Commitment and strikes in wage bargaining

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Abstract

This paper analyzes the long-run strategic relationship between a firm and a union as a repeated bargaining game, where there is incomplete information on the player’s motivation on both sides and each party has a fall-back position. The firm and the union will engage in a reputation-building activity, that will produce a limited number of strikes over time. The bargainer that succeeds in building up a reputation for toughness and obtains a favorable payoff in the long-run is, either the more patient (or alternatively the more centralized), or the party with a higher initial probability of stubbornness, or the party with a smaller fall-back position. Our model also offers predictions on the dependence of strike incidence over time on several parameters. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

There has been much research in recent years trying to account for the occurrence of strikes and other disputes in a context of private information. Most of this literature deals with the case of a single contract negotiation between firms and unions, in which at least one party (usually the firm) has private information about some relevant variables (profitability, value of the labor product, etc.) and

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the uninformed party (the union, generally) uses the strike as a screening device to achieve a better wage from more profitable firms.

This paper is a first attempt to apply a new approach, quite well known in game theory, but surprisingly, not very used in labor economics. We analyze the long-run strategic relationship between a firm and a union as a repeated bargaining game, where there is incomplete information on the player’s motivation on both sides and where each party has a fall-back position. In this context, the firm and the union will have incentives to engage in reputation-building activity trying to convince each other that they are “tough” bargainers. This dynamic struggle between the bargaining pair will produce a number of strikes over time whose chief motivation is not to obtain a high wage in a particular period but, instead, to determine the long-term wage and profit regime.

We have chosen this kind of modelling for several reasons. Firstly, it is known that firms and unions negotiate wage increments periodically (for example, every year in Spain or every 2 or 3 years in the USA), and therefore, it is important to incorporate the linkage of these periodic encounters over time. We take a first step to study this dynamic bargaining by modelling negotiations as a repeated game in a stationary environment. Obviously, this is an important simplification because certain variables, such as the profitability of the firm or the fall-back positions of the players, remain fixed over time. But in any case, we believe that even in this simplified framework, our model can highlight some new issues about strikes and wage determination absent from the usual static analysis. In this sense, we contemplate our approach as complementary to the one-shot asymmetric information models.

Secondly, our model contains a different kind of incomplete information. It is not about the profitability of the firm or the value of the labor services, but about the behavior (or type) of a player. In particular, there is a small probability that the union and the firm are of a tough type, committed to demanding the highest possible payoff for itself, and to strike if the other party does not concede. We believe that this kind of uncertainty is widespread at the beginning of any relation.

And finally, our model also incorporates an important characteristic in real life, namely the “fall-back” position of the players. That is, what they can guarantee for themselves if they abandon the long-term relationship in which they are involved. In our model, the fall-back position is a certain wage for the union and a minimal profitability for the firm. These payoffs will be key factors determining the number of strikes that a party is able to carry out.

Our analysis follows the standard approach on reputation effects started by Kreps and Wilson (1982) and Milgrom and Roberts (1982) and continued by Fudenberg and Levine (1989). In particular, it relies technically on the work of Schmidt (1993), who deals with the case of two long-term players in a class of games of conflicting interests.

The main insight of the model is the following. If the firm and the union struggle to build up a reputation by making high demands and threatening with
strikes, any deviation from this tough strategy will be very harmful. The reason is that if, say the firm, reveals itself as a normal (rational) type, only the union can then be stubborn. And, provided he is sufficiently patient, the union will succeed in building a reputation for toughness obtaining a high average wage in the long run. Thus, if a component of a bargaining pair decides to mimic the commitment type, he must start at the beginning of the relation. But, on the other hand, the strike activity can only be maintained for a finite number of times for normal types, because in equilibrium, they have to guarantee their fall-back positions. As a result, we would observe a limited number of strikes for reputation motives at the beginning of the relation in a bargaining pair. The party with the greatest bound on its strike or reputation-building activity, that is, the party who is more willing to bear the costs of strikes will be able to obtain a favorable payoff.

Since reputation building is a long-term phenomenon, our main results focus on long-run variables. Namely, our model offers predictions on strike incidence over time and on long-run average wages and profits in an established bargaining pair. Nevertheless, the analysis has also implications for the aggregate empirical evidence. Its main contribution is the explanation of a series of short strikes, observed in every period, and which are independent of the economic variables of that particular period. There is some evidence of this type of strikes for the Spanish case Jimenez-Martin, 1995. Our model also provides an explanation for "strike waves", as for example, those observed in the first part of the 20th century in some countries.

Let us now explain, in more detail, the main predictions about strike incidence over time in a particular bargaining pair, about the determinants of which party will succeed in building a reputation, and finally, about the factors that influence in how close the average wages and profits are to the preferred payoffs.

Firstly, strike incidence increases with the profitability of the firm and decreases with the fall-back positions opportunity costs of the firm and the union. A higher profitability increases the incentives to build a reputation for toughness for both players because the prize is higher. Thus, strike incidence per unit of time rises unequivocally. On the other hand, an increase in its minmax or fall-back position makes it more costly for a bargainer to endure strikes since by conceding right away, the bargainer can secure this increased payoff immediately. Therefore, given two bargaining pairs with the same parameters except profitability, we would expect more strikes in the bargaining pair with a higher profitability, mostly at the beginning of the relationship. And, in the same way, we would expect less strikes in the bargaining pair in which at least one party has a higher fall-back position.

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2 Formally, as we work with a repeated game with an infinite horizon, there is a multiplicity of equilibria. Our results characterize bounds on strike incidence and on average payoffs in equilibrium.
Strike incidence is weakly increasing with the patience of the players. Nevertheless, it is typically difficult to justify why one party is more patient than the opponent. However, there is an alternative interpretation that considers that a higher degree of centralization is equivalent to a higher discount factor. A bargaining pair in our model can be a single union and a single firm, but it could also be a national union bargaining in each period with a sequence of small, long-lived and equally patient firms or, a big firm negotiating in each period in turn, with a series of unions each representing some fraction of its employees. In this sense, given two bargaining pairs, with the same remaining parameters, our model would predict a higher strike incidence in the bargaining pair in which there is a bigger difference in the degree of centralization between the parties. Finally, strike incidence is weakly decreasing with any of the initial assessments about the toughness of the players.

Secondly, our model shows that the party who succeeds in building up a reputation for toughness and obtaining a relatively high payoff in the long-run is, either the more patient (or alternatively, the more centralized), or the party for which there is a higher initial probability of stubbornness, or the party with a smaller fall-back position. A more patient player (or more centralized) can distribute the entire cost of the strike activity among a greater number of periods. A bargainer with a smaller fall-back position wins because its opportunity cost of the strike is now smaller.

