Becker’s assortative assignments: stability and fairness

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Abstract

Inspired by Roth and Sotomayor we make a deeper mathematical study of the assortative matching markets defined by Becker, finding explicit results on stability and fairness. We note that in the limit, when the size of the market tends to infinity, we obtain the continuous model of Sattinger and retrieve his characterization of the core of the game in this limit case. We also find that the most egalitarian core solution for employees is the employer-optimal assignment.

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1. Introduction

The aim of this paper is to analyse an important special case of the assignment game of Shapley and Shubik (1972), where so called \( P \)-agents and \( Q \)-agents are to be matched, subject to the matrix of potential productivities for each pair. The special case in question is when the agents can be ordered according to some trait so that the optimal assignment is ‘mating of likes’, i.e. the best agent of both sides are matched to each other, then the next best are matched, etc. In ‘A Treatise on the Family’, Becker (1981) argues that this case is generic for the marriage market. Whereas Becker’s model is discrete, Sattinger (1979, 1993) analyzes an analogous continuous model (i.e. with continuous distributions of agents) for the matching of workers to machines. Sattinger obtains expressions for the set of stable outcomes, i.e. the core of the game.

Our agenda is the following:

- We give two characterisations of assortative markets, one of which is due to Becker
and a new one that describes the set of assortative markets in a more constructive way as the positive hull of a family of fundamental assortative markets.

- For the core of an assortative market, we show that the payoff must be a monotonic function of the trait, and we give explicit expressions for the $P$-optimal and $Q$-optimal outcomes. In the limit, Becker’s assortative market becomes Sattinger’s continuous one, and our results imply Sattinger’s characterization of the core.

- We then discuss the problem of finding an outcome in the core that is fair in the egalitarian sense. Notions of fairness that may seem sensible often fail to be stable. In particular, we show that fairness among the $P$-agents in the sense of minimizing their envy is obtained in their worst possible (i.e. $Q$-optimal) outcome.

2. Assortative markets in the assignment model

We start by recalling the assignment model of Shapley and Shubik (1972). For all further background needed for this section, we refer to the book by Roth and Sotomayor (1990).

Let $P = \{p_1, \ldots, p_n\}$ be a set of agents (e.g. workers or males) and $Q = \{q_1, \ldots, q_n\}$ a set of counterparts (e.g. employers or females). Assigning $p_i$ to $q_j$ is denoted by $p_i \leftrightarrow q_j$ or, equivalently, by $\mu(i) = j$ where $\mu$ is a permutation of $\{1, 2, \ldots, n\}$. Every pair $(p_i, q_j)$ has a potential productivity $\alpha_{ij}$ which must be non-negative.

The optimal assignment problem is to find an assignment $\mu$, such that the total productivity

$$S_\mu = \sum_{i=1}^{n} \alpha_{\mu(i)}$$

is maximal. This $S_\mu$ is distributed as payoffs $u_i$ to $p_i$ and $v_j$ to $q_j$ for $i = 1, \ldots, n$. An outcome is an assignment $\mu$ combined with a pair of payoff vectors $(\bar{u}; \bar{v})$. The outcome is stable if it is individually rational (no payoffs are negative) and if it contains no ‘blocking pairs’ of agents that are not matched but who both have an incentive to reject their current matches in order to join each other instead. It is well-known that for stable outcomes the underlying assignment must be optimal, and no side payments will occur (i.e. in every matched pair $p_i \leftrightarrow q_j$, the payoffs satisfy $u_i + v_j = \alpha_{ij}$, so that the agents share their joint productivity). Therefore, if the agents are numbered such that $p_{\mu(1)} \leftrightarrow q_1, \ldots, p_{\mu(n)} \leftrightarrow q_n$ is an optimal assignment, the stable outcomes are given by the payoff vectors $\bar{u}$ and $\bar{v}$ that satisfy:

$$u_i \geq 0, \quad v_j \geq 0, \quad u_i + v_j = \alpha_{ij}, \quad u_i + v_j \leq \alpha_{ij} \text{ for all } i, j = 1, \ldots, n.$$

In this context, the set of stable outcomes coincides with the more general concept of the core of the game.

