The labor market effects of non-wage compensations

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Abstract

Contrary to the argument that non-wage compensation is a tax on labor reducing employment, we find that employment may increase in response to an increased demand for benefits (a decreased cost of providing benefits or increased government-mandated benefit levels), under the assumption of strong cross-economies of scale. When there are strong cross-diseconomies of scale, employment and hours both decrease. The secular increase in employer-provided insurance and the growth in U.S. employment may well reflect the role of cross-economies of scale, which seems to exist in larger firms with lower marginal non-wage benefit costs. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Non-wage compensation measured as a percentage of total labor compensation has risen significantly in the United States and other developed countries over the past few decades (Hart et al., 1988). For example, in the firms surveyed by the...
Chamber of Commerce, fringe benefits grew from 16.0% of total compensation in 1951 to 31.6% in 1994 (see Table 1). Two fringe benefit items have experienced particularly large increases over this period: Social Security has increased from 1.1 to 5.5% of total compensation, while the insurance component (life, health, and dental premiums; death benefits) has risen from 1.1 to 8.4%.¹ Existing economic analyses suggest that, to the extent that these non-wage payments are quasi-fixed costs, such increases in fringe benefits should reduce employment.² The argument is that non-wage compensation is a tax on labor that increases total labor cost inducing employers to reduce employment.

Because they have been conducted at the firm level, previous analyses tend to assume non-wage payments to be exogenous.³ As Fig. 1 demonstrates, both the legally required (exogenous) and the voluntary (endogenous) components of fringe benefits have grown substantially over the past 40 years, with the voluntary component far outpacing the legally required component after the early 1980s. Clearly, a comprehensive analysis would require that non-wage compensation be modeled as endogenous since the voluntary increases in these payments surely are induced by the market.

Accordingly, we present a market level analysis to see how an increase in non-wage benefits affects equilibrium employment, wages, and benefit levels. We focus on three exogenous sources of increased benefits; an increase in the demand for benefits, a decrease in the cost of providing benefits, and an increase in the mandated benefits. Contrary to the conventional view, our analysis shows that employment may increase, rather than decrease, in response to increased non-wage payments.

Our predictions depend critically on whether there exist cross-scale effects. There exist cross-economies of scale if an increase in benefits (employment) lowers the marginal non-wage cost of employment (benefits). The opposite case exhibits cross-diseconomies of scale.

Our main findings are as follows. When there are strong cross-economies of scale, employment and benefits rise and hours of work and the wage fall.

¹ The U.S. Bureau of Labor Statistics (BLS) (United States Department of Labor, Bureau of Labor Statistics, 1977, 1995) is another source of benefit data, but its data is available only for the years 1966–1974 and 1987–1995. The available BLS data show similar trends in the growth of the components of fringe benefits. For example, social security climbed from 3.2 to 6.0% of total compensation from 1966 to 1995; insurance rose from 2.0 to 6.7% over the same period. Overall, fringe benefits grew from 16.9 to 28.3% of total compensation. [See Hashimoto, 1994 for an update in Table 1 of Woodbury, 1983.]

² This analytical result seems robust to various assumptions. In particular, when the model of employment-hours decisions is expanded to allow for changes in capital, many of the results concerning hours become ambiguous; however, the fixed-cost effect on employment remains intact (Hart, 1984; Hamermesh, 1993).

³ See, for example, Oi (1962), Rosen (1968), Ehrenberg (1971), Hart (1984), and Hamermesh (1993).
Table 1
Components of fringe benefits as a percent of total compensation (firms surveyed by the Chamber of Commerce). 1951 = 100 (Chamber of Commerce of the United States, 1981)

The numbers in parentheses are the percentage of fringe benefits in total compensation.

Legally required benefits are contributions to social security, federal and state unemployment insurance, and workers’ compensation.

Benefits provided voluntarily by the employer include private insurance (life, health, and accidental), privately sponsored retirement and savings plans (pensions, savings and thrift plans), as well as other items (severance pay, supplemental unemployment benefits, and other miscellaneous benefits).


<table>
<thead>
<tr>
<th>Total Fringe benefits</th>
<th>Legally required</th>
<th>Voluntary</th>
<th>Inside payroll</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Social security</td>
<td>Workers’ compensation</td>
<td>Other</td>
</tr>
<tr>
<td>1951</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>(16.0)</td>
<td>(1.1)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>1955</td>
<td>113.7</td>
<td>174.8</td>
<td>87.4</td>
</tr>
<tr>
<td>1959</td>
<td>124.4</td>
<td>172.7</td>
<td>138.2</td>
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<tr>
<td>1963</td>
<td>140.6</td>
<td>241.0</td>
<td>120.5</td>
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<tr>
<td>1967</td>
<td>143.9</td>
<td>315.9</td>
<td>157.9</td>
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<tr>
<td>1971</td>
<td>163.0</td>
<td>360.7</td>
<td>160.3</td>
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<td>1975</td>
<td>182.4</td>
<td>439.5</td>
<td>198.5</td>
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<td>1979</td>
<td>186.1</td>
<td>435.7</td>
<td>249.1</td>
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<tr>
<td>1984</td>
<td>182.7</td>
<td>494.4</td>
<td>179.5</td>
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<tr>
<td>1988</td>
<td>187.2</td>
<td>522.6</td>
<td>166.7</td>
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<tr>
<td>1992</td>
<td>199.4</td>
<td>517.5</td>
<td>221.8</td>
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<tr>
<td>1993</td>
<td>197.9</td>
<td>495.4</td>
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<tr>
<td>1994</td>
<td>198.0</td>
<td>514.3</td>
<td>166.0</td>
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<tr>
<td></td>
<td>(31.6)</td>
<td>(5.5)</td>
<td>(0.9)</td>
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</tbody>
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there are strong cross-diseconomies of scale, however, employment and hours both fall, but the effects on benefits and wage are ambiguous. It is worth noting that, under reasonable assumptions about cross-economies, an increase in fringe benefits can raise equilibrium employment. Note that, at least in the United States, insurance has emerged as not only the fastest growing component of fringe benefits but also the largest. It is possible that increases in insurance and other benefits may have contributed to the growth of U.S. employment during the past several decades.

