Modification of the productivity index model

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Abstract

A soil erosion–productivity model which considers the effect of soil water storage capacity, crop evapotranspiration, soil chemical and physical properties important for crop growth has been modified. The model is shown to give good predictions and promises to be an improvement over the former productivity index (PI) because it accounts for weather and cropping conditions. It also promises to give more reliable results than the currently used insufficient models which consider only soil water storage capacity and crop evapotranspiration. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

Many models have been developed for predicting loss in soil productivity as caused by erosion (Laflen et al., 1985; Pierce et al., 1983; Pierce and Lal, 1994; Stocking and Pain, 1983; Timlin et al., 1986; Williams et al., 1983). However, all the models still need further validation and/or modification (Pierce and Lal, 1994).

The sophistication of the existing soil erosion–productivity models is quite variable. They vary from deterministic mathematical models like the erosion productivity impact calculator (EPIC) (Laflen et al., 1985; Williams et al., 1983), simulating storm-based soil erosion, solute movement over and through soil and soil fertility-crop growth, to models that only simulate reduction of soil water storage capacity by continued soil erosion (Stocking and Pain, 1983; Timlin et al., 1986). Sophisticated models such as EPIC require data base that are not readily available in many developing countries (Biot, 1988; Kiome, 1992). Simple models like those simulating the effect of soil erosion on soil water storage capacity cannot always give accurate predictions of the effect of soil erosion on soil–crop productivity because they do not take into account the physical and chemical properties of soils important for crop growth and affected by soil erosion.

Soil erosion affects many soil characteristics which are related to crop growth and yield (Stocking, 1984, 1994; Pierce and Lal, 1994). Continued soil erosion results in reduced rooting depth and soil water storage capacity, crusting, soil compaction, change in root zone cation exchange capacity (CEC), aluminium and manganese toxicity, soil acidity, soil alkalinity and deterioration of soil biological properties (Pierce and Lal, 1994; Payton and Shishira, 1994; Stocking and Pain, 1983). Unless a model takes into account most of the important factors affecting crop growth.

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that are affected by soil erosion its accuracy will remain unreliable. The modified productivity index (MPI) model presented in this paper takes into account most of the important parameters affecting crop growth and that are affected by soil erosion. The parameters considered in the model are also readily available in developing countries (Mulengera, 1996).

2. The development of the soil erosion–productivity index model

The modified soil productivity index (MPI) model is a hybrid of the productivity index (PI) (Kiniry et al., 1983; Pierce et al., 1983) and the models simulating soil moisture availability to plants and yield relationships (Stocking and Pain, 1983; Timlin et al., 1986). The PI model as originally developed (Kiniry et al., 1983) and subsequently modified (Pierce et al., 1983) is

$$PI = \sum_{i=1}^{n} (A_i \times C_i \times D_i \times E_i \times R_{I_i}),$$

(1)

where $A_i$ is sufficiency of soil water holding capacity in the $i$th layer (Fig. 1), $C_i$ is sufficiency of soil bulk density (and aeration) in the $i$th layer (Fig. 2), $D_i$ is sufficiency of soil pH of the $i$th layer, $E_i$ is sufficiency of soil electrical conductivity (salinity) in the $i$th layer, $R_{I_i}$ is root weighting factor of the $i$th soil layer, and $n$ is the number of soil layers of the root zone depth.

With the exception of $A_i$ sufficiency which is based on assumption only, all the other parameters in the PI model are based on research findings (in the USA) of the effects of respective soil properties on root growth which in turn is assumed to be related to crop growth and production as has been established from research (Kiniry et al., 1983). The $A_i$ sufficiency in the PI model is not linked to crop evapotranspiration requirements. Although all the factors of the model remain researchable for its improvement (Gantzer and McCarty, 1987; Kiniry et al., 1983), the link of available soil water to crop evapotranspiration requirement should provide one of the improvements required to address the variable performance of the PI model in predicting the effects of erosion on soil productivity reported by many authors (Gantzer and McCarty, 1987; Pierce et al., 1983; Kiniry et al., 1983).