The problem of finding optimal assignments and stable outcomes is relatively complicated. It can be done by linear programming or by sophisticated auction algorithms. But we shall see that the solution can be found very easily in the assortative markets that were introduced by Becker.
2.1. Definition of an assortative market

We will assume that the agents can be sorted according to some trait, so that higher indexed agents are ‘better’, where better mates yield higher output. In other words, the matrix \((a_{ij})\) of potential productivities has increasing rows and columns:

\[
\text{If } j < k \text{ then } a_{ij} \leq a_{ik} \text{ and } a_{ji} \leq a_{kj}, \text{ for all } i, j = 1, \ldots, n. \quad (1)
\]

Such a market is **assortative** if, for any submarket defined by a subset of the agents, the optimal assignment is ‘mating of likes’, i.e. matching the two best agents, then the two next best, etc. Becker showed that this is equivalent to every \(2 \times 2\) submatrix having the main diagonal heavier than the opposite diagonal:

\[
\text{If } i < k \text{ and } j < \ell \text{ then } a_{ii} + a_{jj} \leq a_{ik} + a_{jk}, \text{ for all } i, j = 1, \ldots, n. \quad (2)
\]

**Example.** Consider the following two matrices:

\[
\begin{pmatrix}
2 & 4 & 5 \\
3 & 6 & 8 \\
4 & 7 & 10
\end{pmatrix}
\]

is an assortative matrix, while

\[
\begin{pmatrix}
2 & 4 & 6 \\
3 & 6 & 7 \\
4 & 8 & 10
\end{pmatrix}
\]

is not.

By Becker’s mating of likes, the optimal assignment in the first matrix must be the main diagonal, yielding total productivity \(2 + 6 + 10 = 18\). The other five possible assignment give total productivities \(2 + 8 + 7 = 17, 4 + 3 + 10 = 17, 4 + 8 + 4 = 16, 5 + 3 + 7 = 15\) and \(5 + 6 + 4 = 15\). The latter matrix fails to be assortative in that the upper right \(2 \times 2\) submatrix has the main diagonal sum \(4 + 7\) while the opposite diagonal \(6 + 6\) is heavier. Hence, for the submarket \(P = \{p_1, p_2\}, Q = \{q_2, q_3\}\) the optimal assignment is not mating of likes.

To begin with, we shall give an alternative characterization of assortative markets.

2.2. Characterization and generation of assortative matrices

We shall describe a characterization of assortative matrices as sums of certain fundamental matrices. Assuming a fixed matrix size \(n\), let \(M_{ij}\) denote the \(n \times n\) matrix that has ones in all positions \((i', j')\) satisfying \(i' \geq i, j' \geq j\); all other entries are zero. For example, with \(n = 3\) we have

\[
M_{12} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad M_{22} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad M_{32} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}
\]

**Proposition 2.1.** The set of assortative matrices of a given size \(n\) is the positive hull of the fundamental matrices \(M_{ij}\), i.e.

\[
\left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} m_{ij} M_{ij} : m_{ij} \geq 0 \right\}.
\]

It is quite obvious that the \(M_{ij}\) are themselves assortative, and that positive linear combinations of assortative matrices are always assortative. This proves that the above
set of matrices contains assortative matrices only. For the other direction, we refer to the following algorithm.

**Proposition 2.2.** Any matrix \((\alpha_{ij})\) can be (uniquely) decomposed as a sum of the form \(\sum_{ij} m_{ij} M_{ij}\). The algorithm runs in \(n^2\) steps, indexed by \((i, j)\) for \(i = 1, \ldots, n\) and \(j = 1, \ldots, n\). It starts by setting \(H = (h_{ij}) = (\alpha_{ij})\). In step \((i, j)\) it sets \(m_{ij}\) to \(h_{ij}\) and resets \(H' = H - m_{ij} M_{ij}\). All \(m_{ij}\) will be non-negative if, and only if, \((\alpha_{ij})\) is assortative.

**Proof.** The ‘only if’ part of the statement follows from the previous discussion. In order to give an inductive proof of the ‘if’ part, assume that in step \((i, j)\) the matrix \(H\) is assortative. (By construction, rows 1, \ldots, \(i - 1\) of \(H\) have zeros in all positions, and row \(i\) has zeros in all positions 1, \ldots, \(j - 1\).) Assortative matrices are non-negative, so \(m_{ij} = h_{ij}\) becomes non-negative. It remains to prove that the new matrix \(H - m_{ij} M_{ij}\) must be assortative.

