An integration of Schumpeterian and classical theories of growth and distribution

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Abstract

Starting out from a classical perspective on income distribution and growth, the paper reconsiders this issue in the framework of a basic (deterministic) Schumpeterian model with ongoing technical change. On a long-run equilibrium path of this economy, the forces of technological innovation and diffusion balance such that always various techniques of different efficiencies coexist. A consequence of this equilibrium notion is that prices and income distribution can no longer be determined independently of quantities, and that distribution is affected by behavioural parameters beyond the scope of the classical analysis. On the other hand, wages and profits continue to be inversely related. It is also indicated that the present approach casts new light on the internal consistency of the Marxian law of the falling rate of profit. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Classical and neoclassical theories of growth and income distribution have one thing in common: even in models with technical progress, both theories suppose that all firms have free access to the most efficient technological knowledge, or that the competitive process works so fast as to drive old production methods out of the

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market immediately when a new and more profitable technique enters the stage. Apart from the wide gap between this idealized world and the industrial structure to be observed in reality, the assumption raises a fundamental problem at the theoretical level, namely, that there would be no one within the system who has any motivation to change the reached position. Strictly speaking, the long-run equilibrium notion of the classical and neo-classical theory, if it is to apply to a capitalist economy, is a contradiction in terms (Metcalfe, 1997, p. 6).

With respect to the dynamic character of a capitalist economy, a Schumpeterian view seems more meaningful. What Schumpeter envisaged as ‘the essential fact about capitalism’ (Schumpeter, 1950, p. 83) is the process of ‘creative destruction’, a process that ‘incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one’. According to this view economic theory should seriously take into account ‘the ceaselessly changing pattern of economic activity, expressed over time by the emergence of new activities, the demise of existing ones and the changing relative importance of those that currently compete for markets’ (Metcalfe, 1997, p. 6).

Limiting the discussion to technical change and the introduction of unambiguously more efficient production methods in a one-good world, these features can already be captured by a Schumpeterian prototype model as it has been put forward by Iwai (1984). The basic forces are here technological diffusion and innovation. Their interplay suggests another and more advanced equilibrium notion, which can be described as a balance of the centripetal forces of diffusion with their tendency toward the technological frontier, and the centrifugal forces of a never ceasing inflow of new technological knowledge. Since diffusion is gradual and not instantaneous, the economy has in each period a non-degenerate spectrum of coexisting techniques, a multitude of diverse production methods with different efficiencies. As the dynamic process unfolds, the least productive vintages are wiped out through evolutionary pressure, while at the other end of the spectrum new and more profitable production methods are continuously added.

The present paper focusses on this equilibrium notion and its direct implications. To make the argument as simple as possible, technical progress and also the rates of diffusion are assumed to be exogenous. If furthermore the innovations improve uniformly and are introduced deterministically at regular intervals of length $T$, the economy is in long-run equilibrium when the capacity shares of the techniques reproduce themselves over time — only at an increasing technological scale. This means more precisely that the role of technique $i$ at time $t$ is taken over by its more

1 Although endogenous rates of diffusion and technical progress are an urgent problem, it is still a useful research strategy to confine the analysis, in a first step, to such an elementary, perhaps even mechanical model. More elaborate versions, for which a great variety of modelling concepts exist, may be subsequently designed in separate steps. In this way it is possible to distinguish basic effects of innovation and diffusion from more special effects that may be associated with different forms of endogeneization. A premature endogeneization of technical change runs the risk of confusing these effects and their theoretical significance.
productive successor \( i + 1 \) at time \( t + T \). Such a smooth evolution of the economy is called a wave train\(^2\).

Of course, proponents of classical and neo-classical theories are aware of the methodological criticism mentioned above. They argue that their equilibrium concept is nevertheless a convenient instrument that since long has proved worthwhile to study growth and income distribution. The sketchy remarks in the preceding paragraph indicate that alternative modelling devices are available. They can incorporate basic aspects of the Schumpeterian vision and are thus able to overcome (to some extent) the basic methodological problem of the orthodox theories. On the other hand, evolutionary Schumpeterian models usually concentrate on the details of technical change and are not very explicit about income distribution. At this point, the present paper attempts to build a bridge.

The paper starts out from the classical notion of income distribution as it is represented by the Sraffian ‘degree of freedom’, according to which one distributional variable, the real wage or the profit rate, is exogenously given. This treatment of distribution is integrated into the Schumpeterian framework provided by Iwai (1984). An immediate consequence of the new equilibrium concept of a wave train in this model is that the economy’s overall profitability can no longer be expressed by a uniform rate of profit. Since there is always a whole spectrum of techniques of different efficiencies, the place of this variable is now taken by the \( \text{average rate of profit} \). It is important to note that in this way the otherwise so convenient Sraffian dichotomy between prices and quantities is undermined: whereas in the standard production price systems income distribution can be investigated without recourse to quantities, the determination of the average profit rate involves the capacity shares of different techniques. Since the latter are endogenously determined, the quantity side has to be invoked right from the outset\(^3\).

From the fact that the equilibrium capacity shares of the techniques are brought about by the interaction of diffusion and innovation, it moreover follows that the parameters that measure the ‘strength’ of these forces have also a bearing on the characteristics of the wave train. Thus, our approach will be able to identify factors that have an impact on income distribution and growth in long-run equilibrium, which are beyond the scope of the classical analysis. It may be claimed that in this respect the Schumpeterian long-run equilibrium concept here employed, while at a similar level of formalization, is more fruitful than the classical specification.

In elaborating on these points, the remainder of the paper is organized as follows. Section 2 presents the modelling equations of the evolutionary process of diffusion and innovation. The corresponding long-run equilibrium concept of a wave train is introduced in Section 3. Section 4 is devoted to the determinants of income

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\(^2\) In Iwai’s (1984) model, the introduction of innovations is governed by a random process. The appropriate equilibrium notion is then one of stochastic equilibrium, which is constituted by an invariant probability distribution of ‘cost gaps’. (A similar concept of cost gaps is introduced in Section 3).