2. Description of the problem

We conduct the analysis using the social welfare maximization problem, where social welfare refers to the sum of the employer and employee surpluses. Suppose the inverse demand and supply functions of labor are, respectively,

$$W^d = W^d(E) = f(E; h, G),$$
$$W^s = W^s(E) = g(E; h, G).$$

These functions are parameterized by standard hours, $h$, and non-wage benefits, $G$. Social welfare is given by

$$Z = Z(E, h, G) = \int_0^E [W^d(t) - W^s(t)] dt,$$
where $E$ is the equilibrium level of employment. Market forces drive social welfare to its maximum,

$$\max\{Z(E,h,G)|E,h,G \geq 0\},$$

satisfying the first order conditions:

$$\frac{\partial Z}{\partial E} = 0, \quad \frac{\partial Z}{\partial h} = 0, \quad \frac{\partial Z}{\partial G} = 0.$$  \hspace{1cm} (5)

One may analyze the effects of exogenous changes (such as an increase in worker demand for benefits) by totally differentiating the equations summarized by Eq. (5). In order to conduct such analysis, the second order condition, that the Hessian matrix of $Z(E,h,G)$ be negative definite, must hold. Rather than arbitrarily imposing this key condition, we derive the market labor demand from the individual firm’s profit-maximization problem in such a way as to be consistent with the second order condition. In fact, our formulation of the firm’s profit-maximization problem leads to the simplest possible functional form for the market labor demand supporting the second order condition. \(^4\)

2.1. Firm’s decision

Assume that a competitive industry consists of identical firms and that a firm’s labor expenses consist of both wage and non-wage costs as follows:

$$\theta = \theta(n,h,G) = whn + C(G,n)n,$$  \hspace{1cm} (6)

where $n$ is employment, $h$ is hours of work per worker, $G$ is the quantity of fringe benefits per worker, and $w$ is the hourly wage rate. \(^5\) We assume the non-wage cost function, $C$, to satisfy the following conditions:

$$C_1 > 0, \quad C_{11} > 0, \quad C_2 \geq C_{22} = 0, \quad C_{12} = C_{21} \geq 0.$$  \hspace{1cm} (7)

We call $C_1$ the marginal non-wage cost of benefits, and $C_2$ the marginal non-wage cost of employment. We specify the marginal non-wage cost of benefits to be positive ($C_1 > 0$) and rising ($C_{11} > 0$). For simplicity, the marginal non-wage cost of employment is assumed be a constant ($C_{22} = 0$), which may be any real number ($C_{21} \geq 0$).

The expression $C_{12}$ (or $C_{21}$) represents cross-scale effects. If $C_{12} < 0$, an increase in benefits ($G$) lowers the marginal non-wage cost of employment ($C_2$). Symmetrically, an increase in employment ($n$) lowers the marginal non-wage cost

\(^4\)Note that the market labor demand and supply functions cannot both be linear. If this were the case, the Hessian matrix for the social welfare function, $Z$, would not be negative definite. Under the simplification of linear labor supply, the simplest demand form is quadratic.

\(^5\)We assume for simplicity that non-wage labor costs are quasi-fixed, or that they are independent of hours of work. Although some non-wage labor costs do depend upon hours of work, approximately 20% of total labor costs in both the US and OECD countries are quasi-fixed (Hart et al., 1988).
of benefits \((C_i)\). We define these phenomena as cross-economies of scale. In contrast, the case of \(C_{12} > 0\) represents cross-diseconomies of scale.

We argue that cross-economies are empirically plausible, since larger firms are likely to have lower marginal non-wage cost of benefits for the following reasons. First, larger firms can negotiate better deals with benefit providers. Second, larger firms may have more specialized and higher skilled benefit administrators. Third, larger firms may have better technologies for administering benefit programs, e.g., processing claims by using computers rather than manually.

The firm’s production function is given by

\[ Q = F(n, h), \]

where \(Q\) denotes output and \(F_1, F_2, F_{12} > 0\) and \(F_{11}, F_{22} < 0\), implying that the marginal products are positive and decreasing and that the two inputs are complements.

The firm’s profit function is given by

\[ \pi = \pi(n, h, G) = pF(n, h) - \left[ wh + C(G, n) \right]n, \]

where \(p\) is the product price and \(\pi\) is assumed to be a concave function. The firm maximizes its profits subject to a fixed reservation level of workers’ utility. Assume that all workers are identical and have a utility function \(U = U(Y, X, G)\), for which \(Y = wh\) denotes a worker’s earnings, \(X\) denotes his leisure, and \(G\) denotes his benefits. The firm’s decision problem then has the form

\[
\begin{align*}
\max & \quad \pi(n, h, G) = pF(n, h) - \left[ wh + C(G, n) \right]n \\
\text{s.t.} & \quad n \geq 0, \quad h \geq 0, \quad G \geq 0, \quad \text{and} \quad U = U_0.
\end{align*}
\]

For any given fixed combination of output price, the wage rate, and reservation utility \((p, w, U_0)\), the firm selects the optimal combination of employment, hours, and benefits \((n, h, G)\) by satisfying the first order conditions associated with Eq.

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6 A study by the US Congressional Budget Office (Congress of the United States — Congressional Budget Office, 1992, Table 4, p. 32) reports that, in employment-based health insurance programs, larger firms enjoy greater cost advantages in billing, advertising, sales commissions, claims administration, and general overhead. Thus, total administrative expenses for health insurance as a percent of benefit costs are 40% for firms with 1–4 employees, 18% for firms with 50–99 employees, 8% for firms with 250–999 employees, and 5.5% for firms with 10,000 or more employees.

7 As in Nadiri and Rosen, 1969, Brechling, 1977 and Hart, 1984, capital, which is assumed to be fixed, is ignored in our analysis. Alternatively, one could include \(G\) as an argument in the production function, but such an extension is left for future study.

8 Let \(L(\lambda; n, h, G) = \pi(n, h, G) - \lambda(U - U_0)\) denote the Lagrangian. The first order conditions are:

\[
\begin{align*}
L_n &= pF_1 - \left[ wh + C(G, n) \right] + \lambda_n = 0, \quad L_h = pF_2 - wh - \lambda(\frac{dU}{dh}) = 0, \quad L_G = -C_1 n - A U_G = 0, \quad L_\lambda = -(U - U_0) = 0.
\end{align*}
\]
The first order condition with respect to employment (i.e., \( L_n = 0 \) in Footnote 8) generates the firm’s inverse demand for labor, parameterized by \( h \) and \( G \) as follows:

\[
w = w(n; h, G) = (pF_i - C - nC_z)/h. \tag{10}
\]

The next task is to derive the market demand for labor. \(^9\)

2.2. The market demand for labor

The simplest model specification satisfying the second order condition is a negative-definite quadratic form for the welfare function \( (3) \). Assuming a linear labor supply, this functional form is equivalent to having the following market labor demand function:

\[
w^d = f(E; h, G) = \tau(h, G) + \beta(h, G)E
\]

\[
= a_0 + (a_1 h + a_2 h^2) + (a_3 G + a_4 G^2) + \beta(h, G)E, \tag{1}^9
\]

where the intercept \( \tau(h, G) \) is quadratic and the slope \( \beta(h, G) \) is linear. Note that Eq. (1) is linear in \( E \) and quadratic in \( (h, G) \). \(^10\)

The derivative \( \partial \beta/\partial G \) is critical to our analyses. Eq. (A7) shows that \( \partial \beta/\partial G = -2C_{12}/(Kh) \), indicating that the sign of \( \partial \beta/\partial G \) depends solely on the sign of \( C_{12} \). As noted earlier, the term \( C_{12} \) defines cross-scale effects. Therefore, \( \partial \beta/\partial G \geq 0 \) represents cross-economies of scale \( (C_{12} \leq 0) \) while \( \partial \beta/\partial G < 0 \) represents cross-diseconomies of scale \( (C_{12} > 0) \).