Clearly, when subtracting a multiple of \(M_{ij}\) from \(H\), the only possibility of violating the monotonicity condition Eq. (1) is in the border between zeros and ones in \(M_{ij}\). A violation would hence have to look like \(h_{i',j'} - h_{i',j} = h_{i',j} - h_{i',j'}\) for some row \(i' > i\). But \(H\) obeys condition Eq. (2), so \(h_{i',j} + h_{i',j-1} \geq h_{i',j-1} + h_{i',j}\). Substitution of \(h_{ij} = m_{ij}\) and \(h_{i',j-1} = 0\) implies that violation of Eq. (1) is impossible.

Similarly, a violation of the assortativity condition Eq. (2) must look like \(h_{i',j'} - h_{ij} \leq h_{i',j'} - h_{i',j}\) for some rows \(i' < i \leq i''\) and columns \(j' < j \leq j''\). But then \(h_{i',j'} = h_{i',j'} = h_{i',j} = 0\) and by condition Eq. (1) we cannot have \(h_{i',j'} - h_{ij} \leq 0\).

Hence the new matrix is assortative, so by induction every step will generate a non-negative coefficient.

**Example.** The example of an assortative matrix from Section 2.1,

\[
\begin{pmatrix}
2 & 4 & 5 \\
3 & 6 & 8 \\
4 & 7 & 10
\end{pmatrix},
\]

is decomposed as

\[
2M_{11} + 2M_{12} + 1M_{13} + 1M_{21} + 1M_{22} + 1M_{23} + 1M_{31} + 0M_{32} + 1M_{33}.
\]

An economic interpretation of the fundamental matrices is that a \(P\)-agent of index at least \(i\) and a \(Q\)-agent of index at least \(j\) can perform some task that contributes to productivity, but that this task is impossible to perform in pairs where at least one of the agents is of lower rank in the underlying trait of Section 2.1.

### 2.3. Results about the core

#### 2.3.1. Assortative payoff

First, we claim that independently of which stable outcome we have chosen, the payoffs must mirror the assortative matrix. In other words, if \(p_i\) is better than \(p_j\), then he must always get at least as high income; similarly for the \(Q\)-agents. Here the first assortative matrix condition is sufficient.
Proposition 2.3. In an assortative market, or in any market satisfying Eq. (1), the payoff vectors $\bar{u}$ and $\bar{v}$ will in every stable outcome satisfy $u_1 \leq u_2 \leq \cdots \leq u_n$ and $v_1 \leq v_2 \leq \cdots \leq v_n$.

Proof. Suppose that the payoff vectors $\bar{u}$ and $\bar{v}$ yield a stable outcome together with an assignment $\mu$. It is known that in stable outcomes there are no side payments, so $u_i + v_j = \alpha_{ij}$ for all $j = \mu(i)$. Also, stability implies $u_i + v_j \leq \alpha_{ij}$ for all $i$, $j$. Now suppose, in order to obtain a contradiction, that $u_i \geq u_{i+1}$ for some $i$. Take $j = \mu(i)$. Then

$\alpha_{ij} \leq \alpha_{i+1,j} \leq u_{i+1} + v_j < u_i + v_j$

contradicting the absence of side payments in a stable outcome. $\square$

2.3.2. Explicit expressions for optimal outcomes

Shapley and Shubik showed that the set of stable outcomes has a unique $Q$-optimal outcome. This means that no other stable outcome is better for any $q_j$. Dually, there is also a unique $P$-optimal outcome. Since the $P$-agents and the $Q$-agents share the total payoff, it is necessarily so that the $Q$-optimal outcome is the worst possible outcome for the $P$-agents, and vice versa. For the general assignment game, the optimal outcomes can be found using quite sophisticated methods, such as the auction algorithm of Demange et al. (1986). For assortative markets we can explicitly compute these optimal outcomes.

