A model of labour supply with job offer restrictions

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Received 28 April 1997; accepted 6 January 2000

Abstract

We specify a model of labour supply with job offer restrictions. A job offer is defined by a wage rate and working hours. The number of job offers is restricted, and follows a Poisson distribution. Individuals choose the alternative with the highest utility level. Three specifications are estimated: a basic specification, a specification in which the number of job offers depends on characteristics, and a specification in which the wage rate depends on working hours. Although the properties of the sample distribution of hours observed are replicated well for all of the specifications, the implications for the underlying behaviour are different. © 2000 Elsevier Science B.V. All rights reserved.

JEL classification: J22; C51
Keywords: Labour supply; Econometric modeling

1. Introduction

In the mid-eighties it was recognized that the standard neo-classical labour supply model (see, e.g. Heckman, 1974; Hausman, 1980) was not able to explain
the patterns observed in the empirical distribution of weekly working hours. In particular, it is unable to explain the high peaks at levels like 40 h/week. Dickens and Lundberg (1985) introduced hours restrictions into the labour supply model assuming that hours arrive from a discrete offer distribution. Tummers and Woitiez (1991) extended the model by making the wage rate dependent on hours. Van Soest et al. (1990) compared the neo-classical labour supply model with the model with hours restrictions using a Dutch data set on labour supply, taking into account the tax system. Their estimation results are relevant empirical evidence in favour of the hours restrictions model. In Blundell et al. (1987), a different approach to restrictions on the labour market is followed. Here, the emphasis is on the modeling of involuntary unemployment.

Following Tummers and Woitiez (1991), and Van Soest et al. (1990), we specify a model of labour supply with job offer restrictions: Individuals are restricted in their choice of working hours by the limited availability of jobs. In the previous studies, job offers depend on hours only, but not on the wage. This is in contrast with job search models (see Mortensen (1986) for an overview), in which job offers are characterized by wages, arriving from a wage offer distribution. We thus make the assumption that a job offer consists of two components: a wage component and an hours component. An individual chooses the job yielding the highest utility level. If all jobs offered generate a utility level that is less than the utility of not working, the individual prefers to be jobless. It is possible that an individual will receive no job offer at all, so involuntary unemployment may arise.

We compare three different model specifications: a basic model and two extensions. The emphasis is on comparing the implications of the different specifications. The distribution that generates the number of job offers is allowed to depend on individual characteristics, and we test whether the average number of job offers depends on individual characteristics. Moreover, the model is estimated both with and without hours dependence of the wage equation, which enables us to test for the significance of hours in the wage equation. Because of the flexible specification of the distribution of hours offered, the distribution of weekly working hours looks similar to the empirical distribution for all of the three specifications. However, the different specifications turn out to have quite different implications for the underlying behaviour. For example, the estimates of the specification without hours in the wage equation show that few jobs with weekly working hours above 40 are offered, whereas the model in which the wage equation includes hours suggests that individual choice is the reason for observing low frequencies of weekly working hours above the full time level.

We basically use the same data as Tummers and Woitiez (1991) and Van Soest et al. (1990).

In Section 2, the model is presented. Section 3 includes the likelihood contribution for estimating the model parameters by maximum likelihood. Section 4 presents the data, as well as the estimation results for various model specifications. Simulation experiments are performed. Section 5 concludes.
2. The model

Individuals are constrained in their choice of labour by the availability of job offers. The number of job offers, \( n \), follows a Poisson distribution with parameter \( \lambda \). An advantage of this Poisson specification, as opposed to the binomial specification of Tummers and Woitiez (1991) and Van Soest et al. (1990), is that it can easily be made dependent on individual characteristics. In these studies, the effect of individual characteristics has typically been ignored. Moreover, the binomial distribution requires the choice of a fixed maximum number of offers. A job offer has two determinants, the wage \( w \) and a weekly number of working hours \( h \). A job offer is modelled as a simultaneous draw of a wage rate \( w \) and a weekly number of working hours \( h \) from a joint wage-hours offer distribution \( f(w,h) \). Hours are assumed to be distributed according to a discrete distribution: \(^1\)

\[
P(h = h_l) = p_l, \quad l = 1, \ldots, m
\]

in which \( m \) is the number of categories of hours, and \( h_l, l = 1, \ldots, m \), are the different values that the hours offer can take. The advantage of this approach is that no heavy restrictions, like single peakedness or symmetry, are placed on the shape of the distribution. The wage offer distribution is assumed to be log-normal:

\[
\ln w = x_l' \eta + v, v \sim N(0, \sigma_v^2)
\]