The models simulating soil moisture availability to plants and yield relationships as affected by soil erosion, e.g. the soil life and the water budget models (Stocking and Pain, 1983; Timlin et al., 1986), use the equation developed by Doorenbos and Kassam (1979):

$$Y_a/Y_m = 1 - k_y(1 - ET_a/ET_m),$$

(2)

where $Y_a$ is actual crop yield (t/ha), $Y_m$ is potential crop yield under water constraint – free conditions (t/ha), $k_y$ is empirical yield response factor for a given crop type and stage of development, $ET_a$ is actual crop evapotranspiration for the crop development stage under consideration, and $ET_m$ is potential evapotranspiration.
of a disease free crop under water constraint – free conditions.

The PI model was modified by removing parameter $A_i$ from the PI model as it has a weak scientific basis, followed by combining the equation formed by the remaining parameters in Eq. (1) with Eq. (2) as shown in Eq. (3):

$$PP = (1 - k_y(1 - ET_a/ET_m)) \sum_{i=1}^{n} (C_i \ast D_i \ast E_i \ast RI_i),$$

where $PP$ is productivity potential of a soil $(0 \leq PP \leq 1.0)$.

Eq. (2) or the first factor of Eq. (3) involves determining actual and potential evapotranspiration values and the crop response factor. Many equations and/or procedures are available for estimating crop evapotranspiration values for different climatic conditions (Doorenbos and Kassam, 1979). The yield response factor varies from crop to crop and between different stages of development (Doorenbos and Kassam, 1979; Timlin et al., 1986). When insufficient data are available, estimates given by Doorenbos and Kassam (1979) are reasonably sufficient.

The second factor of MPI uses parameters evaluated as in PI model (Pierce et al., 1983; Mulengera, 1996). To determine the sufficiency of bulk density, $C_i$, one has to obtain nonlimiting, critical and root-limiting bulk densities which depend on soil family texture classes (Kiniry et al., 1983; Pierce et al., 1983). These are given in Table 1. For each soil textural class with a given bulk density, one uses the nonlimiting, critical and root-limiting bulk densities from Table 1 to determine the relative position of the soil’s density on the $x$-axis of Fig. 2 and then reads its bulk density sufficiency value on the $y$-axis. The bulk density sufficiency value determined from Fig. 2 is adjusted to take into account permeability rates (for water and air) by equation:

$$C_i = 1 - (1 - \text{SUFF}_g) \ast \beta,$$

where SUFF$g$ is sufficiency of bulk density from Fig. 2, and $\beta$ is adjustment factor determined from Table 2.

The pH sufficiency, $D_i$, is determined using the following equations (Pierce et al., 1983):

$$D_i = \begin{cases} 0.75 & \text{for pH} > 8.0 \\ 2.086 - 0.167 \text{pH} & \text{for 6.5 < pH} \leq 8.0 \\ 1.0 & \text{for 5.0 < pH} \leq 6.5 \\ 0.12 + 0.16 \text{pH} & \text{for pH} = 5.0 \text{ to 5.5} \\ 0.44 \text{pH} - 1.31 & \text{for pH} = 2.9 \text{ to 5.0} \\ 0.0 & \text{for pH} < 2.9. \end{cases}$$

The sufficiency of electrical conductivity, $E_i$, for soils affected by salinity is determined using equation:

Table 1
Nonlimiting, critical and root-limiting bulk densities for different family texture classes (source: (Pierce et al., 1983))

<table>
<thead>
<tr>
<th>Family texture class</th>
<th>Nonlimiting bulk density (g/cm$^3$)</th>
<th>Critical bulk density (g/cm$^3$)</th>
<th>Root-limiting bulk density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy</td>
<td>1.60</td>
<td>1.69</td>
<td>1.85</td>
</tr>
<tr>
<td>Coarse loamy</td>
<td>1.50</td>
<td>1.63</td>
<td>1.80</td>
</tr>
<tr>
<td>Fine loamy</td>
<td>1.46</td>
<td>1.67</td>
<td>1.78</td>
</tr>
<tr>
<td>Coarse silt</td>
<td>1.43</td>
<td>1.67</td>
<td>1.79</td>
</tr>
<tr>
<td>Fine silt</td>
<td>1.34</td>
<td>1.54</td>
<td>1.65</td>
</tr>
<tr>
<td>Clay</td>
<td>1.40 (35–45%)</td>
<td>1.49</td>
<td>1.58</td>
</tr>
<tr>
<td>&gt;45%</td>
<td>1.30</td>
<td>1.39</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Table 2
Adjustment factors ($\beta$) for sufficiency of bulk density used in Eq. (4) (source: (Pierce et al., 1983))

<table>
<thead>
<tr>
<th>Permeability (mm/h)</th>
<th>&lt;1.5</th>
<th>1.5–5.1</th>
<th>5.1–15.2</th>
<th>15.2–50.8</th>
<th>&gt;50.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine loamy</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Coarse silt</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Fine silt</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Clay</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>35–60%</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>&gt;60%</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>
(Kiniry et al., 1983):

\[ E_i = 1.14 - 0.07 \text{EC}, \]  

(6)

where EC is the electrical conductivity (dS/m).