\(^3\) To be precise, within the paper’s one-good economy (but only here) this holds true if the position of workers is characterized by the (deflated) real wage rate, rather than the share of wages in national income. Our methodological stance on this issue will be made clearer at the end of Section 3.
distribution, where the exogenous distribution variable is the real wage rate deflated by a measure of labour productivity. A main result is that the average rate of profit on the resulting wave train is influenced by two parameters grounded in the quantity side: one is the responsiveness of investors to the profit rate differentials of the existing techniques, the other is related to the initial capacity share at which a new technique is installed. If, on the other hand, these parameters are fixed, it is seen that the inverse relationship of the classical theory between the (deflated) real wage and the (average) rate of profit carries over, though the mechanism is now more complex that in a one-good world with just one technique. The basic arguments are intuitive, while a more careful analysis to confirm this type of reasoning has to resort to numerical simulations. Section 5 briefly deals with the Marxian law of the falling rate of profit if technical change is capital-using and labour-saving, and (deflated) real wages remain fixed. In the classical production price systems the profit rate cannot reasonably fall owing to Okishio’s theorem, but in the Schumpeterian framework these problems of internal consistency no longer prevail. Section 6 concludes. An appendix collects some further details in the formulation of the model.

2. Formulation of the evolutionary process

The dynamic process begins with \( i = 1, 2, \ldots, N_o \) production methods at starting time \( t = 0 \). Technical progress is exogenous and deterministic. A new technique is introduced into the economy every \( T \) years, let us say at dates

\[
J = \{mT: m = 0, 1, 2, \ldots\} \quad (1)
\]

We consider a one-good world where the same good, besides serving as a consumption good, is installed as a capital good in different production processes. Technical progress is assumed to be Harrod neutral. Correspondingly, the capital coefficient \( b \) is the same for all techniques, as well as the rate of depreciation \( \delta \). The efficiency of a technique is therefore unambiguously characterized by its labour coefficient. Denote the unit labour requirements of technique \( i \) by \( a_i \), and by \( q_i \) its labour productivity, \( q_i = 1/a_i \). The productivity of the new techniques is supposed to grow exponentially at an annual rate \( \lambda \). Since the adjustment mechanisms below are specified in continuous time, it is convenient to define \( \lambda \) as the compound rate of growth, so that

\[
q_i = q_{i-1} \exp(\lambda T), \quad \text{or} \quad a_i = e^{-\lambda T} a_{i-1}, \quad i = 1, 2, \ldots \quad (2)
\]

Let \( N_t \) be the index of the best-practice technique (BPT) at time \( t \), the most efficient technique currently in use or just about to be set up. The discontinuous changes in this variable follow from (1),

\[
\dot{N}_t = 0 \quad \text{if } t \notin J
\]

\[
N_t = N_{t-T} + 1 \text{if } t \in J \quad (3)
\]
The time index of $N_t$ will be omitted in the running text whenever this seems possible. The capacity share of technique $i$ at $t$ is denoted by $k_i(t)$. To build up the capacity share $k_N$ of the BPT when it comes into existence at time $(N - N_o)T$, we assume a (short) set-up phase of length $T_u$. Designate this span of time by

$$U(N_t) : = [(N_t - N_o)T, (N_t - N_o)T + T_u]$$

Within the set-up phase, the capacity share $k_N$ of the BPT is supposed to increase autonomously at a given rate $\kappa$,

$$\dot{k}_N(t) = \kappa \quad \text{if} \quad t \in U(N), \quad \text{where} \quad N = N_t$$

Certainly, $k_N(t) = 0$ for $t \leq (N - N_o)T$. By the end of this transitional stage, at $t = (N - N_o)T + T_u$, the BPT has established a share $k_N(t) = \kappa T_u$ in total capacity. From then on, the BPT is treated like any other technique, whose changes are governed by the diffusion equation that is to be derived next$^4$.

Given the real wage rate $w = w(t)$, the rate of profit of technique $i$ at time $t$ is

$$r_i(t) = \frac{1 - w(t)a_i}{b} - \delta$$

Average values across techniques are denoted by a bar over the variable. The techniques themselves are weighted by their capacity shares. Thus the average rate of profit at time $t$ is

$$\bar{r}(t) = \sum_{i=1}^{N_t} r_i(t)k_i(t)$$

and analogous for average unit labour requirements, $\bar{a} = \sum a_i k_i$, which is equal to $L/Y$ ($L$, total labour; $Y$, aggregate output). An exception is average labour productivity $\bar{q}$, where the coefficients $q_i$ are weighted by the employment shares $L_i/L$, or directly

$$\bar{q}(t) = 1/\bar{a}(t)$$

Technological diffusion is a gradual adjustment process such that the more efficient techniques grow faster than the older vintages$^5$. In the present context this means that the capital stock growth rates $g_i$ are determined by differential rates of profits. With respect to a propensity to save out of profits $s$, we draw on the formulation of investment in classical gravitation processes and specify the growth rate of the capital stock of technique $i$ as

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$^4$ Since (in Eq. (10) below) the diffusion process yields a growth rate formulation for the changes of the capacity shares, $\dot{k}_i/k_i$, some functional expression, the BPT can only be subjected to the diffusion equation after a positive capacity share $k_N > 0$ has been erected. The concept of the set-up phase $T_u > 0$ was introduced to make the time path of $k_N$ continuous (rather than assuming a jump from zero up to a certain positive level at times $t \in J$).

$^5$ Retardation factors that prevent firms from adopting the most productive technique immediately are, in particular, adjustment costs, uncertainty, and the often proprietary nature of the newest technology (cf. Soete and Turner 1984, p. 617).
\[ g_i(t) = s\ddot{r}(t) + \rho [r_i(t) - \ddot{r}(t)] \] (9)

Eq. (9) is an investment function where causality runs from profit rates to growth rates and any problems of effective demand are neglected (an explicit formulation of consumption demand is thus dispensable). The coefficient \( \rho \) measures the responsiveness of investors to differential profits. It will be an important factor in the overall speed of diffusion, though not the only one. Obviously, (9) directly implies the standard Cambridge equation in the aggregate, i.e. \( \ddot{g}(t) = s\ddot{r}(t) \) (\( \ddot{g} \) the average capital growth rate). If \( \rho \) or the differential profits are sufficiently large, net investment may also be negative, \( g_i(t) < 0 \), for the least productive techniques. Furthermore, some equipment might even be scrapped if a technique is to shrink faster than the capital stock depreciates\(^6\).