The wage depends on individual characteristics \( x_l \). The subscript \( l \) indicates possible dependence on the hours offer. Tummers and Woitiez (1991) specified a normal wage distribution, thereby not restricting the range of possible wages to positive values. The equation is formulated in terms of the net wage rate. Individuals have a utility function \( u(h, y) \), defined over labour supply \( h \) and income \( y \). The utility function is specified according to Hausman (1980). The Hausman (1980) utility function yields a linear labour supply function when maximized subject to a linear budget constraint.

\[
u(h, y) = - \ln(\gamma - \beta h) - \frac{\beta (h - X \delta - e - \beta y)}{\gamma - \beta h}
\]

where

- \( \beta, \gamma, \delta \) are parameters, \( \beta < 0, \gamma > 0 \)
- \( y \) is disposable income
- \( h \) is the number of working hours
- \( e \) is an unobserved random taste variable, \( e \sim N(0, \sigma_e^2) \)
- \( X \) is a vector of individual characteristics

---

\(^1\) A discrete hours distribution was introduced by Dickens and Lundberg (1985).
For any given number of working hours the budget constraint is\(^2\)
\[
y = \omega h + \mu
\]  
where \(\mu\) is non-labour income.

At a given point in time, an individual receives \(n\) (possibly zero) job offers, each of them consisting of a wage \(w\), \(0 < w < \infty\) and a number of working hours \(h \in \{h_1, \ldots, h_m\}\). Furthermore, an individual can always choose not to work. The alternative which yields the highest level of utility will be chosen. An individual will be observed to be non-working if the utility level of not working exceeds the utility level of all of the \(n\) job offers.

3. Maximum likelihood estimation

The model parameters will be estimated by maximum likelihood. For working individuals, let the observed wage–hours pair be denoted by \((w_l, h_l)\), \(l \in \{0,1,\ldots,m\}\), and \(u_0\) denotes utility at 0 hours. In Appendix A, it is shown that the likelihood contribution of a working individual, given the random preferences \(e\), is

\[
l(w_l, h_l | e) = \lambda \exp\left( -\lambda \left[ 1 - P(w_l, h_l | e) \right] \right) k(w_l, \eta_l x_l, \sigma_l) p_l, \\
\text{if } u(h_l, w_l, h_l + \mu) > u_0, \text{ zero otherwise}
\]

The function \(k()\) denotes the log-normal wage offer density function. \(P(w_l, h_l | e)\) is the probability that the observed wage–hours combination has a utility level at least as high as that of any other possible combination of wages and hours.\(^3\) Let \(g_l(e)\) denote the wage at which the individual is indifferent between the observed combination \((w_l, h_l)\) and working hours of \(h_l\), at random preferences \(e\), i.e. \(u(h_l, g_l(e) \ h_l + \mu) = u(h_l, w_l, h_l + \mu)\). Then \(P(w_l, h_l | e)\) can be written as

\[
P(w_l, h_l | e) = \sum_{l=1}^{m} p_l P(w \leq g_l(e) | e)
\]

This is the probability that any other combination of wages and hours \(h_l, l = 1,\ldots,m\), has a wage at most as high as its indifference wage \(g_l(e)\). If wages follow the log-normal distribution, the probability can be expressed in terms of the standard normal distribution function \(\Phi()\):

\[
P(w \leq g_l(e) | e) = \Phi\left( \frac{\ln g_l(e) - \eta_l x_l}{\sigma_l} \right) \text{ if } g_l(e) > 0 \\
= 0 \text{ if } g_l(e) \leq 0
\]

\(^2\) Non-linearity in hours arises if the wage rate \(w\) depends on hours.

