Matching and competition for human capital

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Abstract

A simple model of discretionary worker investment in human capital is developed in which worker productivity is affected by a firm-specific match and employers bid strategically for workers. The labor market returns a share of specific capital productivity to workers without Nash bargaining power and without recourse to long-term contracts, because efficient turnover transforms a worker’s former employers into her outside options. When the cost of specific investment falls, wage profiles become less steep and turnover is reduced. Perversely, an increase in the probability of turnover increases the (privately) optimal investment in specific capital.

JEL classification: J24; J31

Keywords: Human capital investment; Matching

1. Introduction

Human capital theory emphasizes the distinction between specific skills, productive at only one firm, and general skills, productive at many firms, because of the different incentives for accumulation of the two types of skill. Since more than one firm will bid for a worker with general skills, “the cost as well as the return from general training would be borne by trainees, not by firms” (Becker, 1962, p.13). In contrast, the return from specific investment is a quasi-rent, available

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only when the trainee works for the specific firm, so alternative employers cannot be relied upon to bid up wages. As the cost of investment must be borne before the returns are available, the investment decision depends on the expected resolution of a bilateral monopoly. In the absence of credible promises of future compensation, workers exposed to the potential of being 'held-up' by their employers are unwilling to invest in specific capital. Nevertheless, the best evidence suggests that wages do rise with job tenure as well as experience, and that the source of the wage gain is training (Brown, 1989; Topel, 1991). However, training alone does not explain all of the wage growth among workers. Topel and Ward (1992), e.g., find that job changes also account for a substantial fraction of wage growth, and conclude that the labor market systematically sorts workers into firms at which they have better matches.

If the labor market sorts workers into better matches, then a worker's present employer may in the future become her best alternative. Even if employers can extract all of the relationship-specific surplus, the ability of workers to change firms means that specific capital may be transformed from surplus at one firm into the worker's rival opportunity at the other, and hence, into wages. This paper develops a simple two-period model of human capital accumulation in the presence of worker–firm matching to explore the potential of efficient turnover for providing workers a share in the return from their specific investments. To focus attention on the role of the spot labor market in inducing specific investments by workers, long-term contracts are assumed to be unavailable. The principal finding is that matching and training explanations of wage growth reinforce each other: when matching is important, workers are sorted across firms and so have an incentive to invest in specific as well as general human capital.

There remain important differences between the effect of general and specific human capital investment in the model. Since workers capture all of the returns from general capital, the socially efficient levels of these skills are accumulated. Not so with specific capital: workers ignore the benefit these skills confer on their employer, so they underinvest relative to the social optimum. Less obvious is the fact that workers also invest in specific skills insufficiently to maximize their own lifetime incomes. This occurs because a worker enters the labor market twice, once before and once after her investment in human capital. The anticipated investment in specific skills increases the expected profit of her employer, and so intensifies wage competition for young, uncommitted workers. Thus, part of the benefit to specific capital is incorporated in the first period wage. At the time the specific investment is actually made, this wage is fixed, and unable to commit ex

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1 Grout (1984) provides a formal model of the hold-up problem in the absence of contracts.
2 A recent paper by Parent (1996) finds empirical support for the complementarity between matching and human capital investment.
ante, the worker ignores this effect and underinvests in specific capital. More surprising is that if a worker were able to commit to the wage-maximizing specific investment, she would overinvest in these skills, relative to the socially optimum.

I examine the comparative static effects of changes in the cost of investment. In the limit as specific capital becomes very inexpensive, turnover vanishes and the wage of workers who leave their first-period employer exceeds that of those who stay. However, the relationship between the cost of investment and the relative wages of movers and stayers is not monotonic. Increases in specific capital can be correlated with either a rise or fall in the returns to tenure at the firm.

The assumption that differentiates this work from the extensive literature on matching and specific human capital investment is not that outside options exist, or even that the outside option may exceed the value of continuing the relationship in which a specific investment has been made. It is that the bidding for the worker’s services by the outside option is responsive to the valuation of the worker in her initial match. For example, Mortensen (1988) and Jovanovic (1979b) assume that workers are paid their marginal product, while MacLeod and Malcolmson (1993) allow a worker’s current employer to renegotiate her wage based on the value of her outside option, but assume that the outside firm makes just one take-it-or-leave-it offer, independent of the worker’s current match value.

The bidding structure in the model is a generalization of the ‘offer matching’ model of Mortensen (1978) in which workers search for outside offers and the present employer is able to make a counter-offer to retain the worker. This is extended in the present paper to allow the competitor a similar chance to make multiple offers. In Mortensen’s model, specific capital refers to the match value itself, and discretionary investment is not considered. Hashimoto (1981) and Hall and Lazear (1984) point out that if the transaction costs associated with verifying outside offers are high, offer matching procedure is less efficient, perhaps even than a fixed wage. Like Mortensen, I assume that wage bargains are implementable at a negligible cost. But as long as transaction costs are not so high as to make wages totally unresponsive to competition, the results of the model hold qualitatively.

