Contracts for health care and asymmetric information

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Abstract

The paper presents a stylised model of contracting for a specific health service. The benefit of this service differs across patients. A purchaser, National Health Service (NHS, insurer) offers a contract to providers (hospitals, GP’s), under the constraint of limited information about the provider’s costs and the contract specifies the payment as a function of the number of cases treated. A number of features of the optimal contract are derived. Some of these are surprising: for example, the price per case generally increases with the efficiency of the provider. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The reforms in the UK National Health Service which began in the late 80’s have formally separated providers and purchasers of health services, and therefore created the need for formal contractual relationships between them (see the papers in Culley et al., 1990; Propper 1995 gives a preliminary assessment). In this paper, I apply the standard model of the modern theory of regulation (Baron and...
Myerson, 1982) to obtain a straightforward stylised analysis of these contractual relationships. This is useful, because it has implications which might, at first sight, appear to run counter to conventional wisdom. For example, one would naturally expect that a purchaser should pay a lower price, for a given service on a given patient, to a more efficient provider, viz., a hospital with a lower cost of providing that service. And yet, Proposition 3 below shows that, for a range of parameters, the price of health services is unambiguously higher in more efficient hospitals. This is due to the asymmetric information, and while the model used here is simple, the intuition underlying Proposition 3 is general, and is likely to hold in more realistic settings: efficiency dictates that low cost providers treat relatively more expensive patients; to the extent that providers have both a different objective function from purchasers and superior information about their own cost (or, more generally, about relevant parameters), they must be given an incentive to treat these more expensive cases. This incentive is created by a contract linking a higher price per case to a higher throughput. A high cost hospital would not want to pretend to be more efficient, because the increase in revenues that would be generated in this way is insufficient to compensate it for its higher cost of providing the service.

The plan of the paper is as follows: the model is presented in Section 2 and the main results in Section 3. Section 4 derives conditions under which the decision on the number of cases to be treated can be delegated to the provider through the offer of a menu of fixed payments plus a payment per case treated. Section 5 contains concluding remarks. The mathematical derivations of the results are assembled in Appendix A.

2. The model

I consider the provision of one specific and unambiguously defined service. The demand for the service is exogenously given and known to all parties; without the loss of generality, it can be normalised to 1. A health care purchaser (a Health Authority in the National Health Service (NHS)) offers a contract to a provider
Patients differ according to their ability to benefit from the treatment; some patients may benefit more than others, for example because they are younger and not affected by other pathologies, and hence they are likely to live longer. Specifically, I assume that each patient is characterised by a parameter $\beta \in \mathcal{B}$, which denotes her ability to benefit perhaps the increase in QALYs which the treatment determines. $\beta$ is distributed in the population according to a function $H(\beta)$, with $H'(\beta) = h(\beta)$ and $\lim_{\beta \to +\infty} H(\beta) = 0$, $\lim_{\beta \to -\infty} H(\beta) = 1$.

The cost of treatment also varies, depending on two parameters, $\beta$ and $\gamma$. The intrinsic efficiency of the provider, and reflects the hypothesis that efficiency varies across hospitals: different hospitals incur a different cost to treat the same patient. I assume that cost is decreasing in $\beta$: patients who benefit most from the treatment are also the cheapest to treat. In the realistic simplification where $\beta$ measures the life expectancy, this implies that younger patients (who are expected to survive longer) are cheaper to treat than older ones. Formally, the provider’s monetary cost of treating a given patient is $c(\beta, \gamma)$, satisfying: $c_{\beta}(\beta, \gamma) < 0$, $c_{\gamma}(\beta, \gamma) > 0$, $c_{\beta\gamma}(\beta, \gamma) \geq 0$, where subscripts denote partial derivatives.

$\gamma \in [\gamma, \bar{\gamma}]$ is distributed according to a differentiable function $G(\gamma)$, with $G(\gamma) = 0$, $G(\bar{\gamma}) = 1$, $\gamma < \gamma$, $G'(\gamma) = g(\gamma)$, and $[(d/d\gamma) G(\gamma)/g(\gamma)] > 0$ (the standard assumption of a monotonic hazard rate). Moreover, $\beta$ and $\gamma$ are independently distributed: the distribution of the potential benefit in the population does not depend on the hospital’s intrinsic efficiency, which seems natural.