Note that Eq. (1) is derived from the \( \pi \)-max problem \( (9) \), and it is the simplest form yielding the second order condition for welfare maximization. Appendix A provides a detailed derivation \(^11\) of the market demand for labor from the firm’s demand for labor (10). The demand function (1) is a linear approximation of the inverse market demand with a quadratic intercept and a linear slope (see Eqs. (A4))

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\(^9\) The firm’s demand for labor is a function of the full labor cost (= wage+benefit cost). The inverse demand functions plotted against the wage, therefore, generate a family of curves, where each curve is parameterized by \( h \) and \( G \).

\(^10\) One could further simplify Eq. (1) by removing the first order terms \( h \) and \( G \), but such a change does not affect the analysis.

\(^11\) The derivation assumes a product-price feedback given by \( p = p(E) \), \( p' < 0 \) (i.e., an increase in \( E \) raises aggregate output and therefore reduces the product price), and proceeds in two steps. We first obtain the second order Taylor approximations for the firm’s production and cost functions. This is equivalent to assuming that both the cost and the production functions are quadratic. Using more general functional forms would make the model too complex to handle. We then linearly approximate the market demand function in terms of \( E \) assuming that \( (h, G) \) affect the intercept quadratically and the slope linearly.
and (A5)). As noted earlier, these approximations yield the simplest social welfare function needed for comparative statics. 12

Fig. 2a portrays the labor demand \( W = f(E; h, G_0) \) with the value of \( G \) fixed at \( G_0 \), and Fig. 2b portrays a special case of a two-dimensional demand curve \( W = f(E) \) obtained by fixing \( h \) at \( h_0 \) as well. Note that an increase in \( G \) always reduces the intercept, because \( \partial \beta / \partial G < 0 \). 13 With cross-economies of scale (i.e., \( \partial \beta / \partial G > 0 \)), an increase in \( G \) makes the demand curve flatter (Fig. 2b) and with cross-diseconomies, steeper (Fig. 2c).

12 The negative definiteness of the welfare function is usually assumed. In our model, this condition almost follows from the standard conditions on the cost and production functions. Since our derivation shows that \( a_0 > 0 \), \( \beta < 0 \), and \( a_0 < 0 \), we only need to assume that \( a_2 < 0 \). See Footnote 16 for a related discussion.

13 This follows from Eq. (A4) and \( C_i > 0 \).
2.3. Market supply of labor

We abstract from the complexities of the worker’s labor participation decision by specifying the inverse market labor supply function to be of the following form, which is constant with respect to $E$ and parameterized by $(h, G)$:

$$w^* = g(E; h, G) = \alpha_0 + \alpha_1 h - \alpha_2 G,$$

(2)*

where $w^*$ is the workers’ asking wage and $\alpha_0$, $\alpha_1$, $\alpha_2 > 0$ are constants. The simplifying assumption that the supply curve is horizontal, shifting downward as either $G$ increases or $h$ decreases, is consistent with the worker’s utility function given earlier in the context of the firm’s problem. For example, when $U(Y, X, G)$ is linear and a worker takes $w$, $h$, and $G$ as fixed, $U_{\text{max}}$ leads to the same relationship among $w$, $h$, and $G$ as in Eq. (2)*.

Fig. 3a shows the linear labor supply function $W = g(E; h, G_0)$ with $G$ fixed at $G_0$. If one also fixes $h$ at $h_0$, one obtains a horizontal supply curve. This supply curve shifts down when $G$ increases from $G_0$ to $G_1$ (Fig. 3b).

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8 Our main predictions (i.e., effects in the presence of strong cross-scale effects) continue to hold with upward sloping curves. This result can be demonstrated as follows. Suppose the horizontal supply curves in Fig. 6 are replaced by upward sloping curves. By assuming that an increase in benefits has no effect on the slope of the supply curve and that it only shifts down the curve, one derives the same predictions about employment because of the large changes in the slope of demand curves caused by the strong cross-scale effects.
2.4. Market equilibrium

The competitive market equilibrium is the solution that maximizes the sum total of the surpluses of employers and employees. The process of reaching this equilibrium is illustrated in three steps. First, the market optimizes on $E$ for each fixed $(h, G)$ by setting $\partial Z/\partial E = 0$. In the second step, the market chooses the optimum quantity of $G$ by setting $\partial Z/\partial G = 0$. In the third step, the market chooses the optimum value of $h$ by setting $\partial Z/\partial h = 0$.

The first step generates a map of locus points for all potential equilibrium combinations $(W, E)$ as a function of $G$ and $h$. The result is shown in Fig. 4, where each curve traces $(W, E)$ for all $G$ and a fixed $h$. As one moves down on the curve, $G$ increases and $w$ decreases. Fig. 5 depicts a special case of Fig. 4 obtained by fixing $h$ and $G$ at $(h_0, G_0)$; it shows the equilibrium point $(w_0, E_0)$.
where the demand and supply curves intersect each other. The second step chooses the optimal point on each curve by equating the marginal social cost of $G$ with its marginal social value. The third step selects the optimum hours $h^*$, or the optimum curve $L_2$ in Fig. 4.

To understand the curvature in Fig. 4, let us start at a very small value of $G$ (i.e., at the top end point of $L_1$). As $G$ increases, more workers decide to work and they are willing to work at lower wages than before. As a result, the equilibrium employment increases and the wage decreases, and the curve moves down and to the right. After a certain level of $G$ is reached (i.e., beyond point A), benefits become less attractive than before because of the rising marginal non-wage cost of benefits, so a further increase in benefits reduces both the equilibrium employment and the wage, and the curve moves down and to the left.

The above process is equivalent to solving the social welfare maximization problem summarized by Eq. (4), with the welfare function (3) specified as

$$Z(E, h, G) = \left[ (a_0 - \alpha_0) + \left( \frac{a_1}{a_2} \right) h + a_2 h^2 \right] + \left[ \left( \frac{a_3}{a_4} + a_4 G + a_4 G^2 \right) h \right] E + \beta(h, G) E^2 / 2 + \text{constant.}$$

(3)*

The optimality conditions appear in Eqs. (B1.1), (B1.2), (B1.3), (B2.1), (B2.2) and (B2.3). As discussed earlier, the derivative $\partial \beta / \partial G$ represents cross-scale effects. The effect of $\partial \beta / \partial G$ on equilibrium is illustrated in Fig. 4, where $h_1 > h_2 = h^* > h_3$ denote, respectively, the upper bound, the equilibrium level, and the lower bound of hours. The three corresponding $L$ curves are $L_1$, $L_2$, and $L_3$. Moving down the $L$ curve increases $G$ and decreases $w$. For convenience, Fig. 4 depicts three potential equilibrium points on the curve $L_2$. Readers are advised, however, that a unique map of locus points is associated with each value of $\partial \beta / \partial G$, so that only one equilibrium point is relevant on a particular curve. Note also that Fig. 4 indicates the maximum employment (for each fixed $h$) as being attained at equilibrium only when $\partial \beta / \partial G = 0$ (i.e., when the slope of the labor demand curve is constant with respect to $G$); otherwise, equilibrium employment is less than the maximum feasible level.