Several of the paper’s findings are related to other work in the literature. In a model of non-discretionary human capital accumulation (learning-by-doing), Bernhardt and Timmis (1990) also found that specific capital accumulation can cause intertemporal wage profiles to become flatter. Because human capital cannot serve as loan collateral, firms are able to act as financial intermediaries, providing smoothed consumption to workers. Felli and Harris (1994) propose a model in which worker–firm matches are experience goods (i.e., the value of a particular relationship is revealed slowly over time) that also allows for the possibility of their present employer being their future outside option. Interpreting the information on match value as specific capital, Felli and Harris find that wages may rise with tenure in a firm if the information gained is valuable in both the worker’s present and alternative matches. In contrast, I assume that specific capital is only
of value in the relationship in which the worker is engaged during its accumu-
lation.

Section 2 develops the basic model of worker investment. Section 3 contains
several extensions. Section 4 concludes.

2. The model

The model contains two firms and a continuum of workers. All agents are
risk-neutral. To focus attention on the labor market, I assume that firms are
price-takers in the output market, and normalize the price of output to 1. Workers
have careers lasting for two periods, during which they supply their labor services
inelastically. The two firms compete for every worker twice, once at the start of
each career period. The firm employing a worker in the first period is referred to
as the employer, and denoted firm \( e \); the other firm is referred to as the
competitor, and denoted firm \( c \). Competition is restricted to wages, which are
assumed to be set by an ascending bid auction. In each period, firms make
successively higher wage offers until one of the firms is no longer willing to raise
its bid. This implies that all of surplus from a particular match go to the firm.
Workers and firms are unable to sign long-term contracts. In particular, at the start
of period 1, firms are unable to commit to a period 2 wage, and workers are
unable to commit to any level of investment in either specific or general human
capital.

In the absence of investment in human capital, each worker’s productivity is
determined by the realization of an idiosyncratic worker–firm match. The match
between worker \( i \) and firm \( j \) is denoted by \( \theta_i \in [\theta, \theta] \), and is drawn according to
the distribution function \( M(\theta) \), with density \( \mu(\theta) \), and expectation \( E\theta \). For
simplicity, I assume that the quality of a worker’s match at one firm is independ-
ent of her match at the other firm and of all other workers’ matches.

Workers can augment the value of their match by making costly investments in
specific and general human capital. These investments become productive one
period after they are made. General capital is identically productive at both firms,
and specific capital is productive only at the firm employing the worker while it is
accumulated. Let \( s \) and \( g \) denote the workers’ investment in specific and general
human capital. The human capital production function, \( H(s, g) \), and investment
cost function, \( C(s, g; \alpha, \beta) \) have the following properties:

\[
H(s, g) = s + g,
\]

\[
C(s, g; \alpha, \beta) = c(s; \alpha) + c(g; \beta) \quad c' > 0, \quad c'' > 0,
\]

where \( \alpha \) and \( \beta \) are exogenous variables that increase the marginal cost of
investment (i.e., \( \alpha > 0 \) and \( \beta > 0 \)). A restriction implicit in this assumption is
that the productivity and the cost of human capital investment are the same at both firms. This is discussed in Section 3.

Because each worker’s productivity is independent of all others, it is possible to consider the career of individual workers separately. To reduce notation, I will suppress the index $i$. The timing of the game involving a representative worker is depicted in Fig. 1. At the start of period 1, with the value of the worker’s matches unknown, the two firms compete for her first-period services. The winner, firm $e$, pays the worker $w_1$. During period 1, before her match at either firm is revealed, the worker invests in firm-specific and general human capital. At the end of period 1, her matches and level of investment in human capital are revealed, and second-period wage competition ensues. Finally, the worker chooses a firm for which to work, works out her career, and retires.

Notice that this model is a hybrid of the two types of model in the matching literature. The first is that found in a standard search model, where the value of any particular match is discernible immediately when the parties meet. The other type treats matches as an experience good, in that the value of a given match is revealed over time. The present model combines elements of both of these variants. For young workers, match is an experience good; some work experience must be accumulated before anyone can evaluate the match at any firm. Once experienced though, a worker’s match at all firms is revealed without further experience.

2.1. Equilibrium

I restrict attention to the subgame perfect equilibrium, in which the beliefs and actions of all agents are consistent with optimal play on every possible continuation of the game. Workers always act to maximize their expected future income by working for the firm offering the highest wage, and undertaking the income maximizing investments in human capital. Firm strategies specify wage bidding behavior at two points in each worker’s career. At the start of period 1, because the worker’s productivity is unknown, the best that the firms can do is to offer an initial wage that equates the expected profit of firm $e$ and firm $c$. At the start of period 2, with the match information revealed, both firms have a dominant strategy, familiar from ascending-bid auctions, to continue to improve upon their opponent’s wage offer up to the total productivity of the worker.

