Wage bargaining and turnover costs with heterogeneous labor and asymmetric information

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Abstract
We study a model of individual wage bargaining between heterogeneous workers and firms, with instantaneous matching, free firm entry, workers’ individual productivities are discovered by firms only after being hired, and it is expensive for firms to hire and fire workers. We show that inefficiencies due to bargaining and externalities in the matching process lead firms to employ too few worker types. Employment among employed worker types is also inefficiently low when workers have high bargaining power, but may be too high when workers’ bargaining power is low. The government can correct these inefficiencies by reducing or increasing firms’ hiring and firing costs. This implies that the costs of firing tenured workers ‘almost always’ should be reduced. We argue that the model gives a good description of recent labor market phenomena in advanced economies. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction
The purpose of this paper is to study the properties of a labor market where heterogeneous workers bargain individually with their employers over the wage.
We extend the standard bargaining/matching model (e.g., Pissarides, 1985, 1987, 1990; Mortensen and Pissarides, 1994, 1998) in two new directions. First, workers are no longer identical but instead have different but given productivities, known to each worker but not to the firm at the time the worker is hired. With heterogeneous labor, the issue of firing then becomes relevant even when there are no (idiosyncratic or general) shocks, since firms may wish to replace their initially hired workers with other, more productive, ones. Secondly, we assume that there are both hiring costs (paid for by the firm and corresponding to recruiting costs in Pissarides), and costs of firing already engaged workers. Hiring costs do not directly affect firms’ firing decisions; firing costs, however, do. We will assume that firms observe individual workers’ productivities only after the time at which they are hired, which implies that any hiring costs must then already be sunk. Firms then wish to retain those workers who have the highest productivities, and may wish to fire those workers whose productivities fall below some minimum level. We then find it relevant to distinguish between three categories of firing costs: (a) the cost to the firm of immediately getting rid of a worker that is just hired, but the firm does not wish to keep; (b) a pure cost paid by the firm (which consequently ‘vanishes’ from the firm–worker relationship) upon the separation of a ‘tenured’ (or initially wanted) worker from the firm; and (c) a redundancy payment from the firm to a tenured worker upon separation. Since firms are identical, all choose the same cutoff level for productivity, $z_1$, beyond which workers are retained. A simplification relative to the standard matching model is that our process of matching workers and firms involves no frictions, and that active jobs suffer no vacancies. In addition to simplifying the analysis considerably, such an approach also has the attractive feature that a standard competitive solution now arises when workers’ relative bargaining strength goes to zero, making it possible to investigate the issue of market efficiency in this important special case.

The paper integrates a modified version of the Pissarides–Mortensen matching/bargaining theory for the labor market, with recent literature on turnover costs. It makes a first step in the direction of endogenizing simultaneously worker hiring standards and overall employment when workers have unobservable productivity differences. Several of our results are novel; in particular, those describing how firms’ hiring and firing decisions imply externalities in the market for matching of heterogeneous workers, and how firms’ minimum hiring standards and overall employment depend on workers’ bargaining strength and on the costs of firing workers immediately or later. We also point out in Section 5, how the model may help us to understand and interpret important and recently observed labor market phenomena in advanced economies. In particular, the model yields a coherent theory of observed differences in wages and in hiring and unemployment rates for workers at different productivity levels, and predicts how these variables will change when hiring and firing costs change. By deriving the optimal solution (given the informational constraints imposed) we are also able to discuss possible
room for government policies that affect firms’ costs of hiring and firing, and indicate whether, and in case how, recent labor market developments in advanced economies may deviate from an approximate (constrained) social optimum.

We present the basic model in Sections 2 and 3. In Section 4, we derive the solution chosen by a social planner subject to the same technological and cost conditions as market agents, and compare this to the market solutions. Conclusions and interpretations are offered in the Section 5, where we also point out some potential directions for future research.

2. The basic model

Consider an economy with a large exogenous number of workers, normalized to one, and a large (endogenous) number of active firms (or jobs), each employing exactly one worker. All firms and workers are risk-neutral. We will assume perfect matching, by which we mean that either active jobs, or actively searching workers, are matched perfectly, i.e., suffer no unemployment. From the solutions derived in the following, we must then have that jobs are always matched perfectly, since equilibrium in the market always requires some unemployment among workers. The number of active jobs then equals the number of employed workers, $L$. All jobs are identical and have fixed productivities over time, i.e., we consider a stationary steady state. There are no capital costs.

Labor is heterogeneous, and workers’ productivities denoted by $z$, distributed according to a continuous distribution $G(z)$, with support $[0, \ z_m]$. For a given worker, $z$ is fixed and known to the worker himself, but not to the firm at the time he is hired. When the worker is hired, the firm incurs a hiring cost $H$, after which the worker’s productivity is immediately revealed to the firm. The firm chooses

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2 This assumption corresponds to that made, e.g., by Shapiro and Stiglitz (1984) in a related efficiency wage model, and sets our model apart from the Pissarides framework. It is, however, made mainly for analytical tractability; no essential changes in the basic results; in particular, those deriving from labor heterogeneity per se, would follow if we instead adapted the Pissarides framework. The main difference is that firms’ hiring cost would become endogenous, while here it is an exogenous variable.

3 A trivial extension would be to assume a given rental cost of capital per job, as in (Pissarides, 1990), with no consequences for the main conclusions in the following.

4 Since we assume that no part of $H$ is paid for by the worker, the most straightforward interpretation of $H$ in our model is as recruiting or set-up costs expended by the firm prior to the arrival of the worker. If $H$ instead consisted mainly of costs incurred after the arrival of the worker (e.g., training costs), the logic of the model should otherwise dictate that these costs be shared by the two parties, according to their bargaining strengths. One might then conceivably explain the lack of worker payments by limitations on workers’ assets or access to credit markets, or by legislation requiring firms to cover any such costs. Possible implications of the sharing of hiring costs will not be pursued here, but are an interesting topic for future research.
to retain the worker given that his productivity falls in the domain \([z_1, z_\infty]\), where \(z_1 \geq 0\). When \(z_1 = 0\), all worker types are retained and none consequently separated immediately. When \(z_1 > 0\), workers whose productivities are in the domain \([0, z_1]\) are separated immediately. \(^5\) \(^6\) There is free entry of jobs and, apart from \(H\), no establishment costs for jobs. Assume that all workers remain in the market for an infinite period of time. For a wanted worker (with \(z \geq z_1\)) who is currently unemployed, his lifetime discounted value of labor market participation, \(U(z)\), is given by the following Bellman equation:

\[
ru(z) = b + h\left[W(z) - U(z)\right],
\]

where \(b\) is the level of income (or income-equivalent utility) in the unemployed state, \(h\) is the continuous rate of transition from unemployment to employment for such a worker, and \(W(z)\) is the expected discounted lifetime utility in the employed state.

