Is tax progression really good for employment?
A model with endogenous hours of work

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Abstract

This paper discusses the effect of tax progression on wage setting and employment in a unionised labour market. Recent contributions to this field argue that tax progression paradoxically enhances employment if wage setting is subject to collective bargaining. In this literature, individual hours of work are usually assumed to be exogenously given. We show that the positive employment effect of tax progression can be generalized to a model with a positive labour supply elasticity of individual workers. However, the wage-moderating effect of tax progression does not unambiguously carry over to a world where the union may fix both wages and individual hours of work. In this framework, the union reacts to tax progression by cutting individual working time. The wage rate, however, may decrease or increase. If the wage rate increases, the number of employed workers may decline despite the reduction in hours of work. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The issue of mass unemployment, which is a key policy problem in many European countries, has raised interest in the effect of different tax systems and tax reforms on employment. The most striking result that has emerged from this debate concerns the role of tax progression. According to the traditional view, tax progression reduces employment because high marginal tax rates discourage labour supply. This result, however, is based on the assumption of competitive labour markets. A number of recent contributions discuss the effects of tax progression on employment in unionised labour markets. In this literature, it turns out that higher tax progression reduces wages and boosts employment. The underlying argument may be put as follows: unions face a trade-off between higher wages and employment. Progressive taxes make wage increases less attractive relative to increases in the number of employees. Consequently, more tax progression induces wage moderation. This yields what may be called an employment paradox of tax progression.

It is characteristic of the existing literature on tax progression and unemployment that individual working time is assumed to be exogenously given. Collective bargaining is usually restricted to wages alone or wages and aggregate employment if efficient bargaining models are considered. This paper extends the literature as follows. For the model with bargaining over wages only, we show that the employment paradox of tax progression holds not only for a given per worker labour supply but also for a positive labour supply elasticity of individual workers. We then turn to the case where trade unions and firms bargain over both wages and individual hours of work. Recent contributions by Holm et al. (1997) and Sorensen (1997) also analyse the employment effects of tax progression in such a framework. Holm et al. (1997) show for the case of a monopoly union that, if utility is separable between consumption and leisure, an increase in tax progression, departing from a proportional tax system, enhances employment. Sorensen (1997) derives the same result in a Right-to-Manage-model, where the workers’ utility is linear in consumption. Our paper generalises these results and demonstrates that, in a Right-to-Manage-model, separability is sufficient for a positive employment effect of tax progression. Our analysis also shows, however, that it may be misleading to think of separability as a useful benchmark case. We present

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4 The question of how individual work hours are set under union-firm bargaining is discussed in Calmfors (1985), Calmfors and Hoel (1988), Holmlund (1989), Booth and Ravallion (1993) and Oswald and Walker (1993).
a simple example of non-separable preferences where tax progression raises wages and therefore reduces overall employment.

Our analysis proceeds as follows. In Section 2, we introduce a simple model of an economy with union-firm bargaining. In Section 3, we consider a situation with bargaining over wages only, where the employed workers may set their work hours individually. In Section 4, we assume that there is bargaining over both wages and individual hours of work. For both cases, the tax reform we consider is a revenue-neutral increase in tax progression. Section 5 gives our conclusions.

2. The model

The following analysis is based on a variant of the Right-to-Manage-model, which is standard in the literature on unionised labour markets. 5 We deviate from the standard set-up of this model by endogenizing individual hours of work per employed worker. Table 1 summarizes the structure of our model.

Eq. (1) represents the labour demand schedule of firms which gives labour demand \( L \) as a function of the gross wage \( w \). As in much of the union literature, we assume that labour demand elasticity \( \varepsilon \) is constant and greater than one. 6

We assume that labour demand is simply demand for manhours (Eq. (2)). 7 Total manhours \( L \) are given by the number of employed workers \( N \) times individual hours of work \( h \). Eq. (3) gives a representative worker’s utility \( U \), which depends on his/her consumption \( C \) and hours of work \( h \). \( U(C, h) \) is assumed to have the usual neoclassical properties. Eq. (4) is the budget constraint of an employed worker. Labour income is subject to a proportional income tax \( t \). In addition, the worker receives a government transfer \( T \), which may be interpreted as an income tax credit. The two parameters \( t \) and \( T \) determine, in a very simple way, the characteristics of a progressive income tax schedule, where increases in \( t \) or \( T \) c.p. raise the degree of tax progression. The parameter \( k \) is a

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5 For the theory of trade unions and collective bargaining, see McDonald and Solow (1981), Oswald (1985) or Booth (1995).