Finally, we also characterize the conditions in which the average wage or profit gets close to the preferred payoff of the winning party. Again patience plays a key role in obtaining a high or low average equilibrium wage or profit. However, it is not only that the “winner” has to be patient, but he must also be sufficiently patient as compared with the opponent. Recall that a high degree of centralization is equivalent to having greater patience. Therefore, our model captures quite an intuitive result: a “big” union (firm) is in a strong position when confronted, in a long-term relation, with a sequence of “small” firms (unions) in each period. The big party is more able to bear the costs of strikes that are necessary to endure a tough position and impose a favorable payoff regime.

The rest of the paper is organized as follows. Section 2 describes the repeated wage bargaining game. Section 3 analyzes the reputation and strike activity of the parties. In Section 4, we obtain and comment on the main result of the paper on strike incidence over time and average wages and profits. In Section 5, we discuss in more detail the results and the different predictions of the model. Finally, Section 6 concludes by discussing some related literature and advances future lines of research.

2. The wage bargaining game

Consider a bargaining pair, a union and a firm (as unitary actors we will call them, he and she, respectively), engaged in a repeated relationship in a stationary
They bargain over a surplus $b$ each period $t$, in $T$ periods, where $T$ may be finite or infinite. Most of the paper deals with the infinite horizon although the results hold if there is a sufficiently long finite horizon.\footnote{The interpretation of the infinite horizon is that there is always a positive probability that the relation continues for an additional period. This is clearly the more realistic case.} We consider, for simplicity, a single union and a single firm, but it is possible to extend the model to a wide range of situations such as, for example, a national union bargaining with a sequence of long-lived firms in each period, or alternatively, a single firm negotiating in each period, with several of her unions, each representing a fraction of her employees.\footnote{These different types of bargaining pairs reflecting different levels of centralization will play an important role in our analysis, as we will show in Section 5.}

We suppose that both parties have a fall-back position that determines the interval in which the offers can be made. We assume the firm has to guarantee for herself a minimal profitability. If the shareholders obtain a lower profitability, then they will leave the firm because they can get this benefit elsewhere. This minimal profitability, which we call $\pi_0$, will determine the biggest wage that the union can get. That is, $w^* = b - \pi_0$.

We also assume that the union has his reservation utility: the firm cannot enforce a lower wage because the workers would leave the firm and would go to an alternative sector. This wage, $w_0$, can be considered, for example, as the wage that is paid to the workers in the non-unionized sector. This wage also determines the maximal profit that the firm can get, that is, $\pi^* = b - w_0$.

The constituent game $G$ is as follows: in each period $t$, both players, the firm and the union, simultaneously submit offers, $w_i$ and $w_u$, respectively, where both offers have to belong to the interval $W = [w^*, w_0]$, where $w^* > 0$ and $w^* < b$. If the firm’s offer is greater than or equal to the union’s offer, the agreed wage is $w_i$ and the negotiation is over. If the firm’s offer is smaller than the union’s offer, then the union rejects the offer with a strike, where both players get zero.\footnote{This is a simplification and excludes the possibility of considering the duration of strikes. A more realistic stage game, for example, an alternating offers procedure, could allow endogenizing strike duration, but the analysis of such a repeated game would become very complex.} Obviously, this simple simultaneous demand game is not a realistic model; however, it is adopted for simplicity.\footnote{This stage game could be justified in terms of delegation of representatives of the firm and the union, when it is not possible to react to the other’s offer.} Nevertheless, an advantage is that it provides the same bargaining power to the parties.\footnote{Other stage games are also quite simple, such as a take-or-leave-it extensive form, but these games give too much bargaining power to one party.} Notice that in this stage game, the participation constraints coincide with the minmax payoff for each player. In fact, this is all we need to obtain our main results. In other words, our conclusions...
would hold for any stage bargaining game in which the worst agreement for each party (minmax payoff) is better than the disagreement (strike) payoff.8

Let \( g_i(w_i, w_o) \) denote the payoff function of player \( i \) (where \( i = f, u \), the firm or the union) in the stage game \( G \). We also denote \( d_i \) as the minmax payoff of player \( i \), that is, \( d_i = \pi_o \) and \( d_u = w_o \).

Let \( G^T \) denote the stage game \( G \) repeated \( T \) times, where \( T \) may be finite or infinite. The game \( G^T \) represents the repeated collective negotiations between a firm and a union involved in a long-run relationship. The firm (union) discounts her (his) payoff in future periods with the discount factor \( \delta_i < 1 \) (\( \delta_u < 1 \)). In the repeated game, the overall payoff for player \( i \) (\( i = U, F \)) from period \( t \) onwards (including period \( t \)) is given by

\[
V_i^t = \sum_{\tau=t}^{\infty} \delta_i^{\tau-t} g_i^\tau.
\]

The results are stated in terms of average discounted payoffs \( v_i \), where

\[
v_i = (1 - \delta_i) V_i^t = (1 - \delta_i) \sum_{\tau=t}^{\infty} \delta_i^{\tau-t} g_i^\tau.
\]

Let \( h' \) be a history of the repeated game out of the set \( H' = (W \times W)^T \) of all possible histories up to period \( t \). A pure strategy for the repeated game of the firm is a sequence of functions, \( s'_f = H'^{t-1} \rightarrow [w_o, w^*] \). Similarly, a pure strategy for the union in the repeated game would be a sequence of functions, \( s'_u = H'^{t-1} \rightarrow [w_o, w^*] \).

Let \( \sigma'_u \), \( \sigma'_f \) denote a mixed (behavioral) strategy of the union and the firm, respectively. The set of all pure mixed strategies is denoted by \( S_U \) or \( S_F \) (\( \Sigma_U \) or \( \Sigma_F \)).

Let us introduce some additional concepts that will play an important role in what follows. We define the Stackelberg action of a player, in terms of the stage game, as the action that yields the highest possible payoff, provided his opponent plays a best response. The Stackelberg action of the firm \( (w_i^t) \) would be “Offer \( w_o \)”, which would give to the firm a “Stackelberg” payoff of \( b - w_o \), if the union plays his best response “Demand \( w_o \)”. In this case, the union’s share would be \( w_o \), his minmax payoff. The union plays a non-best response to the Stackelberg action of the firm, when he demands a wage bigger than \( w_o \), obtaining both players a payoff of zero because of the strike.

The Stackelberg action of the union \( (w_o^u) \) is “Demand \( w^* \)”. If the firm plays her best response, “Offer \( w^* \)”, to this action, the union would get \( w^* \) and the firm \( b - w^* \), her minmax payoff. If the firm does not play a best response, that is, she offers a wage different to \( w^* \), then the union will strike and the payoff will be zero for both parties.

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8 Recall that the minmax payoff would be the payoff that a player can guarantee himself if he thinks that the opponent’s intentions are to play an action to harm him.
In this stage game, the maximal payoff the players can get is the same as their Stackelberg payoff. That is, \( w^* \) for the union and \( b - w_o \) for the firm.