We will now find it convenient to define the following three components of firing costs.

- \(F_0\) is the cost of immediately getting rid of a worker who is just hired, and whose productivity is too low to be kept.

- \(F_1\) denotes the dissipative cost associated with firing a ‘tenured’ worker, i.e., one which the firm has initially decided to keep but who later is fired when the job in question is closed down. By ‘dissipative’ we here mean that this is an expense that benefits neither the firm nor the worker (e.g., a legal or administrative cost, or a tax).

- \(F_2\) denotes a final redundancy payment from the firm to a ‘tenured’ worker, upon dismissal. This cost is thus not dissipative, it is instead a pure transfer from the firm to the worker.

We will, in the following, utilize the definition \(F = F_1 + F_2\), which denotes the firm’s total expense related to firing a tenured worker. We will throughout assume that firing costs are paid fully by the firm; see the comment to this assumption in \(^4\) above.

For an employed worker with productivity \(z\) the discounted lifetime value is determined by a relationship similar to Eq. (1):

\[
rW(z) = w(z) + s\left[U(z) + F_2 - W(z)\right].
\]

Here, \(w(z)\) is the wage earned by a worker of ability \(z\), \(s\) is an exogenous rate of job exit. We will assume, throughout, that the only reason why a worker at

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\(^5\) The assumption that workers’ productivities are discovered immediately can be relaxed without altering major results. One alternative is to assume that workers go through an initial traineeship or test period, whose length is stochastic and exponentially distributed.

\(^6\) We will demonstrate below that the profitability of employing workers in our model always increases strictly in \(z\), for all firms.
equilibrium loses his job, is because his job ceases to exist. From Eqs. (1) and (2), we find
\[ W(z) = \frac{1}{r+s} \left[ w(z) + sF_2 + sU(z) \right] = \frac{(r+h)[w(z) + s(F_2 + b)]}{r(r+s+h)} \]
(3)
\[ U(z) = \frac{h[w(z) + sF_2] + (r+s)b}{r(r+s+h)}. \]
(4)
Denote the present discounted value to the firms, of having a job filled with a worker of quality \( z \), by \( J(z) \). Consider a position filled with a worker of quality \( z \), where the hiring cost \( H \) is sunk and the worker screened. Given that the worker is not fired immediately (i.e., \( z \geq z_i \)), and the firm is required to make the redundancy payment \( F_2 \), \( J(z) \) is given by
\[ rJ(z) = z - w(z) + s[-F - J(z)], \]
(5)
yielding
\[ J(z) = \frac{1}{r+s} \left[ z - w(z) - sF \right]. \]
(5a)
Denote by \( \phi \) the probability that the firm samples a desirable worker, when drawing among the mass of unemployed workers. This is given by (from Appendix A)
\[ \phi = \frac{s[1 - G(z_i)]}{(s+h)G(z_i) + s[1 - G(z_i)]}. \]
(6)
The density in the firm’s sampling distribution over \( z \) levels among desirable unemployed workers, \( g_s(z) \), is given by (also from Appendix A)
\[ g_s(z) = \frac{s}{(s+h)G(z_i) + s[1 - G(z_i)]} g(z), \ z \in [z_1, z_m]. \]
(7)
and where \( g_s(z)/\phi \) is the conditional density for workers who are not immediately fired. Calling the cumulative sampling distribution \( G(z) \), by definition \( \phi = 1 - G(z_i) \). Note also that \( \phi < 1 - G(z_i) \). Thus, the probability of sampling an acceptable worker from the pool of the unemployed is lower than the fraction of acceptable workers in the entire labor force, since the unemployment rate is lower among the former.

The cost of the firm’s first sampling given a vacancy is \( H \). Provided that the first engaged worker does not have the required quality (i.e., \( z < z_i \)), the cost of the next (and possible following) sampling(s) is \( H + F_0 \), where \( F_0 \) as noted is the
\footnote{This requires that \( z - w(z) \) be strictly increasing in \( z \), which holds in all cases studied below.}
cost to the firm of firing a worker immediately, after being hired. \( F_0 \) may be small, and will be assumed smaller than \( F_1 \). The total expected cost of filling the job with a worker of ability \( z \geq z_1 \) is now given by

\[
C = H + (1 - \phi)(H + F_0) + (1 - \phi)^2(H + F_0) + \ldots
= \frac{(H + (1 - \phi)F_0)}{\phi}.
\]  

(8)

Define the expected value to the firm of a filled job (with a worker of productivity \( z \geq z_1 \)) by \( EJ \). We then find, integrating Eq. 5a over \( z \),

\[
EJ = \frac{1}{\phi} \int_{z=0}^{z_m} J(z) g_z(z) dz = \frac{1}{r + s} \frac{1}{\phi} \int_{z=0}^{z_m} \left[ z - w(z) \right] g_z(z) dz - \frac{s}{r + s} F.
\]

(9)

At equilibrium \( EJ = C \), implying the condition

\[
\frac{1}{r + s} \int_{z=0}^{z_m} \left[ z - w(z) \right] g_z(z) dz = H + \frac{s}{r + s} F + (1 - \phi)F_0.
\]

(10)

We assume that each firm unilaterally selects the level of \( z(= z_1) \) beyond which a worker is retained, and below which he is immediately fired. The solution for \( z_1 \), and for the wage schedule \( w(z) \), will be derived in Section 3.