6 The assumption \( \varepsilon > 1 \) is standard in the literature on taxation and trade unions (see, e.g., Sorensen, 1997) and emerges from a Cobb–Douglas technology. It is problematic insofar as most empirical studies suggest \( \varepsilon < 1 \) (see, e.g., Hamermesh, 1993, pp. 61–104). For the following analysis, it would be sufficient to assume that \( \varepsilon \) is locally constant and that \( \varepsilon > 1 - \theta \) holds, where \( \theta \) is a parameter capturing the union’s relative bargaining power which will be discussed further below. One may also note that, whether or not \( \varepsilon \) is assumed to be constant may sometimes matter for the impact of taxation in unionised labour markets, as is shown in a different context by Koskela et al. (1998).

7 We abstract from possible effects of working hours on labour productivity. Empirical evidence which justifies using this specification has been provided by Leslie and Wise (1980).
Table 1

\[
\begin{align*}
L &= w^{-\varepsilon} \quad (1) \\
L &= Nh \quad (2) \\
U &= (C,h) \quad (3) \\
C &= w(1-t)h + T - k \quad (4) \\
R &= twhN - TN \quad (5) \\
\Omega &= N + (M - N)U_0 \quad (6) \\
\pi &= Y(L) - wL \quad (7)
\end{align*}
\]

Parameters: \( \varepsilon > 1; 0 < t < 1 \).

fixed cost of being employed; \( k \) is independent of the amount of hours worked and may be interpreted, for instance, as the cost of commuting or training cost.  

Given the tax parameters \( t \) and \( T \), Eq. (5) describes total income tax revenue \( R \). Eq. (6) is the objective function of the trade union \( \Omega \). \( M \) denotes the overall number of workers. The \( M - N \) unemployed workers attain the utility level \( U_0 \) which we assume to be fixed. \( ^9 \) Eq. (7) gives the firm’s profits \( \pi \). Eqs. (1)–(6) can be simplified to Eqs. (1), (3a), (5a) and (6a) in Table 2.

Eqs. (1), (3a) and (6a) are the building blocks for our analysis of the labour market. In what follows, we will assume throughout that the union bargains with the firm over the wage rate while, along the lines of the ‘right-to-manage’ approach of the union literature, firms preserve the right to choose employment in terms of man-hours \( L \). However, we will distinguish two ways of determining individual hours of work \( h \). In the first case, each employed worker chooses the individually optimal working time \( h \), given the wage set by the union and the tax parameters \( t \) and \( T \). This case is analysed in Section 3. In the second case, we assume that the union and the firm bargain not only over wages but also over

\( ^8 \) As will become clear below, some fixed cost of employment is required to exclude a corner solution in the case where unions and firms bargain over both wages and hours. For the role of fixed employment cost in the theory of labour markets, see, for instance, Ehrenberg and Smith (1994).

\( ^9 \) This utility may be considered as the utility when enjoying leisure or working in the underground economy.
working time per employed worker. This is to take account of the empirical fact that wage bargaining agreements often stipulate not only on wages but also individual working time. This second case is discussed in Section 4.