Solving backwards, these strategies are as follows.

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>worker $i$ hired by $e$</td>
<td>worker $i$ invests</td>
</tr>
<tr>
<td>matches revealed</td>
<td>competition with firm $c$</td>
</tr>
<tr>
<td>worker $i$ retires</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1.
2.1.1. Period 2

In period 2, with the investment in human capital made, all that awaits resolution is the worker’s second-period wage and place of employment. Wage competition results in the worker being employed at the firm where she is most productive at a wage equal to her value at the losing firm. The knowledge of match quality is valuable to the firms, as they are able to employ only those workers who yield a positive profit. For given investments in specific and general human capital, \( s \) and \( g \), the worker’s expected period 2 wage is:

\[
E_{w|g}(w_2(\theta_e, \theta_c, s, g)) = E_{w|g}(\min\{\theta_e + s + \theta_c + g\}).
\]

2.1.2. Period 1

The first period comprises two subperiods: the initial competition for young workers, and the human capital investment decision. In the second subperiod, the first period wage is set so it plays no role in the worker’s investment choices. The worker solves the following problem:

\[
\max_{s, g} E_{w|g}(w_2(\theta_e, \theta_c, s, g) - C(s, g; \alpha, \beta)).
\]

Let \( s^* \) and \( g^* \) denote the optimal choices. At the start of period 1, firms anticipate the investment decisions of the workers and bid up the first-period wage until at least one firm is indifferent to the first-period employer or the second-period competitor. As the two firms are symmetric, this causes \( w_1 \) to rise until the expected profit of the employer and competitor are equal:

\[
w_1 = E\theta + E\pi^e_2 - E\pi^c_2.
\]

where \( E\pi^j_2 \) is firm \( j \)'s expected period 2 profit, given by:

\[
E\pi^e_2 = E_{w|g}(\theta_e + s^* + g^* - \min\{\theta_e + s^* + g^*, \theta_c + g^*\});
\]

\[
E\pi^c_2 = E_{w|g}(\theta_c + g^* - \min\{\theta_e + s^* + g^*, \theta_c + g^*\}).
\]

The following propositions characterize the equilibrium. (All proofs are in Appendix A).

**Proposition 1:** Workers invest in both general and firm-specific human capital. The worker chooses a level of general capital investment that exceeds that of specific capital, i.e.:

\[
g^* > s^* > 0.
\]

Workers are willing to invest in both types of human capital despite the absence of contracts because they expect the market to provide them a share in the return from both investments. In the event that the worker is more valuable at firm \( e \), the second-period wage is equal to the workers value at firm \( c \), which increases one for one with the investment in general capital, but is unaffected by the specific
investment. However, in the event that the worker is more valuable at firm $c$, the second-period wage increases with both general and firm-specific human capital. When choosing her investments, the worker weighs the marginal gain in productivity with the probability of receiving a return. Since the worker always gets the return from a general investment, but gets the return from specific investments only in the event that she is more valuable at firm $c$, she chooses less specific than general human capital. The key to specific investment is the variability in productivity due to matching: were the matches at firm $e$ and firm $c$ perfectly correlated, $s^*$ would be zero.

2.2. Efficiency

**Proposition 2:** Workers choose the socially efficient level of general capital, but less than the socially efficient level of specific capital.

Finding underinvestment in specific capital is not surprising, since when choosing her investment, the worker disregards the benefits accruing to the firm. The nature of the inefficiency is particularly stark in the present model: the worker values and hence invests in specific capital proportionately to the probability of separating from her current employer; efficiency, in contrast, requires her to invest in proportion to the probability of remaining with her current employer. Because for positive levels of the specific investment, the former probability is larger than the latter, in the present context, there is an underinvestment. In a more general context, e.g., in which firms also can make specific investments, this misalignment of incentives raises the possibility that there will be an overinvestment in specific capital.

Much of the literature on contracting is devoted to rectifying the hold-up problem leading to underinvestment. For example, MacLeod and Malcomson (1993) propose as a solution that parties write a contract that specifies a high second-period wage. In their model, specific investments by workers reduce the loss of leisure associated with working. If workers can count on the wage not to change, they can make efficient investments without the worry of being held-up by their employer. Since the wage contract is verifiable and enforceable, and both parties must agree to renegotiate it, if separation does not occur, the gains from worker’s specific investments accrue to the workers. This solution would not work in the present model, because the specific investment increases output which, in the absence of renegotiation, is the property of the employer. The wage must be

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3 Of course, this begs the question of why it is that the firm ‘owns’ the worker’s output. The property rights approach to incomplete contracts would suggest that workers in this model become self-employed (see, e.g., Hart, 1995). I am implicitly assuming that this is impossible. This might be due to wealth constraints, for example.
flexible to both provide and be an incentive to invest, i.e., prevent the worker from holding-up the firm, and at the same time transfer the benefit of specific investments to workers.