3. Market equilibrium

We now assume that the wage for each retained worker is determined in an asymmetric Nash bargain between the worker and the firm, with relative bargaining strengths \( \beta \) and \( 1 - \beta \), where \( \beta \in (0, 1) \). Assume also that the threat point for the bargaining solution, relevant both to workers and firms, involves no redundancy payment \( F_z \) to a worker who leaves the firm while the firm is operating. This implies that the worker would receive a final redundancy payment only in the case where the job in question is shut down.  

\[8\]

We assume that such a condition can be fulfilled for \( h \in (0, \infty) \) and with \( z_1 \) given from Eq. 15. This implies that \( H \) and \( F_z \) are all not in excess of certain levels. In the opposite case, no firms could profitably enter the market, and there would be no employment.

\[9\]

We here assume that the event of closing down a job is publicly observable, e.g., because only then no new worker is subsequently employed by the firm. Thus, the payment of \( F_z \) would be conditioned on a publicly observable event. Note also that the firm would have no incentive in the model, to fire a worker just for the purpose of avoiding making the final redundancy payment, since the firm will always have a positive current surplus from a retained worker, which is lost when the worker is fired. A reasonable belief to be held by an outside observer may then be that in the (out-of-equilibrium) event of a worker separation from the firm prior to job closure, this is always due to the worker quitting, thus never meriting a final redundancy payment.
Consider an ongoing relationship, i.e., one that was not broken up immediately, whereby \( W(z) \) and \( U(z) \) are given by Eqs. (3) and (4). The net utility of the worker from the match, \( S(z) \), is given by \( W(z) - U(z) \), i.e., by

\[
S(z) = \frac{1}{r + s} \left[ w(z) + sF_2 - rU(z) \right] = \frac{w(z) - b + sF_2}{r + s + h}. \tag{11}
\]

Likewise, the net surplus, \( Q(z) \), of the firm is given by the surplus over the default utility in the case of a worker quit, \( J(z) + F_1 \):

\[
Q(z) = \frac{1}{r + s} \left[ z - w(z) + rF_1 - sF_2 \right]. \tag{12}
\]

Defining \( E \_z \) as the conditional expectation of \( z \) given \( z \geq z_1 \), we may now characterize market equilibrium, as follows.

**Proposition 1**: Given individual and asymmetric Nash bargaining over wages, market equilibrium is characterized by the following equations, solving for \( w(z), z, h \) and \( L \):

\[
w(z) = \beta \frac{(r + s + h)}{r + s + \beta h} (z + rF_1) + \frac{(1 - \beta)(r + s)}{r + s + \beta h} b - sF_2 \tag{13}
\]

\[
z_1 = b + \frac{s + \beta (r + h)}{1 - \beta} F_1 - \frac{r + s + \beta h}{1 - \beta} F_0 \tag{14}
\]

\[
E \_z = b + \frac{1}{1 - \beta} \left\{ (s + \beta (r + h))F_1 + (r + s + \beta h) \left( \frac{H}{\phi} + \frac{1 - \phi}{\phi} F_0 \right) \right\} \tag{15}
\]

\[
L = \frac{h}{s + h} \left[ 1 - \frac{1}{\beta} \right]. \tag{16}
\]

Provided that \( z_1 \in [0, z_\text{m}] \).

**Proof**: Eq. (13) is derived directly, setting \( w(z) \) to maximize the Nash product \( S(z)^{\beta}Q(z)^{1-\beta} \), where \( U(z) \) is taken as given and determined from Eq. (11). In deriving Eq. (14), we use Eq. (13) and recognize that firms set \( z_1 \) unilaterally, given an internal solution for \( z_1 \). The condition \( J(z_1) = -F_0 \) then makes the firm exactly indifferent about immediately firing a newly hired worker, leading to Eq. (14). Eq. (15) is then found, from Eq. (10). Eq. (16) defines \( L \). Q.E.D.

From Eq. (13), \( w(z) \) (apart from the term \(-sF_2\)) is a weighted sum of the terms \( z + rF_1 \) (the net current value to the firm of continuing rather than ending the employment relationship) and \( b \) (the current utility of the worker’s outside option), with weights that tend to \( \beta \) and \( 1 - \beta \) as \( h \) tends to zero, and (for given \( \beta > 0 \)) to 1 and 0 as \( h \) tends to infinity.
Note that an increase in \( F \) reduces \( w(z) \) by \( sF \). A higher \( F \) here implies no improvement in the worker’s expected attachment value, only that more of this value takes the form of a final redundancy payment, and less the form of wages. This follows because neither the worker’s nor the firm’s threat point for the bargain involves the redundancy payment \( F \). In such a case, the final redundancy payment has no real effect on the bargain between the two parties; it only has a purely distributional effect, related to the distribution of the worker’s money compensation over time.\(^7\)

For Proposition 1 to hold, we had to assume that the solution value for \( z_1 \) from Eq. (14) lies in the domain \([0, z_m]\). A sufficient condition for \( z_1 > 0 \) is \( rF_0 < b \), which will be assumed here and in the following. With regard to \( z_m \), a sufficient condition for \( z_1 < z_m \) is

\[
z_m > b + \frac{1}{1-\beta} \left[ (\beta r + s) F_1 - (r + s) F_0 \right],
\]

for all relevant values of \( \beta \) and \( F_i \), which will also be assumed to hold.

A marginal worker as viewed by firms, is defined by \( z = z_1 \). For such a worker,

\[
S(z_1) = \frac{\beta}{1-\beta} (F_1 - F_0).
\]

Whenever \( F_0 < F_1 \), there is a positive net surplus associated with, and going to, a marginal employed worker. Thus, a worker with productivity slightly below \( z_1 \) would now have been able to enjoy a positive net surplus from not having his match broken immediately but rather continuing it. The reason why the match is still immediately broken (unilaterally by the firm) is that when \( F_0 < F_1 \), it is advantageous for the firm to break the match with a marginally profitable worker immediately rather than later, since this reduces dissipative firing costs.\(^{11}\)

To justify our assumption that \( F_0 < F_1 \), note first that firms’ costs of getting rid of newly hired workers are likely to be low. Such workers have little legal job protection, and any disruption of the general production process of the firm that may follow from their departure, is bound to be minimal. A main component of

\(^{10}\) This is in accordance with the ‘efficient contracts’ case of Lazear (1990), which he later questions on the grounds of realism. One could in principle think of alternative formulations giving more of an allocative effect to redundancy payments, in particular when such payments are made in cases of premature termination of the employment relationship (at the initiative of either party). Saint-Paul (1995) explores a model where it is assumed that redundancy payments are made when the worker is fired, but not when the worker quits, and shows that this leads to inefficient employment decisions by firms. The advantage of our formulation, relative, e.g., to that of Saint-Paul, is that we here do not require the reason for such a separation (whether it is a quit or a fire) to be observable, something that is required in his formulation.