\(^3\) Derivations and further details can be found in Appendix A.
For non-working individuals, the conditional likelihood function is

\[ l(h = 0 \mid e) = \exp\left( -\lambda \left[ 1 - P(0 \mid e) \right] \right) \tag{9} \]

with \( P(0 \mid e) \) as in Eq. (6), evaluated at 0 hours, being the probability that the utility of not working is at least as high as the utility level of an arbitrary job offer.

After integrating over random preferences, the likelihood contribution for workers is

\[
l(w, h_{l*}) = \int_{B} l(w, h_{l*} \mid e) \frac{1}{\sigma_e} \phi\left( \frac{e}{\sigma_e} \right) \, de, l* \in \{1, \ldots, m\}, 0 < w < \infty
\]

\[ \phi(.) \text{ being the standard normal density function, and } B \text{ being the region of random preferences in which utility of not-working is lower than the utility of the observed wage-hours combination:} \]

\[ B := \{ e \mid u_0 \leq u(h_{l*}, w, h_{l*} + \mu) \} \tag{11} \]

For non-workers, the likelihood contribution is

\[ l(h = 0) = \int \exp\left( -\lambda \left[ 1 - P(0 \mid e) \right] \right) \frac{1}{\sigma_e} \phi\left( \frac{e}{\sigma_e} \right) \, de \tag{12} \]

4. Empirical results

4.1. The data

The sample is a cross-section data set from OSA. It is a sample of 849 married females, drawn from the Dutch population in 1985. Table 1 contains sample statistics. There are five levels of education. The highest (educ5) serves as reference level. Moreover, three types of education are distinguished: non-technical and non-commercial type of education (sec1), semi-technical and semi-commercial type of education (sec2), and technical- and commercial-type of education, the reference level.

4.2. Estimation results

The number of hours categories \( m \) in the hours offer distribution (Eq. 1) is chosen to be 15, and hours are classified in groups of four:

\[ P(h = h_l) = p_l \text{ with } h_l = 4 \times l, l = 1, \ldots, m \tag{13} \]

As a result, the maximum number of hours that can be offered is 60. This choice has been made on the basis of the range of observed hours. Moreover, in the

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4 Organisatie voor Strategisch Arbeidsmarktonderzoek (Organization for Strategic Labor Market Research; a government agency in the Netherlands).
Table 1
Sample statistics, sample size = 849

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours: weekly working hours</td>
<td>27.3</td>
<td>12.5</td>
</tr>
<tr>
<td>Net wage rate: guilders/h</td>
<td>12.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Family size: persons</td>
<td>3.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Age</td>
<td>37.2</td>
<td>9.9</td>
</tr>
<tr>
<td>Non-labor income (guilders/week)</td>
<td>713.5</td>
<td>301.3</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor market state: number of employed</td>
<td>331</td>
</tr>
<tr>
<td>Educ1</td>
<td>217</td>
</tr>
<tr>
<td>Educ2</td>
<td>223</td>
</tr>
<tr>
<td>Educ3</td>
<td>322</td>
</tr>
<tr>
<td>Educ4</td>
<td>71</td>
</tr>
<tr>
<td>Children with age below 6</td>
<td>221</td>
</tr>
<tr>
<td>Sector 1</td>
<td>4</td>
</tr>
<tr>
<td>Sector 2</td>
<td>121</td>
</tr>
</tbody>
</table>

The distribution of observed hours, peaks at 20, 32 and 40 h/week occur, whereas the observed frequency of intermediate levels is low.\(^5\) Therefore, some equality restrictions are placed on probabilities of hours categories with a low sample frequency. These restrictions are

\[
p_1 = p_2 = p_3 = p_4 \quad (1-16 \, h/week) \\
p_6 = p_7 \quad (24-28 \, h/week) \\
p_{12} = p_{13} = p_{14} = p_{15} \quad (48 \, h/week or more) \\
\]

(14)