3. Wage bargaining and individual choice of working hours

3.1. Wage bargaining

In this section, we analyse the case where the employed workers choose their work hours, i.e., labour supply \( h \), while the union only sets the wage rate per work hour. We plausibly assume that individual workers act competitively and take the wage rate set by the union as given. The union’s objective function can then be written as

\[
\Omega = NV( w(1 - t), T - k ) + ( M - N ) U_0 \\
= \frac{L(w)}{h(w(1 - t), T)} \left( V(w(1 - t), T - k) - U_0 \right) + MU_0
\]

where \( V(w(1 - t), T - k) \) is the indirect utility function and \( h(w(1 - t), T) \) is the individual labour supply function. For further use, we denote the labour supply elasticity by

\[
\eta = \frac{\partial h}{\partial (w(1 - t))} \frac{w(1 - t)}{h}
\]

The outcome of wage bargaining is derived using the Nash Bargaining Solution (Nash, 1950). While the threat point of the firms is plausibly assumed to be zero, the threat point of the union is to have all workers unemployed, i.e., \( MU_0 \). The
Nash maximand can then be written as
\[ \Psi = \theta \ln \left( \frac{L}{h} (V - U_0) \right) + (1 - \theta) \ln \pi \]  
(8)

The parameter \( \theta \) (with \( 0 \leq \theta \leq 1 \)) represents the union’s relative bargaining power. The bargaining outcome is derived by maximizing Eq. (8) over \( w \). Using the properties of the labour demand function (1) and making some rearrangements allows to write the first-order condition as
\[ V_t w (1 - t) - (V - U_0) (\varepsilon + \eta + \gamma) = 0 \]  
(9)
where
\[ \gamma = \frac{(1 - \theta)}{\theta} (\varepsilon - 1) \geq 0 \quad \text{for} \quad 0 < \theta \leq 1 \]

Eq. (9) implicitly defines the wage rate emerging from the bargaining process as a function of the tax parameters: \( w = w(t, T) \).

3.2. Revenue-neutral change in tax progression

In this section, we consider the effects of a revenue-neutral change in tax progression on wages (\( w \)), overall employment (\( L \)), and the number of employed workers (\( N \)). The change in tax progression is induced by a variation of the income tax credit (\( T \)) and an adjustment of the marginal tax rate (\( t \)) such that overall labour tax revenue (\( R \)) remains constant. We assume throughout that the Laffer curve has a positive slope, i.e., that an increase in \( T \) can only be financed by an increase in \( t \). From Eq. (5a), one obtains:
\[ dR = R_d \, dt + R_t \, dT + R_w \, dw = 0 \]  
(10)
If the initial tax system is proportional with \( t > 0 \) and \( T = 0 \), which we assume to be the case in what follows, we have
\[ R_d = wL > 0; \quad R_t = - \frac{L}{h} < 0; \quad R_w = tL(1 - \varepsilon) < 0 \]  
(11)
The effect of a revenue neutral change in tax progression on the wage rate follows from Eq. (10) and the total differential of the first-order condition of the bargaining problem (9). Under the assumption that the labour supply elasticity is locally constant \(^{10} \) and nonnegative, we can state

\[^{10}\] An example for preferences satisfying this assumption is \( U = (C - (h^{1+\beta} / 1 + \beta))^\sigma \), with \( \beta > 0 \) and \( 0 < \sigma < 1 \). It is easy to check that the labour supply elasticity is then \( 1 / \beta \). One may also note that this type of utility function implies that the income effect on labour supply is zero. While this assumption is often used in theoretical analysis (see, e.g., Keen and Marchand, 1997 or Fuest and Huber, 1999), it is clear that, empirically, the income effect on labour supply will usually be different from zero.
Proposition 1: A revenue-neutral increase in tax progression, departing from a proportional tax system \((T = 0, \ t > 0)\), and assuming that the labour supply elasticity is locally constant and nonnegative, reduces the wage rate and raises both employment \((L)\) and the number of employed workers \((N)\).

Proof: See Appendix A.

Proposition 1 extends the literature by generalizing the finding that tax progression raises employment to a situation where the labour supply of employed workers is endogenously determined. The effect on the bargaining outcome reflects that both bargaining parties have an interest to keep tax payments to the government as low as possible. In a progressive tax system, this can be done by spreading total income over more workers by reducing the wage rate. This raises labour demand and triggers a reduction in individual labour supply and hence, the income of each individual worker. As a result, the number of employed workers increases.