\(^{11}\) A consequence of this is that some workers with \( z \) in some range below but close to \( z_1 \) will have an incentive to make an up-front payment to the firm upon joining, in order for the firm not to fire them immediately. Here we rule out such up-front payments.
could be required salary to the worker in an initial test period. In contrast to tenured-worker salary, such required salary would be ‘dissipative’ in the eyes of the firm, since it would go to workers who are productionwise worthless. For retained workers it is different. Such workers will over time enjoy greater real and formal job protection, and shedding them may involve significant legal costs. Secondly, the departure of workers who have been in the firm for some time may disrupt the production process due to established productive interaction with other, remaining, workers (although this is not explicitly modelled). There is no reason to expect \( F_0 \) and \( F_1 \) to be the same, and the latter should in most circumstances be greater, perhaps significantly so.

An important question is whether the derived equilibrium is unique, in particular whether the solutions for \( z_1 \) and \( h \) from Eqs. (14) and (15) are unique. Define then \( \theta(z_1) = E_z z - z_1 \), given by

\[
E_z z - z_1 = \frac{1}{1 - \beta} \frac{r + s + \beta h}{\phi} (H + F_0). \tag{19}
\]

We may then formulate the following result.

**Proposition 2:** Equilibrium as characterized in Proposition 1 is unique given that

\[
\frac{1 - G(z_1)}{g(z_1)} > E_z z - z_1 - \frac{(r + s)(s + h)}{(1 - \beta)s} \frac{H + F_0}{1 - G(z_1)} \tag{20}
\]

holds everywhere.

**Proof:** We have from Eq. (6) that

\[
\frac{1}{\phi} = 1 + \frac{s + h}{s} \frac{G(z_1)}{1 - G(z_1)}. \tag{6a}
\]

Inserting from Eq. (6a) in Eq. (19) and differentiating Eq. (19) with respect to \( z_1 \) yields \( d\theta/dz_1 < 0 \) everywhere if and only if Eq. (20) holds everywhere. Moreover, for a given \( z_1 \) we have from Eq. (14) that there is a unique equilibrium value of \( h \), implying that \( w(z) \) and \( L \) are given uniquely from Eqs. (13) and (16). Thus, Eq. (20) is sufficient for equilibrium to be unique. Q.E.D.

With a general productivity distribution function \( G(z) \), the possibility of multiple solutions is in principle open. When we have multiple equilibria, high levels of \( z_1 \) go together with high levels of \( h \). Since higher \( z_1 \) for given \( h \) implies lower employment, and higher \( h \) for given \( z_1 \) higher employment, employment levels cannot generally be ranked among such equilibria. In the following discus-
sion, we will however generally assume that Eq. (20) holds, and that equilibrium is unique.

The effects of changes in the cost variables \( H, F_0, \) and \( F_1 \) on \( h \) and \( z \) can be found from differentiating Eqs. (14) and (15) with respect to \( h, z \) and the cost variables (inserting for \( L \) from Eq. (16)). Note in line with the comment above, that changes in \( F_0 \) have no effects on \( h \) and \( z \). One may readily show that \( z \) is increased when \( H \) and \( F_1 \) increase. The reason is partly that the general cost level then increases. Moreover, a higher \( F_1 \) raises workers’ bargaining threat point and thus the wage. Firms then become more selective with respect to what workers to keep at the time of recruiting. Differentiating Eq. (16) yields

\[
\frac{dL}{dL} = \frac{s}{(s+h)^2} [1 - G(z)] dh - \frac{h}{s+h} g(z) dz.
\]

Total employment must then drop when \( H \) or \( F_1 \) increases. We find, in Appendix B, that \( z \) decreases with \( F_0 \), and \( h \) most likely decreases as well. As will be elaborated in Section 4, when all firms reduce \( z \) this leads to a reduction in screening costs, in two different ways. First, screening costs are reduced in individual firms since a smaller fraction of all potential worker types are immediately dismissed. Secondly, the pool of undesirable workers is reduced, thus lowering average screening costs in all firms. The latter is an externality which each firm does not consider individually, but which generally raises firm entry and thus also possibly \( h \). It also implies that an increase in \( F_0 \) has a more favorable effect on employment when screening is imperfect.

4. Efficient solutions

The assumption of individual bilateral bargaining over wages, and the externalities involved in the screening mechanism, imply that there is no prior reason to expect the market solution to be efficient. In this section, we will derive a ‘constrained efficient’ solution and compare it to the market solution. By ‘constrained efficient’ we mean that the total steady-state output of the economy is maximized, taking the assumed informational constraints as given. Here we take as given that the authorities have no way of circumventing the problem that workers’ productivities cannot be observed prior to hiring. Moreover, no distributional considerations are made when constructing the government’s social welfare function (the level of welfare of workers is simply counted as their salaries). Our objective is to derive the levels of \( z \) and \( h \), and consequently \( L \), that would be set by a social planner who could set these directly, given that the planner faces real hiring costs \( H_0 \), real dissipative firing costs \( F_{00} \) for workers to be dismissed immediately, and \( F_{10} \) for workers to be kept, and is subject to the same screening
technology as that facing firms. Define the total match value for a given \( z \), once the worker is employed, \( H_0 \) sunk and the worker retained (i.e., \( z \geq z_1 \)), by \( M(z) \), where 
\[
M(z) = \frac{1}{r + s} (z - b - s F_{10} + s(-M(z)),
\]
implies
\[
M(z) = \frac{1}{r + s} (z - b - s F_{10}). \tag{22}
\]
A match breakup involves a social firing cost of \( F_{10} \) and loss of the match value \( M(z) \). Denote by \( EM \) the expected ex ante value of a successful match (i.e., a match where \( z \geq z_1 \) is realized), including costs sunk in order to accomplish such a match. We find 
\[
EM = E[M(z)] - \left[ H_0 + G(z_1)F_{10} \right]/[1 - G(z_1)],
\]
where as before \( G(z) \) is the sampling distribution over \( z \) for hiring firms, from the pool of unemployed workers. Define \( T \) as the ex ante value of all successful matches \( L \) in existence at a given time. As before \( L \) is given by Eq. (16). \( T \) may be expressed by:
\[
T(z_1, h) = \frac{h}{s + h} \left[ \int_{z_1}^{z_\infty} \frac{z - b - s F_{10}}{r + s} g(z) dz - \left( \frac{s + h}{s} \right) G(z_1) \right]
+ \left[ 1 - G(z_1) \right] \left( H_0 - \frac{s + h}{s} G(z_1) F_{10} \right). \tag{23}
\]
The government’s objective is to maximize Eq. (23) directly with respect to \( z_1 \) and \( h \).\(^{12}\) The solution to this problem can be formulated in the following proposition.