Three model specifications have been estimated: the basic version (number of job offers does not depend on characteristics, the wage rate does not depend on hours), a second version with the number of job offers depending on individual characteristics, and a third version with a wage equation that depends on working hours. For the second version, the Poisson distribution is parameterized as

\[
\lambda_i = \exp(\theta' z_i), \ i = 1, \ldots, N \\
\]

(15)

The wage equation for the third specification depends on hours, by specifying the wage equation

\[
\ln w = \eta' x + \tau_1 h + \tau_2 h^2 + \nu \\
\]

(16)

The equation is formulated in terms of the net wage rate. Barzel (1973) argues that for the gross wage rate, lower wages will be paid at low hours of work, due to fixed cost of work for the firm, whereas at sufficiently high hours of work, the marginal wage rate will be lower as well, due to decreasing marginal productivity.

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\(^5\) See, e.g. Fig. 1.
This suggests an inverted U-shaped relation between the wage rate and hours. If the wage equation were formulated in terms of levels, in which case the model allows for negative values of the wage rate, Moffit (1984) shows that this hypothesis leads to an S-shaped budget constraint.

Another reason for the wage rate to depend on hours is that at institutionally determined numbers of hours like 20, 32 and 40, the offered wages may be higher and, therefore, hours restrictions need not be the only explanation for observing peak levels in the empirical distribution of working hours. Including dummy variable for these peak levels in the wage equation allows for testing the latter hypothesis. However, inspection of the data led us to abstain from it because no such relation appears to be present.

Alternatively, the appearance of hours in the wage equation may be caused by the absence of any demand side conditions, like unemployment rates. Times of low unemployment possibly go together with both higher wages and larger proportions of people working full time. To be able to separate this relation between hours and wages from other effects, business cycle variables, such as the (regional) unemployment rate or vacancies in the economy, could be included in the model. The importance of business cycle variables in labour supply models is emphasized in a study by Blundell et al. (1998). The present paper, however, is based on a single cross-section data set. Identification of business cycle effects in a cross section would have to be based on regional differences in variables like the unemployment rate. Whether a country like The Netherlands is large enough for regional differences in the unemployment rate to cause differences in the wage level is questionable. The availability of data over several years instead of a single cross section would be desirable, if not necessary, for the identification of business cycle effects.

Table 2 shows the estimation results for the three specifications. Family size and the number of children younger than 6, influence the preference for working negatively. Note that the size of the estimates of the preference parameters is different for the three specifications. For the second specification, the (absolute) values of the estimates of the preference parameters, including that of the standard deviation (S.D.) of random preferences, are rather large and, moreover, they are imprecisely determined. The standard deviation of the distribution of random preferences is lowest in the third specification.