I further assume that hospitals can dump patients (Ma, 1994, pp. 103–108): before choosing to treat a patient, the hospital observes her specific value of $\beta$, and can then decide to turn her away. If a patient is not treated during the contractual period, then she does not go back to the same hospital in subsequent contractual periods. This appears to happen in the NHS (see Lenaghan, 1996 for a detailed description of the way treatment is rationed by the British Health Authorities). From a theoretical point of view, refusing treatment to some patients must inevitably follow from the fact that technology in health care is far more advanced than the taxpayer’s willingness to pay for it. The assumption that if a

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3 Factors affecting the efficiency of providers could be the availability of qualified nurses in the areas where the hospital is located, the intrinsic ability of the consultants, the organisational skill of the management, and so on.

4 Including some measure of “fitness” would strengthen the case: fitter patients are likely to recover more fully (benefiting more) and more quickly (occupying a hospital bed for a shorter period). One can always think of situations where the opposite holds: for example, when the treatment relieves pain or discomfort, the patients who benefit less are those who are mildly affected, and hence, those who can probably be treated more cheaply.
patient is refused treatment by a hospital, then she is not treated by the purchaser simplifies the analysis, and could be relaxed by further research.

The purchaser’s objective is the maximisation of a utility function given by:

\[ V(T, x(\beta)) = \int_{-\infty}^{+\infty} x(\beta) v(\beta) h(\beta) d\beta - (1 + \lambda)T + \alpha U(\gamma) \quad (1) \]

where:

- \( x(\beta) \in [0,1] \) is the proportion of the patients whose benefit is \( \beta \) who receive the treatment.
- \( v(\beta) \) is the social benefit of treating a patient with benefit \( \beta \). \( v \) satisfies \( v'(\beta) > 0 \), but, in general, it need not satisfy \( v(\beta) = \beta \), which would imply that the social benefit equals the individual benefit. Thus, the first term on the RHS of Eq. (1) is the total social benefit of the service.
- \( T \) is the total contractual payment made to the provider.
- \( \lambda \geq 0 \) is the shadow cost of public funds. It reflects the fact that raising funds to pay for public services free at the point of consumption involves a distortionary and administrative cost, so that the overall cost of raising £1 is £(1 + \lambda). This should be determined endogenously as the multiplier of the purchaser’s overall budget constraint (imposed by the ultimate authority, e.g., the Department of Health). However, if, as seems realistic, the contract considered covers a sufficiently small proportion of the purchaser’s overall budget, then \( \lambda \) can be treated as approximately exogenously given for a single contract. 5
- \( U(\gamma) \) is the utility of a provider whose cost parameter is \( \gamma \). This is weighted by \( \alpha \in [0,1] \): if \( \alpha = 0 \), the extreme situation is obtained where the purchaser does not include the provider’s utility into its objective function. \( \alpha = 1 \), is the standard approach in regulation and procurement where the purchaser of public services maximises the sum of the utilities of all the participants in the industry, including the provider (Laffont and Tirole, 1993). With regard to the utility function of the provider, I assume hospitals to be profit maximisers. This is a useful analytical device, which is well understood in the health economics literature (Dranove and White, 1994, pp. 198ff).

Eq. (1) implies that the effect on the utility of the purchaser of the patients not treated in the hospital (whether because they receive no treatment or because they are accommodated privately) is independent of their value of \( \beta \).

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5 This also implies that if the purchaser buys services for a given treatment from several providers (e.g., hospitals located in different cities within the purchaser’s administrative region), each contract is written independently of all others. In other words, it is not possible to transfer patients from one hospital to another. Further extensions of the model will obviously need to relax it. The paper by Gal-Or (1994) suggests an elegant avenue for the analysis of such situations.
3. The optimal contract

The purchaser offers the provider a schedule \( T(n) \), specifying a total payment, \( T \), depending on the number of cases treated, \( n \), and allows the hospital to choose a point on this schedule. To find the optimal contract, I use the revelation principle. This says that the optimal contract is the same as that which would result if the purchaser offered the provider a pair of schedules \( (T, \beta^*(\gamma)) \), giving the relationship between \( \gamma \) and the total payment \( T \) and the cut-off point \( \beta^* \), which satisfy the truth-telling (or incentive compatibility) constraint: the hospital prefers the combination \( T(\gamma), \beta^*(\gamma) \), which corresponds to its true value of \( \gamma \), to any combination \( T(\hat{\gamma}), \beta^*(\hat{\gamma}) \) with \( \hat{\gamma} \neq \gamma \). The first step is therefore the determination of the hospital’s truth-telling constraint. This is done in the following proposition.