There is another sense in which the worker underinvests in specific capital.

**Proposition 3:** Workers choose the lifetime income-maximizing level of general capital, but less than the lifetime income-maximizing level of specific capital.

Since all of the returns to general capital accrue to the worker in the second period, her general skills investment decision is income-maximizing. Specific capital investments raise second-period wages, but because of its anticipated effect on firm $e$’s profit, the first-period wage also rises, by an amount equal to the productivity of the specific capital. Since first-period wages are set at the time of investment, the worker’s investment in specific skills does not reflect her entire benefit, and the worker underinvests.

Were the worker somehow able to commit at the time of hiring to making a particular specific investment, she would choose to make that which maximizes her lifetime wages. It is interesting to compare this with the socially efficient level of investment.

**Proposition 4:** If workers could commit to making the lifetime wage-maximizing specific investment, they would overinvest relative to the social optimum.

Since the lifetime wage incorporates the whole of the productivity of the specific investment into the first-period wage, plus a portion of this value again into the second-period wage, the weight-specific investment received in the lifetime wage maximization problem exceeds the probability that it is ultimately productive. For an extreme example, consider an investment that acted solely by destroying potential value at firm $c$. This might be, e.g., a process of corporate enculturation at firm $e$ that, unproductive in itself, made the workers less useful to firm $c$. It is clear that the socially optimal investment is zero, but there remains a private incentive for the worker to commit to invest, and thereby increase first-period wage competition.

To summarize, socially efficient specific investment requires the worker to invest proportionately to her probability of remaining at her first-period employer. Instead, she maximizes future income by investing proportionately to her probability of separating from her employer, which leads her to underinvest. If a the time of hiring the worker were able to commit to her lifetime wage-maximizing investment, she would invest more than the socially efficient level. This suggests that in a more general model allowing for some commitment and investment by both parties, solving the hold-up problem by introducing a third party and strategic price determination could merely replace the problem of underinvestment in relationship-specific assets with one of overinvestment.
2.3. Comparative statics

The comparative static predictions from a decrease in the marginal cost of general and specific capital provide additional insight into the relationship between the effects of matching and human capital on wage profiles, tenure effects and turnover.

**Proposition 5:** A reduction in $\beta$ lowers the marginal cost of investment in general capital and:

(a) increases the worker’s investment in general capital;
(b) increases the wages of old workers relative to young workers, i.e., steepens wage profiles; and
(c) has no effect on firm profits, the rate of turnover, or the relative wages of movers and stayers.

None of these predictions is at variance with the standard conception of general capital. In particular, Proposition 5c demonstrates that simple matching alone interacted with general capital is insufficient to explain the ‘tenure effect’ on wages as measured by the differential between the wage of movers, workers who separate from their initial employer, and stayers, those who do not.\(^4\)

**Proposition 6:** A reduction in $\alpha$ lowers the marginal cost of investment in specific capital and:

(a) increases the worker’s investment in specific capital;
(b) decreases the wages of old workers relative to young workers, i.e., flattens wage profiles; and,
(c) reduces firm profits and the rate of turnover.

That wage profiles are flatter as a result of increased investment in specific capital is somewhat surprising. Typically, the first-period wage is assumed to be unaffected by anticipated specific investments. But when $w_1$ adjusts to equate the expected profits of the two firms, it rises with an increase in anticipated specific investment. The second-period expected wage also rises, but by less than the first-period wage. Notice that the increase in the first-period wage is not a result of firms ‘financing’ specific investment to ensure that the worker undertakes it, as no such commitment by the worker is extracted at the time the wage is negotiated. Rather, higher wages are paid to attract the worker to the firm, and are secured by the knowledge that the worker will invest in specific capital, and thereby increase

\(^4\) In this model, if there were no investment, wages would fall between periods. This is because employers are willing to bid the wage up to a young worker’s expected value, while for old workers, the wage is equal to the expected minimum of the two valuations.
expected future profit. In fact, as the above discussion of lifetime wage maximization implies, the increase in the first-period wage exceeds the expected gain in second-period profit, so expected profits fall. Wages rise in the first period to offset the expected differential between firm profits which widens both because firm $e$ period 2 profits rise and firm $c$ period 2 profits fall.

**Proposition 7:** (a) If the cost of specific investment is prohibitively high, so $s^* = 0$, the expected second-period wage conditional on remaining with firm $e$ equals the expected second-period wage conditional on moving to firm $c$.(b) As the cost of specific investment goes to zero, $s^*$ approaches the maximum match differential. In the limit, the expected wage of stayers equals $E\theta + g^*$, which is exceeded by the expected wage of movers, $\theta + g^*$.(c) For intermediate values of the cost of specific investment, the expected wage of movers may be less than or greater than the expected wage of stayers.