The estimate of the Poisson parameter $\lambda$ for the third specification is about half the value of the estimate of the basic model. For the basic model, the estimate value of 36.7 is rather high. A possible reason for this is that both variance and mean of the Poisson distribution are determined by the same constant parameter $\lambda$. The second specification, which allows $\lambda$ to depend on characteristics, gives the model more flexibility in this respect. Indeed, the value of $\lambda$, evaluated at sample averages of the characteristics, is 1.86. In Tummers and Woitiez (1991), the fixed maximum number of job offers of the binomial distribution that they specified was set equal to 10. The present results show that choosing a fixed, a priori maximum
### Table 2
Parameter estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Basic model</th>
<th>Job offers with characteristics</th>
<th>Hours dependent wages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>S.E.</td>
<td>Est.</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\beta$</td>
<td>-0.029</td>
<td>0.0069</td>
<td>-0.16</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>6.47</td>
<td>1.61</td>
<td>22.6</td>
</tr>
<tr>
<td>$\delta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>18.9</td>
<td>9.6</td>
<td>409.5</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-56.3</td>
<td>10.4</td>
<td>-421.3</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-27.2</td>
<td>7.4</td>
<td>-264.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>36.7</td>
<td>32.3</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td></td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-3.4</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>1.4</td>
<td>1.43</td>
<td></td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>0.40</td>
<td>0.17</td>
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</tr>
<tr>
<td>$\theta_5$</td>
<td>-0.57</td>
<td>0.53</td>
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</tr>
<tr>
<td>$\theta_6$</td>
<td>-0.97</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>$\theta_7$</td>
<td>-0.45</td>
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</tr>
<tr>
<td>$\theta_8$</td>
<td>-0.12</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>Days</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.46</td>
<td>0.06</td>
<td>0.27</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>1.45</td>
<td>0.29</td>
<td>1.86</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>1.37</td>
<td>0.24</td>
<td>1.91</td>
</tr>
<tr>
<td>$\eta_4$</td>
<td>-1.02</td>
<td>0.17</td>
<td>-1.1</td>
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<td>$\eta_5$</td>
<td>-0.48</td>
<td>0.07</td>
<td>-0.40</td>
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<tr>
<td>$\eta_6$</td>
<td>-0.45</td>
<td>0.07</td>
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<td>$\eta_7$</td>
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<td>$\eta_8$</td>
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<tr>
<td>$\tau_1$</td>
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<td>$\tau_2$</td>
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<td>-0.00036</td>
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<tr>
<td>Offer hours</td>
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</tr>
<tr>
<td>$p_1 = p_2 = p_3 = p_4$</td>
<td>0.074</td>
<td>0.001</td>
<td>0.12</td>
</tr>
<tr>
<td>$p_5$</td>
<td>0.14</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>$p_6 = p_7$</td>
<td>0.051</td>
<td>0.008</td>
<td>0.048</td>
</tr>
<tr>
<td>$p_8$</td>
<td>0.12</td>
<td>0.018</td>
<td>0.089</td>
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<tr>
<td>$p_9$</td>
<td>0.060</td>
<td>0.013</td>
<td>0.039</td>
</tr>
<tr>
<td>$p_{10}$</td>
<td>0.23</td>
<td>0.028</td>
<td>0.13</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.032</td>
<td>0.010</td>
<td>0.015</td>
</tr>
<tr>
<td>$p_{12} = p_{13} = p_{14} = p_{15}$</td>
<td>0.0056</td>
<td>0.0024</td>
<td>0.0019</td>
</tr>
<tr>
<td>Log-likelihood value</td>
<td>-2002.5</td>
<td>-1960.7</td>
<td>-1985.9</td>
</tr>
</tbody>
</table>
number of job offers in a one-parameter distribution may place too heavy a restriction on the specification. The likelihood ratio test statistic for testing the null, whether $\lambda$ depends on characteristics based on the log-likelihood value of the first two specifications, is 83.6, which is above the critical value at the 5% level of 14.07.

For all of the three specifications, the mean of the log-wage increases with the level of education. Although the age pattern is similar in the sense that all models have a positive parameter estimate for log-age and a negative for log-age squared, the highest wage is reached at age levels of 33, 41 and 28 for the three respective models.

The estimates of the distribution of hours offered also show differences between specifications. The model with covariates in $\lambda$ places higher probabilities on weekly working hours of 20 or less, compared to the basic model, and lower probabilities on higher levels of hours. The model with hours dependent wages, on the contrary, places higher probabilities on higher working hours than the basic model. In particular, the offer probabilities of weekly working hours greater than 40 are much higher. Consequently, the explanation for the low frequency of observed hours above 40 in the basic model is that these values of hours are hardly offered. The model with hours dependent wage offers, however, suggests that the probability of receiving hours levels of 44 or higher is not so low at all, but the marginal increase in income as a result of working an additional hour is so low, compared to the effect on the marginal utility of leisure, that the individuals are in general not willing to supply these high levels of hours.

The likelihood ratio test statistic to test the hypothesis $\tau_1 = \tau_2 = 0$ has the value 16.6. The critical value at the 5% level is 6.0, so the hypothesis is rejected.