**Proposition 1.** The hospital’s truth-telling constraint is given by:

\[
\hat{U}(\gamma) = -\int_{\beta^*(\gamma)}^{\infty} c_s(\beta, \gamma) h(\beta) d\beta \quad \text{and} \quad \beta^*(\gamma) > 0
\]  

The proofs of all the results are confined to Appendix A. I use the standard notation \( \hat{U}(\gamma) = dU(\gamma)/d\gamma \). Note that \( \hat{U}(\gamma) < 0 \), that is, more efficient hospitals

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Proof: Suppose not, that is, suppose that given \( \beta_1 < \beta_2 \), patients with \( \beta \in [\beta_1, \beta_1 + \epsilon_1) \) are treated, and patients with \( \beta \in [\beta_2, \beta_2 + \epsilon_2) \) are not treated, for some “small” \( \epsilon_1 \) and \( \epsilon_2 \) satisfying \( \beta_1 + \epsilon_1 < \beta^* \) and \( \beta_2 + \epsilon_2 > \beta^* \). By treating the latter and omitting to treat the former, the hospital could increase its profit by reducing its cost without modifying the total number of patients treated. Both the provider and the purchaser are strictly better off as a consequence. All statements in the paper should be qualified by noting that they hold except at most in a set of measure zero: in what follows I leave this qualification implicit.
are better off. A further constraint which the purchaser must satisfy is the participation constraint, ensuring that under no circumstances the hospital refuses to sign the contract: \( U(\gamma) \geq 0 \) for all \( \gamma \in [\underline{\gamma}, \overline{\gamma}] \). In view of the fact that \( U(\gamma) \) is decreasing and that rent is costly, the participation constraint simplifies to:

\[
U(\overline{\gamma}) = 0
\]  

The purchaser’s payoff can be written as:

\[
\int_\gamma^\overline{\gamma} \left[ -(1 + \lambda) T(\gamma) + \int_{\beta^*(\gamma)}^\infty \nu(\beta) h(\beta) d\beta + \alpha U(\gamma) \right] g(\gamma) d\gamma.
\]

It is convenient to use the definition of \( U(\gamma) \) to eliminate \( T(\gamma) \); an optimal control problem is thus obtained, whose solution can be found with the standard techniques and is given in Proposition 2.

\[
\max_{U(\gamma), \beta^*(\gamma)} \int_\gamma^\overline{\gamma} \left[ -(1 + \alpha - \lambda) U(\gamma) \right.
\]

\[
+ \int_{\beta^*(\gamma)}^\infty \left[ \nu(\beta) - (1 + \lambda) c(\beta, \gamma) h(\beta) d\beta \right] g(\gamma) d\gamma
\]

s.t.: (2) and (3).

**Proposition 2.** Let

\[
c_{\beta, \gamma}(\beta, \gamma) < \left[ (g(\gamma))/(G(\gamma)) \right]
\]

\[
\times \left[ (\nu(\beta) - (1 + \lambda) c_{\beta}(\beta, \gamma))/(1 + \lambda - \alpha) \right],
\]

for every \( \gamma \in [\underline{\gamma}, \overline{\gamma}] \), \( \beta \in [\beta, \overline{\beta}] \). The optimal contract offered by the purchaser in conditions of asymmetry of information satisfies:

\[
\nu(\beta^*(\gamma)) - (1 + \lambda) c(\beta^*(\gamma), \gamma) = (1 + \gamma - \alpha) \frac{G(\gamma)}{g(\gamma)} c_{\beta}(\beta^*(\gamma), \gamma)
\]

\[
T(\gamma) = \int_{\beta^*(\gamma)}^\infty c(\beta, \gamma) h(\beta) d\beta + \int_{\beta^*(\gamma)}^\overline{\beta} c(\beta, \gamma^*(\beta)) h(\beta) d\beta
\]

where \( \gamma^*(\beta) \) is the inverse function of \( \beta^*(\gamma) \).

