Discussion

Correction note on “The demand for health with uncertainty and insurance”

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Abstract

This paper proves the non-existence of an optimal solution under the Liljas [Liljas, B. (1998). The demand for health with uncertainty and insurance. J. Health Econ., 17, 153–170] type of insurance. The reason for the non-existence is that the insurance induces the individual to increase his time input, relative to medical expenditure in the household production of health investment. Hence, it distorts the balance of inputs in the production of health investment. Moreover, it also distorts the household production for consumption goods through time constraints. Therefore, this paper proposes an alternative insurance that covers the time loss due to illness and has an optimal solution. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The Grossman model, originally developed by Grossman in 1972, is very valuable in order to analyze the individual lifetime health investment behavior. Unfortunately, there are few papers that introduce uncertainty and insurance into...
the Grossman model in spite of their importance in the real world. Liljas (1998) is probably the first to do so, thus contributing significantly to the development of the Grossman model. Unfortunately, there is one crucial error in this paper. The optimal solution does not exist under the Liljas (1998) type of insurance.

This paper proves the non-existence of an optimal solution under the Liljas (1998) type of insurance and proposes an alternative insurance that has an optimal solution. A more careful treatment on the Liljas' model is necessary for the development of the Grossman model and health economics. This paper is organized as follows. Section 2 reformulates the Liljas model so as to make the non-existence proof much clear. Section 3 proves the non-existence of an optimal solution under a Liljas (1998) type of insurance and Section 4 proposes an alternative type of insurance which has an optimal solution. Finally, Section 5 summarizes the results.

2. Solution to the optimization problem

Although the original work by Liljas (1998) solves the model following Grossman (1972a,b), this paper uses the method of maximum principle to make the non-existence proof more tractable. Almost all the notations and assumptions in this paper are the same as in Liljas (1998) to facilitate comparisons. Thus, for a more precise explanation of the assumptions and notations in this model, see Appendix A of Liljas (1998).

In Liljas (1998), uncertainty is introduced into health capital such as

\[ H_t = H_t^E + \sigma_t \varepsilon, \tag{1} \]

\[ H_t^E = \int_{H_{\text{min}}}^{H_{\text{max}}} H_t g(H_t) dH_t, \tag{2} \]

Since \( H_{\text{min}} \) is assumed to be positive, then

\[ \Pr \left( \varepsilon \geq \frac{-H_t^E}{\sigma_t} \right) = 1 \tag{3} \]

This means that \( H_t \) is never negative.

\footnote{Dardanoni and Wagstaff (1987), Selden (1993), and Chang (1996) also introduced uncertainty into the Grossman model. However, these studies treated only one or two period model.}

\footnote{It is well known that the dynamic optimization problem can be solved through several methods (see Kaimen and Schwartz, 1991). Wagstaff (1986) also solves the Grossman model with the method of maximum principle.}

\footnote{It is shown from Eq. (3) that the initial health capital must be more than zero for the health capital during their lifetime to be more than zero.}
Thus, the individual utility maximization problem under uncertainty is reformulated as

\[
\max \int_0^T \int_{H_{\text{min}}}^{H_{\text{max}}} u(\phi_t, H_t, Z_t) e^{-\gamma t} g(H_t) dH_t dt,
\]

subject to

\[
\int_0^T \int_{H_{\text{min}}}^{H_{\text{max}}} (\pi_t I_t + q_t z_t + w_t T_t) e^{-\gamma t} dH_t dt,
\]

\[
H_t = I_t - \delta_t (H_t, Q_t) H_t.
\]

The Hamiltonian reads

\[
J = \int_{H_{\text{min}}}^{H_{\text{max}}} \left[ \int_0^T u(\phi_t, H_t, Z_t(X_t, T_t)) e^{-\gamma t} dt 
- \lambda \int_0^T (\pi_t I_t (M_t, TH_t) + q_t Z_t(X_t, T_t) + w_t (\Omega - \phi_t H_t)) e^{-\gamma t} dt 
+ \mu_t (I_t (M_t, TH_t) - \delta_t (H_t, Q_t) H_t) \right] g(H_t) dH_t
\]

where \( E[\] is defined on the health capital. This can be rearranged as

\[
J = \int_0^T E \left[ u(\phi_t, H_t, Z_t(X_t, T_t)) e^{-\gamma t} - \lambda (\pi_t I_t (M_t, TH_t) + q_t Z_t(X_t, T_t)
+ w_t (\Omega - \phi_t H_t)) e^{-\gamma t} + \mu_t (I_t (M_t, TH_t) - \delta_t (H_t, Q_t) H_t) \right] dt
\]