4.3. Simulations

More insight in the performance of the three models is obtained by simulating the distribution of weekly working hours. For each individual, a random preference parameter $e$ and a number of job offers $n$ are drawn from their assumed distributions. Next, $n$ wage–hours pairs are drawn, the utility levels are calculated, and the highest utility level is compared with the utility of not working to make the participation decision. This procedure is repeated 10 times for each individual. Table 3 shows the simulated frequencies for each of the three specifications. Apart from the simulated frequencies, the table presents desired frequencies. These are based on the number of working hours the individual would have chosen if she were not restricted in hours: the tangency point of the indifference curve and the budget constraint. For a linear budget constraint, the utility maximizing number of hours of utility function (3) is generated by $h$ with

\[
\begin{align*}
    h = 0 & \quad \text{if } h^* \leq 0 \\
    = h^* & \quad \text{if } h^* > 0 \\
    h^* & = \beta \mu + \gamma w + X \delta + e
\end{align*}
\]  

(17)
Table 3
Simulated hours frequencies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
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<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.610</td>
<td>0.609</td>
<td>0.527</td>
<td>0.603</td>
<td>0.484</td>
<td>0.600</td>
<td>0.137</td>
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<tr>
<td>4</td>
<td>0.0153</td>
<td>0.0200</td>
<td>0.0321</td>
<td>0.0177</td>
<td>0.00441</td>
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<td>8</td>
<td>0.0318</td>
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<td>0.0316</td>
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<td>12</td>
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<td>0.0627</td>
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<td>0.0249</td>
<td>0.0306</td>
<td>0.0315</td>
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To simulate desired hours, a random preference parameter \( e \) and a number of job offers \( n \) are drawn. As the individual is not restricted in her working hours, the job...
Fig. 2. Distribution of working hours per week, variation in $\lambda$.

offer is determined by the wage rate: (i) draw $n$ wage rates and choose the highest; (ii) simulate desired hours according to Eq. (17).

The participation decision, and the peaks at 20, 32 and 40 h/week are predicted well by the three model specifications.

For the basic model, desired participation is somewhat higher than the actual participation: the frequency of observed participation is 39%, whereas the fre-
frequency of desired participation is 47%. Moreover, desired participation at 40 h/week is about three times smaller than the actual participation at 40 h/week. For the basic model, the hours distributions are plotted in Fig. 1 (positive hours only). The peaks in the observed hours distribution at the levels of hours 20, 32, and 40 are not reproduced by the distribution of desired hours.

For the second specification, in which λ depends on characteristics, the distributions of simulated and desired hours are depicted in Fig. 2. For simulated hours, the figure is similar to Fig. 1. However, the distribution of desired hours is very flat. This is due to the large estimate of the standard error (S.E.) of random preferences. The flat distribution of desired hours suggests that there are equal number of people who desire hours in each category.

The distribution of simulated and desired hours for the specification with hours dependent wages are depicted in Fig. 3.

5. Conclusions

Following Tummers and Woitiez (1991) and Van Soest et al. (1990), we specified a model of labour supply with job offer restrictions. A job offer is characterized by a wage rate and a weekly number of working hours, and the number of job offers is Poisson distributed. Individuals choose the job offer with the highest utility level. They may, however, prefer not to work, or receive no job offer.

As opposed to previous model specifications, we specify the wage equation in terms of log-wages, so that the wage distribution can generate positive wages only. Moreover, the wage is modelled as part of the job offer, conforming to job search theory. Instead of focussing on the difference between the neo-classical labour supply model and the model with demand side restrictions, we allow for different specifications of the model with restrictions itself. Due to the flexible functional form of the distribution of hours offered, all of the three specifications can replicate the patterns observed in the empirical distribution of working hours, like in the previous studies. However, the different specifications have quite different implications for the underlying behaviour. If the number of job offers does not depend on individual characteristics, variation in random preferences dominates the utility function. As a consequence, the distribution of desired hours is very flat. If the wage rate depends on hours, the distribution of desired hours shows a peak at 28 h/week. For levels of hours above 40, the probabilities of hours offered are highest in the latter specification, providing a different explanation for observing low sample frequencies for these levels of hours than the other specifications.