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I study the optimal regulation, given the (informational) constraints faced by the purchaser: thus the present model differs from Fenn et al.’s (1994) analysis of contracts for the NHS, in that they consider two possible regimes of price regulation in symmetric information.
The condition in the statement of Proposition 2 ensures that the cost of treating a patient with a slightly higher ability to benefit, \( c_\beta(.) \), does not increase too rapidly as the provider becomes less efficient, \( \gamma \) increases. It is satisfied if \( c_\beta(\beta, \gamma) \leq 0 \), and for every value of \( c_\beta(.) \) for efficient providers (because for \( \gamma \) close to \( \gamma \), \( G(\gamma) \) is close to 0). If it is violated, then the first order conditions do not identify a local maximum.

In the case of symmetric information, Eq. (4) reduces to \( v(\beta^s(\gamma) = (1 + \lambda)\epsilon(\beta^s(\gamma), \gamma) \) for every \( \gamma \in [\gamma, \bar{\gamma}] \), where \( \beta^s(\gamma) \) is the optimal cut-off point with symmetric information: marginal social benefit must equal marginal cost (including the distortionary cost of taxation). The term on the RHS of Eq. (4) is therefore the cost of asymmetric information, and it is zero when \( \gamma = \gamma \) (efficiency at the top), and when \( \alpha = 1 + \lambda \), that is when transfers to the provider have no utility cost. In all other cases, the purchaser’s attempt to reduce the costly transfer to the provider entails a reduction in the number of patients treated. Specifically, since \( \beta^s(\gamma) \) is an increasing function (from Eq. (2)), more patients are treated in an efficient hospital: put differently, a patient with low ability to benefit may be refused treatment by a high cost hospital, when a low cost hospital would treat her.\(^8\) While this is also the case with symmetric information, Corollary 1 shows that the asymmetry of information exacerbates this effect.

Corollary 1. \( \beta^*(\gamma) = \beta^s(\gamma) \) and \( \beta^*(\gamma) > \beta^s(\gamma) \) for all \( \gamma \in (\gamma, \bar{\gamma}) \).

That is, the cut-off point is the same under symmetric and asymmetric information for most of the efficient hospital. In all other cases, asymmetry of information entails a reduction in the number of patients treated.

Consider now the second statement in Proposition 2. With symmetric information, the hospital would just break even, simply receiving a transfer equal to its cost: \( T^s = \int_{\beta^s(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta \). With asymmetry of information, the payment to the hospital is made up of two parts. A first part, independent of \( \gamma \), is a lump sum equal to the total cost of the least efficient hospital. The second part is an incentive payment in the following sense: the hospital is offered a payment for treating patients in addition to the number treated by the least efficient hospital; each of these additional patients carries a payment equal to the cost that would be incurred if this patient were treated at the hospital where she would be the last (most serious) patient to be treated. Because the cost of treating inframarginal patients is below this ‘valuation’, an efficient hospital is allowed to keep the cost savings that result from its superior efficiency.

Corollary 2 studies the relationship between the provider’s efficiency and its total budget.

\(^8\) This is a hotly debated point in the NHS, known with the catchy label of “postcode rationing”.
Corollary 2. \( \check{T}(\gamma) = -\beta^* (\gamma) c(\beta^*(\gamma), \gamma) h(\beta^*(\gamma)) < 0 \).

That is efficient hospitals always receive more money than higher cost ones (in the case of symmetric information an increase in \( \gamma \) has an ambiguous effect on the provider’s total budget). This would seem surprising. It is however due to the fact that they are required to treat more patients and that they receive more ‘‘profit’’; these two effects outweigh the fact that hospitals with lower cost can receive less money to treat a given number of patients and still break even. Although \( T \) could also be a decreasing function with symmetry of information, Corollary 3 shows that in the presence of asymmetric information, \( T \) is on average steeper.

Corollary 3. Let \( T^g(\gamma) \) denote the payment schedule in the symmetric information case. Then: \( T(\gamma) > T^g(\gamma) \) and \( T(\gamma) < T^g(\bar{\gamma}) \).

That is, the budget of the most efficient hospital is always increased by asymmetric information, the budget of the least efficient hospital always decreased. Thus an effect of the asymmetry of information is an increase in the variability of the budget allocation across different hospitals with respect to the allocation which would result in the conditions of symmetry of information. Note also that, since the most efficient provider receives more cash in conditions of asymmetric information, but treats the same number of patients, it follows that asymmetric information increases the cash per patient received by efficient hospitals.