These properties are summarized in the following proposition.

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15 By Eqs. (B1.2) and (B1.3), the conditions, $\partial Z / \partial h = 0$ and, $\partial Z / \partial G = 0$ can be rewritten as $(a_1 + 2a_2 h) E + (E^2 / 2)(\partial \beta / \partial h) = \alpha_1 E$, $(a_3 + 2a_4 G) E + (E^2 / 2)(\partial \beta / \partial G) = - \alpha_2 E$. The left-hand (right-hand) sides in both expressions represent the effects of hours and benefits on labor demand (supply). Because the surplus integrates the difference between the demand and supply curves, the left-hand (right-hand) side is interpreted as the marginal social value (marginal social cost).

16 The second order condition is consistent with our assumptions on the cost and production functions. When, $\partial \beta / \partial h = \partial \beta / \partial G = 0$, the second order condition is equivalent to $\beta < 0$, $a_2 < 0$, and $a_4 < 0$, where $\beta < 0$ and $a_4 < 0$ follow directly from the firm’s decision problem.
Proposition 1. (a) In the absence of cross-scale effects (i.e., \( \partial \beta / \partial G = 0 \)), competitive equilibrium employment is at its maximum feasible level for a given \( h \); however, in the presence of cross-scale effects, equilibrium employment is less than its maximum feasible level; (b) In the presence of cross-economies of scale (i.e., \( \partial \beta / \partial G > 0 \)), the equilibrium wage lies above that wage associated with maximum employment (point B in Fig. 4); (c) In the presence of cross-diseconomies of scale (i.e., \( \partial \beta / \partial G < 0 \)), the equilibrium wage lies below that wage associated with maximum employment (point C in Fig. 4).

Under competition, the market maximizes the sum of the employer and employee surpluses, not employment. In (a) there are no cross-scale effects and maximizing surpluses happens to maximize employment. In (b) and (c), however, there are cross-scale effects and maximizing surpluses does not maximize employment.

We proceed to evaluate the effects of an exogenous increase in non-wage benefits on employment, hours, and wages. Increases in \( G \) are traced to an increase in the demand for \( G \) (i.e., an increase in \( \alpha_2 \) in Eq. (2)) , to a decrease in the cost of providing \( G \) (i.e., an increase in \( \alpha_3 \) in Eq. (1)) , and an increase in the mandated benefits.

3. Effects of increased demand for \( G \)

The demand for benefits may increase if, for example, a new law is introduced that taxes fringe benefits less heavily than wage earnings, or real income grows and fringe benefits are income elastic. An increase in the demand for \( G \) is represented by a worker’s willingness to reduce his asking wage, i.e., by an increase in \( \alpha_2 \) in Eq. (2). Proposition 2 summarizes our predictions.

Proposition 2. Consider the effects on \( E, G, h \) of an increase in the demand for \( G \) (i.e., an increase in \( \alpha_2 \)).

(a) If \( \partial \beta / \partial G \geq 0 \), then \( \partial E / \partial \alpha_2 > 0 \), \( \partial G / \partial \alpha_2 > 0 \), and \( \partial h / \partial \alpha_2 \) > 0 ;

(b) if \( \partial \beta / \partial G \) is close to \( +\infty \), then \( \partial E / \partial \alpha_2 > 0 \), \( \partial G / \partial \alpha_2 > 0 \), and \( \partial h / \partial \alpha_2 < 0 \).

Note that \( - \alpha_2 = \Delta w / \Delta G \) is the marginal value of \( G \). If \( G \) is increased by one unit, a worker is willing to reduce his asking wage by \( \alpha_2 \) dollars. Thus, an increase in the demand for \( G \) results in a downward shift of the horizontal supply curve (see Fig. 6). Similarly, a decrease in the cost of providing \( G \) increases the employer’s willingness to pay a higher wage resulting in an upward shift of the negatively sloped demand curve. This change is represented by the dominant factor (i.e., \( \alpha_3 \)) through which affects the demand function (1). The precise expressions for \( \partial E / \partial \alpha_2 \), \( \partial G / \partial \alpha_2 \) and \( \partial h / \partial \alpha_2 \) appear in Eqs. (B6.1), (B6.2) and (B6.3).

The expression ‘close to \( +\infty \) or \( -\infty \)’ means that the variable is such a large (positive or negative) finite number that it outweighs all other constants in the corresponding expressions. For example, the sign of \( \partial \beta / \partial h \) in Eq. (A7) is solely determined by \( C_{12} \); however \( C_{12} \), cannot be equal to \( +\infty \) or \( -\infty \) if the second order conditions are to hold.
(c) if $\frac{\partial \beta}{\partial G}$ is close to $-\infty$, then $dE/d\alpha_2 < 0$, $dh/d\alpha_2 < 0$, and the sign of $dG/d\alpha_2$ is ambiguous;
(d) if $\frac{\partial \beta}{\partial G} < 0$ is not very large, then the effects are ambiguous.

In (a), there may be zero or positive cross-scale effects. In either case, both employment and benefits increase. The intuition behind this result is straightforward. An increase in the demand for benefits is an increase in the marginal value of benefits. An increased demand for benefits makes it welfare enhancing to increase benefits and to decrease the wage in the compensation package. As a result, the equilibrium benefit level rises. This change in the compensation package in turn raises the value of employment to both employers and employees, resulting in an increased employment.

We illustrate employment effects in (b) and (c) using Figs. 2–6. In (b), there exist strong cross-economies of scale and employers now prefer giving employees more benefits and lower wages, so an increase in $G$ lowers the intercept and flattens the demand curve (Fig. 2b). At the same time, an increase in $G$ shifts down the supply curve, as workers trade off wages for benefits (Fig. 3b). As a result, the equilibrium employment increases (Fig. 6a). Hours of work decrease, however.

The intuition behind the above result is that increases in benefits and employment reinforce each other because of strong cross-economies of scale, causing benefits and employment to increase more than they do in (a). Hours decrease because employers substitute employment for hours and workers reduce hours of work in response to a decline in the wage (cf. Proposition 3).

In case (c), there exist strong cross-diseconomies of scale, so the increase in $G$ steepens the demand curve and lowers the intercept (Fig. 2c), resulting in a decrease in equilibrium employment (Fig. 6b). Here, cross-diseconomies of scale

![Diagram](image)

Fig. 6. Effects on $(W, E)$ of an increase in benefits $(G_i > G_j)$. (a) Employment increases when $d\beta/dG > 0$; (b) Employment decreases when $d\beta/dG < 0$. Note that $h$ is fixed at $h_0$ in both cases.
generate forces that are opposite to the positive influence on employment associated with case (a), and employment ends up falling.