As the cost of specific investment decreases, the level of investment increases, and eventually the expected wage of movers exceeds that of stayers: sorting across firms leads to substantial wage gains for movers. Ever since the work of Jovanovic (1979a), the question, of what portion of the estimated coefficients on firm tenure variables in wage regressions is due to true ‘tenure effects’ — such as specific human capital accumulation — and what portion is simply the effect of better matches being more durable, has been vigorously debated. Typically in the underlying theory, specific capital accrues mechanically. One might think that had an empirical measure of specific capital investment been available, a finding that the wages of movers rose more than those of stayers would show that workers do not share in the returns to specific capital, and preclude this explanation of firm tenure effects. This proposition cautions against such an interpretation: the relative wage of movers and stayers can fall or rise when the cost of specific investment falls. A full examination of tenure effects requires a multiperiod model.

3. Extensions

3.1. Cross-firms variations in investment productivity

So far, I have assumed that the two firms are symmetric in the opportunities they offer workers for acquiring human capital. Consider instead what would happen if the productivity of a worker’s investment were different across firms. (Cross-firm differences in the cost of investment would have a similar effect.) If employment at one firm allows a worker to accumulate more general skills per unit of investment than employment at the other, all workers would choose to work at the more productive firm. Workers discount the initial wage offer from the less productive firm. In equilibrium, the initial wage the high productivity firm
must pay to attract the worker is equal to the wage that makes the low productivity firm indifferent to hiring the worker, minus the additional net income the worker expects to earn in period 2 from her higher general skill productivity. Compared to the symmetric case of Section 2, the ‘losing’ firm \( c \) and the worker are no better or worse off, and the ‘winning’ firm, \( e \), earns higher profits. The additional profit to firm \( e \) is simply the market value of its superior technology for providing general skills to workers. The principle difference is that all workers choose the same initial employer.

If instead specific capital investment is more efficient at one firm, but general investments are symmetric, a similar situation results. All workers choose employment at the more efficient firm, the initial wage is set by the indifference condition of the losing bidder, and so the additional surplus from more productive investment technology accrues to the firm which owns it. This highlights the fact, obscured by the symmetry assumed in Section 2, that it is the potential value of the specific investment at the losing firm which causes wages to rise in the first period.

3.2. Informed investment

Consider the effect of a worker learning her match at firm \( e \) before she undertakes any investment. Such knowledge does not affect the general investment: because the expected second-period wage for all matches of worker rises dollar for dollar with the value of general skills, match quality does not enter the worker’s decision. On the other hand, the return to specific capital is match-dependent, and therefore, so is the investment decision. In Section 2, I assumed that firm \( c \) learns its match before bidding for the worker in the second period. Alternatively, one might assume that this information is available only after winning the worker. In this case, firm \( c \) can do no better than to raise its bid up to the expected value of the worker. In either case, the following proposition holds.

**Proposition 8:** If at the time of investment workers know their match quality at firm \( e \), then workers invest more in general capital than in specific capital, and the specific investment is a non-increasing function of match quality, i.e.:

\[
g^* > s^*(\theta_e) > 0;
\]

and

\[
\frac{\partial s^*(\theta_e)}{\partial \theta_e} \leq 0.
\]

The worker weighs her investment in specific capital by the probability of turnover, which is always less than one, so she invests more in general than specific skills. Now, however, the weight differs across matches: since the because
the probability of turnover decreases in match quality, so does investment in specific capital. This result is the opposite of what one would expect in a model with exogenous turnover in which the workers least likely to leave invest most in specific capital (see, e.g., Jovanovic, 1979b).  

A worker with a poorer match at firm e invests more in specific capital than one with a better match, but not so much more as to fully offset the difference in match quality. This makes sense: if a poorly matched worker did fully compensate for the difference in match quality, she would face the same probability of turnover as the good worker. But then the marginal benefit of the last unit of specific capital would have yielded the same expected return to the two workers, and yet have entailed a higher marginal cost for the poorer-matched worker. Thus, if the better worker were investing optimally, the poorer worker would be overinvesting. As the general investment does not affect the probability of turnover, this leads to the following result.  

**Proposition 9:** If at the time of investment workers know their match at firm e, the probability of turnover and the worker investment in human capital are positively correlated.

### 3.3. Free entry

Suppose that there are an infinite number of employers, each of which can for a cost \( k \) participate in auctions for the worker’s services. Then there is a mixed strategy equilibrium in which each potential competitor enters with sufficient probability that the expected gross profit for every competitor is equal to \( k \). Let \( n \) denote the expected number of firms competing for worker in the second period. Notice that \( n \) decreases in \( k \).