The first-order conditions of this problem are

\[
\frac{\partial J}{\partial X_t} = E \left[ \frac{\partial u}{\partial Z_t} \frac{\partial Z_t}{\partial X_t} e^{-\gamma t} - \lambda q_t \frac{\partial Z_t}{\partial X_t} e^{-\gamma t} = 0, \right.
\]

\[
\frac{\partial J}{\partial T_t} = E \left[ \frac{\partial u}{\partial Z_t} \frac{\partial Z_t}{\partial T_t} e^{-\gamma t} - \lambda q_t \frac{\partial Z_t}{\partial T_t} e^{-\gamma t} = 0, \right.
\]

\[
\frac{\partial J}{\partial M_t} = -\lambda \pi_t I_t e^{-\gamma t} + \mu_t I_t = 0,
\]
\[ \frac{\partial J}{\partial T_i} = -\lambda \pi_t \frac{\partial I_i}{\partial T_i} e^{-rt} + \mu_t \frac{\partial I_i}{\partial T_i} = 0, \]
\[ -\mu_t = \frac{\partial J}{\partial H_i} = \phi_i \left( E \left[ \frac{\partial u}{\partial h_i} e^{-\theta t} + \lambda w e^{-rt} \right] - \mu_t E \left( \delta_t + \frac{\partial \delta_t}{\partial H_i} H_i \right) \right). \]

Since \( Z(X, T) \) and \( I(M, T, H) \) are independent of \( H \), they are not evaluated by the expectation. From Eq. (11),
\[ \mu_t = \lambda \pi_t e^{-rt}. \]

Note that \( \lambda \) is constant, since it is the multiplier for the individual life time budget constraint and not for the variable financial asset in each period. Thus, we obtain
\[ \frac{\dot{\mu}_t}{\mu_t} = \frac{\dot{\pi}_t}{\pi_t} - r, \]
and rearranging Eq. (13)
\[ \frac{\dot{\mu}_t}{\mu_t} = -\phi_i \left( E \left[ \frac{\partial u}{\partial h_i} e^{-\theta t} + \lambda w e^{-rt} \right] \frac{1}{\mu_t} + E \left( \delta_t + \frac{\partial \delta_t}{\partial H_i} H_i \right) \right). \]

Substituting Eqs. (14) and (15) into Eq. (16), the Euler equation becomes
\[ \phi_i \left( \frac{1}{\lambda} E \left[ \frac{\partial u}{\partial h_i} \right] e^{(-\theta) t} + w_i \right) = \left( r + E \left( \delta_t + \frac{\partial \delta_t}{\partial H_i} H_i \right) \right) \pi_t - \dot{\pi}_t. \]

This is the same as the Euler equation under the uncertainty presented by Liljas (1998). Hence, the change in the resolution procedure does not alter the solution in any way.

3. Non-existence of the optimal solution

This section presents the nonexistence of an optimal solution when the Liljas (1998) type of insurance is introduced. Liljas (1998) introduces the social insurance as
\[ \eta w_i (\text{TW} (H_{\max}) - \text{TW} (H_i)) \eta \in (0, 1). \]

In order to express \( \text{TW} \), as \( \text{TW} (H_i) \), the author implicitly assumes the existence of an optimal solution under this type of insurance. Indeed, if there is an optimal solution, then all endogenous variables should only be the function of \( H_i \).
Unfortunately, this assumption is not so obvious since the existence of an optimal solution is not proven in Liljas (1998). Therefore, this insurance should be reformulated as

$$\eta_w(TW - TW_i), \eta \in (0,1).$$

(19)

where $TW$ is the maximum attainable working time. The Hamiltonian of the optimization problem with insurance reads as

$$J = \int_0^T E \left[ a(\phi_i H_i, Z_i(X_i, T_i))e^{-\theta t} - \lambda \left( \delta_t I_i(M_i, TH_i) \right) + q_i Z_i(X_i, T_i) 
+ w_i(H_i - T_i - TH_i) + \eta_w(TW - (\phi_i H_i - T_i - TH_i))e^{-\theta t} 
+ \mu_u(I_i(M_i, TH_i) - \delta_i(H_i, Q_i) H_i) \right] dt$$

(20)

Note that we use the relation

$$TW_i = \phi_i H_i - T_i - TH_i.$$  

(21)