By the assumption of Harrod neutral technical progress, the techniques’ capacity shares \( k_i \) coincide with their shares in the total stock of fixed capital. It is then easily checked that the changes in \( k_i \) are determined by the differential equations

\[ \dot{k}_i = (g_i - \ddot{g})k_i = \rho (r_i - \ddot{r})k_i \] (there is no implementation lag in this model). These equations also ensure that the identity \( \Sigma k_i = 1 \) is maintained. It has, however, to be noted that the economy-wide application of Eq. (9) is limited to the time intervals where the BPT does not expand autonomously according to Eq. (5). For the set-up phase itself, the growth rates in Eq. (9) have to be modified in two ways: the BPT \( i = N \) is excluded from (9), and at the same time the aggregate relationship across all techniques, \( \ddot{g} = s\ddot{r} \), is supposed to be preserved. The variations in the capacity shares \( k_i \) may thus be briefly summarized as follows:

\[
\begin{align*}
\dot{k}_i(t) &= \rho [r_i(t) - \ddot{r}(t)]k_i(t) \\
\text{for } i &\leq N_i \text{ if } t \notin U(N_i) \\
\dot{k}_i(t) &= \rho [r_i(t) - \ddot{r}(t)]k_i \\
\text{for } i &\leq N_i - 1 \\
\text{and subsequent remormalization} &\text{ if } t \in U(N_i)
\end{align*}
\] (10)

(the precise details of the renormalization, within the discrete-time framework of the numerical simulations, are given in the appendix).

To close the model it remains to specify the time path of the real wage rate \( w = w(t) \). If the income shares of wage and profit receivers are not to drift apart over time, the wage rate must grow with labour productivity. Since we will concentrate on the long-run equilibrium evolution of the economy, it may conveniently be assumed that wages grow at a constant rate \( g_w \)\(^7\). Thus, from the specification of the rate of technical progress in Eq. (2),

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\(^6\) A referee has pointed out two assumptions implicit in this diffusion model. First, there is no market for used capital goods of older vintages that may be bought at lower prices, so that their prospective profit rate could equal the profit rate on the most productive equipment. Second, the price of the machines used by the older techniques continues to be determined by their production cost, since they are being produced.

\(^7\) In more general models demanding less knowledge from the agents, \( g_w \) may be plausibly replaced by expectations about future productivity growth, possibly combined with a Phillips curve relationship as in Silverberg and Lehnert (1993). We here neglect these elements since they are mainly a matter of disequilibrium adjustments and the economy’s equilibrium evolution would only be marginally affected by such alternative specifications. (Wages would rise less smoothly between two innovations, while their average growth rate over time would remain the same.)
Given an initial number of techniques $N_0$, a frequency distribution of their capacity shares $(k_1, \ldots, k_N)$ at $t = 0$ (with $k_{N_0} = 0$) and also an initial real wage rate $w(0)$, the evolution of these and the newly arriving techniques is now fully described.

### 3. Wave trains

Eqs. (1)–(11) specify an evolutionary process that captures the basic Schumpeterian hypotheses on technological diffusion and innovation. While diffusion constitutes an equilibrating force that tends to steer the economy’s state of technology toward an equilibrium in which all firms use the most efficient production method, the function of innovation lies precisely in upsetting this equilibrating tendency. The dynamic interaction between the continuous and equilibrating forces of diffusion, and the discontinuous and disequilibrating forces of innovation, may nevertheless exhibit a certain regularity. In our deterministic setting for the technological innovations, the forces of diffusion and innovation balance if, over regular intervals of time, they reproduce exactly the same spectrum of coexisting techniques, only at a higher scale. This means that the role of technique $i$ at time $t$ is taken over by technique $i + 1$ at time $t + T$. Formally, the capacity shares constitute an equilibrium trajectory, denoted by $\{k^*_i(t)\}$, if over time and across techniques they are related by

$$k^*_{i+1}(t + T) = k^*_i(t), \quad i \in \mathbb{N}, t \geq 0$$

This time path of capacity shares corresponds to what is known in the literature on partial differential equations (with their continuous state space) as a travelling wave, or a wave train. In the present framework with the discrete unit costs $a_i$ of successive techniques, a solution of system (1–11) is a wave train if there exists a real function $\phi(\cdot)$ such that

$$\phi(\ln a_i + g_w t) = k^*_i(t), \quad i \in \mathbb{N}, t \geq 0$$

(cf. Henkin and Polterovich, 1991, p. 556). In fact, compatibility of Eq. (12) and Eq. (13) follows from the definition of $g_w$ in Eq. (11) and reformulation of (2) as

$$\ln a_{i+1} + g_w(t + T) = \ln a_i + g_w T + \ln a_0, \quad \text{and the relationship} \quad k^*_{i+1}(t + T) = \phi[\ln a_{i+1} + g_w(t + T)] = \phi[\ln a_i + g_w T + \ln a_0 + g_w T] = \phi[\ln a_i + g_w t] = k^*_i(t).$$

By introducing the concept of cost gaps, the frequency distributions of techniques can be studied independently of time progressing. The cost gap of technique $i$ is given by its unit cost in relation to the unit cost of the best-practice technique $N$. Denoting this gap by $z_{N_i}$, we have

$$z_{N_i} = a_i/a_N$$
Clearly, the ratios \( a_i/a_N \) remain unaltered if the same number is added to \( i \) and \( N \), while the cost gap of a given technique \( i \) increases as new techniques are introduced. The frequency distribution of these cost gaps is directly represented by the wave train function \( \phi(\cdot) \). To see this suppose a given cost gap \( z = z_{N-i} \) is brought about by technique \( i \) at a time \( T \) years after the introduction of the most recent technique \( N = N_t \). Accordingly, let \( 0 \leq t < T, N_i = N_o + m \) for a suitable integer number \( m \), and \( t = mT + \tau \). Then it is easily verified that with respect to some constant number \( C \),

\[
\phi[\ln z_{N-i} + g_{w\tau} + C] = k_i^*(t), \quad \text{where } N = N_i,
\]

(see the appendix). Eq. (15) together with Eq. (12) tells us that it does not matter whether the cost gap \( z \) is brought about by technique \( i \) at time \( t \) or by technique \( i + 1 \) at time \( t + T \): the capacity share will be the same\(^8\). Hence, the distribution of cost gaps remains invariant over time. More precisely, the distribution of the variable \( \ln z \) is slightly shifted between two innovations as the term \( g_{w\tau} \) increases from 0 to \( g_{w\tau} \), but identical values are assumed if we look at the distribution every \( T \) years.

Fig. 1 presents a typical example of the equilibrium evolution of an economy (the underlying numerical details are given in the next section). The diagram clearly shows how the frequency distribution of the techniques’ capacity shares moves regularly over time toward the techniques with lower unit labour requirements (ULR). Correspondingly, as indicated by the bold line on the surface running parallel to the time axis, it is seen how the capacity share of a given technique

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\( ^8 \) Formally, with \( N + 1 = N_{t+1} \) and \( z = z_{N-i} = z_{(N+1)-(i+1)} \), the relationship reads

\[
\phi[\ln z + g_{w\tau} + C] = \phi[\ln z_{(N+1)-(i+1)} + g_{w\tau} + C] = \phi[\ln z_{N-i} + g_{w\tau} + C] = k_i^*(t) = k_i^*_{i+1}(t+T).
\]
steadily declines in the later stages of its life-cycle. (Its rise in the early stages, when it was still among the most efficient techniques, is hidden by the crest of the distribution surface.) The wave train function \( \phi(\cdot) \) can be read off along the ULR-axis at, for example, \( t = 20 \).

