Information Technology and Multiple Representations: new opportunities – new problems

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ABSTRACT Computer environments that employ multiple representations have become commonplace in the classroom. This article reviews the arguments and evidence for the benefits of such software and describe what the associated learning demands are likely to be. By describing the results of two evaluation studies in primary mathematics, the authors show that children as young as six can, in the right circumstances, benefit from multi-representational software. The authors discuss the features of the learning environments that influenced performance and consider how teachers could support learning with these types of environments.

Multiple Representations in the Classroom

Classroom teaching has always employed multiple external representations (MERs) in the pursuit of learning. Teachers use MERs explicitly in order to make abstract situations more concrete. For example, children are given a percentage such as 33% alongside a drawing of a pie chart with one third shaded. They also use MERs more implicitly when a diagram is presented with accompanying text. Teachers also set questions which require children to translate from one representation to another, e.g. algebra word problems.

In addition, software that employs MERs has become increasingly available in the classroom. For example, geometry packages such as Geometry Inventor (LOGAL/ Tangible Math) allow tables and graphs to be dynamically linked to geometrical figures. New multi-media technologies abound with MERs including video and spoken text. Traditional classroom uses of MERs, such as using an equation to produce a table of values which can then be plotted as a graph, have been altered by the introduction of graphical calculators.

We will review the claims made for learning with MERs, specifically in relation to computer based learning, and discuss their associated costs for the learner. Two multi-representational learning environments will be presented and we shall discuss how they have been evaluated in order to examine under what conditions MERs aid learning. Finally, we shall briefly examine the implications of the results for classroom teaching.

Benefits of Multiple Representations

Recently, many claims have been made for the advantages of MERs for learning and problem solving. These can be divided into three broad claims:
that MERs support different ideas and processes;
that MERs constrain interpretations and;
that MERs promote a deeper understanding of the domain.

We shall briefly review the arguments and evidence for each of these claims.

**Different Processes**

One of the main advantages proposed for the use of MERs is that by using combinations of representations, we can exploit their different properties to aid learning. Larkin & Simon (1987) propose that diagrams which contain the same information as equivalent written descriptions will still differ in their computational properties. For example, diagrams exploit perceptual processes, by grouping together relevant information, and hence make processes such as search and recognition easier. Further research has shown that common mathematical representations differ in their inferential power (e.g. Cox & Brna, 1995; Kaput, 1989). For example, tables tend to make information explicit, emphasise empty cells (thus directing attention to unexplored alternatives), and highlight patterns and regularity. The quantitative relationship that is compactly expressed by the equation $y=x^2+5$ fails to make explicit the variation which is very evident in an equivalent graph. Hence, by combining representations with different computational properties, we are no longer limited by the strengths and weaknesses of one particular representation.

**Constraints**

MERs are beneficial because they can be used to clarify a situation by constraining learners’ interpretations. Here, a second representation may be provided to support interpretation of a more complicated or less familiar representation. For example, a child who believes that a flat line on a velocity-time graph represents a stationary object, or that negative gradient must entail a negative direction can re-examine these misconceptions when this information is presented within a simulation environment (e.g. the DM3 microworld, Hennessy et al, 1995).

**Deeper Understanding**

Kaput (1989) proposes that multiple linked representations may allow learners to perceive complex ideas in a new way and to apply them more effectively. By providing a rich source of representations of a domain, one can supply learners with opportunities to build references across these representations. Such knowledge can be used to expose underlying structure in the domain represented. On this view, mathematics knowledge can be characterised as the ability to construct and map across different representations.

**Costs of Multiple Representations**

The potential learning advantages of MERs do not come without associated costs. Learners are faced with three tasks when they work with MERs.
Firstly, they must learn to understand each representation. They must understand how a representation encodes and presents information (the ‘format’). In the case of a graph, this would be attributes such as lines, labels, and axes. They must also learn what the ‘operators’ are for a given representation. For a graph, operators to be learnt include how to find the gradients of lines, maxima and minima, intercepts, etc. At least initially, such learning demands will be great and will obviously increase with the number of representations employed.

Secondly, learners must come to understand the relation between the representation and the domain it is representing. To return to the graph example, children must learn when it is appropriate to examine the slope of a line, the height or the area under a line. For example, when attempting to read the velocity of an object from a distance-time graph, children often examine the height of a line, rather than the gradient. In addition, the operators of one representation are often used inappropriately to interpret a different representation. This results in common mistakes such as viewing a graph as a picture. A velocity-time graph of a cyclist travelling over a hill should be U shaped, yet children show a preference for a hill shaped curve (Kaput 1989).