The first-order conditions are

$$\frac{\partial J}{\partial X_i} = E \left[ \frac{\partial u}{\partial Z_i} \frac{\partial Z_i}{\partial X_i} e^{-\theta t} - \lambda q_i \frac{\partial Z_i}{\partial X_i} e^{-\theta t} = 0, \right.$$  

$$\frac{\partial J}{\partial T_i} = E \left[ \frac{\partial u}{\partial Z_i} \frac{\partial Z_i}{\partial T_i} e^{-\theta t} - \lambda q_i \frac{\partial Z_i}{\partial T_i} e^{-\theta t} - \lambda \eta_w e^{-\theta t} = 0, \right.$$  

$$\frac{\partial J}{\partial M_i} = -\lambda \pi_i \frac{\partial I_i}{\partial M_i} e^{-\theta t} + \mu_i \frac{\partial I_i}{\partial M_i} = 0, \right.$$  

$$\frac{\partial J}{\partial H_i} = -\lambda \pi_i \frac{\partial I_i}{\partial H_i} e^{-\theta t} + \mu_i \frac{\partial I_i}{\partial H_i} - \lambda \eta_w e^{-\theta t} = 0, \right.$$  

(24)

$$-\dot{\mu}_i = \frac{\partial J}{\partial H_i},$$  

(26)

$$= \phi_i \left(E \left[ \frac{\partial u}{\partial H_i} e^{-\theta t} + \lambda w_i e^{-\theta t} \right] - \mu_i E \left[ \delta_t + \frac{\partial \delta_i}{\partial H_i} H_i \right] \right)$$  

$$+ \lambda \eta_w E \left[ \frac{\partial TW_i}{\partial H_i} \right] e^{-\theta t}$$

For the existence of an optimal solution, all first-order conditions should be satisfied. From Eq. (24), it follows that

$$\mu_i = \lambda \pi_i e^{-\theta t},$$  

(27)

Substituting this expression into Eq. (25), we obtain

$$\lambda \eta_w e^{-\theta t} = 0.$$  

(28)
As \( r \) and \( w_t \) are given and positive and \( \lambda \) should not be zero, unless the lifetime income constraint is satisfied, \( \eta \) must be zero so as to satisfy Eq. (28). This means that there is no insurance. Thus, there is no optimal solution under the Liljas (1998) type of insurance.

Since Liljas (1998) implicitly assumed the existence of an optimal solution, the insurance proposed in the paper may be inadequate. Moreover, the reason for the non-optimal solution under this insurance is that it just covers the time input \( TH_t \) to produce gross health investment, through the relation of \( TW_t = \Omega - T_t - TH_t - TL_t \). Thus, this insurance affects individuals by increasing their time input \( TH_t \), relative to medical expenditure \( M_t \). As a result, the balance of inputs for health investment is substantially distorted. Moreover, this insurance also distorts the balance of inputs in the household production function for consumption goods through the time constraint.\(^5\)

This can easily be proved using Eqs. (22) and (23). For the existence of the optimal solution, the insurance should not distort the balance of the medicare expenditure \( M_t \) and time inputs \( TH_t \) in the production of health investment. This indicates the insurance which only covers medical care expenditure also distorts the balance of inputs and has no optimal solution, even though this type of insurance is the most likely in actual world.

4. An alternative way to introduce insurance

This section proposes an alternative insurance to that of Liljas type, under which the first order conditions for optimization problem are satisfied simultaneously. That is the insurance with an optimal solution. The proposed insurance is

\[
\eta_t w_t TL_t, \quad \eta_t \in (0,1),
\]

which includes an insurance for the time loss due to illness. Introducing this type of insurance, the Hamiltonian is formulated as

\[
J = \int_0^T E \left[ u(\phi_t, H_t, Z_t, X_t, T_t) e^{-\theta t} - \lambda (\pi_t I_t(M_t, TH_t) + q_t Z_t(X_t, T_t)) \\
+ w_t (1 - \eta_t) (\Omega - \phi_t H_t) e^{-\theta t} + \mu_t (I_t(M_t, TH_t) - \delta_t (H_t, Q_t) H) \right] dt
\]

(30)

Using the expression

\[
TL_t = \Omega - \phi_t H_t,
\]

(31)