For two reasons, we believe that (a) and (b) may be the most relevant cases in understanding the U.S. experience. First, insurance has been the fastest growing component of fringe benefits in recent years with the result that by 1994 it has become the largest benefit component (see Table 1). Second, since large firms enjoy cost advantages in providing insurance, an additional employee reduces the marginal cost of benefits (i.e., $C_{12}$ is negative). These considerations suggest that the secular increase in fringe benefits, to the extent that it is attributable to an increase in the demand for benefits, may have contributed to employment growth.

Because $w^d = w^s = w$ in equilibrium, the effect of a demand driven change in $G$ on the wage rate $w$ is ascertained from the following equation:

$$\frac{dw}{d\alpha_2} = \alpha_1 \frac{dh}{d\alpha_2} - \left[ \alpha_2 \left( \frac{dG}{d\alpha_2} \right) + G \right].$$

(11)

Thus, the effect on the wage consists of two components — the term $\alpha_1 \left( \frac{dh}{d\alpha_2} \right)$, which represents the effect operating through work hours, and the term $\left[ \alpha_2 \left( \frac{dG}{d\alpha_2} \right) + G \right]$, which represents the effect working through benefits. The wage could increase or decrease, depending on cross-scale effects. Predictions about the wage effect are as follows:

**Proposition 3.** Consider the wage effect of an increase in the demand for $G$ (i.e., an increase in $\alpha_2$).

(a) If $\partial \beta / \partial G = 0$, then $\frac{dw}{d\alpha_2} < 0$;

(b) if $\partial \beta / \partial G$ is close to $+\infty$, then $\frac{dw}{d\alpha_2} < 0$;

(c) the wage effect is ambiguous in all other cases.

In (a), the wage falls because substituting benefits for the wage is mutually beneficial to employers and employees. The wage falls even more in (b) because strong cross-economies of scale make it attractive to substitute benefits for wages more than in (a). Our model does not predict the wage effects in other situations.

4. Effects of decreased cost of $G$

Non-wage benefits may increase if the cost of providing fringe benefits decreases. A decrease in the cost of $G$ is represented by the employer’s willingness to increase the wage rate, i.e., an increase in $\alpha_1$ in Eq. (1). Intuitively, a decrease in the cost of $G$ should have the same effect as an increase in its demand. Our analysis shows that our intuition is indeed correct. By totally differentiating the first order condition, we obtain the following results:

$$dE/d\alpha_1 = dE/d\alpha_2, \ dh/d\alpha_1 = dh/d\alpha_2, \text{ and } dG/d\alpha_1 = dG/d\alpha_2.$$

(12)
In other words, the effect of a decrease in the cost of \( G \) is equivalent to the effect of an *increase* in the demand for \( G \).

The wage effect is evaluated via the following equation:

\[
\frac{dw}{da} = \alpha_1 \frac{dh}{da} - \alpha_2 \frac{dG}{da}.
\]

(13)

We, therefore, have the following proposition:

**Proposition 4.** Consider the effects on \((E, G, h)\) and on \( w \) of a decrease in the cost of providing benefits (i.e., an increase in \( a_3 \)).

(a) If \( \frac{\partial \beta}{\partial G} = \frac{\partial \beta}{\partial h} = 0 \), then \( \frac{dE}{da} > 0 \), \( \frac{dG}{da} > 0 \), \( \frac{dh}{da} = 0 \), and \( \frac{dw}{da} < 0 \);

(b) if \( \frac{\partial \beta}{\partial G} \geq 0 \), then \( \frac{dE}{da} > 0 \), \( \frac{dG}{da} > 0 \), and \( (\frac{dh}{da})(\frac{\partial \beta}{\partial h}) > 0 \), the wage effect is ambiguous;

(c) if \( \frac{\partial \beta}{\partial G} \) is close to \( +\infty \), then \( \frac{dE}{da} > 0 \), \( \frac{dG}{da} > 0 \), \( \frac{dh}{da} < 0 \), and \( \frac{dw}{da} < 0 \);

(d) if \( \frac{\partial \beta}{\partial G} \) is close to \( -\infty \), then \( \frac{dE}{da} < 0 \) and \( \frac{dh}{da} < 0 \), the effects on the wage and benefits are ambiguous;

(e) if \( \frac{\partial \beta}{\partial G} < 0 \) is not a large number, all effects are ambiguous.

The interpretations of these predictions are similar to those in Propositions 2 and 3, so we will not repeat them. As previously, we posit that (b) and (c) may be the most relevant to understanding the U.S. experience. In particular, the postwar growth in insurance, to the extent it is attributable to a decrease in its cost, may well have contributed to employment growth.

5. Effects of government-mandated increase in \( G \)

Government mandated benefits have been one of the major contributors to the growth of non-wage payments (cf. Fig. 1). If the government stipulates the minimum quantity of fringe benefits provided by the employer, then \( G \) in the previous model is replaced by the mandated level \( G \). As a result, only two endogenous variables, \( E \) and \( h \), remain in the maximization problem (4). In the revised maximization problem, the first and second order conditions are unchanged except that their dimension is reduced from three to two. 20 Totally
differentiating the first order conditions with respect to \( \overline{G} \), we obtain the following:

\[
d E / d \overline{G} = \left( -2 \alpha_2 \frac{E}{|H^+|} \right) \left[ \alpha_2 + (a_3 + 2a_4 \overline{G}) + E(\beta \bar{\beta} / \bar{\partial G}) \right],
\]

\[
d h / d \overline{G} = \left( E/2 |H^+| \right) \left( \beta \bar{\beta} / \bar{\partial h} \right) \left[ \alpha_2 + (a_3 + 2a_4 \overline{G}) + E(\beta \bar{\beta} / \bar{\partial G}) \right] \]

\[
= \left( -1/4a_2 \right) \left( \beta \bar{\beta} / \bar{\partial h} \right) \left( d E / d \overline{G} \right),
\]

where \( |H^+| \) is the determinant of the new Hessian matrix \( H^+ \) (Footnote 20).

Note that the sign of \( d E / d \overline{G} \) is the same as

\[
\xi(G) \equiv \left[ a_3 + 2a_4 \overline{G} + E(\beta \bar{\beta} / \bar{\partial G}) \right] - ( - \alpha_2 ),
\]

where the term in brackets is the effect of \( G \) on labor demand, and the term \( (-\alpha_2) \) is the effect on labor supply. When there are strong cross-economies or cross-diseconomies of scale, \( \beta \bar{\beta} / \bar{\partial G} \) is a very large number so that it dominates in determining the signs of \( d E / d \overline{G} \) and \( d h / d \overline{G} \) in Eqs. (14a) and (14b). Now, define \( \overline{G}_0 \) as the value of \( G \) associated with the maximum feasible \( E \). In other words, \( \overline{G} \) is defined by

\[
\xi(\overline{G}_0) = 0, \text{ or } \left[ a_3 + 2a_4 \overline{G}_0 + E(\beta \bar{\beta} / \bar{\partial G}) \right] - ( - \alpha_2 ) = 0.
\]

Then, by

\[
\xi'(\overline{G}_0) = 2a_4 + (d E / d G) \beta \bar{\beta} / \bar{\partial G} \bigg|_{\overline{G}_0} = 2a_4 < 0,
\]

we \( d E / d \overline{G} = \xi(\overline{G}) < 0 \) if \( \overline{G} > \overline{G}_0 \) and \( d E / d \overline{G} = \xi(\overline{G}) > 0 \) if \( \overline{G} < \overline{G}_0 \). Finally, the following equation evaluates the wage effect of an increase in \( \overline{G} \):

\[
d w / d \overline{G} = \alpha_4 (d h / d \overline{G}) - \alpha_2,
\]

which depends critically on the sign of \( d h / d \overline{G} \). Proposition 5 summarizes the above predictions.