The weighting factor, \( R_i \), is based on the seasonal distribution of plant water uptake from different horizons within the root zone. The equation predicting the profile of fractional water uptake from a moist soil is given by (Gantzer and McCarty, 1987):

\[
R_i = 0.152 \log\left\{ R + (R^2 + 6.45)^{0.5} \right\} - 0.152 \log\left\{ D + (D^2 + 6.45)^{0.5} \right\},
\]

(7)

where \( R_i \) is the fractional seasonal water uptake from a given soil depth, \( D \). Summation of \( R_i \) values for all root zone depth sections under consideration add up to 1.0, and \( R \) is the maximum plant rooting depth.

3. Evaluation of the MPI

3.1. Materials and methods

Data for evaluating the modified productivity index model were obtained from run-off plots set up by Soil Erosion Research and Water Harvesting Research Programmes at the Agricultural Research Institute (ARI), Hombolo, located at 35 km north east of Dodoma in the central semi-arid regions of Tanzania (Mulengera, 1996; Hatibu et al., 1995; Mahoo et al., 1995) and the soil survey report by De Pauw et al. (1983). The run-off plots set up by the WaterHarvesting research programme were on a soil classified as Typic Ustorthent (Soil Survey Staff, 1992) or Dystric Regosol (FAO, 1998). The soil on which the run-off plots were set up by the Soil Erosion Research was classified as Haplic Alisol (FAO, 1998) or Arenic Kandustult (Soil Survey Staff, 1992). Both of the research programmes are run by the Department of Agricultural Engineering and Land Planning, Sokoine University of Agriculture, Morogoro, Tanzania. Data collected by the two research programmes and used to evaluate the MPI were rainfall storm volumes, run-off volumes, soil pH, soil texture, soil bulk density, soil permeability and sorghum crop growth and yields from the run-off plots for the 1994 and 1995 rainy seasons. The run-off plots from which the data were obtained for both research programmes had tillage treatments as shown in Table 5 and similar inorganic fertilizer applications at planting and after first weeding. The data obtained from the soil survey report by De Pauw et al. (1983) and used in the MPI evaluation were soil water holding capacities in the crop root zone depths and potential evapotranspiration values, \( ET_m \).

The rainfall storm volumes and run-off volumes obtained from the two research programmes and the root zone water storage capacities of the soils at the programmes’ research sites were used to calculate the effective rainfall volumes (i.e. rainfall storm volumes minus run-off volumes and percolation volumes in excess of available soil water storage capacities in the root zones). Ten days interval effective rainfall volumes and Fig. 3 were used to calculate actual crop (sorghum) evapotranspiration values. The 10 days interval was selected to shorten the calculations while maintaining accurate estimates of the actual crop evapotranspiration values. Potential evapotranspiration values used were those calculated by De Pauw et al. (1983) using Penman (1948) method and meteorological data from Dodoma Airport. As shown in Fig. 3, actual crop evapotranspiration rate, \( ET_a \), depends on maximum crop evapotranspiration, \( ET_m \), and water available in root zone. Water available in root zone for each time interval of computation, \( AW_i \), was obtained using water balance Eq. (8) shown below. The \( AW_{i-1} \) in the equation was obtained by assuming that root zone soil moisture available at the start of each rainy season was negligible. The assumption was based on the soil moisture monitoring results by Hatibu et al. (1995) which are reinforced by the fact that the area of study experiences seven to eight months of dry seasons having potential evapotranspiration rates above 6.5 mm/day:

\[
AW_i = AW_{i-1} + PR_i - RO_i - PEL_i - ET_{a_{i-1}},
\]