4. Implementation of the optimal contract

I now study the relationship between the total payment received and the number of cases treated. This is important in view of the importance attributed by politicians and popular opinion to the issue of prices for health services. I focus on two concepts: the ‘‘average’’ price, and the ‘‘marginal price’’. The average price, \( t(\gamma) \), is simply the total budget divided by the number of cases treated, \( t(\gamma) = [T(\gamma)]/(1 - H(\beta^*(\gamma))) \). Its relationship to \( \gamma \) is described in Proposition 3.

Proposition 3.

\[
\frac{dt(\gamma)}{d\gamma} = \frac{\check{\beta}^*(\gamma) h(\beta^*(\gamma))}{[H(\beta^*(\gamma))]^2} \left[ \int_{\beta^*(\gamma)}^{\infty} e_\beta(\beta, \gamma) [1 - H(\beta)] d\beta + U(\gamma) \right].
\]

Therefore there exists \( \gamma_0 \in [\underline{\gamma}, \bar{\gamma}] \), such that \( t(\gamma) \) is decreasing for \( \gamma \geq \gamma_0 \).

In words, Proposition 3 says that, at least for \( \gamma \) high, the budget per patient treated, that is the price per treatment, is increased as the hospital becomes more...
efficient. Surprising as it may appear, the finding of a positive relationship between the price charged and the efficiency of the hospital has however a natural explanation: efficient hospitals treat more patients, and therefore they treat more expensive cases. This is socially efficient, because they are better hospitals, but, of course, is more costly, and the hospital must be given an incentive to do so.

Another important concept is the marginal price. This is the change in the monetary reward that would be received by a hospital if it chose to treat a further patient in addition to those it already treats (see Wilson, 1993, p. 48). Because the price schedule is non-linear, it generally differs from the average price. The marginal price is given by the slope of the function, say \( \tau(n) \), which relates the total payment received by the provider to the number of cases treated: \( \tau(n) = T(\gamma^*(B(n))) \), where \( \gamma^*(\beta) \) is defined above as the inverse of \( \beta^*(\gamma) \), and \( B(n) \) is the inverse of \( [1 - H(\beta)] \); in words, \( B(n) \) is the cut-off ability to benefit when \( n \) patients are treated. Propositions 4 and 5 illustrate the shape of this relationship.

**Proposition 4.** \( \tau(n) \) is increasing.

That is, hospitals that treat more patients are allocated a greater budget. This implies that it is not necessary to specify both the number of patients to be treated and the total payment as in block contracts, but it is possible to implement the contract by conditioning the payment to the hospital on the number of patients it chooses to treat. Even though contracts may be made to depend on the hospital total cost, there is no benefit in doing so.

**Proposition 5.**

Let

\[
\frac{d}{d\gamma} \left( \frac{G(\gamma)}{c_{B(\beta,\gamma)}} \right)^2 \geq \frac{c_{B(\beta,\gamma)}}{c_{\gamma}(\beta,\gamma)},
\]

then \( \tau(n) \) is concave.

The condition given in Proposition 5 has a very natural interpretation, as it implies that the cost of the most expensive patient increases as the exogenous cost parameter increases. Suppose a hospital experiences a small increase in \( \gamma \). Its costs are now higher, but it will now be required not to treat its most expensive patients. Having fewer patients implies that the hospital has a lower total cost, but, if the condition in Proposition 5 holds, this reduction in cost is not sufficient to offset the exogenous increase in the cost. Very loosely put, it says that cost savings from improved efficiency are more important than cost savings due to reduced throughput.

Propositions 3–5 can be used to determine the shape of the relationship between the total budget and the number of patients treated. This is illustrated in
Fig. 1. Total payment as a function of the number of patients treated. (A) Price always increases as cost decreases; (B) price first increases then decreases as cost decreases.

Fig. 1. Panel A is the case in which $\gamma_0 = \gamma$, panel B, the case where $\gamma_0 > \gamma$. The slope of the thin line from the origin to the curve $\tau(n)$ is the average price and the slope of the tangent to the curve is the marginal price. The marginal price is always decreasing as the efficiency increases. The average price, on the other hand, increases as $\gamma$ is reduced: all the way in Panel A, up to $\gamma_0$ in Panel B.