As we have seen above, our stage game has the feature that the Stackelberg action of the union holds down the firm to her minmax payoff, and the Stackelberg action of the firm holds the union to his minmax payoff. Therefore, the game consists of two-sided conflicting interests (Schmidt, 1993).

In what follows, we will assume that there is a particular kind of incomplete information. In real life situations, there is always some uncertainty about the actual motivation of the opponents. The players can be of many different types. For example, the union can be a “normal” union, in the usual sense in economics, that is, concerned only about his own payoff. But there can also be other types of unions concerned, for example, by considerations of fairness, inequality aversion or even by political goals, where each one of these types of union might have a different target wage and strike policy. On the other side, the firm could also have different types. For example, there can be a “normal” firm, but there can exist uncertainty about its personnel policy, about its profitability, its costs, etc. We could assume formally that each party can be one of a finite number of types. Nevertheless, we are interested in the equilibrium behavior of the normal types of union and firm in this context of incomplete information. There is a well-known result in perturbed repeated games, stating that if there is, for example, a positive probability of the existence of a type of union committed to demand \( w^* \), the highest possible wage, then the best the normal type can do, in case it is worth mimicking the behavior of other types of union, is to imitate this commitment type. Therefore, in what follows to simplify the model, we will work with a simple set consisting of two types for each player. In particular, there is a positive probability, \( \mu_u^\star \), that there is a commitment type of union, which we will call a “fighting union”, \( U^\star \), for which the dominant strategy in the repeated game is: “Demand \( w^* \)”, in every stage game.9

On the other hand, there is also a probability, \( \mu_f^\star \), that there is a type of firm, which we will call a “tough firm”, \( F^\star \), for which the dominant strategy in the repeated game is “Offer always \( w_o \).” These are the “commitment types” in the same sense of Fudenberg and Levine (1989). Both types of players are “committed” to play their “Stackelberg actions”. We will call \( G(\mu_u^\star,\mu_f^\star) \) the perturbed repeated bargaining game. The problem we want to investigate is, as we said before, whether the normal firm and union can establish a reputation for being “strong” players in order to obtain a higher payoff than they would get in a complete information situation.

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9 Note that the strategy of this union is to demand the highest possible wage compatible with the firm obtaining her minimal profitability. Thus, there will be a different “target” wage depending on the size of the surplus, that is, if \( b \) increases, \( w^* \) will also increase and vice versa.
3. Reputation and strike activity

In this section, we want to analyze the incentives of the components of a bargaining pair (firm and union) to engage in a reputation-building activity by calling and/or enduring strikes. This activity may involve some losses in each period due to the strike but might yield a higher payoff to one party in the long run.

We will initially consider, for simplicity, just one-sided incomplete information. The firm, for example, does not know the true type of union, which may be of two types: a fighting or a normal union. In this context, the normal union has an incentive to build a reputation for “toughness” mimicking the behavior of the fighting union, that is, demanding a high wage and striking in case the firm does not concede. But the firm clearly understands the incentives of the normal union and will also be interested, in turn, in trying to discover the true type of union, offering low wages and resisting strikes. Notice that from the firm’s point of view, these strikes act as a screening device.

The main question we want to address is whether there is an upper bound on the number of strikes in equilibrium. To obtain this, it is crucial that the firm has a positive fall-back position (her minimal profitability or minmax payoff) that she has to guarantee in equilibrium.

Let us provide some intuition into the arguments behind the following lemma and proposition. Consider a history of $t$ negotiations in which the union has always demanded $w^*$ and has gone on strike whenever the firm has not conceded. Suppose that the firm tries to test whether she faces the fighting union (or a normal union mimicking the fighting one) by offering a wage different to $w^*$ in $t + 1$. That is, she will play a non-best response against the union’s Stackelberg action in period $t + 1$, yielding a loss to the firm of $\pi_0 = b - w^* > 0$, because of the strike. On the other hand, the maximum possible gain that the firm can obtain, by discovering that the union is the normal type, is $b - w_c$. But this gain should not be delayed too much in the future, because in equilibrium, the expected payoff of the firm has to be at least the discounted minmax payoff for the rest of the game. More formally, the firm has to expect that the union will not strike with a probability higher or equal to $\eta > 0$, in one of the next $t + 1, t + 2, \ldots, t + M_f$ periods, otherwise, the firm would get less than her minmax payoff in equilibrium, which is a contradiction. This is what Lemma 1 states.

**Lemma 1.** Let $G^t(\mu_0^*, \mu_f^*)$ be a perturbed repeated bargaining game, and let $\mu_0^* > 0$ and $\mu_f^* = 0$. Consider any Nash equilibrium $(\bar{\sigma}_n, \bar{\sigma}_f)$ of the repeated game and any history $h^*$ consistent with this equilibrium in which the union has always demanded $w^*$. (This history exists because $\mu_0^* > 0$.) Suppose that, given this history, the firm offers a wage lower than $w^*$ (following her equilibrium strategy) in period $t + 1$, then the probability that the firm assigns to the event that the
union will not demand \( w^* \), given that he has always demanded \( w^* \) before, must be at least \( \eta \), in at least one of the periods \( t + 1, t + 2, \ldots, t + M_f \).

Where, for any \( \delta_t \), \( 0 < \delta_t < 1 \), there is a finite integer
\[
M_f \geq N_f = \frac{\ln(1 - \delta_t) + \ln \pi_o - \ln(b - w_o)}{\ln \delta_t} > 0
\]
and a positive number \( \eta \),
\[
\eta = \frac{(1 - \delta_t)^2 \pi_o}{(b - w_o)(1 - \delta_t^M_f)} - \frac{\delta_t^M (1 - \delta_t)}{1 - \delta_t^M} > 0.
\]

Proof: See Appendix A.

Every time that the firm offers a wage different from \( w^* \) in a block of \( M_f \) periods (in a history in which the union has always demanded \( w^* \)) and observes that the union strikes, she has to update her beliefs about the fact that she is confronted with the fighting union. This updating is done using the probability computed in Lemma 1. But as a probability is at most one, this updating can only last a finite number \( (K) \) of periods and after these \( K \) periods (Proposition 1), the firm will concede the wage \( w^* \). Consequently, the firm will only “induce” strikes a bounded number of times when confronted with a union that always demands \( w^* \).

The following proposition characterizes such a bound.

**Proposition 1.** Let \( G^f(\mu_o^*, \mu_f^*) \) be a perturbed repeated bargaining game, let \( \mu_o^* > 0 \) and \( \mu_f^* = 0 \) and let \( [m] \) be the integer part of \( m \). Consider any equilibrium and a history consistent with such equilibrium, in which the union has always demanded \( w^* \). Then there exists an upper bound, \( K_f(\mu_o^*, \delta_f, b, w_o, \pi_o) \), on the number of periods in which the firm offers a wage lower than \( w^* \) and there is a strike.

\[
K_f = K(\mu_o^*, \delta_f, b, w_o, \pi_o) = M_f \ln \mu_o^* = M_f \left( \frac{\ln \mu_o^*}{(1 - \delta_f)^2 \pi_o} + \frac{\delta_f^M (1 - \delta_f)}{1 - \delta_f^M} \right)
\]

where \( M_f = \lfloor N_f \rfloor + 1 \).

