes through the self-selection impact and the influence on the relative wage.

Let us move on to the interpretation of the dynamics of the proposition. Basically, the rule measures the sum of the marginal benefits over time. The parameter \( \beta^{t-1} \) in the rule captures the physical depreciation of the stock of the public good that, if evaluated at time \( t \), starts at period \( t+1 \). The intuition emerging from the term \( \rho^t \rho^t \) proves, maybe, more interesting. As noticed in Section 3 above, the multiplier \( \rho \) is associated with the shadow price of the public funds. Now suppose that this shadow price is for some reason higher at period \( t+1 \) than at period \( t \). This means that the weight for the MSB of the public funds become more expensive at next period, it is useful to increase the
stock already at period $t$, and take advantage of the preservation of the public good as a stock to the next period.

In second-best tax models where the tax system has redistributive objectives, the interpretation of the shadow price of public funds, $\rho$, depends on the social value of redistribution and on the efficiency losses of taxation.\(^\text{13}\) Therefore, under the most general second-best circumstances, the actual realised value of $\rho$ remains unknown. Consider next a special case, where the tax system may operate in the first-best world within each generation (that is, the terms referring to self-selection vanish). Then it may be deduced from Eq. (10) that the ratio of the shadow prices at different periods is determined by

$$\frac{\rho^{t+1}}{\rho^t} = \frac{\partial W/\partial U^{t+1}}{\partial W/\partial U^t}.$$

This expression highlights the intergenerational redistribution aspect present at the dynamic public good provision rule. It states that if the social marginal value of an extra income for generation $t+1$ is higher than that of generation $t$, the sum of the marginal rate of substitution of generation $t+1$ has a higher weight on the public good provision rule. This implies that the public good stock may be employed as a intergenerational redistributive device: in the described case, the increase of the public stock should be high at this period, since it increases the utility of the people living in the future. Note that similar kind of intertemporal considerations are present in a second-best case as well, though in a muted form, as the evaluation of $\rho^{t+1}/\rho^t$ becomes more complicated.

To sum up, the modified, dynamic, Samuelson rule encompasses both inter- and intragenerational distributional concerns. These concerns arise from the fact that the redistributitional capacity of the government is restricted. Given these circumstances, it is usual in the public economics literature to ask if there are some special conditions, under which the second-best optimality rule reduces to the simple, first-best one. Were this the case, the public good provision rule would become much more simplified. Christiansen (1981) and Boadway and Keen (1993) show that if consumer preferences are weakly separable between goods and leisure, the first-best rule for the public good provision remains valid also for the second-best case under non-linear income taxation.

In the present model with the public good affecting the production function and

\(^{13}\)This issue comes close to the notion of marginal cost of public funds (MCPF). In its usual form, as a division of shadow price of public funds by the shadow price of private income, it does not show up in non-linear tax models that follow the self-selection interpretation, such as Boadway and Keen (1993). Christiansen (1998) incorporates the notion into such a regime and shows that public funds are distortionary (MCPF > 1) if the self-selection constraint is tightened, i.e., the tax increase of the high-ability type is large enough. The present set-up is more general than that of Christiansen in that it also encompasses capital income taxation. Examining the level of MCPF in our framework would certainly be interesting, but worthy of a separate analysis.
the level of wages as well, these separability conditions do not hold. The following corollary summarises this:

**Corollary 2.** Under the optimal income and savings taxation scheme, if consumer preferences are weakly separable between goods and leisure, the dynamic analogue of the public good provision rule does not reduce to the first-best intragenerational Samuelson principle.

The notion of the first-best rule in the intragenerational sense refers to a situation where only the direct effects of the public good are taken into account in the social marginal benefit of the public good, given in expression (23). To put it more rigorously, the social marginal value of an incremental public in the first-best situation would then be given by \( \text{SMB}^*_G = \sum_i \text{MRS}^i_{1,G} + \sum_i \text{MRS}^i_{2,GX} + F^i_G \). The modified Samuelson rule of Proposition 3 would still depend on intergenerational concerns, captured by the discount factors — hence, the notion of ‘intragenerational Samuelson principle’. While the separability structure mentioned in Corollary 2 makes the self-selection term disappear from the social marginal valuation of the public good (expression (23)), the last term referring to the impact of public good on wages remains in the formulation. In contrast to the earlier results cited above, it is not possible to separate efficiency and equity concerns in the public good provision in conditions where standard separability assumptions are valid.

To sum up, both the capital income tax and public good provision rules depend crucially on the implications of endogenous wages. In particular, the standard rules required for decentralisation of equity and efficiency concerns need to be augmented by imposing separability also into the production side of the economy. In other cases, first-best rules do not remain valid in the second-best circumstances.

### 5. Conclusion

This paper examined the problem of optimal taxation and government expenditure in a dynamic setting. We combined a Mirrlees type of an optimal income tax model and an overlapping generations model, such as used by Diamond (1965). While it is not surprising, therefore, that most of the properties of these models carry over to the present set-up, combining them makes it possible to derive some properties of optimal taxation of income from capital, which have been neglected earlier. In particular, it is possible to characterise the shape of an optimal tax on capital income similar to the way the properties of optimal labour income taxation have been highlighted before in a static framework. Allowing the wages to be endogenously determined in the model, we also provide a new rationale for distorting capital income taxation: the marginal tax rate on income from capital depends on the relative valuation between present and future consumption by true
low-ability households and mimickers, and on the impact of capital stock on the income earning abilities of future generations. The latter impact seems to have interesting implications: it calls for violation of production efficiency and means that the separability result derived by Ordover and Phelps (1979) does not hold within the present setting.

As opposed to much of the other literature, this paper also examined the optimal provision of a public good which is treated as a stock variable. This was addressed by deriving a dynamic analogue of the Samuelson rule of public good provision in a second-best world with distortionary labour and capital income taxation. The rule was shown to capture not only efficiency considerations, but also inter- and intragenerational redistributional concerns, in a way that allows some novel intuition to be gained. As in the case of capital income taxation, the standard separability rules do not remain valid in the present set-up with endogenous wages.

The paper seems to contain some attractive areas of further work. In particular, we plan to address the interrelation of public good provision and dynamic efficiency of the OLG model within the present set-up. Another area of further work might concern the optimal public provision of private goods, which is rather well-understood in a static framework, but unexplored in a dynamic world.

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