When the condition in Proposition 5 holds, $\tau(n)$ is concave. This implies that it is possible to offer the hospital a menu of contracts, each giving the price per case (given by the slope of $\tau(n)$), plus a lump sum payment (given by the intercept of the tangent with the vertical axis), and allowing them to choose among these, thus delegating also the decision on the number of patients to be treated to the hospital. This is exactly analogous to Laffont and Tirole (1986).

5. Concluding remarks

The paper considers a stylised market for the provision of a specialist health service. Unlike other markets such as those for standard generic drugs, or generic cleaning or catering services, in these markets there are typically few agents on each side. This implies that it will be governed by complex individual contractual relationship between each purchaser and each provider. Any price list made public is likely to be considered by the parties just a starting point for negotiation (Dawson, 1994). A sound theoretical understanding of the implications for health services of the established theoretical literature on optimal contracts, and in particular of the role of asymmetric information, is therefore important.
My results can be used to interpret the following scenario: in a sample of hospitals, data on caseloads, budgets, and “prices” for a given service are collected. This data will show differences among the hospitals (for one such example, see Propper, 1996): how can these differences be interpreted? The analysis of the paper suggests that some of the immediate interpretations one would give are not appropriate. In fact, if the providers are offered the optimal contracts, the relationship between exogenous efficiency and observable variables can be surprising. In my stylised model, low cost hospital have a high case load, which is intuitive, and are allocated a higher budget, which is also intuitive. But the implied price per case is higher than in high cost hospitals (possibly at least up to a point, see Fig. 1B). This is counterintuitive, but does have a natural interpretation, as discussed in Section 4. This variable, the price per case, has received much attention; for example, the NHS guidelines require the price to be based on cost. It has been argued (e.g., Dawson, 1994), that this requirement is probably so vague as to be meaningless. The paper reflects this view: both because low cost hospitals treat more expensive patients, with higher $\beta$, and because it can be increased in ways that are not easily observed, cost should be treated as endogenous, and cost and price determined simultaneously, together with the number of patients treated.

The aim of the paper is to draw attention to the role of the provider’s informational advantage, therefore it should be seen as a building block for a richer analysis which includes other facets of the provision of health services, for example, issues relating to the quality of care, the possibility of competition among providers, and the potential for conflict between agents within providers (e.g., between consultants and managers).

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Appendix A

Proof of Proposition 1. Let $U(\gamma)$ devote the hospital’s utility if it truthfully reports cost $\gamma$.

$$
U(\gamma) = T(\gamma) - \int_{\beta^*(\gamma)}^{\infty} c(\beta,\gamma)h(\beta)d\beta
$$

(A1)
Let \( \varphi(\gamma, \tilde{\gamma}) \) be the hospital’s utility when it has cost \( \gamma \) but makes a false report \( \tilde{\gamma} \). \( \varphi(\gamma, \tilde{\gamma}) = T(\tilde{\gamma}) - \int_{\beta^{-1}(\tilde{\gamma})}^{\infty} c(\beta, \gamma) h(\beta) d\beta \). The first order condition for truth-telling requires that \( \left[ \frac{\partial \varphi}{\partial \tilde{\gamma}} \right]_{\tilde{\gamma} = \gamma} = 0 \) for all \( \gamma \in [\gamma, \tilde{\gamma}] \), that is:

\[
\frac{d}{d\gamma} \left[ T(\gamma) + \beta^*(\gamma) c(\beta^*(\gamma), \gamma) h(\beta^*(\gamma)) \right] = 0
\]

(A2)

Differentiate Eq. (A1) to get

\[
\dot{U}(\gamma) = \frac{d}{d\gamma} \left[ T(\gamma) + \beta^*(\gamma) c(\beta^*(\gamma), \gamma) h(\beta^*(\gamma)) \right] - \int_{\beta^{-1}(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta
\]

Using Eq. (A2), the first statement is obtained.