\(^5\) This insurance also affects the individual's workable time \( TW_t \) through the time constraint. In the short run, an increase in time input for health investment \( TH_t \) will reduce the workable time \( TW_t \) since the increase in \( TH_t \) will not increase health capital \( H_t \) immediately. Needless to say, in the long run, the effect will be indefinite due to the increase in healthy time \( \phi_t H_t \).
the first-order conditions are defined by

$$\frac{\partial J}{\partial X_t} = E \left[ \frac{\partial u}{\partial X_t} \right] \frac{\partial Z_t}{\partial X_t} e^{-\theta t} - \lambda q_t \frac{\partial Z_t}{\partial X_t} e^{-rt} = 0, \quad (32)$$

$$\frac{\partial J}{\partial T_t} = E \left[ \frac{\partial u}{\partial T_t} \right] \frac{\partial Z_t}{\partial T_t} e^{-\theta t} - \lambda q_t \frac{\partial Z_t}{\partial T_t} e^{-rt} = 0, \quad (33)$$

$$\frac{\partial J}{\partial M_t} = -\lambda \pi_t \frac{\partial I_t}{\partial M_t} e^{-rt} + \mu e \frac{\partial I_t}{\partial M_t} = 0, \quad (34)$$

$$\frac{\partial J}{\partial TH_t} = -\lambda \pi_t \frac{\partial I_t}{\partial TH_t} e^{-rt} + \mu e \frac{\partial I_t}{\partial TH_t} = 0, \quad (35)$$

$$-\mu_t = \frac{\partial J}{\partial H_t} = \phi_t \left( E \left[ \frac{\partial u}{\partial h_t} \right] e^{-\theta t} + \lambda w_t (1 - \eta_t) e^{-rt} \right) + \mu e \left[ \delta_t + \frac{\partial \delta_t}{\partial H_t} + \frac{\partial \mu_t}{\partial H_t} \right]. \quad (36)$$

Eq. (34) implies

$$\mu_t = \lambda \pi_t e^{-rt}. \quad (37)$$

Since \( \lambda \) is constant, we obtain

$$\frac{\mu_t}{\mu_t} = \frac{\pi_t}{\pi_t} - r. \quad (38)$$

and so Eq. (36) can be rearranged as

$$-\mu_t = \phi_t \left( E \left[ \frac{\partial u}{\partial h_t} \right] e^{-\theta t} + \lambda w_t (1 - \eta_t) e^{-rt} \right) \frac{1}{\mu_t} + E \left[ \delta_t + \frac{\partial \delta_t}{\partial H_t} + \frac{\partial \mu_t}{\partial H_t} \right]. \quad (39)$$

Substituting Eqs. (37) and (38) into Eq. (39), the Euler equation, obtained under this alternative insurance, is given by

$$\phi_t \left( \frac{1}{\lambda} E \left[ \frac{\partial u}{\partial h_t} \right] e^{-\theta t} + w_t (1 - \eta_t) \right) = \left( r + E \left[ \delta_t + \frac{\partial \delta_t}{\partial H_t} + \frac{\partial \mu_t}{\partial H_t} \right] \right) \pi_t - \pi_t. \quad (40)$$

Obviously, under this alternative insurance, these first order conditions can be satisfied simultaneously. Hence, the existence of an optimal solution is assured.

Of course, the key point of this alternative insurance is that it does not create a distortion between time input \( TH_t \) and medical expenditure \( M_t \). It has an effect...
only through $H_t$. Needless to say, changes in $H_t$ would affect all control variables, including $TH$, and $M_t$, but its effects are along the optimal policy function for non-insurance model. Therefore, it does not distort the balance of inputs in both household production functions.

Unfortunately, this type of insurance has no foothold in the actual world. It suggests that insurers should insure against all time loss, including time loss in housekeeping due to illness. However, it is well known that making individuals express their real time loss is difficult due to the moral hazard based on asymmetric information. On the contrary, as it is shown, a more realistic insurance, which insures only medical care expenditure, does not have an optimal solution. Therefore, this may imply that an insurance with optimal solution is difficult to maintain in the real world.

5. Concluding remarks

This paper proves the non-existence of an optimal solution under the Liljas (1998) type of insurance and proposes an alternative insurance with an optimal solution. The reason for the non-existence of such an optimal solution is that it creates a distortion between time input and medical expenditure in the health investment function. As a result, it distorts the balance of inputs in both household production functions for health investment and consumption goods. Hence, we propose an alternative insurance that does not distort the balance of inputs for health investment. A more careful treatment of the Liljas model is significant for the development of the Grossman model.