**Proposition 3**: The government’s constrained optimal solution for \( z_1 \) and \( h \) is given by
\[
\begin{align*}
z_1 &\geq b + s F_{10} - \frac{(r + s) h}{s} H_0 - \frac{(r + s)(s + h)}{s} F_{10} \tag{24} \\
E_z z &\geq b + s F_{10} + (r + s) H_0 + \frac{(s + h)^2(r + s)}{s^2} \frac{G(z_1)}{1 - G(z_1)} (H_0 + F_{10}), \tag{25}
\end{align*}
\]
where both Eqs. (24) and (25) hold with inequality for \( z_1 = 0 \), and both with

\(^{12}\) As opposed to in, e.g., Hosios (1990) and Pissarides (1990), such a maximization is meaningful here even when there is positive discounting (\( r > 0 \)), since we assume instantaneous matching of firms. This implies that we can assume that the optimal stock of workers is hired at a given instant of time. Eq. (23) expresses the social value of such initial hirings. Given that employment is to be kept constant from then on, maximizing Eq. (23) is then the same as maximizing the discounted value of all future hirings as well.
equality for $z_1 \in (0, z_m)$. When $z_1 = 0$, $h = \infty$, while when $z_1 > 0$, $h > 0$ and finite.

**Proof**: Maximizing Eq. (23) with respect to $z_1$ and $h$ yields

$$\frac{dT}{dz_1} = \frac{h}{s + h} g(z_1) \left( -z_1 + b + s F_{10} \right) + H_0 - \frac{s + h}{s} \left( H_0 + F_{10} \right) \leq 0$$

(26)

$$\frac{dT}{dh} = \frac{s}{h(s + h)} T - \frac{h}{s + h} G(z_1) \left( H_0 + F_{10} \right) \geq 0.$$ 

Eq. (26) holds with equality if and only if $z_1 \in (0, z_m)$. Eq. (26) is thus identical to Eq. (24). When $z_1 > 0$, there always exists a positive value of $h$ yielding equality in Eq. (27). When $z_1 = 0$, $G(z_1) = 0$ and Eq. (27) is fulfilled with inequality. We then have the limit solution $h = \infty$. Inserting for $T$ from Eq. (23) in Eq. (27) yields Eq. (25). Q.E.D.

The system Eqs. (24) and (25) yields two possible sets of solutions. One set implies that $z_1 > 0$ and that $h$ is positive but finite, corresponding to equalities in Eqs. (24) and (25). The other set implies that $z_1 = 0$ and $h = \infty$, corresponding to a corner solution with inequalities in both. It is easily realized that the latter is always a local optimum and thus a mathematical solution to this system: given $z_1 = 0$, $h = \infty$ is optimal; and given $h = \infty$, $z_1 = 0$ is optimal. This may or may not be the globally optimal solution. In general, it will be globally optimal whenever no internal solution to the first-order conditions can be found, or whenever a corner solution dominates an internal solution. Since $h$ is in most practical cases likely to be (perhaps significantly) larger than $s$, we see from Eq. (24) that an internal solution (with $z_1 > 0$) requires that $H_0$ and $F_{10}$ both be small compared to $b/r$.

We will now assume that an internal solution to Eqs. (24) and (25) exists and is optimal. We may then study how the costs of sorting out unwanted workers, in the form of increased firing costs $F_{10}$ and subsequent hiring costs $H_0$, affect the constrained and second-best optimal solution. In Eq. (24), higher $H_0$ and $F_{10}$ both tend to reduce, and a higher $F_{10}$ to increase, the constrained optimal level of $z_1$.

In interpreting these effects, consider first the effect of $F_{10}$. An increase in this variable has two separate effects on the constrained optimal value of $z_1$, which both go in the same direction. First, it becomes socially more expensive to immediately shed workers, leading to lower optimal $z_1$ and thus less immediate shedding. Secondly, a high $z_1$ implies a negative externality for other firms, since there will then be a large group of unwanted workers among the pool of the unemployed, and thus many who will be immediately shedded. This externality is
greater the greater is $F_{00}$. As a consequence, when $F_{00}$ is greater, the social cost associated with this externality is reduced by more when $z_1$ is lowered, implying that the optimal $z_1$ is lower. Note that individual firms consider only the first of these effects, and not the second. The first (direct cost) effect is however greater in private firms’ calculations, in Eq. (14), than in those of the planner. This is because the cost of $F_0$ is compared to the firm’s private net return from keeping the worker, which is lower than the social return when workers have positive bargaining power (since workers then reap part of the marginal social value of the match ex post). Setting $F_{00} = F_0$, we then cannot in general say whether the effect of increased $F_0$ on $z_1$ is greater or smaller in the market solution than in the optimal solution.

Consider next the effect of $F_{10}$. From Eq. (24) an increase in $F_{10}$ raises $z_1$, since the required productivity of retained workers must be higher when the deadweight loss associated with future firings of such workers increases. Notice, however, from Eq. (14) that an increased $F_1$ increases $z_1$ by even more in the market solution. This is because when workers have bargaining power, firms do not reap the entire social return from retaining a worker, but must still pay the entire dissipative firing cost $F_1$. High dissipative firing costs then tend to make firms’ minimum hiring standards too high.

Consider finally the effect of $H_0$ in Eq. (24). In the constrained optimal solution, there are now two separate effects involved. The first is an externality cost effect, in much the same way as for $F_{00}$ above. Increasing the pool of unwanted unemployed workers has the effect of contaminating this pool by leading to more immediate fires and thus more hirings in equilibrium. It may then be socially efficient to select a low value of $z_1$ when $H_0$ is high, in order to reduce this contamination effect. The second, opposite, effect is that a higher $H_0$ must be viewed as a greater cost of investing in the hiring of workers. This effect draws in the direction of requiring a higher return on such an investment, and thus a higher required $z_1$. Overall, the effect on the socially optimal value of $z_1$ of an increase in $H_0$ is still always negative, from Eq. (24). Notice also that an increase in hiring costs has no effect on $z_1$ in the market solution (14), since firms have no incentives to consider either of these two effects (they clearly do not consider the first externality effect, and neither the capital cost effect, since this cost is sunk at the time the decision is made about possibly shedding the worker immediately). An increase in $H_0$ then always biases the market solution for $z_1$ upwards, i.e., makes firms too choosy about whom to retain.