The unit costs of the active techniques at that date range from a high of \( a_{12} = 52.7 \) to a low of \( a_{12} \times 27 = 12.2 \) (see the bold line at \( t = 20 \)). That is, the new techniques are more than four times as productive as the old vintages. Though the only purpose of Fig. 1 is to illustrate the properties of a highly stylized economic model, these differences in efficiencies might not be too extreme. Thus, to emphasize the economic significance of his general approach, (Iwai, 1984, p. 324) gives a representative example of the frequency distribution of an industry in the USA (the metal stamp industry) where the ratio of payroll to value added covers a range from 0.15 to 0.85.

Applying the concept of a wave train to system (1–11) raises three questions: Does a wave train \( \{k^*_i(t)\} \) exist? Is it uniquely determined? Is it globally stable? The latter means that starting from an arbitrary initial frequency distribution \( \{k_i(0)\} \), the resulting solution \( \{k_i(t)\} \) of (1–11) converges toward the wave train capacity shares \( \{k^*_i(t)\} \) as \( t \) gets large.

These problems have been subjected to a rigorous mathematical analysis by Henkin and Polterovich (1991) in another, similarly elementary Schumpeterian model. Under a reasonable set of conditions, the authors prove existence and uniqueness of a wave train as well as its global stability. Unfortunately, the (quite elaborated) analysis cannot be readily carried over since the present model differs in a number of specification details. However, existence and the powerful global stability result were fully confirmed in all our numerical simulations. As we have found no reason to harbour any doubts, we do not pursue the mathematical issue any further.

The property of global stability establishes the economic significance of the long-run equilibrium notion of a wave train. Actually, the global stability was also exploited to compute the wave trains in the numerical investigations below. Accordingly, we initialized the dynamic process (1–11) with an arbitrary distribution of capacity shares \( \{k_i(0)\} \) at \( t = 0 \), and let it run for so long until Eq. (12) was approximately satisfied. Then, setting the wage rate back to \( w = w(0) \), the process was rerun with the capacity shares just found as the initial frequency distribution.

Against the background of the classical theory of income distribution, another feature of interest are the wages and profits on a wave train. Since there is now no longer a uniform rate of profit, we refer to the average profit rate in order to characterize the position of profit receivers by a single number. Denoting by \( v \) the share of total wages in gross national income \( (v = wL/Y) \), we have

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9 Of course, these differences cannot be directly compared to that of Fig. 1 since we have no information about the probably varying capital-output ratios in Iwai’s example.

10 Strictly speaking, our results could be compatible with the existence of another, but unstable, wave train, although such a phenomenon would have no practical relevance even if it existed.
\begin{equation}
v(t) = w(t) \hat{a}(t)
\end{equation}
\hspace{1cm} (16)

(cf. the remark on Eq. (7)). Using Eq. (6), the average rate of profit can be written as

\begin{equation}
\bar{r}(t) = \frac{1 - v(t)}{b} - \delta = \frac{1 - w(t) \hat{a}(t)}{b} - \delta.
\end{equation}
\hspace{1cm} (17)

Since by hypothesis real wages \( w(t) \) grow at the same rate at which the unit labour requirements of the BPT decline, \( w(t) \hat{a}(t) \) does not systematically rise or fall in the long-run. That is, \( v(t + T) = w(t + T) \hat{a}(t + T) = w(t) \hat{a}(t) = v(t) \) for all \( t \), while between two innovations \( v(t) \) may fluctuate on a small scale (these intermediate fluctuations are neglected in the following discussion). By the same token, it is ensured that the average rate of profit remains constant in the course of the process. Hence, if an initial real wage rate \( w(0) \) is exogenously given, it has associated with it a wave train solution \( \{k^*(t)\} \) of the evolution of capacity shares, and these in turn determine the average rate of profit \( \bar{r} \) as well as the overall growth rate of the economy, \( \bar{g} = \bar{r}. \) In this sense, the degree of freedom from the classical theory of income distribution is maintained.

On the other hand, also the average rate of profit \( \bar{r} \) may be chosen as the exogenous variable. Observe that, by virtue of Eq. (17), this is tantamount to fixing the wage share \( v. \) In the computation of the corresponding wave train, we can then replace Eq. (11) with Eq. (16) and set directly \( w(t) = v/\hat{a}(t). \)

Knowledge of the real wage rate \( w(0) \) on a wave train is not yet sufficient to assess the position of workers on this long-run equilibrium growth path; we also have to know the general level of technology at that time, \( t = 0. \) To discuss variations of income distribution, it is therefore appropriate to deflate the real wage by an index of labour productivity. However, the weights constituting such an index should be independent of the frequency distribution of techniques, which will generally change from one wave train to another. This requirement rules out average productivity \( \bar{q} = 1/\hat{a} \) as a deflator. Instead, it appears natural to employ the labour productivity of the BPT for this purpose. Looking at wages in the middle between two innovations, define

\begin{equation}
\omega = w(T/2)/q_N = w(T/2) a_N, \quad \text{where } N = N_o = N_{T/2}
\end{equation}
\hspace{1cm} (18)

It will be clear by now that on a wave train we have \( w(t_m)/q_{N(m)} = \omega \) for all \( m \in \mathbb{N}, \) where \( t_m := m T + T/2, \) \( N(m) := m T + T/2. \) This means that if we study changes in income distribution, we exogenously fix the deflated real wage \( \omega \) and set the initial real wage \( w(0) \) such that, on the equilibrium path, \( w(T/2) = \omega q_{N_o} \) comes about \( (N = N_o). \) The wave train distribution function \( \phi(\cdot) \) is subsequently used to compute the corresponding average rate of profit\(^12.\)

\(^11\) In all what follows it will be understood that averages like \( \bar{r}, \hat{a} \) and \( \bar{z} \) (the average cost gap) are based on the capacity shares \( k^*_i \) of a wave train.