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Appendix A

By introducing financial asset dynamics explicitly, the more familiar type of dynamic optimization problem can be formulated. Furthermore, through this formulation, it is easily shown that the condition where $\lambda$ should be constant over time is not essential in the Liljas (1998) model.
The individual utility maximization problem is reformulated as

$$\max \int_0^T E\left[u\left(\phi_i, H_t, Z_t\right)\right] e^{-\theta t} dt$$  \hspace{1cm} (a1)$$

$$\dot{H}_t = E\left[I_t - \delta_i(H_t, Q_t) H_t\right]$$  \hspace{1cm} (a2)$$

$$\dot{A}_t = E\left[r A_t + w_i \phi_i H_t - q_i Z(X_t, T_t) - \pi_i I(M_t, TH_t)\right]$$  \hspace{1cm} (a3)$$

where $A_t$ is a financial asset. Then, the Hamiltonian is formulated as

$$J = E\left[a(\phi_i, H_t, Z(X_t, T_t)) e^{-\theta t} + \tilde{\lambda}_i(r A_t + w_i \phi_i H_t - q_i Z(X_t, T_t)$$

$$- \pi_i I(M_t, TH_t)) + \mu_i(I(M_t, TH_t) - \delta_i(H_t, Q_t) H_t)\right]$$  \hspace{1cm} (a4)$$

where $\tilde{\lambda}_i$ is the costate variable on the transition equation for financial assets. Obviously, it expresses the individual budget constraint at each period. The first-order conditions are

$$\frac{\partial J}{\partial X_t} = E\left[\frac{\partial u}{\partial Z_t}\right] \frac{\partial Z_t}{\partial X_t} e^{-\theta t} - \tilde{\lambda}_t q_t \frac{\partial Z_t}{\partial X_t} = 0.$$  \hspace{1cm} (a5)$$

$$\frac{\partial J}{\partial T_t} = E\left[\frac{\partial u}{\partial Z_t}\right] \frac{\partial Z_t}{\partial T_t} e^{-\theta t} - \tilde{\lambda}_t q_t \frac{\partial Z_t}{\partial T_t} = 0.$$  \hspace{1cm} (a6)$$

$$\frac{\partial J}{\partial M_t} = -\tilde{\lambda}_t \pi_i \frac{\partial I_i}{\partial M_t} + \mu_i \frac{\partial I_i}{\partial M_t} = 0.$$  \hspace{1cm} (a7)$$

$$\frac{\partial J}{\partial TH_t} = -\tilde{\lambda}_t \pi_i \frac{\partial I_i}{\partial TH_t} + \mu_i \frac{\partial I_i}{\partial TH_t} = 0.$$  \hspace{1cm} (a8)$$

$$- \dot{\tilde{\lambda}}_i = \frac{\partial J}{\partial A_t} = \tilde{\lambda}_i r.$$  \hspace{1cm} (a9)$$

$$\mu_i = \frac{\partial J}{\partial H_t} = E\left[\frac{\partial u}{\partial h_i}\right] \phi_i e^{-\theta t} - \mu_i E\left[\frac{\partial \delta_i}{\partial H_t} + \delta_i\right] + \tilde{\lambda}_i w_i.$$  \hspace{1cm} (a10)$$

Eq. (a7) implies

$$\mu_i = \tilde{\lambda}_t \pi_i.$$  \hspace{1cm} (a11)$$

and we can obtain

$$\frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{\pi}_i}{\pi_i} = \frac{\dot{\tilde{\lambda}}_i}{\tilde{\lambda}_t}.$$  \hspace{1cm} (a12)$$
On the other hand, Eq. (a9) implies
\[ \frac{\dot{\lambda}_t}{\lambda_t} = -r, \]  
(a13)

and so
\[ \lambda_t = e^{-rt} \lambda_0 \]  
(a14)

and Eq. (a12) can be reexpressed by
\[ \frac{\dot{\mu}_t}{\mu_t} = \ddot{\pi}_t - r. \]  
(a15)

As from Eq. (a10)
\[ \frac{\dot{\mu}_t}{\mu_t} = -\frac{1}{\mu_t} E \left[ \frac{\partial u}{\partial h_t} \right] \phi_t e^{-\theta_t} + E \left[ \frac{\partial \delta_t}{\partial H_t} H_t + \delta_t \right] - \frac{\dot{\lambda}_t}{\lambda_t} w_t \phi_t, \]  
(a16)

substituting Eqs. (a11), (a12) and (a14) into Eq. (a16), we obtain
\[ \phi_t \left( \frac{1}{\lambda_0} E \left[ \frac{\partial u}{\partial h_t} \right] e^{\theta_t} + w_t \right) = \left[ r + E \left[ \delta_t + \frac{\partial \delta_t}{\partial H_t} H_t \right] \right] \pi_t - \ddot{\pi}_t. \]  
(a17)

Eq. (a17) is the same as Eq. (17), if \( \lambda \) is \( \dot{\lambda}_0 \). It implies that \( \lambda \) is just the marginal utility of initial wealth. Thus, it is constant by definition. Therefore, this condition is not essential in the model.

References