Following Laffont and Tirole (1993, p. 63), I derive the following condition for truth-telling. Let \( f(\beta, \gamma) = \int_{\beta^{-1}(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta \). Incentive compatibility implies that for any pair \( \gamma_1 \) and \( \gamma_2 \), the following must hold:

\[
T(\gamma_1) - \phi(\beta^*(\gamma_1), \gamma_1) \geq T(\gamma_2) - \phi(\beta^*(\gamma_2), \gamma_2)
\]

Adding up and integrating:

\[
\int_{\gamma_1}^{\gamma_2} \beta^*(\gamma) f_{12}(x) \, dx \, dy \geq 0
\]

Now, \( f_{12}(x, y) = -c(\gamma, \gamma) h(x) < 0 \), and hence \( \beta^*(\gamma) \) must be non-decreasing. It can also be shown (see again Laffont and Tirole, 1993, p. 121) that if \( \beta^*(\gamma) \) is non-decreasing, then the first order condition is sufficient for optimality.

**Proof of Proposition 2.** The Hamiltonian of the problem is:

\[
\mathcal{H} = \left\{ -(1 + \lambda - \alpha) U(\gamma) - \int_{\beta^{-1}(\gamma)}^{\infty} \left[ (1 + \lambda) c(\beta, \gamma) - \nu(\beta) \right] h(\beta) d\beta \right\}
\]

\[
\times g(\gamma) - \mu(\gamma) \int_{\beta^{-1}(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta
\]

From which \( \mu(\gamma) = (1 + \lambda - \alpha) G(\gamma) \), and Eq. (4) is obtained. Totally differentiate Eq. (4):

\[
\frac{d\beta^*}{d\gamma} = \left[ \frac{1 + \lambda}{1 + \lambda - \alpha} c'_{\beta}(\cdot) + \frac{\nu'(\beta)}{1 + \lambda - \alpha} + \frac{G(\gamma)}{g(\gamma)} c_{\gamma}(\cdot) \right] \frac{d\gamma}{d\beta}
\]

\[
= \left[ \frac{1 + \lambda}{1 + \lambda - \alpha} c_{\beta}(\cdot) + \frac{G(\gamma)}{g(\gamma)} c_{\gamma}(\cdot) \right] c_{\beta}(\cdot) + \frac{G(\gamma)}{g(\gamma)} c_{\beta}(\cdot) c_{\gamma}(\cdot)
\]

\[
\frac{d\gamma}{d\beta} = \left[ \frac{1 + \lambda}{1 + \lambda - \alpha} c_{\beta}(\cdot) + \frac{\nu'(\beta)}{1 + \lambda - \alpha} + \frac{G(\gamma)}{g(\gamma)} c_{\gamma}(\cdot) \right] c_{\beta}(\cdot) + \frac{G(\gamma)}{g(\gamma)} c_{\beta}(\cdot) c_{\gamma}(\cdot)
\]
c_{\gamma s}(\cdot) \geq 0$ and, if the condition in the statement is satisfied, then $\beta^*(\gamma) > 0$ and the second order condition for truth-telling is satisfied. To derive Eq. (5), note that

$$T(\gamma) = U(\gamma) + \int_{\beta^*(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta$$

(A3)

with

$$U(\gamma) = \int_{\beta^*(\gamma)}^{\infty} c_{s}(\beta, \tilde{\gamma}) h(\beta) d\beta d\tilde{\gamma}$$

(A4)

Developing Eq. (A4):

$$U(\gamma) = \int_{\beta^*(\gamma)}^{\infty} c_{s}(\beta, \tilde{\gamma}) h(\beta) d\beta d\tilde{\gamma} + \int_{\beta^*(\tilde{\gamma})}^{\infty} c_{s}(\beta, \tilde{\gamma}) h(\beta) d\beta d\tilde{\gamma}

= \int_{\beta^*(\gamma)}^{\infty} c_{s}(\beta, \tilde{\gamma}) h(\beta) d\beta d\tilde{\gamma}

+ \int_{\beta^*(\tilde{\gamma})}^{\infty} c_{s}(\beta, \tilde{\gamma}) h(\beta) d\beta d\tilde{\gamma}$$

which gives Eq. (5), from Eq. (A3).

**Proof of Corollary 1.** This follows immediately from Eq. (4), noting that the second term on the LHS is positive, except at $\gamma = \gamma^*$, and is zero in symmetric information.

**Proof of Corollary 2.** From Eq. (A3), above $T(\gamma) = U(\gamma) - \beta^*(\gamma) c(\beta^*(\gamma), \gamma) h(\beta^*(\gamma)) + \int_{\beta^*(\gamma)}^{\infty} c_{s}(\beta, \gamma) h(\beta) d\beta d\beta$, and the Corollary follows from Eq. (2).