\(^12\) Conversely, we may fix \( \bar{r} \) (or \( \omega = 1 - b(\bar{r} + \delta), \) respectively), obtain the associated wave train including \( \hat{a}(t), \) and then from \( w(t) = v/\hat{a}(t) \) compute the deflated real wage as \( \omega = w(t_m)/q_{N(m)} \) for some integer number \( m. \)
Before we illustrate the construction of a wage-profit frontier in this way, we should stress an important conceptual difference from income distribution in the production price systems of the classical theory. In this type of analysis, the study of income distribution, given the ‘book of blueprints’ of techniques, requires the computation of relative prices, while at least in single production systems quantities do not interfere. In the present context of a one-good economy, relative prices are no problem (they would reappear if we tried to set up a two-sectoral version of the model). However, given the path of technical progress, the determination of the distributional variables \( \omega \) and \( \bar{r} \) requires the computation of the equilibrium frequencies of the techniques’ capacity shares. As already indicated in the introductory section, the classical dichotomy between prices and quantities therefore no longer obtains. Despite the highly simplified assumptions on technology, a study of income distribution in an evolutionary economy has to integrate the quantity side right from the outset.

It has already been noted above that if, instead of the deflated real wage \( \omega \), the wage share \( \bar{r} \) is taken as the exogenous variable, the average rate of profit can be directly determined by Eq. (17) without further knowledge of the capacity shares. However, quantities will then interfere as soon as the assumption of a one-good economy is dropped. More importantly, the interdependence between the price and the quantity side under a given wage share occurs, almost trivially, already at a much more elementary level, in basic (multisectoral) production price systems with a uniform rate of profit and best-practice techniques only (cf. Franke, 1998b, 1999). This inconvenience is certainly one of the reasons why classical analysis normally chooses real wages, rather than the wage share, to characterize the position of workers. As we wish to discuss the classical theory within its own preferred setting, we here follow this tradition.

4. Determinants of income distribution

To study the evolution of the economy on a wave train by means of computer simulations, we now have to specify the numerical parameter values. One set of parameters is given by familiar macroeconomic magnitudes: the capital-output ratio \( b \), the rate of depreciation \( \delta \) and the propensity \( s \) to save out of profits. On the basis of Simon (1990), they are fixed as follows,

\[
\begin{align*}
  b &= 1.70 \\
  \delta &= 0.0461 \\
  s &= 0.2670
\end{align*}
\]  

(again, see the appendix for further details). The other set of parameters governs technological diffusion and innovation: the annual rate of technical progress \( \lambda \), the innovation time \( T \) (the interval between two innovations), the length of the set-up phase \( T_u \), the autonomous rate of change \( k \) at which the capacity share of the BPT increases over this period, and the responsiveness \( \rho \) of investors to differential profits. Here we put \(^{13}\)

\(^{13}\) Actually, \( \lambda \) was derived from a discrete-time rate of growth \( \bar{\lambda} = 3\% \). That is, \( \lambda \) was set such that \( \exp(\bar{\lambda}T) = q_i/q_{i-1} = 1 + \lambda T \), which yields \( \lambda(\ln(1 + \lambda T))/T \).
\[ \lambda = 0.029 \quad T = 2.00 \quad T_u = 0.10 \quad \kappa = 0.02 \quad \rho = 1.00 \] (20)

The innovation parameters \( T \), \( T_u \), and \( \kappa \) can be encapsulated in a single parameter \( v \) defined as
\[ v = \kappa T_u / T \] (21)

The motivation for this definition and its relationship to Iwai’s (1984) stochastic innovation hypothesis is discussed in Franke (1998a). The remarkable point is that (essentially) the same wave train distribution function \( \phi(\cdot) \) is brought about by different combinations of \( T \), \( T_u \) and \( \kappa \) that give rise to identical values of \( v \) in Eq. (21)\(^{14}\).

Finally, we take the deflated real wage rate \( \omega \) as the exogenous variable for income distribution and set
\[ \omega = 0.307 \] (22)

(Recall \( \omega = w(T/2)/q_N \) from Eq. (18), with \( N = N_0 = N_{T=2} \). Eqs. (19) and (20) and Eq. (22) may serve as a base scenario. In particular, the special choice of \( \omega = 0.307 \) results in the following values of the wage share, the average profit rate, and the capital growth rate,
\[ \nu(T) = 70.02\%, \quad \bar{r}(T) = 11.22\%, \quad \bar{g}(\tau) = s\bar{r}(\tau) = 3.00\% \] (23)

which prevail on the wave train in the middle between two innovations, at time \( \tau = T/2 \). Eq. (23) shows that the model can be calibrated such as to be roughly compatible with empirical values of the macroeconomic key variables here involved.

The remainder of the paper is concerned with comparative dynamics. Accordingly, some of the exogenous parameters are varied and the characteristics of the corresponding equilibrium growth paths are studied. In this respect, note first that variations in the exogenous rate of technical progress \( \lambda \) have trivial effects on income distribution, since by construction \( \lambda \) determines directly the growth rate of real wages. In the present setting it is therefore meaningless to compare two economies with different values of \( \lambda \).

A most important case, however, is a change in the responsiveness \( \rho \) of investors to differential profits. Although \( \rho \) certainly has its institutional foundations, it is also of a psychological nature. Given the economy’s institutions, \( \rho \) may be regarded as a psychological coefficient. Now, it is a centrepiece of the classical theory that the long-run equilibrium position is independent of such a reaction intensity. In this theory, the only significance of \( \rho \) would be in the disequilibrium adjustment process toward the steady state. For example, a mathematical analysis of a gravitation process might reveal that this parameter must not be too large if stability is to be ensured. By contrast, in the present Schumpeterian framework it turns out that the

\(^{14}\) For example, an increase in \( T \) together with the corresponding reduction of \( \kappa \) or \( T_u \) means a smaller number of ‘active’ techniques \( i \) on the wave train. However, the graph connecting the points (ln \( z_i \), \( \phi(\ln z_i) \)) remains (essentially) the same, and so will remain the average rates of profit and growth, etc. That the graphs do not exactly coincide is due to the discrete state space of the technological coefficients.
responsiveness $r$ already has a bearing on the long-run equilibrium itself. For example, maintain the other parameter values in (19, 20, 22) and increase the responsiveness to, say, $r = 1.20$. Income distribution and growth on the new wave train are then given by

$$v(T) = 66.31\%, \quad \bar{r}(T) = 13.40\%, \quad \bar{g}(\tau) = 3.58\% \quad (\tau = T/2) \quad (24)$$

Comparing these values with Eq. (23), it is seen that a higher responsiveness of investors, or intensified competition, improves the position of profit earners and also leads to faster growth if workers receive the same real wages.