**Proposition 5.** Consider the effects on \( (E, h) \) and \( w \) of an increase in the mandated benefit levels, \( \overline{G} \).

(a) If \( \beta \bar{\beta} / \bar{\partial G} \) is close to \( + \infty \), then \( d E / d \overline{G} > 0 \), \( d h / d \overline{G} < 0 \) and \( d w / d \overline{G} < 0 \);

(b) if \( \beta \bar{\beta} / \bar{\partial G} \) is close to \( - \infty \), then \( d E / d \overline{G} < 0 \), \( d h / d \overline{G} < 0 \) and \( d w / d \overline{G} < 0 \);

(c) let \( \overline{G}_0 \) be given by Eq. (16). Then \( d E / d \overline{G} < 0 \Leftrightarrow \overline{G} > \overline{G}_0 \); and the effects on hours and the wage are ambiguous.

Employment effects of a mandated increase in benefits depend on the initial benefit level as well as the cross-scale effects. If there exist strong cross-economies (diseconomies) of scale, a mandated increase in \( G \) lowers (raises) the marginal non-wage cost of employment. This flattens (Fig. 2b) or steepens (Fig. 2c) the demand curve, resulting in an increase (Fig. 6a) or decrease (Fig. 6b) in equilibrium employment.
In (a), there exist strong cross-economies of scale, so the mandated increase in benefits reduces the marginal non-wage cost of benefits, causing employment to increase at the expense of hours. This prediction is the opposite of what one might expect using the traditional analysis because the traditional analysis ignores the influence of cross-economies of scale.

In (b), there exist strong cross-diseconomies of scale, and the mandated increase in benefits causes employment and hours to decrease because of a large increase in the marginal non-wage cost of employment. On the other hand, if the initial benefit levels are “high” (“low”), employment decreases (increases) regardless of cross-scale effects, but the effect on hours cannot be predicted.

In both (a) and (b), the strong cross-scale effects cause \( dG/dh = \) to be a large negative number. As a result, Eq. (17) is negative, i.e., the wage falls. This result suggests that the recent decline in work hours and the slow wage growth may reflect the effects of increases in mandated benefits.

As for (c), consider the special case of \( \partial G/\partial E = 0 \). Then \( G_0 \) is the competitive equilibrium level of benefits for which \( E \) is at its maximum feasible level. Therefore, employment decreases regardless of whether the government mandates that \( G \) be increased or decreased. There is nothing that the government can do to increase employment by regulating \( G \).

6. Summary

We have conducted a market level analysis to evaluate the effects of increased benefits on employment, hours of work and the wage rate. Our model facilitates an analysis of how employment, hours of work and the wage change in response to an increase in the demand for benefits, to a decrease in the cost of providing benefits, or to an increase in government-mandated benefit levels.

Our analysis shows that employment may increase, rather than decrease, in response to increased non-wage benefits. Our predictions are found to depend critically on cross-scale effects. When there are strong cross-economies of scale, employment increases in response to an increased demand for benefits, to a decreased cost of providing benefits, or to increased government-mandated benefit levels. Furthermore, hours of work and the wage both decrease. If there are zero or small cross-economies of scale, employment and benefits increase, but no clear predictions emerge either for hours or for the wage. When there are strong

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cross-diseconomies of scale, employment and hours both decrease, but the effects on benefits and the wage are ambiguous. If there are weak cross-diseconomies, the model generates ambiguous predictions.

Cross-economies of scale seems to be a reasonable assumption, at least for some sectors in the U.S. labor market, where employer-provided insurance coverage has emerged as not only the fastest growing fringe benefit but also the largest. Cross-economies in insurance are empirically plausible, because larger firms are likely to have lower marginal non-wage costs of benefits. If so, increased fringe benefits due to a number of different shocks might have contributed to, rather than hindered, employment growth during the past several decades in the US and elsewhere.

Several simplifying assumptions have helped make the analysis manageable. Further refinement of the model would enrich the analysis. First, one might model the worker’s decision on how many hours to work. Second, one might generalize the model by specifying the cost of providing benefits to depend on hours of work. Third, one might model workers as heterogeneous with respect to their tastes for fringe benefits and hours in order to generate an upward sloped supply curve.

Many of our predictions are testable in principle. To test them, one would begin by classifying observations (i.e., firms, industry sectors, or countries) according to cross-scale effects. Then, one would estimate the relationship between the changes in the dependent variables (i.e., employment, non-wage benefits, hours of work and wages) and the changes in the exogenous variables (i.e., the demand for benefits, the costs of providing benefits, and government mandates). Such estimated relationship would enable one to accept or reject our predictions.

Testing our theory by inter-country comparisons would be useful as well. Different countries have experienced different rates of rising non-wage labor costs. Among some of the OECD countries, for example, between the mid-1960s and the early 1990s, the proportion of non-wage labor cost grew by 101% in the UK, 70% in the US, 26% in Japan, 18% in Germany, and 4% in France. A careful examination of the sources of these differences and differences in employment growths would be useful for testing our theory.

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Appendix A. Derivation of market labor demand functions

I. Consider the second order Taylor approximations of $F$ and $C$, or assume that $F$ and $C$ are both quadratic as follows:

$$F(n,h) = F_{11}n^2/2 + 0.5F_{22}h^2 + F_{12}nh + \theta_1n + \theta_2h + \theta_0,$$

$$C(G,n) = C_{11}G^2/2 + C_{12}Gn + \phi_1G + \phi_2n + \phi_0.$$

(Note that $C$ is assumed to be linear in $n$, or that $C_{22} = 0$.) The individual labor demand functions then have the following form:

$$n = (hw - \Omega) / \Delta,$$  \hspace{1cm} (A1)

where

$$\Delta = pF_{11} - 2(C_{12}G + \phi_2),$$
$$\Omega = p(F_n - F_{11}n) - (C_{11}G^2/2 + \phi_1G + \phi_0).$$

Proof: We begin with the following expressions:

$$F_n = \partial F / \partial F = F_{11}n + F_{12}h + \theta_1 > 0,$$
$$F_h = \partial F / \partial h = F_{22}h + F_{12}n + \theta_2 > 0,$$

and

$$C_1 = \partial C / \partial G = C_{11}G + C_{12}n + \phi_1 > 0,$$
$$C_2 = \partial C / \partial n = C_{12}G + \phi_2 \geq 0.$$