The effect of the general human capital investment is unchanged from the model above: the identity of the winner and runner-up in the competition for the worker is immaterial. When workers invest in specific capital, expected second-period wages are bid-up in the event that firm e has the second highest valuation, which occurs with positive probability, so some such investment occurs. Similarly, a wedge between the expected profit of the employer and any other second-period

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5 The result that the investment in general capital is independent of match quality arises because of the separability of both the cost and productivity of general and specific capital. If the costs of investment were such that increases in specific investment raised the cost of general investment, say because the worker faces a time constraint, then poorly matched workers would invest less in general skills.

6 Notice that this does not predict that overall investment is positively correlated with turnover, as investment by firms is not included. Firms prefer to invest most in workers for which the probability of turnover is lowest.
competitor exists, so period 1 wage competition intensifies. However, the wedge is less than in Section 2, since firm \( e \) only captures the additional and the surplus when it has the best of the \( n_2 \) matches, and the competitors earn zero in all cases.

The more firms that enter, the smaller are each of these effects. However, for any positive \( k \), the main results from Section 2 are obtained. Workers are willing to invest some positive amount in specific skills, and because the employer also gains from this, period 1 wages are bid-up. Because of the zero-profit constraint, the worker receives the whole of firm \( e \)’s expected second-period profit as a transfer in the first period. The most important difference is that the whole value of specific capital declines as \( k \) gets small. So while from a career perspective workers still invest too little in specific capital, the lifetime optimal amount decreases as \( k \) falls, and so may be less than the optimal general investment. Further characterizations depend on the match distribution.

4. Conclusion

This paper constructs a model in which firms hold all of the bilateral bargaining power, yet worker–firm matching ensures that workers receive a share of productivity increases resulting from relationship-specific investment. Part of the wage increase comes in the initial wage, not as a result of firms financing specific investment to ensure that workers undertake it, but rather from firms competing to attract workers whom they know will yield higher profits when trained. The size of the wage increase is related to the potential investment in specific skills at firms other than that which wins the worker. In an industry for which opportunities across firms for human capital investment are very similar, this would look like an upfront payment for specific investment.

Workers undertake specific investments, looking forward to future competition between their current employer and outside firms. In some cases, workers with poorer current matches may even invest more in specific capital. The fact that wage tenure profiles depend on both realized and potential investments in human capital complicates the testing of human capital models in the presents of matching. This might explain the results in Levine (1993), where doubt is expressed about the usefulness of human capital theory based on the finding that workers, who (according to their own assessment) perform jobs which require extensive training, face wage profiles which are no steeper, and have turnover rates that are no lower than those who work in low training jobs.

It is straightforward to alter the model so that workers invest more in specific than general human capital. The easiest way is to redefine the functions \( H \) or \( C \). Alternatively, Bernhardt and Scoones (1993) develop a model of promotion in which specific capital yields an additional benefit to workers. There firms are asymmetrically informed about the general capital of workers. When workers accumulate specific capital, employers are less reluctant to reveal their general
skills by promoting them. If this promotion effect is strong enough, it may outweigh the higher direct wage incentive of general capital.

Many questions cannot be addressed in this simple two-period model. It would be interesting to know if a multiperiod model could explain the wage and career dynamics reported in Topel and Ward (1992), e.g., how do career mobility and investment in human capital evolve? If employers bidding for experienced workers knew that these workers might leave again, how would their offers be affected? These questions are the subject of future research.

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Appendix A

Proof of Proposition 1: Workers choose human capital to solve:

$$\max_{s,g} E W_s = \int_{-1}^{\bar{\tau}} \left[ \int_{0}^{Q} (\theta_s + s) dM(\theta) \right] dM(\theta_s)$$

$$+ \int_{0}^{\bar{\tau}} (\theta_e + s + g) dM(\theta_e) \right] dM(\theta_e)$$

$$+ \int_{-1}^{\bar{\tau}} \int_{0}^{\bar{\tau}} (\theta_e + g) dM(\theta_e) dM(\theta_e) - C(s,g;\alpha,\beta).$$

The necessary conditions for a maximum are that:

$$g^* \text{solves: } 1 - C_g = 0,$$

$$s^* \text{solves: } \int_{0}^{\bar{\tau}} (1 - M(\theta_e + s) dM(\theta_e) - C_s = 0,$$

where the subscripts denote partial derivatives. Notice that these conditions imply that the problems are separable. Under the assumptions on the cost function, these are also sufficient conditions, i.e.:

$$-C_{ss} < 0,$$

$$-\int_{0}^{\bar{\tau}} \mu(\theta_e + s) dM(\theta_e) - C_{ss} = D < 0.$$ 

The first term in Eq. (A2) is the probability that the worker is more valuable at firm e than firm e, so is less than 1. Since C(·) is convex, this implies that $g^* > s^* > 0$. 