The qualitative changes in the wage share $v$ and the average profit rate $\bar{r}$ are easily explained. A higher responsiveness $r$ means that the more efficient techniques expand more rapidly than before. Average labour productivity $\bar{q}$ is therefore higher, which is tantamount to a lower average cost gap $\bar{z} = \bar{a}/a_n = 1/(\bar{q}a_n)$ (cf. Eq. (8) and Eq. (14)). Recalling Eq. (16) and Eq. (18) and using $w\bar{a} = (w/q_N)(\bar{a}/a_N) = \omega\bar{z}$, it remains to note that the wage share is related to the average cost gap by

$$v = \omega\bar{z}. \quad (25)$$

In the presence of unchanged deflated real wages we therefore have that faster diffusion, which reduces the average cost gap $\bar{z}$, yields a lower wage share and a higher average rate of profit. Again we emphasize that this result and the underlying mechanism have no counterpart in the classical theory of distribution and growth.

In the further discussion of the long-run equilibria it will be useful to refer to the original Iwai (1984) model. Iwai characterizes the techniques $i$ by their unit costs $c_i$ and postulates that the motions of the capacity shares are directly governed by these unit costs:

$$\dot{k}_i = -\gamma (\ln c_i - \ln c)k_i \quad (26)$$

where $\gamma$ is a constant parameter that measures the general speed of technological diffusion (Iwai, 1984, p. 328). In Franke (1998a) the intuition is confirmed that a higher value of $\gamma$ indeed reduces the cost gap (analogously defined) on a wave train of this economy$^{15}$. This result is central to the following discussion of the comparative dynamics.

Now, the profit rate differentials in the present model can be expressed as

$$r_i - \bar{r} = (-v/b)(a_i - \bar{a})/\bar{a}. \quad (27)$$

With Eq. (25), the upper part of the diffusion Eq. (10) can thus be rewritten as

$$\dot{\bar{k}}_i = -\frac{\rho \omega \bar{z}}{b} a_i - \frac{\bar{a}}{\bar{a}} k_i \quad (27)$$

As the percentage deviations of unit labour requirements $a_i$ in (27) correspond to Iwai’s log differences in unit costs $c_i$ in (26), we have the following approximate

$^{15}$By using an approximation procedure, Iwai derives an explicit mathematical formula for his (stochastic) economy that produces the same result. In other respects, however, his approximation procedure is problematic; see Franke (1998a).
relationship between the concept of the technological speed of diffusion and some central parameters in our model,

$$\gamma \approx \gamma' = \frac{\rho \omega \bar{z}}{b} = \frac{\rho v}{b}$$  \hspace{1cm} (28)$$

Let us also call $\gamma'$ the speed of technological diffusion.

Before going on, it is worth mentioning that, had the investment function (9) been specified as

$$g_i = s[\bar{r} + \rho (r_i - \bar{r})]$$  \hspace{1cm} (9a)$$

Eq. (28) would become $\gamma \approx s \rho \omega \bar{z}/b$. In this case, income distribution on the wave trains would also be dependent on the propensity to save out of profits, another result which is alien to the production price systems of the classical theory. Evidently, an increase in $s$ has then the same effect as an increase in $\rho$. Hence, under Eq. (9a) a higher savings propensity would lead to a lower wage share and a higher average rate of profit. It is also worth mentioning that the growth rate of the economy would rise more than proportionately.

While in Iwai’s model the speed of diffusion is an exogenous parameter, Eq. (28) reveals that it is endogenous in our setting if the real wage rate $\omega$ is chosen as the exogenous income distribution variable.\(^{16}\) Nevertheless, despite the interference of the average cost gap $\bar{z}$ a ceteris paribus increase in the responsiveness $\rho$ to profit rate differentials will also cause the general speed of technological diffusion $\gamma'$ to rise. Although, as indicated above, the average cost gap $\bar{z}$ decreases, the direct influence of $\rho$ remains dominant, so that the product $\rho \bar{z}$ in (28) increases. (In fact, if $\rho \bar{z}$ were to decrease, the lower value of $\gamma'$, according to the remark on Eq. (26), would be associated with a higher value of $\bar{z}$, which would be a contradiction).

This line of reasoning can be readily carried over to ceteris paribus changes in the deflated real wage rate $\omega$. Just like $\rho$, a rise in $\omega$ speeds up the diffusion of new techniques and thus lowers the average cost gap $\bar{z}$, but again not so much as to diminish $\gamma'$ in Eq. (28). Since by (25) the higher product $\omega \bar{z}$ means a higher share of wages $v$, we can conclude that an increase in real wages $\omega$ induces a fall of the average rate of profit $\bar{r}$ in the new long-run equilibrium. In this sense, the classical inverse relationship between real wages and profits is re-established.

The income distribution effects just discussed are summarized in Fig. 2, which shows the response surface of the average rate of profit $\bar{r}$ on the respective wave trains under variations of the deflated real wage rate $\omega$ and the responsiveness coefficient $\rho$. Over the range of parameters considered, the relationship between $\bar{r}$ and $\omega$ is nearly linear. On the other hand, $\bar{r}$ is a convex function of $\rho$, i.e. the increments in the profit rate induced by a further rise in $\rho$ become smaller with higher values of $\rho$.

\(^{16}\) $\gamma'$ will again be exogenous if instead the average profit rate, or equivalently the wage share $v$, is chosen as the exogenous distributinal variable.
Fig. 2. Average rate profit ($\bar{r}$) on a wave train under variations of the deflated real wage rate ($v$) and the responsiveness of investors ($\rho$). Note: the reference stick indicates the base scenario.

Having discussed the effects of $\omega$ and $\rho$, the parameter $v$ of Eq. (21) above, which characterizes the introduction of new techniques, should not be neglected. Since a rise in $v$ corresponds to a faster diffusion of the best-practice techniques in their set-up phase, the effect will be expected to be similar to an increase in $\rho$. This is certainly true as regards the average profit rate or the wage share. To give an example, raising $v$ in the base scenario from $v = 0.0010$ to $v = 0.0030$ increases the average profit rate by about 4%, while reducing $v$ to $v = 0.0003$ lowers $\bar{r}$ by almost 4% (vis-a-vis $\bar{r} = 11.22\%$ and $v = 70.02\%$ in the base scenario, the precise figures are $\bar{r} = 15.23\%$ and $v = 63.21\%$ in the first case, and $\bar{r} = 7.32\%$, $v = 76.66\%$ in the second case).