A second-best optimum with $z_1 > 0$ trades off the externality costs discussed here, against the efficiency loss from retaining workers with too low productivities. An important property of the constrained optimal solution as long as $z_1 > 0$ is optimal, is that it can never be efficient to have full employment among desirable workers, i.e., $h$ must be finite. To see this, consider a possible case where $h \rightarrow \infty$. But then the market equilibrium would imply (from Appendix A) that $G_s(z_1)$ is very close to 1, i.e., (almost) all workers in firms’ sampling distribution over
unemployed workers would be unwanted. Clearly this cannot be efficient, since the marginal hiring and firing costs, associated with hiring one additional qualified worker, would go to infinity.

On the other hand, the optimal solution may imply that Eqs. (24) and (25) hold with inequalities and that consequently $z_1 = 0$ and $h = \infty$. This is always the optimal solution whenever $H_0 + F_{0i}$ is high relative to $b/s + F_{1i}$. In such a case, there are no externalities associated with the sampling and immediate shedding of workers. Such avoidance of externalities implies a ‘price’ in the form of inefficiently high employment, whereby some workers will be employed whose current productivities fall below their total current opportunity cost in the particular firm–worker relationship (which equals $b + sF_{1i} + (r + s)H_0$). Such a solution is still constrained optimal, as the externality costs associated with not employing some of these, in terms of greater hiring and firing costs for other firms, would have been even greater, had such low-productivity workers instead been placed in the unemployment pool.

To consider market implementation of the constrained optimal solution, we compare Eqs. (24) and (25) to the equivalent market solutions in Eqs. (14) and (15). The comparison is most straightforward with an internal solution to Eqs. (24) and (25). Assume that the government can freely tax or subsidize hiring and firing costs, and that $H_i - H_0$ and $F_i - F_{0i}$, $i = 0, 1$, represent net government subsidy rates (or tax rates when negative), and that there are no net costs or gains to the government associated with net subsidies or taxes. We impose no prior constraints on $H_i$ or $F_i$, i.e., either of these could be negative as part of an implemented efficient solution. An efficient solution can then in principle always be implemented, by only setting $H_i$ and $F_i$ at appropriate levels. To shed further light on the comparison between the constrained efficient and market solutions, we consider the following two simplified cases.

Case a: $\beta \rightarrow 0$. This case corresponds to a ‘competitive’ solution where firms can be viewed as posting wages, and the same wage to all workers. Eqs. (14) and (15) can now be written as

$$z_1 = b + sF_1 - (r + s)F_0$$

$$E_cz = b + sF_1 + (r + s)H + \frac{(s + h)(r + s)}{s} \frac{G(z_1)}{1 - G(z_1)}(H + F_0).$$

Assuming $H = H_0$ and $F_i = F_{0i}$, $i = 0, 1$, comparing Eq. (14a) to Eq. (24) reveals that $z_1$ is unambiguously higher in the market solution than in the efficient solution. Thus, minimum hiring standards are inefficiently high in the unregulated market solution. The factor driving this result is the negative externality noted above, from dismissing a worker that is initially engaged. Such a dismissal ‘contaminates’ the unemployment pool, and increases the hiring (and subsequent
firing) costs of other firms. Firms have individually no incentive to consider this externality.

Comparing Eq. (15a) to Eq. (25) similarly reveals that \( h \) is too high in the market solution (since \( E_z \) increases strictly in \( z \), for a given \( h \), and must consequently be greater in Eq. (15) than in Eq. (27)). This is a result of a negative externality from firm establishment and subsequent hiring. The basis for this externality is that it is advantageous for hiring firms to have many high-quality workers among the pool of the unemployed. When a firm hires, and keeps, a high-quality worker, the pool of the unemployed is thereby ‘contaminated’, in a similar way as when a low-quality worker is fired, and hiring made less attractive to other firms. This factor makes the constrained optimum rate of hiring, among the group of desirable workers, lower than the market hiring rate (in this particular case, where workers have no bargaining power).

Overall, when social hiring and firing costs equal private costs and workers’ bargaining power is low, firms’ minimum hiring standards (and thus too few worker types employed by firms) and the rate of employment for those types retained, are both too high. These two factors have opposite effects on overall employment, relative to the constrained optimum, and we cannot in general say whether employment is higher or lower than the ‘socially optimal’ level. Unemployment is however too heavily concentrated to low-productivity workers in the market solution.

Implementing the socially optimal level of \( z \) in the market would require that firms either face a higher cost of immediately shedding a worker, and/or a lower dissipative firing cost, than the respective social costs. In addition, \( H \) may have to be adjusted (either upward or downward) relative to \( H_o \), in order for the market to implement the efficient level of \( h \). In general, if \( F_i \) is subsidized by the government, \( H \) will have to be taxed; if instead \( F_o \) is taxed, \( H \) may have to be either taxed or subsidized.

Case b: \( F_i = F_o = 0 \), \( i = 0,1 \). This is the special case with no firing costs. At the market solution, \( z = b \), from Eq. (14), which exceeds the constrained optimal level of \( z \) from Eq. (24). To study the effect for \( h \), note that Eq. (15) now can be written as

\[
E_z = b + \frac{r + s + \beta h}{1 - \beta} \left[ 1 + \frac{s + h}{s} \frac{G(z)}{1 - G(z)} \right] H. \tag{15b}
\]

Again, when \( \beta \) is low, \( h \) must be higher in the market solution than in the constrained efficient solution. When \( \beta \) increases and approaches one for given \( h \), by contrast, \( E_z \) increases and approaches infinity in the limit. Since \( z \) is a constant, from Eq. (14), and \( E_z \) is a strictly increasing function of \( z \), \( E_z \) must also be a constant. Then \( h \) must go to zero as \( \beta \) grows, and in fact hits zero at a level of \( \beta \) between zero and one. Thus, as \( \beta \) is increased parametrically, starting from zero, \( h \) is reduced, from an overoptimal level to a suboptimal one. For a
sufficiently high \( \beta \), there can be no market solution as \( h \) in our model is below zero. Entering the market will then never be worthwhile for any firm.