Proof of Proposition 2: Efficient investment requires \( s^{**} \) and \( g^{**} \) to maximize expected second period net output:

\[
\max_{s, g} \int_{q}^{\theta^*} \left[ \int_{\theta_e + s}^{\theta^*} (\theta_e + s + g) dM(\theta_e) + \int_{\theta_e + s}^{\theta^*} (\theta_e + g) dM(\theta_e) \right] dM(\theta_e) \\
+ \int_{\theta_e + s}^{\theta^*} (\theta_e + s + g) dM(\theta_e) dM(\theta_e) - C(s, g; \alpha, \beta).
\]

Inspection reveals that the problem is unchanged with respect to \( g \), so \( g^{**} = g^* \). For \( s \), the necessary condition is that:

\[
s^{**} \text{ solves } 1 + \int_{q}^{\theta^*} (M(\theta_e + s) - 1) dM(\theta_e) - C_s = 0. \tag{A3}
\]

The first two terms in this expression are the probability that the worker is more productive at firm \( e \) than firm \( c \). For \( s = 0 \), this probability is 0.5, so, initially at least, social- and self-interest coincide. However, once \( s > 0 \), the worker is likely to be more productive at firm \( e \), and thus \( s^{**} > s^* \), the level given determined by Eq. (A2).

Proof of Proposition 3: Substituting Eqs. (2) and (3) into Eq. (1) and simplifying, expected lifetime income is given by:

\[
E \theta + s + Ew_2 - C(s, g; \alpha, \beta).
\]

The necessary conditions for lifetime income maximization are that:

\[
g^{***} \text{ solves: } 1 - C_s = 0, \tag{A4}
\]

\[
s^{***} \text{ solves: } 1 + \int_{q}^{\theta^*} (1 - M(\theta_e + s)) dM(\theta_e) - C_s = 0. \tag{A5}
\]

It is immediate that \( g^{***} = g^* \). The additional effect of specific investment on first-period wages that are ignored by the worker choosing \( s^* \) together with the assumptions on \( C \) implies that \( s^{***} > s^* \). Finally, because the second term in Eq. (A5) is positive, the assumptions on investment cost imply that \( s^{***} > g^{***} \).

Proof of Proposition 4: Immediate from a comparison of Eqs. (A3) and (A5).

Before proving Propositions 5 and 6, it is useful to assemble the following quantities.

Firm \( c \)'s expected profit (and in equilibrium, the profit of firm \( e \) also) is given by:

\[
E \pi_c = \int_{q}^{\theta^*} \left[ \int_{\theta_e + s}^{\theta^*} (\theta_e + s - s) dM(\theta_e) \right] dM(\theta_e). \tag{A6}
\]
The expected wage change between periods 1 and 2 is:

\[ E_{w_2} - w_1 = E_{\theta_2 \theta_1} \{ \min(\theta_2 + s, \theta_1) \} + g - E\theta - s. \]  

(A7)

The probability of turnover is:

\[ \Pr(\theta_2 > \theta_1 + s) = \int_{\theta_1 + s}^{\theta_2} \int_{\theta_1}^{\theta_2} dM(\theta_2) dM(\theta_1). \]  

(A8)

The expected wages of movers and stayers are:

\[ E(\theta_2)_{\text{move}} = g^* + \int_{\theta_1}^{\theta_2} \int_{\theta_1 + s^*}^{\theta_2} \mu(\theta_2 + s^*) dM(\theta_2) dM(\theta_1), \]  

(A9)

\[ E(\theta_2)_{\text{stay}} = g^* + \int_{\theta_1}^{\theta_2} \int_{\theta_1 + s^*}^{\theta_2} \mu(\theta_2 + s^*) dM(\theta_2) dM(\theta_1) + \int_{\theta_1}^{\theta_2} E\theta dM(\theta_1), \]  

\[ 1 - \int_{\theta_1}^{\theta_2} \int_{\theta_1 + s^*}^{\theta_2} dM(\theta_2) dM(\theta_1). \]  

(A10)

**Proof of Proposition 5:**

(a) Differentiating the first-order condition (A1), \( \partial g^* / \partial \beta = -C_{g\beta} / C_{gg} < 0; \)

(b) From Eq. (A7), and the fact that the specific and general investment choices are independent, \( \partial (E_{w_2} - w_1) / \partial \beta = \partial g^* / \partial \beta < 0; \)

(c) Direct inspection reveals that firm profits, turnover, and the relative wages of movers and stayers are unaffected by the level of general capital, and hence, by a decrease in \( \beta. \)

**Proof of Proposition 6:**

(a) Differentiating Eq. (A2) and using the second-order conditions, \( \partial s^* / \partial \alpha = C_{a\alpha} / D < 0; \)