Proof: See Appendix A.

The above formula obtained on the bound on the number of strikes that might arise in any equilibrium, allows us to obtain some results about their dependence
on the different parameters of the model. We will only make some brief comments on these results, but they can be obtained formally with the corresponding derivative.

$K_t$ is an increasing function of $\delta_t$, the discount factor of the firm. If the firm is more patient, then future gains become more important and she may offer low wages and resist strikes to gather information about the type of union. In fact, note that when $\delta_t$ tends to one, $K_t$ tends to infinity.

$K_t$ is a decreasing function of $\pi_u = b - w^*$, the minmax payoff or fall-back position of the firm. The intuition is that if the minmax payoff is higher, the firm has more to lose if she keeps screening and suffering strike costs, when she is confronted with a union that always demands $w^*$.

$K_t$ is an increasing function of the Stackelberg payoff of the firm, $\pi^* = b - w_u$. The intuition is that as $\pi^*$ increases, there is an increment in the potential gains of the firm and raises the incentives to discover the true type of union. As a consequence, $K_t$ is a decreasing function of $w_u$ and an increasing function of $b$.

$K_t$ is a decreasing function of $\mu_u$, the firm’s prior beliefs that she is facing a fighting union. If the firm strongly believes that the type of union is the fighting union, she will screen a smaller number of times. The bigger $\mu_u$, the less updating is needed to reach the point where the posterior probability of being the fighting union would become higher than 1, and the smaller is $K_t$.

So far, we have only analyzed the case in which the firm has incomplete information about the type of union, but we can consider the opposite case in which the firm is the party that has private information and the union tries to discover the true type of firm through strike activity, in a history in which the firm always offers the lowest wage, $w_u$. We would then obtain a similar bound on $K_u$, the number of strikes in equilibrium that the union will incur. The formula on $K_u$ and $M_u$, obtained in the respective Lemma 2 and Proposition 2, are relegated to the Appendix B. We could also formulate some comments on the dependence of this bound $K_u$ on the parameters of the model, but we have omitted them because they are very similar to the previous ones.

We have only worked with one-sided incomplete information merely to simplify the exposition. Notice that when we are deriving the bounds $K_t$ or $K_u$, nothing changes in the proofs if there were two-sided incomplete information. However, an important difference appears in this case. For example, the normal firm, after a history in which the union has always demanded $w^*$, can offer wages different to $w^*$, not only because of the already mentioned screening motive, but also because she wishes to build her own reputation for toughness. But in this case, she is obliged to play her Stackelberg action, “Offer $w_u$”, because if not, she will reveal with certainty her type and will be unable to build any reputation in the future.

The next section analyzes the more relevant and general case of two-sided incomplete information, in which both the firm and the union have incentives to mimic the tough types.
4. Average wage and strike incidence

Intuition suggests that if there is two-sided incomplete information, both parties will wish to build their reputation and there will be a struggle to try to convince one another that they are the commitment types. Obviously, in every period in which there is screening and reputation-building activity, by one or both sides, there will be a strike.

In this section, we investigate three main questions. The first one is strike incidence over time caused by the struggle for motives of reputation in a long-term bargaining pair. The following question refers to the party that eventually is able to build its reputation, i.e., the winning party. And, finally, we want to find out which is the average payoff that the “winner” obtains, that is, the long-run wage and profit regime.

If both parties struggle to build a reputation, any deviation from the tough strategy provokes the probability of the commitment type to fall to zero. Thus, if a bargainer decides to imitate the stubborn type, he must start at the beginning of the relation. This process then becomes a dynamic war of attrition, in which the “loser” eventually obtains its fall-back position. But the tough position can be maintained for a finite number of times, as we know from the results obtained in the above proposition. In this sense, we would observe $K$ strikes, where $K = \min (K_u, K_f)$ and the party with the greatest bound on his strike activity “wins”, and thus, will be able to build his reputation.

Let us analyze in more detail the case in which $K_u > K_f$. Consider first the normal union. This always has the possibility of imitating the fighting union, demanding the highest wage. Which payoff would he get in this situation? There are two possibilities: On the one hand, with probability $\mu^*_u$, he would face the “commitment type” of firm, the one with a “tough labor policy”, obtaining a payoff of zero in every period. On the other hand, with probability $(1 - \mu^*_u)$, he would face the normal type of firm, and we know from Proposition 1, that she will reject the union’s demand of $w^*$ at most $K_f$ times. At worst, this will happen at the beginning of the relation. Therefore, after these $K_f$ periods, the union would get his preferred wage thereafter. Thus, the normal type of union, mimicking the “fighting” union would get at least an average expected equilibrium payoff of $v_u \geq (1 - \mu^*_u) \delta^K_f w^*$.

Now consider the normal type of firm. She also has the possibility of always imitating the “tough” firm, playing her Stackelberg action of offering $w_u$. There are also two possibilities: on the one hand, with probability $\mu^*_f$, she would face the fighting union obtaining a payoff of zero in every strike period. On the other hand, with probability $(1 - \mu^*_f)$, she would face the normal type of union, and we know from Proposition 2 that he will reject the low wage $w_u$ at most $K_u$ times, along a history in which the firm has always offered $w_u$. At worst, this would happen at the beginning of the relation. But these $K_u$ periods in which there are strikes can occur in two different ways: firstly, the union would demand a wage
different to \( w^* \), but greater than \( w_o \) and secondly, he would demand exactly \( w^* \).

In the first case, we are in a history in which the firm has always offered \( w_o \), and after these \( K_o \) periods, the firm can guarantee herself an average payoff of at least 

\[
(1 - \mu^*_o) \delta^K_o (b - w_o).
\]

This is due to the fact that the first time the union demands a wage different from \( w^* \), he reveals that he is a normal union with probability one. ( Recall that we are using a pure Stackelberg strategy.) From these periods onwards, he will be unable to build his reputation and the only reason to go on strike, will be to screen the other party.

But in the second case, the union would play his own Stackelberg action: “Demand \( w^* \)”, trying to build his own reputation. Hence, we would be in a history in which both parties are playing their Stackelberg actions. And, along such a history, we know that the firm cannot play more than \( K_f \) times a non-best response against the union’s Stackelberg action. But, as \( K_o > K_f \), the firm will be the first in giving in against the high demand of the union and will therefore discover her type. This fact means that the firm is unable to build up her reputation and get her Stackelberg payoff. Therefore, this is the worst case for the firm.