4. Union choice of wages and individual hours of work

4.1. Bargaining over wages and individual hours of work

In this section, we consider the case where the union and the firm bargain over both wages \((w)\) and individual hours of work \((h)\). The possibility of collectively setting individual work hours raises the question of why the union, given its egalitarian objective function, would not reduce hours to the point where all workers are employed. In our model, the reason is that there is a fixed cost for each employed worker \((k)\). The union trades off higher employment against raising total cost due to the fixed cost of being employed. If \(k\) is high enough, an interior solution with unemployment emerges. The Nash maximand is:

\[
\Phi = \theta \ln \left( \frac{L}{h} \left( U(w(1-t)h + T - k; h) - U_h) \right) + (1 - \theta) \ln \pi \right) \tag{12}
\]

\(11\) Here, one may raise the question of whether the European unemployment problem is really due to fixed costs of being employed such as commuting costs. Obviously, the actual importance of such fixed costs is an empirical problem. More fundamentally, one might argue that the model underlying our analysis overstates the egalitarianism of trade unions. Of course, these issues are beyond the scope of the present paper.
Assuming that an interior solution emerges, maximizing Eq. (12) over $w$ and $h$ and rearranging yields the first-order conditions:

$$U_w h (1 - t) - (U - U_0) (r + \gamma) = 0$$

(13)

and

$$U_w h (1 - t) + U_h h - (U - U_0) = 0$$

(14)

Note that Eq. (14) implies

$$U_w (1 - t) + U_h = (U - U_0) / h > 0$$

which means that the individual workers are now rationed, i.e., they would prefer to work longer, given the wage rate. Eqs. (13) and (14) now determine the equilibrium wage and working time as functions of $t$ and $T$.

4.2. A revenue-neutral change in tax progression

We now consider again the impact of a revenue-neutral change in tax progression on wages and employment. The difference to the case where the union only sets the wage rate is that the union may now react to the tax change also by adjusting individual hours of work ($h$). Formally, the effects of the change in tax progression are now derived by differentiating Eqs. (13) and (14), and the public sector budget constraint (see Appendix B). This yields the following results.

**Proposition 2:** If the union may set both wages and individual hours of work, and if $U(C,h)$ is separable between consumption and leisure ($U_{12} = 0$) a revenue-neutral increase in tax progression, departing from a proportional tax system ($T = 0$, $t > 0$), reduces wages ($w$) and individual hours of work ($h$) and raises the number of employed workers ($N$) and overall work hours ($L$).

Proof: See Appendix B.

The intuition for this result is similar to that of proposition 1. As the tax system becomes more progressive, the union spreads total labour income over more workers. This is achieved by simultaneously reducing work hours and wages. Proposition 2 generalizes the findings in Holm et al., 1997; Sorensen, 1997. Holm et al. (1997) derive in a monopoly union model that, under the separability

Note that Eq. (13) implies $U > U_0$. The result that the union prevents workers from supplying the amount of hours preferred individually raises the question of whether such a union would not lose the support of its members in the long term. A union model explicitly taking this issue into account is the seniority model Oswald, 1993. In this model, the union’s policy is determined by the interest of the median worker who is not threatened with unemployment and therefore indifferent with respect to the overall number of employed workers.
assumption, tax progression reduces work hours and wages. Sorensen (1997) considers a Right-to-Manage model but only analyses the special case where preferences are linear in consumption.

While proposition 2 thus confirms the employment enhancing effect of tax progression, it is important to note that the separability assumption is critical for the result. To see this, consider the following example of nonseparability. For simplicity, we consider the special case of a monopoly union and normalise $U_0$ to zero.

**Proposition 3:** If the union may set both wages and individual hours of work ($\theta = 1$), and if the utility function takes the form

$$U = \left( \frac{C^{1-\alpha} - h^{1+\beta}}{1-\alpha} \right)^{\alpha}$$

with $0 < \alpha < 1$, $\beta \geq 0$ and $0 < \sigma < 1$, a revenue-neutral increase in tax progression, departing from a proportional tax system ($T = 0$, $t > 0$), reduces $h$ but has the following effect on the wage rate $w$:

$$\frac{dw}{w} > 0 \quad \text{if} \quad \frac{1}{\sigma} > 1 + \beta$$

(15)

**Proof:** See Appendix B.

Proposition 3 shows that the wage-moderating effect of tax progression derived in models with bargaining over wages only is not robust if bargaining over individual hours of work is allowed for.