The final learning demand, unique to multi-representational situations, is that when MERs are presented together, learners must come to understand how representations relate to each other. Without abstraction across representations, the invariances of the domain may remain hidden. Some multi-representational software has been designed expressly to teach such relations. For example, Green Globs (Dugdale, 1982) provides opportunities for learners to relate graphs to equations. A computer displays co-ordinate axes and 13 ‘Green globs’. Students must generate equations that hit as many of these points as possible. More often, however, educational software attempts to use translation across MERs to develop understanding of some aspect of a domain. Thus, students must learn the connections between representations. One example for the primary classroom is that of the Blocks World (Thompson, 1992) which combines Dienes blocks with numerical information. Users act in one notation (such as the blocks) and see the results of their actions in another (numbers).

Consequently, we can see that for users to benefit from multi-representational software, they must meet a number of very complicated learning demands. We will now describe two systems that employ MERs within primary mathematics and discuss studies of how the advantages of MERs can be obtained without being outweighed by the costs.

**Empirical Studies**

Recent approaches to mathematics instruction in the primary classroom emphasise mathematics as flexible, insightful problem-solving. This approach requires understanding that mathematics involves pattern seeking, experimentation, hypothesis testing and active seeking of solutions. This
contrasts strongly with children's beliefs about the nature of mathematics. For example, Baroody (1987) asserts that due to an overemphasis on 'the right answer', children commonly believe that all problems must have a correct answer, that there is only one correct way to solve a problem and that inexact answers or procedures (such as estimates) are undesirable. We have built two systems which address and challenge these beliefs. The first system we discuss examines the idea that there is only one correct way to solve a problem and the second system supports children's learning about inexact or approximate solutions.

Multiple Solutions to Mathematical Problems

As part of a suite of experiments (for more details see Ainsworth 1994), we have assessed a computer based learning environment called COPPERS. We have examined whether children can provide multiple solutions to coin problems. If the concerns of researchers such as Schoenfeld (1992) and Baroody (1987) apply to English primary school children, we would expect that such an apparently simple task will prove difficult. We have also examined whether providing children with MERs of their answers aids this understanding.

COPPERS aims to provide children with some understanding that there can be multiple correct solutions to single problems. Much of the inspiration for the system design comes from Lampert's work (e.g. Lampert 1986). COPPERS operates in the familiar and mathematically meaningful domain of money problems, see Figure 1. Users are posed problems such as 'what is 3 x 20p + 4 x 10p?' They answer these questions by pressing buttons on a 'coin calculator' whose buttons are representations of British coins (see the right side of Figure 1). The coin calculator is designed to provide a simple way of interacting with the system. It is both easy and fun to use and acts to reduce the burden of number facts. So to answer the above problem, a user may select '20p + 20p + 10p + 50p ' or '10p + 2p + 2p + 1p + 5p + 10p + 10p + 10p + 50p'. Either answer is equally acceptable as there is no notion of 'best' answer in the system. Users do not progress to new problems until they have produced a number of different solutions.

One use of MERs in COPPERS is to describe answers in complementary representations. After answering a question, users receive feedback on their solutions. This reveals whether an answer was correct and describes the elements of the solution in detail. This information is displayed in two ways and, by means of highlighting the key elements, the system encourages students to map between the different representations. The first representation is a standard row and column notation (see the right hand side of Figure 2). The user is reminded of how many of each type of coin they used. The second form of representation is a table representation similar to one used by Lampert (left hand side of Figure 2). The answers are displayed in columns. To see how the values in the table correspond to the
final total, the learner must multiply the number in each column with the
number of pence at the top and then add these products together.

Figure 1. COPPERS – a question and the ‘coin calculator’.

The two representations supply different types of information and require
different interpretations. The row and column representation is a familiar
one to children, used consistently once they start working with any
multi-digit sum. Users are likely to need less new knowledge in order to
understand this representation. The operations of multiplication and
addition needed to produce the total are made very explicit in the row and
column notation, making the arithmetical operations one of the most salient
aspects of the representation. In contrast, the summary table is less familiar
to the children and the arithmetical operations are implicit. To understand
and make use of the information, children must decide what processes are
involved and perform them for themselves, hence practising their
multiplication and addition skills. The table also displays previous answers
to the question. This allows children to compare their answers with those
already given and (hopefully) prompts pattern seeking and reflection.

An evaluation study was conducted with 40 six- to seven-year-old
children in a local primary school. We pre-tested their knowledge of multiple
solutions in coin problems with pen and paper tests of questions similar to
those posed by the computer, (see Figure 1 for an example). Children were
required to draw coins to make up the total value of the sums. We found
that their initial performance was, as predicted, surprisingly low for such an
apparently simple task. Although, there are literally hundreds of different
correct answers for these problems, children produced an average of only
2.66 correct answers across three questions, i.e. less than 1 correct answer
per problem. Hence, it would appear that primary school children do not easily produce multiple correct answers to such problems.