The basic model specification is outperformed by the two extensions on the basis of likelihood ratio tests. Therefore, making the number of job offers dependent on individual characteristics is a valuable extension, but the large variance in random preferences reveals that it is difficult to trace down the
underlying behaviour. The message that may be obvious from this is that for an increase in the degree of ‘reality’ of the model we pay a price in terms of identification. The use of additional information can help us to overcome the problem of losing identification. Bloemen (1997) specifies a job search model in which individuals receive job offers over time according to a Poisson process, and decide on the acceptance of a job offer as well as on the number of working hours. The acceptance decision is characterized by a reservation wage, which depends on the individual’s preferences on the current situation and the expectation of the utility of possible future job offers. In determining this expectation, the job offer arrival rate and the wage offer distribution play a role. The model contains the same sets of parameters (preference parameters, parameters of the offer distribution, the job offer arrival rate, the distribution of random preferences) as the present model, but in addition to data on accepted hours and wages, data on unemployment duration are used which enable the (nonparametric) identification of the job offer arrival rate. The hazard rate, which characterizes the distribution of unemployment duration, is defined as the product of the job offer arrival rate and the job acceptance probability (i.e. the probability that the wage offer exceeds the reservation wage).

In the literature, other sources of information have been used to improve on the identification of the labour supply behaviour of individuals. Several studies make use of subjective information on the individuals’ satisfaction with their labour market state or working hours. Other studies follow individuals across time and use information on the change in working hours. Ham (1982) extends the basic information on the individual’s labour market state (i.e. being employed or not) with information on whether or not the individual is underemployed. Underemployed individuals are assumed to be restricted in their choice, whereas those who are not underemployed are assumed to satisfy utility maximizing behaviour in the sense that the difference between the number of hours at which the budget constraint and indifference curve are tangent and observed hours is purely random. In terms of the present paper, a different treatment of underemployed individuals may add to the identification of the model parameters: individuals who are not underemployed can be treated as in the standard neoclassical labour supply model, whereas those who are underemployed would be subject to a framework with offer restrictions. Altonji and Paxson (1986, 1992) use panel data and compare changes in working hours between individuals who do and who do not change jobs. They argue that if the employer determines the working hours, the only way for individuals to get closer to their optimal hours is to change their job. In their analysis, they stress the importance to correct for other job characteristics and unobservables in order to rule out that the higher variation in hours changes between jobs is spurious. Moreover, they emphasize the role that may be played

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6 The utility function specified is the same as in the present paper.
by information on quits and layoffs. Individuals who quit are more likely to be
dissatisfied with working hours than individuals who are laid off. A structural
model based on this idea would be a model describing the behaviour of an
individual who strives for narrowing the gap between actual hours and desired
hours. The availability of desired hours, or at least information on under- and
overemployment would be desirable. If an individual does not manage to narrow
the gap between actual hours and desired hours it can be assumed that no
acceptable job offer arrived.7 Euwals (1997) actually applies the analysis proposed
by Altonji and Paxson (1992) using information on desired hours. Blundell et al.
(1998) use information on search of individuals who are in the state of nonem-
ployment. Thus, they are able to classify the nonemployed as nonparticipants or
participants (those searching). They show how this information enables them to
decompose the probability of being a discouraged worker in the probability of
being discouraged due to search costs and the probability of being discouraged due
to fixed costs. They emphasize the role of demand conditions and they include
estimates a structural model of job search, including information on search. The
search information can be used to obtain information on the costs of search,
whereas job offer arrival rates can be identified from duration data. However, the
model does not include the decision on working hours for those who accepted a
job. Adding the hours decision would result in the incorporation of the model in
Bloemen (1997).

Acknowledgements

The author thanks the Organisatie voor Strategisch Arbeidsmarktonderzoek
(OSA) for kindly providing the data. Thanks are due to Arthur van Soest, Marcel
Kerkhofs, Arie Kapteyn, and three anonymous referees.

Appendix A. Likelihood contributions

In this section, likelihood contributions for working and non-working individu-
als are constructed.