(8)

where \( AW_i \) is water available in the root zone during computation time interval \( i \); \( AW_{i-1} \) is water available in the root zone during computation time interval \( i-1 \); \( PR_i \) is precipitation volume that occurred during the computation interval \( i \); \( RO_i \) is the run-off volume that occurred during the computation time interval \( i \); \( PEL_i \) is percolation volume that occurred during the computation interval \( i \), and \( ET_{a_{i-1}} \) is the actual crop evapotranspiration volume that occurred during computation time interval \( i-1 \).
Crop evapotranspiration depends on root zone water availability as well as the stage of crop development. The root and shoot growth characteristics of the sorghum (sorghum bicolor) ("tegemeo or serena" variety) grown at the two run-off plots research sites used to calculate actual crop evapotranspiration (Doorenbos and Kassam, 1979) are shown in Table 3. During the calculations of the actual crop evapotranspiration values it was assumed that evapotranspiration dropped linearly from maximum evapotranspiration levels in proportion to soil water availability after water depletion has reached 50% of the soil water storage capacity as shown in Fig. 3. It was also assumed that lateral subsurface water flows from outside the run-off plots were negligible. The calculations of crop evapotranspiration values were based on 10 days interval grouped rainfall volumes to shorten the calculations. The actual yield responses for different water consumptive regimes were estimated using the relationships between relative evapotranspiration deficits and relative yield decreases given by Doorenbos and Kassam (1979) (see Fig. 4).

All the parameters in the second factor of Eq. (3), except $E_i$ were determined as outlined above. The $E_i$ parameter was assumed equal to 1.0 because the soils at the water harvesting and soil erosion research sites have no salinity problems.

### 3.2. Results and discussion

The soil properties and the derived parameters of the soil erosion–productivity model(s) are given in Table 4. The soil erosion–productivity calculation results are given in Table 5. The regression analysis performed to evaluate the productivity model(s) resulted in Eqs. (9)–(14). Plots of regression line for Eqs. (10) and (11) are shown in Figs. 5 and 6, respectively.

\[
\text{Yield} = 10.8847 \text{PI} - 0.5974 \\
\text{(for 1994 and 1995 rainy season)} \\
\left( r^2 = 0.47, \ p\% = 0.043, \ n = 9 \right), \ (9)
\]

\[
\text{Yield} = 8.5972 \text{PP} - 1.4871 \\
\text{(for 1994 and 1995 rainy seasons)} \\
\left( r^2 = 0.87, \ p\% = 0.000, \ n = 9 \right), \ (10)
\]

\[
\text{Yield} = 6.996 \frac{Y_a}{Y_m} - 2.9923 \\
\text{(for 1994 and 1995 rainy seasons)} \\
\left( r^2 = 0.93, \ p\% = 0.000, \ n = 9 \right), \ (11)
\]
Fig. 4. Relationship between relative yield decrease \((1 - \frac{Y_a}{Y_m})\) and relative evapotranspiration deficit \((1 - \frac{ET_a}{ET_m})\) for sorghum (Doorenbos and Kassam, 1979).
Yield $= 4.5817 PP - 0.1218$

(for 1995 rainy season)

$(r^2 = 0.81, \ p\% = 0.014, \ n = 6)$, \hspace{1cm} (12)

Yield $= 5.4980 Y_a/Y_m - 2.046$

(for 1995 rainy season)

$(r^2 = 0.81, \ p\% = 0.014, \ n = 6)$, \hspace{1cm} (13)

Yield $= 4.229 \sum_{i=1}^{n} (C_i * D_i * RI_i) - 0.7351$

(for 1995 rainy season)

$(r^2 = 0.81, \ p\% = 0.014, \ n = 6)$, \hspace{1cm} (14)

As shown in Eq. (9), the PI model was able to explain about 47% of the variations in sorghum yields for the two rainy seasons and was significant at 0.043 level. Its prediction is poorer then the MPI model. As shown in Eq. (10), the MPI was able to explain about 87% of the variations in sorghum yields for the two rainy seasons at 0.000 level of significance while the actual evapotranspiration alone explained about 93% of the sorghum yield variations at the same level of significance (see Eq. (9)). Eqs. (9)–(11) show that the effect of soil water storage capacity to yield depends on the way it provides water needed to meet crop water evapotranspiration requirements. If crop evapotranspiration rates can be maintained constant for different cropping seasons the PI model can give good predictions. If this condition is not met the PI model gives poor predictions, hence the variable performances of the model reported by different authors (Gantzer and McCarty, 1987; Pierce et al., 1983; Kiniry et al., 1983).