The key observation is that the payoffs must satisfy

$$u_{i+1} - u_i = u_{i+1} - u_i + \alpha_{ij} - \alpha_{ij} = u_{i+1} + v_i - \alpha_{ij} \geq \alpha_{i+1,j} - \alpha_{ij},$$

where we have used the fact that $\alpha_{ij} = u_i + v_j$ in stable outcomes of assortative markets. From this it follows that the the worst possible payoff for the $P$-agents cannot be worse for them than the payoff $\bar{u}$ obtained by taking $u_1 = 0$ and then choosing the smallest payoff differences between $P$-agents according to the inequality above, i.e. taking $u_{i+1} - u_i = \alpha_{i+1,j} - \alpha_{ij}$ for $i = 1, \ldots, n - 1$. We shall see that this outcome is stable; therefore, it is the $Q$-optimal outcome.

Proposition 2.4. Suppose that $(\alpha_{ij})$ is an assortative matrix. Then the $Q$-optimal stable outcome has payoff vectors $(\bar{u}^Q, \bar{v}^Q)$ given by $u_i^Q = 0$ and

$$u_i^Q = (\alpha_{i,i-1} - \alpha_{i-1,i-1}) + (\alpha_{i-1,i-2} - \alpha_{i-2,i-2}) + \cdots + (\alpha_{2,1} - \alpha_{1,1})$$

for all $i = 2, \ldots, n$, and $v_i^Q = \alpha_{i,i} - u_i$ for all $i$ between 1 and $n$.

Proof. By the foregoing argument, it suffices to show that the stated payoffs $\bar{u}^Q$ and $\bar{v}^Q$ give a stable outcome. In other words, we must show that for all $i$, $j$ we have $u_i^Q + v_j^Q = u_i^Q + \alpha_{ij} - u_i^Q \leq \alpha_{ij}$. We distinguish between two cases: $i > j$ and $i < j$, the case of $i = j$ being obvious.

To begin with, note that for $i > j > k$ we have the following rule:

$$\alpha_{ij} - \alpha_{ik} + \alpha_{jk} = \alpha_{ij} - (\alpha_{ij} - \alpha_{ik}) \geq \alpha_{ij} - (\alpha_{ij} - \alpha_{ik}) = \alpha_{ik}.$$
Then, for \( i > j \), we have
\[
\begin{align*}
    u^Q_{ij} + \alpha_{j-1,i-1}^Q &= (\alpha_{i-1,j-1} - \alpha_{i-1,j-1}) + \cdots + (\alpha_{i+1,j+1} - \alpha_{i+1,j+1}) + \alpha_{i,j}.
\end{align*}
\]
By using Eq. (4), we can replace the last three terms of this equality, \((\alpha_{i+1,j+1} - \alpha_{i+1,j+1}) + \alpha_{i+1,j}\), by \(\alpha_{i+2,j}\), and obtain an inequality. The last three terms in this inequality are \((\alpha_{i+3,j+2} - \alpha_{i+3,j+2}) + \alpha_{i+3,j}\), which we by by Eq. (4) can replace by \(\alpha_{i+4,j}\). Continuing in this way, using Eq. (4) repeatedly (we show the last repetition below), we get
\[
\begin{align*}
    u^Q_{ij} + v^Q_{ij} &\geq \cdots \geq (\alpha_{i+2,j-1} - \alpha_{i+2,j-1}) + \cdots + (\alpha_{i+1,j} - \alpha_{i+1,j}).
\end{align*}
\]
The case when \( i < j \) works in an analogous way. □

By an analogous argument we get the dual result:

**Proposition 2.5.** Suppose that \((\alpha_{ij})\) is an assortative matrix. Then the P-optimal stable outcome has payoff vector \((\bar{u}^P; \bar{v}^P)\) given by \(v^P_i = 0\) and
\[
\begin{align*}
    v^P_i &= (\alpha_{i-1,i} - \alpha_{i-1,i-1}) + (\alpha_{i-2,i-1} - \alpha_{i-2,i-2}) + \cdots + (\alpha_{i+1,i} - \alpha_{i+1,i})
\end{align*}
\]
for \( i = 2, \ldots, n \), and \( u^P_i = \alpha_{i,i} - v_i \) for all \( i \) between 1 and \( n \).