The number of job offers, \( n \), follows the Poisson distribution with parameter \( \lambda \):

\[
p(n) = \frac{\exp(-\lambda) \lambda^n}{n!}, \quad n = 0, 1, \ldots, \infty \quad (A.1)
\]

For a working individual, the observed wage–hours combination is denoted by
\((w_i, h_i)\). If the individual received \( n \) job offers \( (w_{ij}, h_{ij}), j = 1, \ldots, n \), the

\[7\) Note that using information on job changes introduces the concept of cost of turnover (see Van den
Berg, 1992) which, in turn, may require additional information for identification.
observed job \((w, h_{i*})\) is the job with the highest utility level. Moreover, its utility level is higher than \(u_0 = u(0, \mu)\), the utility level of not working ("the reservation utility level"). The relation between the observed job and the \(n\) offers is:

\[
\begin{align*}
(w, h_{i*}) &= (w_{i1}, h_{i1}) & \text{if } & u(h_{i1}, w_{i1}|h_{i1}) + \mu & \geq u(h_{i1}, w_{i1}|h_{i1}) + \mu & j = 2, \ldots, n \\
& & \text{and } & u(h_{i1}, w_{i1}|h_{i1}) + \mu & \geq u_0 \\
(w, h_{i*}) &= (w(2), h_{i2}) & \text{if } & u(h_{i2}, w_{i2}|h_{i2}) + \mu & \geq u(h_{i1}, w_{i1}|h_{i1}) + \mu & j = 1, \ldots, n, j \neq 2 \\
& & \text{and } & u(h_{i2}, w_{i2}|h_{i2}) + \mu & \geq u_0 \\
& & \vdots & & & \\
(w, h_{i*}) &= (w_{iN}, h_{iN}) & \text{if } & u(h_{iN}, w_{iN}|h_{iN}) + \mu & \geq u(h_{i1}, w_{i1}|h_{i1}) + \mu & j = 1, \ldots, n - 1 \\
& & \text{and } & u(h_{iN}, w_{iN}|h_{iN}) + \mu & \geq u_0
\end{align*}
\]

(A.2)

This scheme lists \(n\) possibilities, and for each of them, there are \(n - 1\) job offers with a utility level that does not exceed the utility level of the observed job. The probability that the job \((w, h_{i*})\) is chosen is obtained by \(n\) times, adding up the probability that the utility of \((w, h_{i*})\) is at least as high as \(n - 1\) other jobs:

\[
nl(P(w, h_{i*} \mid e))^{n-1}, \text{ with } P(w, h_{i*} \mid e), \text{ defined in Eq. (6).}
\]

Multiplying by the appropriate wage–hours offer density results in the likelihood contribution, conditional on random preferences \(e\) and the number of job offers \(n\):

\[
l(w, h_{i*} \mid e, n) = n[P(w, h_{i*} \mid e)]^{n-1}k(w, \eta'x, \sigma) p_{ij}, j \in \{1, \ldots, m\}
\]

if \(u(h_{i*}, w_{i*}, h_{i*} + \mu) > u_0\), 0 otherwise

(A.3)

Multiplying (A.3) by (A.1) and summing over all the positive numbers of job offers \(n, n = 1, \ldots, \infty\), results in (Eq. 5), the likelihood contribution conditional on \(e\):

\[
l(w, h_{i*} \mid e) = \lambda \exp\left(-\lambda[1 - P(w, h_{i*} \mid e)]\right)k(w, \eta'x, \sigma) p_{ij}, \\
& j \in \{1, \ldots, m\} \quad u(h_{i*}, w_{i*}, h_{i*} + \mu) > u_0,
\]

(A.4)

Integrating over random preferences results in the likelihood contribution for working individuals (Eq. 10).

For non-working individuals, the probability is determined that not working has a higher level of utility than any of \(n\) possible job offers:

\[
P(h = 0 \mid e, n) = \left[P(0 \mid e)\right]^n
\]

(A.5)

with \(P(0 \mid e)\) defined below (Eq. 9). Multiplying (A.5) by (A.1), and summing over all possible values of \(n\) yields (Eq. 9). After integrating over random preferences, the likelihood contribution (Eq. 12) is obtained.

References