**Proof of Corollary 3.** It is $\beta^*(\gamma) = \beta^s(\gamma)$ and $U(\gamma) > 0$, hence $T(\gamma) = U(\gamma) + \int_{\beta^*(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta > \int_{\beta^*(\gamma)}^{\infty} c(\beta, \gamma) h(\beta) d\beta = T^s(\gamma)$. From Corollary 1, $\beta^*(\tilde{\gamma}) > \beta^s(\tilde{\gamma})$, thus $T(\tilde{\gamma}) = \int_{\beta^*(\tilde{\gamma})}^{\infty} c(\beta, \gamma) h(\beta) d\beta < \int_{\beta^*(\tilde{\gamma})}^{\infty} c(\beta, \gamma) h(\beta) d\beta = T^s(\tilde{\gamma})$. 

.
Proof of Proposition 3. Expand $\dot{i}(\gamma)$:

$$\frac{dt(\gamma)}{d\gamma} = \frac{1}{[1 - H(\beta^*(\gamma))]^2} \left[ \dot{T}(\gamma)[1 - H(\beta^*(\gamma))] ight]$$

$$+ h(\beta^*(\gamma)) \dot{\beta}^*(\gamma) T(\gamma)$$

$$= \frac{h(\beta^*(\gamma)) \dot{\beta}^*(\gamma)}{[1 - H(\beta^*(\gamma))]^2} \left[ -[1 - H(\beta^*(\gamma))] c(\beta^*(\gamma), \gamma) \right]$$

$$+ \int_{\beta^*(\gamma)}^\infty c(\beta, \gamma) h(\beta) d\beta + U(\gamma)$$

Integrate by part the first two terms in the square bracket, to obtain the Proposition.

Proof of Proposition 4.

$$\tau'(n) = \dot{T}(\gamma^*(B(n))) \dot{\gamma}^*(B(n)) B'(n)$$

$$= \frac{T(\gamma^*(B(n)))}{\beta^*(\gamma^*(B(n)))} \frac{1}{h(\beta^*(\gamma^*(B(n))))} \frac{-1}{\beta^*(\gamma^*(B(n)))}.$$ 

Now use Corollary 2 to substitute $\dot{T}(\gamma^*(B(n)))$ and obtain $\tau'(n) = c(B(n), \gamma^*(B(n))) > 0$.

Proof of Proposition 5. Use $\tau'(n) = c(B(n), \gamma^*(B(n)))$ to obtain: $\tau''(n) = \left[c_{\beta}(n) + c_{\gamma}(n)\gamma^{\prime\prime}(n)\right] B'(n) = [c_{\beta}(n) \beta^*(n) + c_{\gamma}(n)] B(n)/\dot{\beta}^*(n)$. Now $B'(n) > 0$, so the sign of $\tau''(n)$ is given by the sign of $c_{\beta}(n) \beta^*(n) + c_{\gamma}(n)$. Expand this term to obtain:

$$c_{\beta}(n) \dot{\beta}^*(n) + c_{\gamma}(n) = \frac{1}{A} \left\{ c_{\beta}(n) c_{\gamma}(n) \frac{d}{d\gamma} \left( \frac{G(\gamma)}{g(\gamma)} \right) + \frac{G(\gamma)}{g(\gamma)} c_{\gamma}(n) c_{\beta}(n) \right.$$

$$+ \frac{u'(\beta^*(\gamma)) c_{\gamma}(n)}{(1 + \lambda)} - \frac{G(\gamma)}{g(\gamma)} c_{\beta}(n) c_{\gamma}(n) \right\}$$

where $A = -c_{\beta}(n) + [u'(\beta^*(\gamma))/(1 + \lambda)] - [G(\gamma)/g(\gamma)] c_{\beta}(n) > 0$. The above can be written as:

$$\frac{1}{A} \left\{ \left[ c_{\beta}(n) \right]^2 \frac{d}{d\gamma} \left( \frac{G(\gamma)}{g(\gamma)} c_{\gamma}(\beta, \gamma) \right) + \frac{u'(\beta^*(\gamma)) c_{\gamma}(n)}{1 + \lambda} \right\}$$

and the Proposition follows.
References