**Example.** Taking once again the example in Section 2.2,
\[
(\alpha_{ij}) = \begin{pmatrix}
    2 & 4 & 5 \\
    3 & 6 & 8 \\
    4 & 7 & 10
\end{pmatrix},
\]
the worker-optimal outcome has worker payoff \(\bar{u}^W = (2, 4, 6)\) and employer payoff \(\bar{v}^W = (0, 2, 4)\). In the employer-optimal outcome we get payoffs \(\bar{u}^Q = (0, 1, 2)\) and \(\bar{v}^Q = (2, 5, 8)\).

2.4. Relation to Sattinger’s model

Returning to Becker’s basic assumption, let the underlying trait of agents be measured by \(g_i \leq g_2 \leq \cdots \leq g_n\) for P-agents and \(k_1 \leq k_2 \leq \cdots \leq k_n\) for Q-agents. Let us remark that given an assortative matrix, such measures can be inferred by factor analysis, cf. Sattinger (1984).

A generic model has the productivity given by \(\alpha_{ij} = f(g_i, k_j)\) where \(f\) is some function. As noted by Becker, this will yield assortative markets under the following conditions: \(f(g, k)\) is any twice differentiable function satisfying
\[
\frac{\partial f}{\partial g} \geq 0, \quad \frac{\partial f}{\partial k} \geq 0, \quad \frac{\partial^2 f}{\partial g \partial k} \geq 0. 
\]

\(^1\)Differentiability of \(f(g, k)\) is not a necessary condition for an assortative matrix; e.g. the famous Leontief function \(\alpha_{ij} = \min(g_i, k_j)\) produces an assortative matrix.
Sattinger (1979, 1993) considers an assignment model, called the Ricardian differential rents model, where there is a continuous distribution of workers’ skills and a continuous distribution of machine sizes. Given this model and the above conditions on the productivity function $f$, Sattinger argues that the equilibrium assignment of workers to jobs will be strictly top-down, i.e. the $n$th worker, in order of decreasing skill, will be assigned to the $n$th machine, in order of decreasing size. From the continuous distributions of skills and machine sizes it then follows that the relationship between them can be described by a monotonic function $k(g)$ determining which machine size will be assigned to a worker of skill $g$.

Then Sattinger proceeds by determining the wage function in equilibrium, i.e. the amount $w(g)$ of the productivity that the employer will pay for a worker of skill $g$. He shows that the wage function must satisfy

$$w'(g) = \left[ \frac{\partial f(g,k)}{\partial g} \right]_{k=k(g)}$$

This equation determines the wage function up to a constant term. The constant term, describing the absolute level of wages, will be determined by the conditions for the last match in order of decreasing skill and machine size, basically by choosing a minimum wage.

Taking assortative matrices in the limit, Eq. (1) says that $f(g,k)$ is increasing in both $g$ and $k$, while Eq. (2) says that the mixed partial derivative $(\partial^2 f)/(\partial g \partial k)$ is positive. Hence, we have obtained the differential rents model as the limit case of assortative markets.

Let us now derive the differential equation Eq. (5) for the wage function $w(g)$ given by $w(g_i) = u_i$. In the worker-optimal outcome we had $u_i - u_{i-1} = \alpha_{i,i} - \alpha_{i,-1}$ while in the employer-optimal outcome we had $u_{i+1} - u_i = \alpha_{i+1,i} - \alpha_{i,i}$. The function $f(g,k)$ being continuously differentiable means that in the limit the left-hand derivative equals the right-hand derivative:

$$\alpha_{i,i} - \alpha_{i,-1} = \alpha_{i+1,i} - \alpha_{i,i}$$

Hence we have that both the employer-optimal and worker-optimal wage functions satisfy the differential equation Eq. (5), so in fact they coincide (except for a constant term) and thus we have a unique wage function in equilibrium, up to the absolute level of wages.

3. **Is there a fair and stable outcome?**

The two most famous guiding principles for choosing outcomes to a game like the assignment game are the **utilitarian** principle, which tells you to maximize the overall productivity, and the **egalitarian** principle which tells you to be as fair as possible (cf. Moulin, 1988), e.g. by minimizing envy or maximizing the outcome of the most unfortunate. These principles are often in conflict with each other. Our approach, as most standard methods in game theory, is utilitarian since stable outcomes maximize
productivity. However, we will now try to analyze how far such outcomes can satisfy the demands of egalitarians. We will take the approach of minimizing envy.