All the previous argumentation shows that we would observe (at most) \( K_f \) strikes in the long-term relation of this bargaining pair and the union would be the party which might build a reputation for “stubbornness”. Therefore, as the normal union may always choose the strategy of mimicking the fighting union, this defines a lower bound for his average expected equilibrium wage.

In general, these arguments hold whenever \( K_i > K_j, j \neq i, i, j = u.f \). Recalling that \( d_i \) is the mimmax payoff for player \( i \), we can now state this result formally in the next proposition:

**Proposition 3.** Let \( G^u(\mu^*_u, \mu^*_f) \) be a perturbed repeated bargaining game, and let \( \mu^*_i > 0, i = u.f \). If \( K_i > K_j \), then

\[
v_i \geq (1 - \mu^*_i) \delta^K_i (b, w_i, \mu^*_u, \mu^*_f, w_o, p, \delta) (b - d_i),
\]

where \( v_i \) is any average equilibrium payoff for player \( i \) in any Nash equilibrium of \( G^u(\mu^*_u, \mu^*_f) \).

Proposition 3 illustrates in a very precise way the restrictions caused by the reputation and commitment effects on long-run wages and profits. We want to stress that our analysis does not yield a unique wage, but establishes a lower bound on the average equilibrium wage which, depending on the value of several parameters, may be tighter than the mimmax payoff. This bound would be a good measure of the value of a reputation for toughness for the winning party.

Therefore, if we observe a high average wage in a long-term relation, it is because the union has gained the reputation of a “tough” bargainer, and, on the contrary, if we see a low average wage regime, then it is the firm who has been able to build a reputation of establishing a tough labor policy.
Notice that the line of reasoning followed in order to obtain these lower bounds resembles some kind of war of attrition argument. The party with a greater bound in his screening and reputation activity, that is, the party more willing to bear the cost of a greater number of strikes, "wins". A natural and interesting case is to analyze under which conditions, the "winner's" payoff converges to the Stackelberg payoff.

Observe also that changes in some parameters influence both $K_u$ and $K_i$, affecting strike incidence. In this sense, it may also occur that a change in some parameters reverses the relation between $K_u$ and $K_i$ and thus, changes the party who obtains the reputational gains. All these questions will be analyzed more deeply in the next section.

5. Discussion of the results

Since reputation building is a long-term phenomenon, our main results focus on long-run variables. Our model offers predictions on strike incidence over time and on long-run average wages and profits in an established bargaining pair in a stationary environment. Recall that a bargaining pair in our model can be a single union and a single firm, as we have assumed so far, but it could also be, for example, a national union bargaining in each period with a sequence of "small" long-lived firms or, even, a firm negotiating in each period, in turn, with a series of unions each representing some fraction of her employees.

As we have pointed out before, our model, unlike one-shot screening models, does not have implications for strike duration or incidence in a particular period. Nevertheless, our model also has implications for the aggregate empirical evidence. Its main contribution is the explanation of a type of strikes without apparent economic motives in the short run, that is, strikes as mistakes according to Hicks' view. Our work would explain a series of short strikes, observed in every period, and that are independent of the economic variables of that particular period. To illustrate our idea, consider for example, the work of Jimenez-Martin (1995) for the Spanish case. He observes "a duality of strikes: short and long. On the one hand, long strikes yield a wage agreement concession on the part of the workers. Thus, they act as a revelation mechanism. On the other hand, short strikes produce a boost to wage settlements. This kind of strike acts as enforcement mechanism (strikes as accidents?)". Note that as long as the effect of a strike on wage level is very small, it suggests that short strikes are much more important in the Spanish case" (p. 38). In our model, the process of building a reputation implies that one party (or both) must strike (or resist strikes) in every period, mostly at the beginning of the relation, to show "toughness" in order to guarantee a favorable long-run payoff regime.

Another important piece of evidence that our paper would contribute to explain are the "strike waves" observed in the first periods of the 20th century in Europe,
Canada and other countries as noted in Huberman and Young (1995). These authors interpret these strike waves as a war of attrition at the beginning of the relation in a bargaining pair, struggling to obtain union representation, better working conditions, etc. Clearly, our "dynamic" war of attrition provides a better explanation of these facts than the usual static war of attrition that considers just a single negotiation.

Let us now develop in more detail the predictions of our model about strike incidence over time and wages and profits in the long-run within a bargaining pair.

As we noted above, our measure of strike incidence is $K = \min(K_u, K_f)$. One can observe that $K$ is a function of the profitability of the firm, the discount factors of the players, the prior probability of toughness, and the fall-back positions of the union and the firm.

Strike incidence is increasing in the profitability of the firm ($b$) and decreasing with the fall-back positions (opportunity costs) of the firm and the union. As we already know, an increase in the Stackelberg payoff of one party rises its incentives to build a reputation for toughness because the prize is higher. Therefore, a higher $b$ implies an increment in both Stackelberg payoffs for the players, thus both $K_u$ and $K_f$ would be higher, and in turn, $K$, strike incidence rises unequivocally. On the other hand, an increase in its minmax or fall-back position makes it more costly for a bargainer to endure strikes since, by conceding right away, this bargainer can secure this increased payoff immediately. Thus if, for example, there is an increment of the fall-back position of the firm, $\pi_f$, $K_f$ will fall, but this increase in the firm’s minmax would also decrease the union’s Stackelberg payoff, $w^* = b - \pi_f$, and $K_u$ will also fall. As a consequence, strike incidence $K$ decreases.

To sum up, given two bargaining pairs with the same parameters except for $b$, we would expect more strikes mostly at the beginning of the relation, in the bargaining pair with a higher profitability $b$. A similar conclusion can be obtained for bargaining pairs with the same parameters but with different fall-back positions. We would expect fewer strikes in the bargaining pair in which one or both parties have a higher fall-back position.

Strike incidence is weakly increasing with any $\delta_i$ (the discount factor of player $i$). As we commented in a previous section, an increase in $\delta_i$ will increase $K_i$, but

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10 Notice that, for simplicity, we have assumed a payoff of zero for both parties in case of a strike. But, obviously, we could suppose a positive and different payoff for both parties in case of a strike (albeit, a payoff smaller than their reservation utilities). For example, in some countries, strikers get an unemployment subsidy $s$. When this is the case, the relevant cost of the strike for the union is the minmax payoff minus the subsidy, that is, $w_u - s$. Therefore, an increase in $s$ would lower the cost of the struggle for reputation and would increase the incentives to strike, i.e., $K_u$ would increase. This prediction of our model is similar to the prediction of the joint-cost hypothesis on unemployment subsidies (see Kennan, 1980; Reder and Neumann, 1980). The only difference is that we are not talking about duration of a strike but about strike incidence per unit of time in a bargaining pair.
it will not affect the incentives to endure the other party’s strikes. Therefore, strike incidence would remain unchanged or increased. Obviously, if both discount factors increase, then strike incidence will increase unequivocally.