Obviously, the role of separability between leisure and consumption must be at the heart of the issue, because this is the crucial difference in assumptions. That non-separability may cause a wage increase is due to the fact that the wage the union sets is higher, the higher the marginal utility of income of employed workers. However, for the utility function under consideration, the marginal utility of income is decreasing in work hours. As $h$ declines in response to the increase in $T$ (and $t$), the marginal utility of income increases, and this has a positive impact on wages\(^{13}\) which may dominate the downward pressure induced by the desire to spread the overall labour income across more workers.

The possibility of an increase in the wage rate raises the question of whether a wage increase may also reduce the overall number of employed workers ($N$), given that individual work hours decline.

\(^{13}\) We gratefully acknowledge that this explanation for the result of proposition 3 has been suggested to us by a referee.
Proposition 4: Suppose that an increase in tax progression raises the wage rate according to proposition 3. The number of employed workers declines if the term $1 - \varepsilon t$ becomes (sufficiently) small, i.e., if the tax rate $t$ is sufficiently high.

Proof: See Appendix C.

That $N$ may decline if the term $1 - \varepsilon t$ is small may be explained as follows. As we show in Appendix C (see Eq. (A.3.9)), an increase in $t$ c.p. raises tax revenue (the slope of the Laffer curve is positive) if and only if $1 - \varepsilon t > 0$. The smaller this term, the larger the increase in $t$ required to replace the revenue lost by a given increase of the income tax credit $T$. In a situation where the marginal tax rate is already quite high such that $1 - \varepsilon t$ is positive but small (which means that tax revenue is close to the maximum of the Laffer curve), the increase in $t$ required to balance the budget is sufficiently high to drive up wages to the point where the effect of the decline in working hours is overcompensated. Thus, if the tax rate $t$ is already quite high before the tax reform, an increase in tax progression will actually reduce the number of employed workers.

5. Conclusions

In this paper, we have discussed the implications of tax progression for employment in a unionized labour market. A number of recent contributions to this literature states that more tax progression may induce unions to cut wages. These contributions have in common that individual hours of work are assumed to be exogenously given. We have shown that the positive employment effect of tax progression can be generalized to a model with a positive labour supply elasticity of individual workers. However, the result does not unambiguously carry over to a world where the union and the firm bargain over both wages and individual hours of work. It turns out that tax progression does reduce wages and raise employment if utility is separable between consumption and leisure. If the separability assumption is relaxed, however, tax progression may raise wages and, albeit under restrictive assumptions, reduce the number of employed workers.

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Appendix A

In this appendix, we give the proof of proposition 1. Differentiating the first-order condition (9) yields
\[ \Psi_{w} dw + \Psi_{w,t} dt + \Psi_{w,T} dT = 0 \]  (A.1.1)
where
\[ \Psi_{w} = V_{11}(1-t)^2w - V_{1}(1-t)(\varepsilon + \eta + \gamma - 1) \]
\[ \Psi_{w,t} = -V_{11}(1-t)w^2 + V_{1}w(\varepsilon + \eta + \gamma - 1) \]
\[ \Psi_{w,T} = V_{12}(1-t)^2w - V_{2}(\varepsilon + \eta + \gamma) \]

Note that
\[ V_{1} = \lambda(w(1-t),T) h(w(1-t),T); \ V_{11} = \lambda_{1} h + h_{1} \lambda; \ V_{12} = \lambda_{2} h + h_{2} \lambda \]

Using
\[ \lambda_{1} = V_{21} = V_{12} \]

and the condition for revenue neutrality (10), it is straightforward to derive
\[ \frac{dw}{dT} \bigg|_{dR=0} = \frac{1}{\Delta} wL \lambda(1 + \eta) < 0 \]  (A.1.2)
where
\[ \Delta = \Psi_{w,w} R_{r} R_{w} - \Psi_{w,w} R_{r} + \Psi_{w,w} \left( R_{r} + R_{w} \frac{dw}{dT} \right) \leq 0 \]

Clearly, the second-order condition of the bargaining problem and the assumption of a positively sloped Laffer curve imply \( \Delta < 0 \). Q.E.D.