An intervention phase followed where children used COPPERS twice. Half the children used a version with only row and column representations and half with both the row and column and tabular representations. A further factor controlled whether children gave one or many answers per question. Hence in some conditions, the table displayed a trace of several answers and in some only one (see Ainsworth, 1994). We found that all children improved their scores significantly on a post-test, producing over four times as many correct solutions per question. However, children who had seen a table in addition to row and column representations produced significantly more, and more correct solutions than those who had not. Hence, children as young as six can learn more effectively to produce multiple solutions when using MERs in comparison to a single representation.

In addition to producing more correct solutions on a post-test, children were also beginning to use the table in a more creative and pattern seeking way. For example, children tried to use as few columns as possible or as many as possible; they might try to get high numbers in particular columns; make patterns across the columns, etc. For example, one subject noted his answer read like a palindrome across the table, ‘its the same backwards as forwards’ and tried to create another palindrome on his next go. Thus, using the table representation facilitated pattern seeking activity by displaying the results of the children’s problem solving in a way that supported the search for patterns.
Computational Estimation

Our research on COPPERS has demonstrated that appropriate combinations of representations lead to increased learning outcomes. In COPPERS the mappings between the representations were signalled by highlighting the salient elements. With the second system, CENTS, we have examined how different combinations of representations affect the translation between representations. The question is how the different learning demands of translation between representations lead to different learning outcomes.

Figure 3. CENTS – a completed problem with pictorial representations.

CENTS (see Figure 3) has been designed to help children learn the basic knowledge and skills required to successfully perform computational estimation (e.g. Reys et al, 1982; Sowder & Wheeler, 1989). The goals of the system are:

- to teach children strategies that they can use to estimate solutions to problems;
- to encourage children’s understanding of how transforming numbers affects subsequent accuracy and;
x to support the development of the required underlying conceptual knowledge.

An example of such conceptual knowledge is that there can be multiple correct estimates for a given problem and that estimates should involve approximate numbers. Children are taught a number of strategies for estimating (at least rounding and truncation) and are given information about the accuracy of their answers using MERs. Knowledge of how to transform numbers in order to achieve accurate results is very difficult for children (Case & Sowder 1990).

CENTS employs a hypothesis testing strategy. Users make predictions about the accuracy of an estimate, then produce an estimate and finally examine the results in the light of their predictions. They produce two different estimates for each problem. This process is supported at a number of stages. CENTS guides the learner’s selection of an appropriate intermediate solution, provides support for place value correction, and monitors number facts, providing help where appropriate. After each problem, children log the results of (at least) two different estimation strategies in an on-line work book. At the end of a session, children are encouraged to review the log book to investigate patterns in their estimates (such as truncation will always produce an underestimate, rounding is often the most accurate strategy).

The prediction and comparison stages are supported by MERs of estimation accuracy which are used for both action and display. We can manipulate how these representations express estimation accuracy using either pictorial representations (see Figure 3), mathematical representations such as numerical displays or histograms or mixtures of mathematical and pictorial representations. The research literature suggests a series of predictions concerning the effectiveness of each of these types of representations. For example, pictures may be beneficial initially and to lower performing children. Mathematical representations will take longer to learn, but will ultimately prove to be a more effective representation. Mixed representations may offer the best situation, in that the pictorial representations can be used to bridge understanding to the more symbolic ones. However, this can only occur if children see how the two representations relate to each other.

In a number of experiments, we examined how best to combine these representations. Measures were taken of children’s domain knowledge. It was reflected in their use of representations and also whether they understood the relation between the representations. For example, in Ainsworth, Wood and Bibby (1996) we examined pictorial, mathematical or mixed representations (plus one non-intervention control) with 48 ten- to eleven-year-old children in a local primary school. At pre-test, we measured the children’s ability to perform estimation and the insight they had into the accuracy of these estimates. We found that these skills were lacking in this
We found that all experimental children improved significantly at performing estimation, becoming more accurate and using more appropriate strategies and that the control group did not improve. However, the insight into the accuracy of estimates (the skill most directly supported by the representations) improved in only the pictures and mathematics groups.

We analysed how the children used the representations during their interactions with the computer in order to determine why these learning outcomes should have occurred. We proposed that the explanation rests in the learning demands of translating between the representations. If learners understand the relation between the representations their actions should be identical over both representations, even if their prediction is wrong with respect to the domain. Given that the pictorial and mathematical groups improved in the accuracy of their judgements and the mixed group did not, we predicted that the former groups would make the same actions on both of the representations and the mixed group would not. We found that over time, the behaviour of mathematical and pictures groups did converge across the representations and that the mixed group did not show this behaviour. We believe that the mixed group did not successfully map between the pictorial and mathematical representations and this failure in translation led directly to the reduced learning outcomes.