However, it is typically difficult to explain changes in the player’s patience and also to justify why firms and unions might face different discount rates. There is a different interpretation, of major interest in our wage bargaining context. Suppose, for example, that firm $F$ bargains in each period against $L$ “small” unions, $(u_1, \ldots, u_L)$. Each union $u_j$, which is long-lived and has the same discount factor as the firm, represents a fraction of the employees of the firm. Divide each period into $L$ subperiods and suppose that the firm bargains sequentially against a small union $u_j$ in subperiod $j$, $j = 1, 2, \ldots, L$. From the perspective of the firm, increasing the number of small unions is equivalent to increasing her discount factor, and would thus be in a better position to gain a reputation for toughness.

Obviously, a completely similar example can be constructed for a national union, which is bargaining in each period with a sequence of “small” firms of an industry. Therefore, different degrees of centralization, on each side of the bargaining pair, become an important element in our model through their influence on the discount factors.

Given two bargaining pairs, broadly defined in the above sense, and similar in all other parameters $(b, \pi, \omega, \mu^*)$, our model would predict a higher strike incidence in the bargaining pair in which there is a bigger difference in the degree of centralization between the parties. In this sense, Kuhn and Gu (1998) in a different context and with a different motivation, also obtain that an increase in the degree of centralization of the union causes an increase in the number of strikes.$^{11}$

Finally, strike incidence is weakly decreasing with any of the initial assessments, $\mu^*$, about the toughness of party $i$. An increment in $\mu^*$ would reduce $K^i$ but will not affect $K^j$. Therefore, $K^i$ would remain unchanged or would fall. Intuitively, this change might be originated by a modification in the leadership of the union or in the management of the firm.

In the process of building a reputation within a bargaining pair, we would expect strikes for a bounded number of periods, early on in the relation. After these periods of strikes, the party more willing to bear the cost of a greater number of strikes (i.e., with the greatest $K^i$) obtains a favorable payoff regime for the rest of the game.

Let us next analyze how the parameters determine which is the winning party. For this purpose, it is useful to think of a completely symmetric situation, that is, $\delta = \delta^i = \delta^j$, $\mu^i = \mu^j$, and $\omega = \pi^*$. Notice that this implies $K^i = K^j$.

$^{11}$ Kuhn and Gu (1998) predict that the formation of a national union (if negotiations are sequential) will lead to more strikes, higher wage demands and higher wage settlements. This is because it implies a sharp increment in the union’s discount factor (from 0 to 1).
Suppose now that the parties have the same fall-back position and the same initial priors \( \mu_i^0 \), then it is quite straightforward to show that the party with a higher discount factor will be the winner.

Recall that an alternative interpretation is that different discount factors reflect different levels of centralization. Consequently, our model will predict, for example, that all other parameters being equal, a national union confronted with a sequence of “small” firms in each period would be able to build a reputation for toughness and obtain a high average wage in the long run. The reason is that this greater patience or level of centralization will allow him to distribute the entire cost of the strike activity over a greater number of periods.

Suppose now that the parties assign different probabilities to the toughness of their opponent, but have the same fall-back positions and the same discount factors. The effect of asymmetric probabilities of being stubborn is clear: the party having a greater ex-ante probability of being tough can achieve a higher average payoff.

However, the issue of how asymmetries at the level of the fall-back positions (minmax payoffs) affect the outcome is less obvious. Suppose that starting from a symmetric situation, there is an increase in the union’s minmax payoff, \( w_a \). As building a reputation for toughness requires one to wait for the opponent to give in, and as the cost of waiting is now larger for the union, \( K_a \) falls. Nevertheless, the increment in \( w_a \) also has the effect of reducing the Stackelberg payoff of the firm \( (\pi^* = b - w_a) \), which, in turn, reduces the incentives of the firm to endure strikes, that is, \( K_f \) falls. It can be proven that the reduction in \( K_f \) is greater than in \( K_a \); therefore, the final effect is that the party with the smallest fall-back position (the firm in this example) gets her reputation of toughness, obtaining high profits in the continuation of the game (provided that she is sufficiently patient).\(^{12}\)

In summary, with all the rest of the parameters being equal between a firm and a union, the bargainer who succeeds in building up a reputation for toughness and obtaining a high payoff in the long-run is either the more patient (or alternatively, the more centralized), the party with the higher initial probability of stubbornness, or the party with a smaller fall-back position. That is, the party with the smallest relative cost for enduring strike activity.

Our model also provides a measure of the value of the reputation, namely, the lower bound on the equilibrium payoff, wage or profit, which was obtained in Proposition 3. How tight this bound is depends, obviously, on the values of the relevant parameters.

A natural and interesting question is to analyze under which circumstances these lower bounds are close to the preferred payoff. It is easy to check that there

\(^{12}\) The intuition of this result relies on the fact that the minmax payoff is a present payoff and, on the contrary, the Stackelberg payoff is a future payoff and the players discount the future.
are basically two possible situations in which one of the players can almost obtain his Stackelberg payoff.

Firstly, if \( \mu_j^* \) goes to zero, while \( \mu_i^* \) is kept fixed, the equilibrium payoff of player \( i \) is bounded below by almost his preferred payoff (for a \( \delta_i \) sufficiently high), as can be easily checked from Proposition 3.

The second and more relevant case has to do with relative patience. Considering again Proposition 3: if \( \delta_i \) goes to 1, while \( \delta_j \) is kept fixed (where \( i \) and \( j \) are the union or the firm, with \( i \neq j \)), the \( K_j \) periods are less important and in the limit, player \( i \) would get his Stackelberg payoff (for a \( \mu_i^* \) small enough). This result reflects the importance of the relative patience of the players, which determines the party most able to take advantage of reputation effects. It is not only that the union, for example, has to be patient, the important point is that he has to be patient enough as compared with the firm. If the firm’s discount factor increases for example, she may continue enduring strikes for a greater number of times when confronted with a “fighting” strategy of the union. That is, \( K_i \) increases. Thus, if the “normal” union wishes to imitate the “fighting” union, he has to wait for a greater number of periods until he can be sure that the firm will concede high wages. Then, to get the same average wage, the union has to be sufficiently more patient.

Recalling again our interpretation of different discount factors between firms and unions as different size of the bargainers (or alternatively, different levels of centralization), our model provides an explanation for the policy of some firms to bargain with the greatest possible number of unions, each one representing a different sector of her employees. In other words, we highlight a particular aspect in a repeated context of the usefulness of the old strategy “divide et impera”. 13

6. Conclusions and extensions

In this paper, we have modelled the strategic relationship between firms and unions as a repeated bargaining game with two-sided incomplete information, showing how the reputation effects can be used to explain strikes and wage determination. Even though these “reputation effects” have been informally discussed in the literature on strikes (Kennan, 1986), our work represents a first step to formalize this idea.