Appendix B

In this appendix, we give the proof of proposition 2. The differentials of the first-order conditions (13) and (14) and of the government budget constraint yield a system of three equations which determine \( dw/dT \), \( dh/dT \) and \( dt/dT \). As the algebra is tedious but straightforward, and given that the proof logically runs along the same lines as the proof of proposition 1, we only report the results here. The calculations are available from the authors on request. Under the assumption of separability between \( C \) and \( h \), \( ^{14} \) a revenue-neutral increase in \( T \) has the following effects on \( h \) and \( w \):
\[ \frac{dh}{dT} \bigg|_{dR=0} = \frac{U_{1}(1-\varepsilon - \gamma) + U_{11}wh(1-t)}{\Xi} < 0 \]  (A.2.1)

\[ ^{14} \text{Note that, in the case of separability, } U_{22} < 0 \text{ is necessary for the second-order conditions to hold.} \]
where
\[ \Xi = U_1 h(1 - \varepsilon - \gamma) \left(U_{22} + U_{11}(1 - t)^2 w^2\right) + U_1 U_{22} w(1 - t) h > 0 \]
and
\[ \left. \frac{dw}{dT}\right|_{aR=0} = \frac{U_{22} U_1 L(1 - t) w^2 h^2 (\varepsilon + \gamma)}{\Gamma} < 0 \] (A.2.2)
where
\[ \Gamma = \Xi \left(R_i + R_i \frac{\partial w}{\partial t} + R_i \frac{\partial h}{\partial t}\right) > 0 \]
The assumption of a positively sloped Laffer curve implies that \( \Gamma > 0 \). Q.E.D.

Appendix C

In this appendix, we finally provide the proofs of propositions 3 and 4. The first-order conditions of the union’s problem are:
\[ \sigma w h (1 - t) C^{-\alpha} - \varepsilon \left( \frac{C^{1-\alpha}}{1-\alpha} - \frac{h^{1+\beta}}{1+\beta} \right) = 0 \] (A.3.1)
and
\[ \sigma h (w (1 - t) C^{-\alpha} - h^{\beta}) - \left( \frac{C^{1-\alpha}}{1-\alpha} - \frac{h^{1+\beta}}{1+\beta} \right) = 0 \] (A.3.2)
Using Eq. (A.3.1), we can transform Eq. (A.3.2) into
\[ \frac{h^{1+\beta}}{1+\beta} = \frac{\varepsilon (1 + \beta)}{\sigma (1 + \beta) + \varepsilon - 1} \left( \frac{C^{1-\alpha}}{1-\alpha} \right) \] (A.3.3)
Substituting Eq. (A.3.3) into Eq. (A.3.1) and making some rearrangements yields:
\[ w (1 - t) h = \frac{\zeta}{\xi - 1} (k - T) \] (A.3.4)
with
\[ \zeta = \frac{\varepsilon (1 + \beta)}{(\sigma (1 + \beta) + \varepsilon - 1)(1 - \alpha)} > 1 \]
Eq. (A.3.4) and \( C = w(1 - t)h - (k - T) \) imply:
\[
C = \frac{1}{\zeta - 1} (k - T) \quad \text{(A.3.5)}
\]

Using Eq. (A.3.5) in Eq. (A.3.3) yields equilibrium individual hours of work \((h)\) as:
\[
h = \rho_1 (k - T)^{\frac{1 - \alpha}{1 + \beta}} \quad \text{(A.3.6)}
\]
with
\[
\rho_1 = \left( \frac{\zeta e - 1}{e} \right)^{\frac{1}{1 + \beta}} \left( \frac{1}{\zeta - 1} \right)^{\frac{1 - \alpha}{1 + \beta}} > 0
\]