However, it should also be noted that a change in $v$ affects the general speed of technological diffusion $\gamma'$ as it was specified in Eq. (28). The decrease of the cost gap $\bar{z}$ that is associated with a rise of $v$ means, perhaps somewhat surprisingly, that a faster introduction of new techniques reduces the overall speed of diffusion $\gamma'$. While $\gamma' = 0.412$ in the base scenario, we obtain, $\gamma' = 0.372$ for $v = 0.0030$ and $\gamma' = 0.451$ for $v = 0.003$\(^{17}\).

\(^{17}\)The possible effects from the set-up phase of innovations tends to be underestimated in the literature. For example, Silverberg and Lehnert (1993) and Silverberg and Verspagen (1997) in their stochastic and otherwise more detailed models do not even make $v$ or a similar parameter explicit. They just mention that the initial capacity share of new techniques is ‘small’ and do not discuss any variations in this respect.
5. A note on the law of the falling rate of profit

In this section we return to the remaining magnitude \( b \) in the specification of the speed of diffusion \( \gamma' \) in (28) that has not been varied yet. Formally, a higher capital output ratio \( b \) has the same effect as a \textit{ceteris paribus} decrease in \( \rho \); it raises the average cost gap \( \bar{z} \) on the new wave train distribution. The corresponding average rate of profit therefore declines for two reasons: one is the direct effect of the higher denominator \( b \) in the definition of profit rates, the other is the indirect effect of the change in the wage share, which owing to Eq. (25) is driven up by the rise in \( \bar{z} \).

The result is reminiscent of the Marxian law of the falling rate of profit that is attributed to a rising trend in the organic composition of capital. In the Sraffian production price systems, this subject is usually studied in the form of capital-using and labour-saving technical change. It is, however, well-known that here Okishio’s theorem holds (Okishio, 1961). When a new method of production replaces an existing one because of higher profitability in a given system of production prices, then, if the real wage remains fixed, the uniform rate of profit in the new equilibrium system will inexorably exceed the original profit rate. There is no scope for a falling rate of profit in this setting. While Okishio’s theorem highlights the salient point in the discussion on the internal consistency of the Marxian law, that capitalists would avoid any technical change that reduces the profit rate, the problem of coming to terms with ‘micro-rational’ behaviour is resolved in the present evolutionary framework. At any point in time, the innovation that yields the highest rate of profit will be introduced, irrespective of the kind of technical change it involves. Clearly, an investor who hesitates to invest in such a technique with a higher capital-output ratio because he is afraid of the consequences for the average profit rate in later years, even if he were able to foresee them, would only earn less profits than his competitors. And of course, the average rate of profit is not his concern.

In the above experiment where two economies with the same growth path of labour productivity but different capital-output ratios \( b \) are compared, there are two magnitudes that affect the organic composition of capital, \( x \). In Marxian terms, \( x \) is the ratio of constant capital to variable capital. Translating this as the capital–wages ratio and decomposing it as \( x = (K/wL) = (K/Y) (Y/wL) = b/v = b/\omega \bar{z} \), it is seen that the higher average cost gap \( \bar{z} \) in the second economy with the

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18 We may note in passing an interesting phenomenon that occurs if alternatively the wage share \( v \) is the distributional variable that is held constant. Then not only does the average profit rate decline by Eq. (17), but, since \( \bar{z} \) rises again, also the deflated real wage rate \( \omega = v/\bar{z} \). Incidentally, this cannot possibly happen in a production price framework where, in the presence of a fixed wage share, an increase in some coefficients of the capital stock matrix \( B \) decreases the (uniform) rate of profit, but raises the real wage rate; cf. Fig. 1 in Franke (1999).

19 Note, however, a difference between the problem studied by Okishio and the present framework, Okishio’s theorem assumes that the real wage stays constant when the labour productivity increases from one economy to the other, while our discussion allows real wages to rise over time in both economies at the same (unchanged) rate as their labour productivity rises over time.
higher value of $b$ has a negative impact on $x$. Nevertheless, the positive effect from $b$ is dominant, so that indeed the organic composition of capital turns out to be higher in the second economy. (The argument is the same as in the previous section for $\rho$: a higher $b$ reduces $\gamma' = \rho \omega z/b$ in (28), despite the rise in $\bar{z}$, so that $x = b/(\omega z)$ increases).

On the other hand, the result that the economy with the higher capital-output ratio exhibits a lower average rate of profit is not particularly exciting. Since labour productivity was assumed to grow at the same rate in this economy, the gains in profitability accruing to an innovation are less than in the economy with the lower value of $b$, i.e. the differences $r_N - r_{N-1}$, and generally $r_i - r_{i-1}$, are smaller. It therefore comes as no surprise that also the average rate of profit is bound to fall.

The discussion so far was based on the *ceteris paribus* assumption of a fixed value of the responsiveness to differential profits, $\rho$. This concept may be considered to be too mechanistic. The lower differences $r_i - r_{i-1}$ in Eq. (10) imply that investors in the profitable techniques wish to expand their market shares less rapidly. By contrast, another basis of comparing two economies with different capital-output ratios may be that one fixes the differences $(k_i/k_i) - (k_{i-1}/k_{i-1})$ in the growth rates of two successive techniques $i$ and $i - 1$ at which they increase their market shares. This hypothesis is captured by variations of $\rho$ proportionally to $b$. In fact, if the ratio $\rho/b$ remains unaltered in the speed of technological diffusion $\gamma'$ in (28), the average cost gap $\bar{z}$ on the wave train would not be affected either. Since the unit labour requirements $a_i$ are identical, Eq. (27) shows that in both economies the market shares of the techniques evolve in the same way. Even if the alternative basis of comparison may be felt to be somewhat arbitrary, the few observations may suffice to indicate that a careful investigation into the effects of capital-using technical change has to include more aspects than just the changes of the capital-output ratio in some definitional relationships.

Note also that if, over longer periods, capital-using technical change is associated with higher growth rates of labour productivity while wages continue to increase at the original growth rate, the wage share would systematically fall and (possibly besides unemployment) the economy would sooner or later run into serious conflicts over income distribution. This is only one point where it becomes clear that, if we go beyond the narrow constraints of the present equilibrium setting, the analysis needs to be complemented by a study of additional dynamic mechanisms.