Our repeated bargaining game captures the dynamic behavior of a bargaining pair (union and firm), although it is admittedly very simplified, basically due to its

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13 A similar comment can obviously be made about an industry union negotiating, sequentially in each period, with the firms belonging to the industry.
stationarity assumptions. Nevertheless, it produces an interesting number of predictions, complementary to those arising from static screening models. Moreover, it also highlights important aspects, not captured in the previous work with one single negotiation, such as the impact of the different levels of centralization of unions and firms and their fall-back positions in the determination of strike incidence over time and long-run average wages and profits.

As we have explained, our model resembles a dynamic war of attrition (in a repeated context). However, it is worth stressing that it is very different from the static war of attrition (concerning a single negotiation). The latter focuses on strike duration and wage settlements in a particular period. By contrast, our repeated model focuses on long-run wages and profits.

To our knowledge, the only work other than ours, that shares the need to deal with a bargaining pair (firm and union) in a dynamic context are the papers of Kuhn and Gu (1998, 1999). These authors also look for insights not provided by existing static bargaining models. They emphasize the learning effect among different bargaining pairs in a sequential context, when the firm’s profitabilities are correlated and analyze its impact on strikes and wages. However, we focus on one long-run bargaining pair, albeit broadly defined, and analyze the reputation activity of the parties and its effects on strikes and wages in the long run. But, many of their results, especially those of Kuhn and Gu (1998) about centralization, are related to ours. In a certain sense, our model generalizes their result showing that the formation of a national union (when negotiations are sequential) leads to more strikes and high wage settlements. In their work, this is because a big union (more centralized) would better internalize the cost of the learning activity. In our case, a big union is more able to bear the cost of the strike activity over time in order to gain a reputation as a hard bargainer.

The simplified model presented in this work can be extended in several ways. The more promising is to relax the stationarity assumption on the profitability of the firm. We could, for example, assume that the parameter \( b \) follows a stochastic process. One possibility is to analyze the case in which there is no private information about \( b \), that is, both parties know the true realization of the value of \( b \) at the beginning of each period. Another, and more complex, extension would be to analyze the case in which only the firm has persistent private information, but not fully permanent, that is, only the firm knows the true realization of \( b \) (in the line of Kennan (1999)). This extension would allow one to integrate both the screening and the reputation motives in a repeated context, in the same model.

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Appendix A. Proofs of Lemma 1 and Proposition 1

Proof 1 (Proof of Lemma 1). Fix a history $h$ up to any period $t$ along which the union has always played $w^*$, such that $h^r$ has positive probability in an equilibrium $(\delta_t, \delta_t)$. Suppose that according to the equilibrium strategy, the firm chooses $s_{t+1}$ with positive probability, which is a non-best response against the Stackelberg action of the union, that is, offers a wage different of $w^*$ (if she wants to build her reputation, she must offer $w_0$ in period $t + 1$).

Define $\pi^T = \text{Prob}(s_u^* = w_0^*|h^{t-1})$ and let $V^T_F(\sigma_t^r, s_u^*)$ be the continuation payoff of the firm from period $\tau$ onwards (and including period $\tau$) given the strategy profile $(\sigma_t^r, s_u^*)$.

Suppose that $\pi^T((w_u^*)) > 1 - \eta$ for all $\tau \in \{t + 1, t + 2, \ldots, t + M_t\}$. It will be shown that this cannot be true in equilibrium, since the firm would get less than her minmax payoff.

The expected payoff of the firm from period $t + 1$ onwards is given by these expressions.

If the union plays $w_u^*$ in all next $M_t$ periods, the expected payoff for the firm would be:

$$\pi^{t+1} g_t(s_{t+1}^r, w_u^*) + \sum_{\tau = t+2}^{t+M_t} \left( \prod_{i=t+1}^{\tau} \pi^r \right) \delta_t^{r-1} g_t(\sigma_t^r, w_u^*)$$

$$+ \prod_{\tau = t+1}^{t+M_t} (\pi^T) \delta_t^M V^T_F^{t+M_t}.$$  \hspace{1cm} (1)

and if in at least one of the next $M_t$ periods, the union does not play his commitment action $w_u^*$, the expected payoff for the firm would be:

$$(1 - \pi^{t+1}) V^T_F^{t+1} + \sum_{i=1}^{M_t-1} \left( \prod_{\tau = t+i+1}^{t+i+1} \pi^T \right) (1 - \pi^{t+i+1}) \delta_t^i V^T_F^{t+i+1}.$$ \hspace{1cm} (2)

Then, $V^T_F(\sigma_t, \sigma_t^r)$ equals the sum of both expressions.

We will use, in what follows, some bounds on the values of the probabilities and the payoffs in the above formula, in order to obtain the contradiction we are looking for.

Firstly, in expression (1), it is obvious that $\pi^T \leq 1$. Then both $(\Pi_{t+1}^T \pi^T)$ and $\Pi_{t+M_t}^T (\pi^T)$ are less than or equal to one.
Secondly, $s^{t+1}_t$ is supposed to be a non-best response against the strategy of the union of "Demanding $w^*$, and if it is not conceded, strike", so that the payoff for both players would be zero since this strategy implies a strike, that is, $g_t(s^{t+1}_t, w_o) = 0$. We also know that if the union plays his Stackelberg action, the maximal payoff the firm can obtain is $\pi_o = b - w^*$, that is, $g_t(\sigma_t, w_o) \leq \pi_o$.

Finally, since $b - w_o$ is the maximal payoff the firm can get, it must be true that:

$$V_{t+1}^f(\sigma^{t+1}_t, w_o) \leq \frac{b - w_o}{1 - \delta_t} \text{ and } V_{t+1}^{t+1} \leq \frac{b - w_o}{1 - \delta_t}.$$ 

Therefore, expression (1) has to be at most:

$$\left( \delta_t + \delta_t^2 + \ldots + \delta_t^{M_t-1} \right) b - w^* + \delta_t^M \frac{b - w_o}{1 - \delta_t}.$$

which is equal to:

$$\left( \frac{1 - \delta_t^M}{1 - \delta_t} \right) (\pi_o) - (\pi_o) + \delta_t^M \frac{b - w_o}{1 - \delta_t}.$$ 

In expression (2), as we have supposed $(1 - \pi^*) < \eta$, and using $(\Pi_{s+1}^{t+1} \pi^*) \leq 1$, we conclude that expression (2) is at most:

$$\eta \left( 1 + \delta_t + \delta_t^2 + \ldots + \delta_t^{M_t-1} \right) b - w_o = \eta - \frac{1 - \delta_t^M}{1 - \delta_t} b - w_o.$$ 