Eq. (A.3.6) shows that the result stated in proposition 2 — the reduction of \( h \) in response to an increase in tax progression — also applies here. Using Eq. (A.3.6) in Eq. (A.3.4) yields the equilibrium wage rate as:
\[
w = \frac{\rho_2}{(1 - t)} (k - T)^{\frac{\alpha + \beta}{1 + \beta}} \quad \text{(A.3.7)}
\]
with
\[
\rho_2 = \zeta^{\frac{\beta}{1 + \beta}} \left( \frac{e - 1}{e} \right)^{\frac{1}{1 + \beta}} \left( \frac{1}{\zeta - 1} \right)^{\frac{\alpha + \beta}{1 + \beta}} > 0
\]

The effects of a revenue-neutral change in tax progression are derived as follows. The differential of Eq. (A.3.7) and the revenue neutrality condition yield:
\[
\begin{align*}
\left( \frac{(1 - t)}{t(1 - \epsilon)} \right)^{\frac{1}{w}} \left( \frac{d w}{d t} \right) & = \left( -\frac{(\alpha + \beta)w(1 - t)}{(1 + \beta)k} \right) \frac{1}{h} \\
\end{align*}
\]
\[
\text{(A.3.8)}
\]

This leads to:
\[
\left. \frac{d w}{d T} \right|_{d R = 0} = \frac{1}{h(1 - \epsilon t)} \left( 1 - \frac{(\alpha + \beta)w(1 - t)h}{(1 + \beta)k} \right)
\]

Note first that a positive slope of the Laffer curve requires \( 1 - \epsilon t > 0 \) since
\[
\frac{d R}{d t} = R_t + R_w \frac{d w}{d t} = \frac{wL}{(1 - t)} (1 - \epsilon t) \quad \text{(A.3.9)}
\]
Note also that:
\[
\frac{\zeta}{\zeta - 1} = \frac{\varepsilon (1 + \beta)}{1 - \sigma + \beta (\varepsilon - \sigma) + \alpha (\sigma (1 + \beta) + \varepsilon - 1)} > 0 \quad (A.3.10)
\]

Using Eqs. (A.3.4), (A.3.7) and (A.3.10) and making some rearrangements shows:
\[
\frac{dw}{dT}|_{dR=0} = \frac{1}{h(1 - \varepsilon t)} \phi_1 \quad (A.3.11)
\]

where
\[
\phi_1 = \frac{(1 - \alpha)(1 - \sigma (1 + \beta))}{1 - \sigma + \beta (\varepsilon - \sigma) + \alpha (\sigma (1 + \beta) + \varepsilon - 1)}
\]

which implies
\[
\text{sgn} \left( \frac{dw}{dT}|_{dR=0} \right) = \text{sgn} \left( \phi_1 \right) = \text{sgn} \left( 1 - \sigma (1 + \beta) \right) \quad (A.3.12)
\]

which proves proposition 3. It is now straightforward to show that a wage increase may also reduce the number of employed workers \(N\), despite the reduction in individual working time as stated by proposition 4. The change in \(N\) is
\[
\frac{dN}{dT}|_{dR=0} = \frac{1}{h^2} \left( hL_w \frac{dw}{dT}|_{dR=0} - L \frac{dh}{dT}|_{dR=0} \right)
\]

which, using A.3.6 (note that \(h\) is independent of \(t\)), A.3.7 and A.3.11, and making some rearrangements, can be written as
\[
\frac{dN}{dT}|_{dR=0} = \frac{1}{h^3 w} \left( \frac{1 - \alpha}{1 - \sigma} \frac{k^2}{(1 + \beta) (1 - \varepsilon t)} \phi_1 \right) \quad (A.3.13)
\]

Of course, a necessary condition for \(N\) to decline is that the wage rate increases, i.e., \(\phi_1 > 0\). A sufficient condition is that the term \(1 - \varepsilon t\) is sufficiently small. Note that \(1 - \varepsilon t\) only affects the second term in Eq. (A.3.13); if \(t\) approaches \(1/\varepsilon\), the right-hand side of Eq. (A.3.13) therefore becomes unambiguously negative. Q.E.D.

References