(b) From Eq. (A7), \( \partial (E_{w_2} - w_1) / \partial \alpha = (\int_{\theta_1}^{\theta_2} \int_{\theta_1 + s^*}^{\theta_2} [1 - M(\theta_2 + s)] dM(\theta_2)] - 1 \times \frac{\partial^2}{\partial \alpha^2} > 0; \)

(c) From Eq. (A8), \( \frac{\partial}{\partial \alpha} \Pr(\theta_2 > \theta_1 + s) = -\int_{\theta_1}^{\theta_2} \mu(\theta_2 + s) dM(\theta_2) \frac{\partial s^*}{\partial \alpha} > 0; \) from Eq. (A5), \( \partial \sigma^*_\alpha / \partial \alpha = -\int_{\theta_1}^{\theta_2} \int_{\theta_1 + s^*}^{\theta_2} dM(\theta_2) \frac{\partial^2}{\partial \alpha^2} > 0. \)

**Proposition 7:**

(a) at \( s^* = 0, \) Eq. (A9) = Eq. (A10).

(b) as \( s^* \rightarrow (\bar{\theta} - \theta), \) the probability of moving goes to zero (Eq. (A8)), and the expected value of \( \bar{\theta} \) conditional on moving approached \( \theta. \) The wage of these workers approaches \( \bar{\theta} + s^* + g^* = \bar{\theta} + g^*. \)
(c) Neither \( \partial E(w_2^{\text{move}})/\partial \alpha \) nor \( \partial E(w_2^{\text{stay}})/\partial \alpha \) can be signed without further restrictions on the match distribution. Hence, the relative wages of movers to stayers may rise or fall with an increase in the cost of specific investment. In fact, even though \( \partial E w_2/\partial \alpha < 0 \), an increase in \( \alpha \) may cause both conditionals to rise. This is a variant of the so-called Simpson’s (or Stein’s) Paradox (see, e.g., Thornton and Innes, 1985).

**Proof of Proposition 8:**

(a) First assume that firm \( c \) learns its match before it bids for the worker. There are two subcases to consider.

(i) As long as \( \theta_c + s < \overline{\theta} \) worker \( i \) chooses human capital to solve:

\[
\text{max}_{s,g} E w_2 = \int_{\overline{\theta}}^{\theta_c + s} (\theta_c + g) dM(\theta_c) + \int_{\theta_c + s}^{\overline{\theta}} (\theta_c + s + g) dM(\theta_c) - C(s, g).
\]

It is clear that the optimal choice \( g^* \) is independent of \( \theta_c \) and identical to that calculated in Proposition 1. \( s^* (\theta_c) \) must satisfy:

\[
[1 - M(\theta_c + s^*(\theta_c))] = C_s(s^*(\theta_c)).
\]

Differentiating Eq. (A11) demonstrates that the optimal investment in specific capital decreases in \( \theta_c \):

\[
\frac{ds^*(\theta_c)}{d\theta_c} = \frac{-\mu(\theta_c + s^*(\theta_c))}{\mu(\theta_c + s^*(\theta_c)) + C_{ss}'} < 0
\]

(ii) If Eq. (A11) is not satisfied for \( \theta_c + s < \overline{\theta} \), the optimal investment solves this as an equality. Further investments in specific capital are of no value, because all workers are retained by firm \( \epsilon \), paid a wage equal to \( \epsilon \theta + g \).

(b) If instead we assume that firm \( c \) must bid on the worker without knowing its match, it will bid up to the point of zero expected profits, offering a maximum wage of \( \epsilon \theta + g^* \). The worker, knowing \( \theta_c \), chooses \( s \) to maximize:

\[
w_s(s, g^*|\theta_c) = \begin{cases} 
\epsilon \theta + g^* & \text{if } \epsilon \theta > \theta_c + s \\
\theta_c + s + g^* & \text{if } \epsilon \theta \leq \theta_c + s.
\end{cases}
\]

The optimal choice is \( s^* = \min[\max((\epsilon \theta - \theta_c), 0), C_{s}^{-1}] \). If \( \theta_c < \epsilon \theta \), the worker invests until either the marginal cost of investment equals 1, or \( \theta_c + s = \epsilon \theta \). If \( \theta_c \geq \epsilon \theta \), specific investments never increase \( w_2 \), so \( s^* = 0 \). Thus, the optimal investment decreases in match quality.

**Proof of Proposition 9:** Consider the first-order condition (A11). The term \( [1 - M(\theta_c + s \theta_c)] \) is the probability that the worker turns over; denote this as \( \Psi(\theta_c) \). Differentiating Eq. (A11) with respect to \( \theta_c \):

\[
\Psi'(\theta_c) - C_{ss}' = 0;
\]
where \( s' \leq 0 \) by Proposition 8, and \( C_{s_3} > 0 \) by assumption. This implies that \( \Psi'(\theta_i) < 0 \), i.e., workers with better matches invest less in specific capital and are less likely to be bid away.

References