6. Conclusion

The model studied in this paper is an elementary extension of an evolutionary process advanced by Iwai (1984). Translating the Schumpeterian ideas on the dynamic interaction of technological diffusion and innovation into a formal lan-

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20 Thus, the Marxian decomposition (with $P = \text{profits}$) $r = P/K = (P/wL)(wL/K) = (P/wL)/x$, which referring to the wage share $\epsilon$ may here be equivalently expressed as $(r + \delta)x = (r + \delta)(b/\epsilon) = (1 - \epsilon)/\epsilon$, is only of limited explanatory power.
guage, Iwai’s main goal was to question the appropriateness of the classical and neoclassical concept of long-run equilibrium. He demonstrated that an economy’s state of technology will indeed be in a perpetual disequilibrium (from the viewpoint of classical and neoclassical model building). Although the evolutionary pressure on profit-seeking firms constitutes a mechanism that steers the economy toward a classical (or neoclassical) equilibrium in which all firms use the most efficient production method available, the function of innovation lies precisely in upsetting this equilibrating tendency. In the long-run, the opposite forces of diffusion and innovation balance, which implies that a multitude of diverse production methods with a wide range of efficiencies will coexist forever.

This general idea comes most clearly to the fore in a deterministic setting where innovations occur at regular intervals every $T$ years. (Iwai’s original hypothesis of stochastic innovations would have complicated our analysis.) While production methods come into existence, run through a life-cycle and eventually die out, the whole spectrum of different techniques reproduces itself, only at an ever increasing scale as time progresses. This means that the role of technique $i$ at some time $t$ is taken over by its more advanced successor at a later point in time, $t + T$. Drawing on Henkin and Polterovich (1991), this smooth evolution of the economy has been called a wave train. Compared to the usual (neo-)classical conception, it may be said that wave trains provide a notion of long-run equilibrium at a higher level.

On the other hand, the balanced growth paths here examined should not be taken too literally. Although they are attractors and, once perturbed, the economy finds back to them, the adjustment process would take a long span of time; typically several decades as shown in Franke (1998a). There are several parameters in the model that one could not reasonably consider to remain fixed over these periods. So we see our paper’s main contribution, not so much in providing an elaborate equilibrium analysis, but in widening the narrow classical and neoclassical perspective on technical progress. Correspondingly, the main purpose of a highly stylized model such as the present one is to serve as a frame of reference.

On this basis, classical and neoclassical theories may be better aware of significant factors in long-run growth and income distribution for which, as they are presently set up, there is no place in the formal analysis. More specifically, the paper has made this point for the contemporary classical theory of the wage-profit frontier. Following this approach and employing the (deflated) real wage rate to parameterize the side of wages, we have demonstrated that, in a wave train equilibrium, distribution can no longer be decoupled from the ‘quantity side’ and is in fact heavily dependent on factors related to the speed of technological diffusion. Thus, it could be seen that distribution depends most prominently on a parameter that measures the general responsiveness of investors to differential rates of profit.

The model has sought to integrate the classical concept of income distribution into the Schumpeterian framework in a straightforward manner. In order to follow the classical tradition and be able to compare directly the level of (deflated) real wages on the growth paths of two distinct economies, it was necessary to treat the rate of technical progress as an exogenous variable, assuming that it is the same in both economies. If one conceives of extensions of the model in the direction of
endogenous technical change (which might also fail to be Harrod-neutral), then this type of comparison will have to be replaced with a comparison of the growth rates of real wages, or with a comparison of the wage shares in national income.

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Appendix A

Generally in the simulations, time is sliced into (arbitrarily) small economic adjustment periods of length $h$ and the discrete-time analogues with step size $h$ of the differential equations are used. This device, while simpler than the Runge–Kutta approximations, is not purely technical but has an obvious economic interpretation.

Thus, in ‘normal’ periods where $t \notin U(N_t)$, we take the capital growth rates of Eq. (9) and determine $\tilde{k}_i(t+h) = k_i(t) + h \left( s\tilde{f}(t) + \rho(r_i(t) - \bar{r}(t)) \right)|k_i(t)$ for $i \leq N$, which corresponds to the level of capital stocks in the next period $t+h$.

Subsequent renormalization yields the (capital and) capacity shares, $k_i(t+h) = \tilde{k}_i(t+h)/\sum_{j \leq N} \tilde{k}_j(t+h)$.

The second part of Eq. (10), which concerns the set-up phase $t \in U(N_t)$, is a shorthand notation for the following procedure. The growth rates of the capital stocks for techniques $i \leq N - 1$ are rescaled such that the growth rate of the total capital stock continues to be equal to $s\tilde{f}$. That is, defining $\tilde{g}_i = s\tilde{f}(t) + \rho(r_i(t) - \bar{r}(t))$ and $x = (s\tilde{f} - s')/\sum_{i \leq N} \tilde{g}_i$, the growth rates $g_i$ for $i \leq N - 1$ are determined by $g_i(t) = xg_i$. For the BPT we put $k_N(t+h) = k_N(t) + h\bar{r}$ by way of Eq. (5), whereas for the ordinary techniques $i \leq N - 1$ we first compute $\tilde{k}_i(t+h) = k_i(t) + h g_i(t)k_i(t)$ and then $k_i(t+h) = \tilde{k}_i(t+h)/[k_N(t+h) + \sum_{j \leq N} \tilde{k}_j(t+h)]$.

Another detail are the old vintages, which tend to disappear in the long-run. For practical reasons we reset the capacity share of the least productive technique back to zero in the computer simulations when it declines below a benchmark of 0.1%.

To derive Eq. (15) note that for a best-practice technique $N = N_m$ one has \[ \ln a_N + g_w mT = \ln a_N + g_w mT = \ln a_N = C. \] Thus, with $t = mT + \tau$, \[ \ln a_i + g_w t = \ln a_i + g_w t = \ln a_i + \ln a_N + \ln a_N + g_w mT + g_w \tau = \ln a_N + g_w mT + g_w \tau + C, \] and Eq. (15) follows from Eq. (13).

The numerical parameters in Eq. (19) are obtained as follows. The capital-output ratio is taken directly from Simon (1990, p. 151). Next, in obvious notation, the ratio of gross investment to GNP is equal to $(\tilde{K} + \delta K)/Y = \tilde{g}b + \delta b$. Adopting Simon’s (p. 153) ratio of 16% and setting $\tilde{g} = 3\%$ yields the value of $\delta$. As for the savings propensity $s$, we take a wage share $v = 70\%$ (cf. Simon, p. 150), compute the

\[ ^{21} \text{Of course, this only makes economic sense } \kappa < s\tilde{f}, \text{ which is checked in the simulations.} \]
profit rate $\bar{r} = (1 - v)/b - \delta$, and thus retrieve $s$ from the Cambridge equation as $s = \bar{g}/\bar{r}$.

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