3.1. Minimizing envy

Envy arises when one values another’s basket of goods more than one’s own, cf. Feldman and Kirman (1974) and Varian (1974). Here, we are only looking at payoffs, so envy refers to inequalities of payoffs. We will make the natural assumption that all envy is between pairs of $P$-agents or pairs of $Q$-agents, so that a $P$-agent is never envious of a $Q$-agent or vice versa. If $u_i > u_j$, then agent $p_i$'s envy of $p_j$ is $u_i - u_j$. The total envy $E$ in a market is the sum of all these differences, i.e.

$$E = \sum_{i,j} (u_i - u_j)^+ + (v_i - v_j)^+,$$

where $[a - b]^+ = a - b$ if $a - b > 0$, and 0 otherwise.

It turns out that assortative markets are particularly well-behaved with respect to envy: every stable outcome will yield the same total envy!

**Theorem 3.1.** In an assortative market $(\alpha_i)$, the total envy in any stable outcome is

$$E = \sum_{k=1}^{n} (2k - n - 1)\alpha_{kk}$$

**Proof.** In any stable outcome of an assortative market we have $u_i + v_j = \alpha_i$, so $u_i - u_j + v_i - v_j = \alpha_i - \alpha_j$. The envy terms $[u_i - u_j]^+$ and $[v_i - v_j]^+$ are non-zero only if $i > j$, by Proposition 2.3. Hence,

$$E = \sum_{i > j} (\alpha_i - \alpha_j) = \sum_{k=1}^{n} (2k - n - 1)\alpha_{kk}$$

by simple verification. □

This invariance implies that the smaller the envy among $P$-agents, the greater the envy among $Q$-agents. If we actually want to minimize the envy among the $P$-agents, we shall minimize the sum $\sum_{i,j} (u_i - u_j)$. But from Section 2.3.2 we already know a way to minimize this sum: just choose the $P$-worst outcome! By symmetry, we also have the analogous result for $Q$-agents.

**Corollary 3.2.** The $P$-worst outcome minimizes the envy among $P$-agents. The $Q$-worst outcome minimizes the envy among $Q$-agents.
This is an illustration of the conflict between the utilitarian and the egalitarian principles.

4. Concluding remarks

This paper has been inspired by the book on two-sided matching markets by Roth and Sotomayor (1990). In their ending remarks on directions for future research (p. 247) they suggest that the mathematical investigations of their book might be profitably combined with the highly structured model of Becker.

We have made a deeper mathematical analysis of Becker’s assortative matching markets. We have further discussed how our results relate to the continuous model of Sattinger. For the economic relevance of this model, we refer to Becker and Sattinger and the references below. As an extra speculation, let us add that under the assumption that the ability to increase one’s skill is positively correlated to one’s current position, a dynamic model of repeated assignments would eventually lead to something like an assortative situation.

We also discussed fairness and found that among stable outcomes there is a maxmin solution that can be found by decentralized bargaining. In economics, taxing is a tool for achieving a higher degree of equality. Let us mention that we have found that a progressive tax will in general distort the assortative structure (while a regressive tax is non-distortionary).

Note that the assortative market can be used as a bench mark when analyzing corrupt and repressive societies.

Most of economic science is about relations between aggregates, and the individual is portrayed as the typical consumer, producer, worker, employer, etc. However, ordering of individuals is crucial in welfare economics, where a tool of analysis is obtained from ordering individuals into an income distribution. Also, a Tayloristic division of labor (Tinbergen, 1959) could be something of the past, especially in the more developed segments of the post-industrial society, cf. Lindbeck and Snower (1996). Another field of research where ordering of individuals is common is in the economics of the organisation, where little can be explained without the concept of hierarchy. The ‘O-Ring Model’ of Kremer (1993) has parallels to our study. Kremer studies the tendency of individuals of the same level of skills to cluster into cooperative units. For empirical evidence, cf. Kremer and Maskin (1996). For clustering in marriage markets, see Bloch and Ryder (1994), Burdett and Coles (1996) and Edlund (1996), to mention a few of the latest contributions.

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