Further regression on six yield data of 1995 was done to check whether the observations in Eqs. (10) and (11) were consistent. The regression results show that, soil parameters affecting crop development and yield, (when soil water holding capacity is excluded) (Eq. (14)), actual evapotranspiration (Eq. (13)) and the MPI (Eq. (12)) can explain about 81% of the yield variations at same level of significance (i.e.

### Table 4

<table>
<thead>
<tr>
<th>Soil erosion research site$^a$</th>
<th>Water harvesting research site$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon (cm)</td>
<td>Texture</td>
</tr>
<tr>
<td>0–10</td>
<td>LS</td>
</tr>
<tr>
<td>10–19</td>
<td>SL</td>
</tr>
<tr>
<td>19–39</td>
<td>SL</td>
</tr>
<tr>
<td>38–100</td>
<td>SCL</td>
</tr>
</tbody>
</table>

$^a$ Root zone water holding capacity for the soils at erosion and water harvesting sites were 70 and 100 mm/m, respectively (De Pauw et al., 1983).

$^b$ pH (CaCl$_2$) determined by multiplying measured pH(H$_2$O) results at the two sites with a factor of 0.84 as estimated from the pH measurements in the report by De Pauw et al. (1983).

### Table 5

<table>
<thead>
<tr>
<th>Site</th>
<th>$\sum C_i * D_i * RI_i$</th>
<th>$\sum_{i=1}^{n} (A_i * C_i * D_i * RI_i)$</th>
<th>$Y/Y_m$</th>
<th>PP</th>
<th>Crop yield (t/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil erosion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plot no. 2</td>
<td>0.47</td>
<td>0.17</td>
<td>–</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>Plot no. 4</td>
<td>0.47</td>
<td>0.17</td>
<td>–</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>Plot no. 5</td>
<td>0.47</td>
<td>0.17</td>
<td>–</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>Water harvest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero tillage</td>
<td>0.6</td>
<td>0.29</td>
<td>0.9</td>
<td>0.7</td>
<td>0.54</td>
</tr>
<tr>
<td>Handhoe tillage</td>
<td>0.6</td>
<td>0.29</td>
<td>0.9</td>
<td>0.7</td>
<td>0.54</td>
</tr>
<tr>
<td>Tractor tillage</td>
<td>0.6</td>
<td>0.29</td>
<td>0.9</td>
<td>0.7</td>
<td>0.54</td>
</tr>
</tbody>
</table>

p% = 0.014). Thus the accuracy difference between the MPI model (Eq. (10)) and that of Doorenbos and Kassam equation (Eq. (11)) in explaining sorghum yield variations can be due to experimental error, especially when considered that limited data were used to test the models and average long term potential evapotranspiration was used to calculate actual crop evapotranspiration for the two cropping seasons.

From the regression results of Eqs. (9)–(14) it can be concluded that a reliable equation predicting effects of soil erosion to crop productivity should consider the effects of soil erosion on all important soil physical, chemical and biological properties affecting crop growth and development. For example, the results have shown that the omission of the effects of soil erosion on crop evapotranspiration requirements makes the equation unreliable (Eq. (9)).

The MPI promises to be an improvement over the PI model (Kiniry et al., 1983; Pierce et al., 1983). It also promises to give more reliable results than the soil life span model (Stocking and Pain, 1983) or the water budget approach proposed by Timlin et al. (1986), both of which only use the reduction of soil water holding capacities by soil erosion in simulating the loss in soil productivity.

The data used to evaluate the MPI model is limited. Further investigation for a wider range of soils, crops and climates are thus needed to ascertain the accuracy of the MPI and possibly improve the methods used to estimate its soil parameter values and/or change its structural form (Gantzer and McCarty, 1987), despite the fact that extensive data collected in the USA and used by Pierce et al. (1983) show the methods to be accurate.

4. Conclusion

A soil erosion–productivity model taking into account important factors affected by erosion and affecting soil productivity has been developed. The model promises to give reliable prediction and requires data base readily available even in developing countries. Data used to evaluate the model were from two soil types and one crop type, thus was limited. More research data are still needed to ascertain its accuracy and possibly modify it.

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References