This evaluation showed that the learning demands of MERs affect how successfully they can be used and hence how they support learning. Each representation in the mixed condition was present in one of the other conditions, where it was used successfully. Hence, in this case, it was the learning costs associated with the building of relations across mixed representations which led to the poorer performance.

**Implications for the Classroom**

We have presented two computer-based mathematical learning environments designed to include MERs. Evaluation studies were conducted and have shown how MERs may be used to help children learn new skills and gain new knowledge. In COPPERS, MERs are used for displaying feedback about solutions and were associated with better performance. However, the system was carefully designed so that the new representation was supported by a familiar complementary representation. In CENTS, this issue was examined further by manipulating the learning demands of mapping between representations. When this cost was too high, children failed to learn the domain material.

Teachers have goals and plans when they enter the classroom. They know what they want their pupils to learn and they usually have clear ideas about how to achieve those learning outcomes. Software that supports such learning, however, is rarely designed with the teacher's specific goals in mind. The studies we have reported here have implications for teachers' decisions about how to use software that employs MERs to assist learning.
At the same time, our research has pointed to the need to monitor the different learning demands associated with MERs and their relationship to the different learning goals that teachers may set.

Let us reconsider the beneficial uses of MERs. First, MERs support different ideas and processes. Secondly, MERs can constrain interpretations and third MERs promote a deeper understanding of the domain. If the teacher’s aim is to exploit the different inferential properties of alternative representations to teach different aspects of domain, then knowing what each representation offers and how to use this representation to achieve the learning goal is essential. Many common packages allow teachers to choose which representation they would like to employ. Spreadsheets, for example, support tables of numerical data which can be displayed in a variety of graphical forms. Hence, in some lessons it may be appropriate to exploit tabular representations to emphasise order and patterns in numbers, in another, graphs may help to show the continuous nature of the phenomenon being examined.

With respect to the learning demands, when the goal involves exploiting different properties of representations then it is necessary to judge whether the learner is familiar with the format and operators of a representation and how these relate to the domain to be learnt.

If the aim of using MERs is to develop better understanding of a domain by constraining the interpretation of a new representation then the translation between representations should be made as transparent as possible. There are two ways in which this can be attained by teachers. First, a familiar representation can be provided alongside an unfamiliar representation. In this circumstance, the interpretation of the unfamiliar representation is limited by the interpretation of the familiar representation. The familiarity of the row and column representation in COPPERS constrained the possible interpretations of the unfamiliar table representation by indicating the appropriate format and operators for the table representation. The second method of controlling the interpretation of one representation is through the exploitation of constraints available in a second representation. In COPPERS coin problems, representations such as ‘5p, 10p, 5p, 10p’ and ‘5p, 5p 10p, 10p’ may appear very different to a young child if they do not understand commutativity. If presented alongside COPPERS table representation of coin values which does not express ordering information, their equivalence is more likely to be recognised.

When the teacher’s goal of MER use is to support abstraction, then the mapping of relations between the representations is crucial. If the representations are too similar then it is likely that abstraction over the representations will provide no additional understanding of the domain when compared with using a single representation. However, as we have demonstrated in CENTS, when the representations are too dissimilar then
abstraction does not occur. Establishing the appropriate level of similarity is an issue that requires further research.

When the teacher’s goals involve either using MERs to constrain the interpretation of a domain or to support abstraction then, then we need to ensure that learners can relate one representation to another. One question that follows is whether it is necessary to explicitly teach relations between the representations. Some multi-representational software such as Green Globs have this as a goal, and such teaching has been found to aid understanding of elementary mathematical ideas (e.g. Lesh, Behr, & Post, 1987). However, Resnick and Omanson (1987) have demonstrated that this does not invariably lead to deeper understanding. We believe that establishing when successful abstraction will occur depends upon the trade off between the benefits of using MERs to support abstraction and the cost of the learning demands.

Conclusions

The systems described in this paper were developed to challenge children’s focus within mathematical problems upon producing a single correct answer. Equally, we have found that to understand learning, one must look beyond answers. We should consider children’s understanding of representations, their relation to domains and to each other. In the classroom of today, teachers are increasingly being offered opportunities to use MERs in new and exciting ways. In this paper, we have argued that these opportunities are also fraught with new problems and that the learning demands of MERs can sometimes outweigh their benefits. Thus, teachers are faced with new challenges in choosing, developing and supporting these fresh approaches to learning.

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