Thus, tying it all together, we get

$$V_{t+1}^f(\sigma_t, \sigma_o) < \frac{1 - \delta_t^M}{1 - \delta_t} (\pi_o) - (b - w^*)$$

$$+ \delta_t^M \frac{b - w_o}{1 - \delta_t} + \frac{1 - \delta_t^M}{1 - \delta_t} b - w_o.$$ 

We can check that $\eta$ (defined in Lemma 1) has been chosen such that

$$\eta \frac{1 - \delta_t^M}{1 - \delta_t} b - w_o + \delta_t^M \frac{b - w_o}{1 - \delta_t} - \frac{\delta_t^M}{1 - \delta_t} (\pi_o) = \pi_o.$$ 

Therefore, we have

$$V_{t+1}^f(\sigma_t, \sigma_o) < \frac{\pi_o}{1 - \delta_t}.$$ 

However, since $\pi_o = b - w^*$ is the firm’s minmax payoff, this is in contradiction with the fact that we are in equilibrium. \qed
Proof 2 (Proof of Proposition 1). Consider the strategy for the normal type of union of imitating the fighting union and playing $w_o^*$ (demanding $w^*$). Take the integer $M_t = [N_t] + 1$, where $[N_t]$ is the integer part of $N_t$, and a real number $\eta > 0$, where $N_t$ and $\eta$ are defined in Lemma 1. Recall that only the normal type of union can accept $w_i < w^*$, thus along such a history where the union has always played his Stackelberg action, the updated probability $\mu_i^*$ of being the fighting union cannot be decreasing.

Divide the $K_i$ (or more) periods in which the firm has offered $w_i < w^*$, and has suffered a strike, in successive blocks of $M_t$, and consider the first of these blocks. By Lemma 1, we know that at least one of them (call it $\tau_i$) must have a probability of calling a strike (that is, of playing $w_o^*$, denoted by $\pi_i^*$) of $(1 - \eta)$ at most. Therefore, the updated probability of being the fighting union (the commitment type), after demanding a wage lower than $w^*$ will be:

$$\mu_{i+1}^* = \frac{\text{Prob}(U = U^* | w_i^* \neq w^* \text{ and } w_o^* = w^*)}{\text{Prob}(w_i^* \neq w^* \text{ and } w_o^* = w^*)} \geq \frac{\mu_i^*}{1 - \eta} \geq \frac{\mu_u}{1 - \eta}$$

where the last inequality holds because $\mu_i^*$ does not decrease.

Now take the second block of $M_t$ periods in which $w_i < w^*$ is offered. Again, at least one of them has a probability of not calling a strike higher than $\eta$. This gives:

$$\mu_{i+1}^* \geq \frac{\mu_i^*}{1 - \eta} \geq \frac{\mu_{i+1}^*}{1 - \eta} \geq \frac{\mu_u}{(1 - \eta)^2}.$$
Thus, we get an upper bound for the number of periods in which the normal firm will offer wages different from $w^*$, that is, the firm is not going to take a best response against the Stackelberg action of the union and there will be a strike. This bound is given by:

$$K_t = k(\mu^*_u, b, \delta_1, w_o, \pi_o)$$

$$= M_t \frac{\ln \mu^*_u}{\ln \left(1 - \frac{(1 - \delta_1)^2 \pi_o}{(b - w_o)(1 - \delta_1^M)} + \frac{\delta_1^M(1 - \delta_1)}{1 - \delta_1^M}\right)}.$$ 

Substituting $M_t$ by $[N_t] + 1$, we get

$$K_t = k(\mu^*_u, b, \delta_1, w_o, \pi_o)$$

$$= ([N_t] + 1) \frac{\ln \mu^*_u}{\ln \left(1 - \frac{(1 - \delta_1)^2 \pi_o}{(b - w_o)(1 - \delta_1^M)} + \frac{\delta_1^M(1 - \delta_1)}{1 - \delta_1^M}\right)}.$$ 

$M_t = [N_t] + 1$ has been chosen because as $K_t$ is increasing with $M_t$, this choice minimizes $K_t$. 

\[\square\]

Appendix B. Lemma 2 and Proposition 2

Recall that $w^*_u$ is the Stackelberg action of the firm that consists of offering $w_o$. If the union does not play a best reply against $w^*_u$, then there will be a strike with a positive probability. Recall also that the maximal possible wage for the union is $w^* = b - \pi_o$.

**Lemma 2.** Let $G^t(\mu^*_u, \mu^*_o)$ be a perturbed repeated bargaining game, and let $\mu^*_u > 0$ and $\mu^*_o = 0$. Consider any Nash equilibrium $(\delta_t, \delta_o)$ of the repeated game and any history $h^t$ consistent with this equilibrium in which the firm has always offered $w_o$. (This history exists because $\mu^*_o > 0$.) Suppose that, given this history, the union demands a higher wage than $w_o$, threatening with a strike if his demands are not met (following his equilibrium strategy) in period $t + 1$, then the probability that the union assigns to the event that the firm will offer $w_o$, given that she has always offered $w_o$ before, must be at least $e$, in at least one of the periods $t + 1$, $t + 2$, ..., $t + M_u$, where, for any $\delta_u$, $0 < \delta_u < 1$, there is a finite integer $M_u$,

$$M_u \geq N_u = \frac{\ln(1 - \delta_u) + \ln w_o - \ln w^*}{\ln \delta_u} > 0.$$
and a positive number \( \varepsilon \),
\[
\varepsilon = \frac{(1 - \delta_N)w_0}{w^* (1 - \delta_N^M)} - \frac{\delta_N^M(1 - \delta_N)}{(1 - \delta_N^M)} > 0.
\]

Proof 3 (Proof of Lemma 2). Similar to proof of Lemma 1. 

Proposition 2. Let \( G^r(\mu_u, \mu_v) \) be a perturbed repeated bargaining game, let \( \mu_u > 0 \) and \( \mu_v = 0 \), and let \( \lceil m \rceil \) be the integer part of \( m \). Consider any \( \varepsilon \)-equilibrium and a history consistent with such equilibrium, in which the firm has always played her Stackelberg action. Then there exists an upper bound, \( K_u(\mu_u^*, \delta_u, w_0, b, \pi_u) \), on the number of periods in which the union demands a wage higher than \( w_0 \) and there is a strike.

\[
K_u = K_u(\mu_u^*, \delta_u, w_0, b, \pi_u) = M_u \frac{\ln \mu_u^*}{\ln \left(1 - \frac{(1 - \delta_u)^2w_0}{w^* (1 - \delta_u^M)} + \frac{\delta_u^M(1 - \delta_u)}{1 - \delta_u^M}ight)},
\]

and \( M_u \geq \lceil N_u \rceil + 1 \), where

\[
N_u = \frac{\ln(1 - \delta_u) + \ln w_0 - \ln w^*}{\ln \delta_u} > 0.
\]

Proof 4 (Proof of Proposition 2). Similar to proof of Proposition 1. 

References