Note that in this case the efficient solution for \( z \) cannot be implemented unless the government has the ability to either tax the immediate shedding of hired workers, or to subsidize the ultimate dismissal of workers upon job closedown. The first of these policy options clearly appears as more attractive and realistic than the second. In addition H may have to be either subsidized (with high worker bargaining power) or taxed (possibly with low worker bargaining power), in order to ensure an optimal level of \( h \) in the market.

To sum up, in both cases a and b, the minimum worker hiring standard in the unregulated market solution is higher than the standard chosen by a social planner. This implies that firms are ‘too choosy’ when selecting whether or not to keep or dismiss a newly hired worker. Employment among those workers with productivities above the minimum standard is higher than the optimal level when workers have very low bargaining power, but lower when workers’ bargaining power is above a certain level. Overall employment may be higher or lower than the constrained optimal level when workers have low bargaining power, but is always too low when workers’ bargaining power is high.

So far we have assumed that the optimal solution for the planner implies \( z = 0 \) and \( h \) finite. As already noted the constrained optimal solution may instead entail a corner solution where \( z = 0 \) and \( h = \infty \), corresponding to the case with inequalities in Eqs. (24) and (25). From Eq. (14), implementing such a solution would require government taxes and subsidies that make \( F_0 > F_1 \), and subsidies to \( H \). Such implementation is, however, in practice impossible: when \( h \) is very large, we have from Eqs. (12) and (13) that the wage of workers eats up the entire surplus of firms ex post, and the solution would require \( F_1 = H = 0 \). We would then have a ‘knife edge’ situation where firms enter even though they would earn no profits after entry, since the government pays all entry costs. This solution can be approximated by making firms retain all worker types (i.e., implement a solution where \( z = 0 \)), and by raising \( h \) to a ‘high’ level.

5. Conclusions

We have studied a model of the labor market where there is instantaneous matching and subsequent individual wage bargaining between each worker and firm, workers have productivity differences that cannot be immediately observed by firms, and it is costly for firms to hire and fire workers. There are two, interacting, aspects of the model which make it novel and rich in structure and, despite many simplifying assumptions, highly relevant for explaining recent labor market phenomena in advanced economies. The first is the assumption of individual wage bargaining in conjunction with heterogeneous labor. Labor heterogeneity is a pervasive phenomenon and arguably the major cause of labor market failure in
advanced economies. Moreover, one may argue that individualized wage bargaining is more realistic and relevant, the more workers differ individually. In particular, centralized wage bargaining or firm wage posting are less stable institutional arrangements when workers’ productivities vary greatly than when they do not. The second point is that when workers are heterogeneous, individual workers’ productivities in many cases cannot be observed by firms before, but only after, the worker has been hired. We show that this may imply serious externalities in the hiring process when there are hiring and firing costs. The reason is that when a firm fires a ‘bad’ worker, or hires a ‘good’ one, it contaminates the pool of the unemployed and increases the hiring (and subsequent firing) cost of other hiring firms. These externalities imply that a planner facing the same informational constraints will choose to employ some ‘unproductive’ workers, and possibly all worker types (even those who produce no output). We demonstrate that firms, in ignoring the externalities just described, ‘almost always’ employ ‘too few’ worker types, i.e., immediately shed ‘too many’ of their initially hired workers to the pool of the unemployed. The rate of employment among ‘desirable’ workers may be ‘too high’ when workers have very low bargaining power, but is ‘too low’ when workers’ bargaining power high. Overall employment may then be either too low or too high when workers’ bargaining power is low, but it is always too low when workers have high bargaining power. In all cases, the composition of the employed labor force is inefficient: firms are too selective, and the average quality of employed workers is too high.

The government may correct these inefficiencies by taxing or subsidizing firms’ hiring and/or firing costs. In cases where firms select too high hiring standards, the government should implement measures that make firms ‘less choosy’ about what workers to keep after having been screened for productivity. Such measures include making it more costly for firms to immediately shed workers, and making it less costly to fire initially retained workers at a later stage when they become redundant. An overall optimal solution will then generally also require that the government either tax or subsidize firms’ hiring. The greater workers’ bargaining power is, the more likely it is that such costs should be subsidized.

On the positive side, our model predicts the unemployment rate to be much higher, and the turnover rate to be lower and thus average unemployment spells

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13 Indersæt (1999) presents a model of wage posting which extends the analysis of Moen (1997) to the case of labor heterogeneity. He also shows that a labor union can be used as a device for committing firms to a high wage, so as to attract high-productivity workers. Ellingsen and Rosén (1997) compare wage posting and individualized bargaining in a related context, while Strand (1999) compares centralized and individualized bargaining. In both cases, it is demonstrated that greater worker heterogeneity makes individualized bargaining a more stable institution, and the other institutions less stable. In both cases it is moreover shown that there will be individualized bargaining only when labor is heterogeneous.
longer, for low-skilled as compared to high-skilled workers. Higher unemployment among the unskilled is found in basically all advanced economies, as reported by Nickell and Bell (1995; 1996) and Jackman et al. (1997), even in the US where the labor market is thought to be much more flexible than in Europe. Average spells of unemployment are also longer for the unskilled in all countries, but this tendency is stronger in Europe than in the US. Our model gives a reasonable description of such phenomena, and thus possibly a relevant (but of course much simplified) starting point for understanding the factors behind them.

In such a context we demonstrate the important role that turnover costs play, both in affecting the unemployment rates of different worker groups and in creating allocative inefficiencies in the labor market. In particular, we demonstrate the potentially very adverse effects of high dissipative firing costs for ‘established’ workers when such costs are paid by firms. Higher firing costs increase the skill-specific level of unemployment, and in addition raise firms’ minimum hiring standards to inoptimally high levels. One may argue that such costs are much higher in Europe than in the US (in particular because the legal obstacles to terminating ‘tenured’ workers are much greater in Europe), and that an obvious policy issue should be to bring these costs down. Final redundancy payments are in our model shown to have fewer such negative effects, and should thus perhaps not be discouraged in the same degree. Another result is that one may visualize cases where getting rid of workers who have no tenure, and thus no established legal rights, should be made more costly to firms. We namely show that high termination costs add to firms’ total labor costs, making firms require inoptimally high productivity levels of workers to be retained. This effect will be counteracted by higher costs of immediately shedding newly hired workers.