The inverse demand for employment (10) is now given by

$$w = \left[ pF_n - (C_n + nC_2) \right] / h$$
$$= (1/h) \left[ pF_n - \left( C_{11}G^2/2 + C_{12}Gn + \phi_1G + \phi_2n + \phi_0 \right) 
+ n(C_{12}G + \phi_2) \right]$$
$$= (1/h) \left[ p(F_n - F_{11}n) - \left( C_{11}G^2/2 + \phi_1G + \phi_0 \right) n \right]$$
$$= (1/h) \left[ p(F_n - F_{11}n) - \left( C_{11}G^2/2 + \phi_1G + \phi_0 \right) n \right]$$
$$= (\Omega + \Delta n) / h$$

so that

$$n = (1/\Delta)(hw - \Omega)$$
$$\left[ h w - p(F_n - F_{11}n) + (C_{11}G^2/2 + \phi_1G + \phi_0) \right]$$
$$\left[ pF_{11} - 2(C_{12}G + \phi_2) \right]$$  \hspace{1cm} (A2)
is the individual labor demand function. Note that
\[ F_n - F_{i1}n = F_{12}h + \theta_1, \]
so that the right hand side of Eq. (A2) is a function of \( W \), parameterized by \( h, G \), and \( p \).

II. If (i) all firms are identical and if (ii) \( p = p(E) \), \( p' < 0 \), then the inverse market labor demand function has the form
\[ w = \tau(h,G) + \beta(h,G) E + \varepsilon(h,G,E), \quad (A3) \]
where
\[ \tau(h,G) = \frac{\text{const.} - C_{11}G^2/2 - \phi_1 G + p_0 F_{12}h}{h}, \quad (A4) \]
\[ \beta(h,G) = \frac{\text{const.} - 2C_{12}G + Kp'(F_{12}h + \theta_1)}{(Kh)}, \quad (A5) \]
and \( \varepsilon(h, G, E) \) is a residual term. [In Eq. (A4), \( p_0 > 0 \) is some constant.]

Proof: Since \( p = p(E) \) is a function of \( E \), the right-hand side of Eq. (A2) contains the term \( E \). Thus, the market labor demand function cannot be derived by multiplying Eq. (A2) by \( K \); it is instead derived by rearranging the expression after moving all \( E \) the terms to the left-hand side. This is done as follows.

Multiplying Eq. (A2) by \( K \), we have
\[ E = nK = K(hw - \Omega)/\Delta \]
from which we derive
\[ Khw = \Delta E + K\Omega \]
\[ = E[pF_{11} - 2(C_{12}G + \phi_2)] \]
\[ + K[p(F_n - F_{i1}n) - (C_{11}G^2/2 + \phi_1 G + \phi_0)] \]
\[ = -(C_{11}G^2/2 + \phi_1 G + \phi_0)K - 2(C_{12}G + \phi_2)E + KF_n p(E). \]
Substituting \( F_n = F_{i1}n + F_{12}h + \theta_1 \) and \( p(E) = p_0 + p'E + \cdots \) into the above expression, we arrive at
\[ w = \frac{\text{const.} - C_{11}G^2/2 - \phi_1 G + p_0 F_{12}h}{h} \]
\[ + \frac{\text{const.} - 2C_{12}G + Kp'(F_{12}h + \theta_1)}{(Kh)} E/(Kh) + \varepsilon(h,G,E) \]
\[ = \tau(h,G) + \beta(h,G) E + \varepsilon(h,G,E), \]
where \( \varepsilon(h, G, E) \) is a residual term containing all other higher order terms of \( E \).

Setting \( \tau(h, G) = a_0 + a(h, G) \), we have \( a_0 = p_0 F_{12} > 0 \) and \( a(h, G) = \text{const.} - C_{11}G^2/2 - \phi_1 G/h \). Thus,
\[ \partial\tau/\partial h = -(C_{11}G + \phi_1)/h, \quad \partial^2\tau/\partial G^2 = -C_{11}/h < 0, \quad (A6) \]
\[ \partial\tau/\partial G = -(C_{11}G + \phi_1)/h, \quad \partial^2\tau/\partial h^2 = 2a(h, G)/h^2, \]
\[ \partial\beta/\partial G = -2C_{12}/(Kh), \quad \partial^2\beta/\partial h = -\left(\text{const.} - 2C_{12}G + Kp\theta_1\right)/K\text{h}^2, \quad (A7) \]
Note that when Eq. (A3) is approximated by Eq. (1)*
\[ w_d = f(E; h, G) \]
\[ = a_0 + (a_1 h + a_2 h^2) + (a_3 G + a_4 G^2) + (b_0 + b_1 h + b_2 G) E, \]
where
\[ \tau(h, G) = a_0 + (a_1 h + a_2 h^2) + (a_3 G + a_4 G^2), \]
\[ \beta(h, G) = b_0 + b_1 h + b_2 G, \]
we have
\[ a_0 = p_0 F_{12} > 0, \quad a_2 = -C_{11} / h < 0, \quad b_0 = p' F_{12} < 0, \]
\[ a_1 + 2 a_2 h = \partial \tau / \partial h, \quad a_3 + 2 a_4 G = \partial \tau / \partial G, \]
and
\[ b_1 = \partial \beta / \partial h, \quad b_2 = \partial \beta / \partial G = -2 C_{12} / (K h). \]
The above expressions show that the signs of \( \partial \beta / \partial h \) and \( \partial \beta / \partial G \) are ambiguous. The assumption that \( a_2 < 0 \) is equivalent to the assumption that \( \alpha(h, G) < 0 \).

Appendix B. Derivation of the Hessian matrix

The first order condition (5) becomes
\[ \partial Z / \partial E = [a_0 - a_0 + [(a_1 - a_1) h + a_2 h^2] + [(a_3 + \alpha_2) G + a_4 G^2]] + \beta(h, G) E = 0, \]
(B1.1)
\[ \partial Z / \partial h = [(a_1 - a_1) + 2 a_2 h] E + (E^2 / 2) \partial \beta / \partial h = 0, \]
(B1.2)
\[ \partial Z / \partial G = [(a_3 + \alpha_2) + 2 a_4 G] E + (E^2 / 2) \partial \beta / \partial G = 0. \]
(B1.3)