In focusing on the effects of turnover costs and labor heterogeneity, we have deliberately played down or disregarded a number of potentially important features. In our model all actual firings among desirable ‘tenured’ workers are fully exogenous, and the economy studied is stationary, firms facing constant demand conditions and productivities over time. Our analysis is thus quite distinct from other recent contributions where worker turnover and turnover costs are central, such as Bertola (1990) Lazear (1990), and Bentolila and Bertola (1990), dealing with turnover costs in a more partial-equilibrium (but not necessarily stationary) setting, and Bertola and Caballero (1994), Mortensen and Pissarides (1994) and Saint-Paul (1995) who consider bargaining models with turnover and turnover costs but where labor is assumed to be homogeneous. In particular, our conclusion that increased dissipative firing costs are always harmful to employment contrasts sharply with that of Bentolila and Bertola, who study a model where firms’ productivities change over time and where the main effect of firing costs is to make firms more choosy about later firings. Extensions of our framework, which are possible avenues for future research, may be to incorporate changing business conditions or productivities over time (as, e.g., in Bentolila–Bertola and Mortensen–Pissarides), on-the-job search (as in (Pissarides, 1994)), stochastic
match values (as in Bertola–Caballero), and the possibility of rehiring laid-off workers. Incorporating such alternative assumptions could also further serve to integrate the theories of contracts and matching when labor is heterogeneous, and clarify their relationships when there are positive turnover costs.

In our model, we have also made the special assumptions that firms have no prior information about individual workers at the time they are hired; and that all productivity differences are purely general. In a related paper (Strand, 1998) I have explored the case where firms have information on whether or not a given worker was last dismissed immediately, or fired because his job was shut down. I find that many of the qualitative conclusions derived here remain to hold, in particular, I still find that employment is always suboptimal when workers’ bargaining strength is not ‘too low’. Other assumptions, with respect to firms’ information about individual workers’ productivities, or with respect to the distribution of workers’ productivities between a general and a firm-specific component, should be explored in future work.

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Appendix A. Derivation of the firm’s sampling distribution over worker qualities

We here wish to derive the sampling distribution $G_z$, for firms’ sampling of workers from the pool of unemployed. First define the distribution over qualities for the entire set of unemployed workers, $G_z$. The fraction of workers in the entire labor force that has $z < z_1$ is $G(z_1)$, and the equivalent fraction among the unemployed is denoted by $G_u(z_1)$. Note that

$$NG(z_1) = U G_u(z_1) = U_1, \quad (A1)$$

where $U$ is the total number of unemployed workers, and $U_1$ is the number of unemployed workers with $z < z_1$, since all workers with $z < z_1$ are unemployed at equilibrium (when each is employed by firms only for an infinitely short period of time after being hired).

For a worker with $z \geq z_1$, the unemployment rate $u_z$ equals the average fraction of the time spent in unemployment, $s/(s + h)$, while the employment rate equals the fraction of the time spent in employment, $h/(s + h)$. Denoting by $U_z$ and $N_z$
the number of workers with \( z \geq z_1 \) in unemployment and employment, respectively. We then have

\[
U_z = N \left[ 1 - G(z_1) \right] \frac{s}{s + h}, \quad (A2)
\]

\[
N_z = N \left[ 1 - G(z_1) \right] \frac{h}{s + h}. \quad (A3)
\]

Since \( U_1 = NG(z_1) \), \( N_1 = 0 \), we then find

\[
G_n(z_1) = \frac{(s + h)G(z_1)}{s + hG(z_1)} \quad (A4)
\]

Since \( G_n(z) \) has density proportional to \( G(z) \), piecewise on \([z_0, z_1]\) and on \([z_1, \infty)\), we find

\[
G_n(z) = \frac{s + h}{s + hG(z_1)} G(z), \quad z < z_1
\]

\[
= \frac{(s + h)G(z_1)}{s + hG(z_1)} + \frac{s}{s + hG(z_1)} \left[ G(z) - G(z_1) \right], \quad z \geq z_1. \quad (A5)
\]

Since the relative frequency with which a worker with \( z < z_1 \) will be sampled by the firm is one, the probability that the firm samples such a worker is

\[
G_n(z_1) = \frac{G_n(z_1)}{G_n(z_1) + 1 - G_n(z_1)} = 1 - \phi. \quad (A6)
\]

which yields, using Eq. \(A4)\),

\[
1 - \phi = \frac{(s + h)G(z_1)}{(s + h)G(z_1) + s[1 - G(z_1)]}. \quad (A7)
\]

The firm’s sampling distribution is then given by

\[
G_s(z) = \frac{(s + h)}{(s + h)G(z_1) + s[1 - G(z_1)]} G(z), \quad z < z_1
\]

\[
= \frac{(s + h)G(z_1)}{(s + h)G(z_1) + s[1 - G(z_1)]}
\]

\[
+ \frac{s}{(s + h)G(z_1) + s[1 - G(z_1)]} \left[ G(z) - G(z_1) \right], \quad z \geq z_1. \quad (A8)
\]
Appendix B. Comparative-static results of changes in $F_0$

The effects of changes in $F_0$ on $h$ and $z_1$ are:

$$\frac{dh}{dF_0} = -\frac{1}{D} \left( r + s + \beta h \right) \left[ \frac{D(z_1)}{1-\beta} - \frac{(s + h)}{s} \frac{r + s + \beta h}{1-\beta} \right] \times \frac{1 - 2G(z_1)}{(1 - G(z_1))^2} \left[ g(z_1)(H + F_0) + \frac{1 - \phi}{\phi} \right]$$

$$\frac{dz_1}{dF_0} = -\frac{1}{D} \left( r + s + \beta h \right) \left[ \frac{\beta}{1-\beta} \phi \left( (1 - \beta + \beta \phi)F_1 \right) + H + \beta(1 - \phi)F_0 \right] + \frac{1}{s} \left( r + s + \beta h \right) \left( H + F_0 \right) \frac{G(z_1)}{1 - G(z_1)}$$

where

$$Dz_1 = (E_\zeta(z - z_1) / [1 - G(z_1)])$$

and

$$D = \frac{\beta}{(1 - \beta) \phi} \left[ H + (1 - \phi + \beta \phi D(z_1))F_0 + (1 - D(z_1))F_1 \right]$$

$$+ \frac{1}{s} \left[ \frac{1}{1 - G} \left( r + s + \beta h \right) \left[ G + \frac{\beta}{1 - \beta} \left( (1 - 2G(z_1)) \right) \right] \times (s + h)(F_1 - F_0) \right] \left( H + F_0 \right)$$

which is positive by virtue of the second-order conditions for an internal optimal solution being fulfilled.

References