It follows from the above equations and the linear assumption \[ \beta(h, G) = b_0 + b_1 h + b_2 G \] that
\[
H = \begin{bmatrix}
\partial^2 Z / \partial E^2 & \partial^2 Z / \partial E \partial h & \partial^2 Z / \partial E \partial G \\
\partial^2 Z / \partial h \partial E & \partial^2 Z / \partial h^2 & \partial^2 Z / \partial h \partial G \\
\partial^2 Z / \partial G \partial E & \partial^2 Z / \partial G \partial h & \partial^2 Z / \partial G^2
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\beta & a_1 - a_1 + 2 a_2 h + E h \beta / \partial h & a_3 + a_2 + 2 a_4 G + E h \beta / \partial G \\
\partial Z / \partial h & (a_1 - a_1 + 2 a_2 h + E h \beta / \partial h) & 2 a_2 E + (E^2 / 2) \partial^2 \beta / \partial h^2 \\
\partial Z / \partial G & (a_3 + \alpha_2 + 2 a_4 G + E h \beta / \partial G) & 2 a_4 E + (E^2 / 2) \partial^2 \beta / \partial G^2
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\beta & (E / 2) \beta \partial \beta / \partial h & (E / 2) \beta \partial \beta / \partial G \\
(E / 2) \beta \partial \beta / \partial h & 2 a_2 E & 0 \\
(E / 2) \beta \partial \beta / \partial G & 0 & 2 a_4 E
\end{bmatrix}
\]
The negative definiteness of $H$ is then equivalent to the following:

$$
\begin{align}
\frac{\partial^2 Z}{\partial E^2} &= \beta < 0, \quad \frac{\partial^2 Z}{\partial h^2} = 2a_2 E < 0, \quad \frac{\partial^2 Z}{\partial G^2} = 2a_4 E < 0; \\
\left(\frac{\partial^2 Z}{\partial E^2}\right) \left(\frac{\partial^2 Z}{\partial h^2}\right) - \left(\frac{\partial^2 Z}{\partial E \partial h}\right)^2 &= 2a_2 \beta E - \left(\frac{E}{2} \frac{\partial \beta}{\partial G}\right)^2 > 0, \\
\left(\frac{\partial^2 Z}{\partial h^2}\right) \left(\frac{\partial^2 Z}{\partial G^2}\right) - \left(\frac{\partial^2 Z}{\partial h \partial G}\right)^2 &= 4a_2 a_4 E^2 > 0; \\
\left|H\right| &= 4a_2 a_4 \beta E^2 - \left(a_4 E^3/2\right) \left(\frac{\partial \beta}{\partial h}\right)^2 - \left(a_2 E^3/2\right) \left(\frac{\partial \beta}{\partial G}\right)^2 < 0.
\end{align}
$$

**Proof of Proposition 1:** By fixing $h$, solving Eq. (B1.1) for $E$, and computing $dE/dG = 0$ to select the point of maximum employment, we obtain

$$
a_3 + a_2 + 2a_4 G + E \frac{\partial \beta}{\partial G} = 0. 
$$

(B3)

We then obtain the point on the $L$ curve corresponding to a competitive equilibrium by combining Eqs. (B1.1) and (B1.3) to arrive at

$$
a_3 + a_2 + 2a_4 G + \left(\frac{E}{2}\right) \frac{\partial \beta}{\partial G} = 0.
$$

(B4)

Clearly, Eqs. (B3) and (B4) are equivalent only when $\frac{\partial \beta}{\partial G} = 0$. Therefore, the competitive equilibrium level of employment corresponds to the maximum level of employment only when $\frac{\partial \beta}{\partial G} = 0$.

We now demonstrate parts (b) and (c) of this proposition. We first rearrange Eq. (B1.3) to obtain

$$
-(a_3 + a_2 + 2a_4 G) = \left(\frac{E}{2}\right) \frac{\partial \beta}{\partial G}.
$$

(B5)

Given that $a_3 < 0$, an increase in $G$ increases both the right- and left-hand sides of Eq. (B5). Because we impose a linearity assumption on $\beta$, $\frac{\partial \beta}{\partial G}$ is a constant. If $\frac{\partial \beta}{\partial G} > 0$, the right-hand-side increases only when $E$ increases. This result implies that we are at point B in Fig. 4 where the current equilibrium wage $w_B$ exceeds $w^*$. Similarly, if $\frac{\partial \beta}{\partial G} < 0$, $E$ falls when $G$ increases. In this case, we are at point C in Fig. 4. This completes the proof of Proposition 1.

**Proof of Proposition 2:** By totally differentiating the first order conditions given by Eq. (5) or Eqs. (B1.1), (B1.2) and (B1.3), we obtain the following comparative statics:

$$
\begin{align}
\frac{dE}{da_2} &= \left(a_2 E^2 / |H|\right) \left[-4a_4 G + E \frac{\partial \beta}{\partial G}\right] \\
\frac{dh}{da_2} &= \left(-E^2 / 4 \left|H\right|\right) \left(\frac{\partial \beta}{\partial h}\right) \left[-4a_4 G + E \frac{\partial \beta}{\partial G}\right] \\
&= -\left(1/4a_2\right) \left(\frac{\partial \beta}{\partial h}\right) \left(dE/da_2\right), \\
\frac{dG}{da_2} &= \left(-E^2 / \left|H\right|\right) \left[2a_2 \beta - \left(E/4\right) \left(\frac{\partial \beta}{\partial h}\right)^2\right] - a_4 G \frac{\partial \beta}{\partial G}.
\end{align}
$$

(B6.1) (B6.2) (B6.3)
where $|H|$ is the determinant of the Hessian matrix, $H$. Because $a_2$ and $|H| < 0$, $dE/d\alpha_2$ has the same sign as
\begin{equation}
-4a_2G + E\partial\beta/\partial G,
\end{equation}

while $dG/d\alpha_2$ has the same sign as
\begin{equation}
2a_2\beta - (E/4)(\partial\beta/\partial h)^2 - a_2G\partial\beta/\partial G.
\end{equation}

It then follows from Eq. (B2.2) that $2a_2\beta - (E/4)(\partial\beta/\partial h)^2 > 0$. Because $a_2 < 0$ and $\partial G/\partial h < 0$, both Eqs. (B7) and (B8) are positive when $\partial\beta/\partial G \geq 0$. The effect on hours follows directly from Eq. (B6.2) and from the fact that $\partial G/\partial h \leq 0$ when $\partial\beta/\partial G$ is a very large positive number (i.e., $C_{12}$ is a very large negative number, see Eq. (A7)). This proves the first two parts of Proposition 2. If $\partial\beta/\partial G$ is a very large negative number (i.e., $C_{12}$ is a very large positive number), Eq. (A7) implies that $\partial\beta/\partial h$ is positive. Thus, by Eqs. (B6.1) and (B6.2), $dE/d\alpha_2 < 0$ and $dh/d\alpha_2 < 0$. This completes the proof of Proposition 2.

**Proof of Proposition 3:** The conclusions follow from Eq. (11) and parts (a) and (b) of Proposition 2.

**Proof of Proposition 4:** The conclusions follow from Eqs. (12) and (13) as well as Propositions 2 and 3.

**Proof of Proposition 5:** When $\partial\beta/\partial G$ is close to $+\infty$ (i.e., $C_{12}$ is close to $-\infty$), Eqs. (A7), (14a) and (14b) jointly imply $dE/dG > 0$ and $dh/dG$ because $a_2 < 0$, $|H^*| > 0$, and $\partial\beta/\partial h < 0$. Similarly, when $\partial\beta/\partial G$ is close to $-\infty$, $dE/dG < 0$ and $dh/dG < 0$. Since $dh/dG < 0$ holds in both cases, it follows from Eq. (17) that $dw/dG$ is negative. Part (c) follows from the discussion preceding the proposition. This completes the proof of our last proposition.

**References**


