Optimal size and location planning of public logistics terminals

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Abstract

The concept of public logistics terminals (multi-company distribution centers) has been proposed in Japan to help alleviate traffic congestion, environment, energy and labor costs. These facilities allow more efficient logistics systems to be established and they facilitate the implementation of advanced information systems and cooperative freight systems. This paper describes a mathematical model developed for determining the optimal size and location of public logistics terminals. Queuing theory and nonlinear programming techniques are used to determine the best solution. The model explicitly takes into account traffic conditions in the network and was successfully applied to an actual road network in the Kyoto–Osaka area in Japan. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

There are many problems concerning the transporting of freight within urban areas such as congestion, negative environmental impacts and high energy consumption. Traffic congestion is becoming worse in urban areas, partly because of increasing truck traffic, and this is causing transportation costs to increase. This is attributed to the fact that small loads of goods are transported frequently to decrease inventory costs and to satisfy consumer needs. Concerning negative environmental impacts, large diesel trucks are major generators of environmental problems emanating from road traffic, such as noise, air pollution, and vibration. To cope with these problems, proposals have been made to construct public logistics terminals in the vicinity of expressway interchanges surrounding large cities in Japan. The concept of public logistics terminals does not intend to strongly restrict the free economic activities of private companies in competitive markets, but it includes the motivation to solve social problems described above

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through promoting more efficient logistics systems for both private companies and society. By consolidating terminals, the implementation of advanced information systems and especially the cooperative operation of freight transportation systems is more practical. Public logistics terminals can be used by third-party logistics providers or companies that have established cooperative contracts.

Public logistics terminals are complex facilities with multiple functions, including transshipment yards, warehouses, wholesale markets, information centers, exhibition halls and meeting rooms, etc. These are designed to meet various needs of the urban logistics system by using advanced information systems. Advanced information systems help implement algorithms and heuristics to develop more efficient routing and scheduling systems for pickup/delivery trucks in urban areas. This would be useful to reduce the number of trucks that are required to provide same or even higher level of service to customers compared with conventional systems. Public logistics terminals also help small and medium size enterprises to implement efficient freight transportation through the mechanization and automation of materials handling. Low-interest funds may be provided by the public sector for this purpose which are not given for private logistics terminals. These terminals can also facilitate the implementation of cooperative freight transportation systems. These are systems in which a number of shippers or freight carriers jointly operate freight vehicles or freight terminals or information systems to reduce their costs for collecting and delivering goods and provide higher level of services to their customers. As Taniguchi et al. (1995) concluded, truck traffic can be reduced by adopting cooperative freight transportation systems.

Similar ideas relating to the planning of public logistics terminals and cooperative operations have been proposed in The Netherlands (Janssen and Oldenburger, 1991) and in Germany (Ruske, 1994). The concept of public logistics terminals is relatively new and needs more intensive investigation in several areas such as their function, size, location, management as well as the role of public sector. This paper describes a mathematical model for determining the optimal size and location of public logistics terminals that will be required when the concept of public logistics terminals is implemented. In the sections below, the phrase “logistics terminals” will be used interchangeably with “public logistics terminals”.

Planning the size and location of facilities are traditional problems (for example, Weber, 1929; Beckman, 1968; Drezner, 1995) and have been studied by applying the methodology of operations research. Optimization problems relating to the location of transportation terminals have been modeled together with the routing of goods (Hall, 1987; Daganzo, 1996). Campbell (1990) developed a continuous approximation model for relocating terminals to serve expanding demand. Noritake and Kimura (1990) developed models to identify the optimal size and location of seaports using separable programming techniques.

This paper focuses on optimization in designing public logistics terminals, explicitly taking into account traffic conditions on the road network. A mathematical model was developed using queuing theory and nonlinear programming. This model deals with the trade-offs between both transportation and facility costs at terminals and aims to minimize the total of these two logistics costs. Within this model, the user equilibrium assignment procedure is used for determining truck and passenger car traffic on the road network under any location pattern of candidate sites of public logistics terminals. In general, this becomes a large-scale nonlinear programming problem and there is much difficulty in obtaining a strict optimal solution. Therefore the model described here adopts genetic algorithms as a solution procedure to obtain an approximate optimal solu-
tion. Genetic algorithms (for example, Goldberg, 1989) are methods that search for a global optimal solution in a short computation time by simulating the generation, selection, and multiplication of individuals as observed in living things. Genetic algorithms are heuristic methods that are useful in practice to obtain approximate optimal solutions of large-scale optimization problems that cannot be solved exactly by conventional methods.

2. The model

The model described here aims to identify the optimal size and location of logistics terminals. Fig. 1 shows the structure of the logistics system that is investigated within this paper. It is assumed that the movement of goods is divided into two parts: line-haul – which is long-distance transportation by large trucks on expressways, and local pickup/delivery – which is transportation over short distances by small trucks on urban roads. Logistics terminals are the connection points between the line-haul and pickup/delivery of goods where transshipments are usually performed. Sometimes goods may be stored at terminals, but no inventory is considered in this study. Points where freight is generated and attracted are set for line-haul and pickup/delivery trucks within the road network. These points are referred to as centroids.

The model developed here has four features: (1) the model determines the optimal location of logistics terminals from candidate nodes that are discretely specified within the road network; (2) the model takes into account trade-offs between transportation costs and facility costs (such as construction, maintenance, land and truck operation costs in the terminals); (3) a planner can determine the optimal size and location of logistics terminals but cannot control the distribution and assignment of truck traffic; (4) the distribution of the movement of goods is determined for each pair of centroids for line-haul trucks and pickup/delivery trucks. Some distribution patterns

Fig. 1. Structure of logistics system investigated.
of goods described in (4) go through a logistics terminal and others go through other logistics terminals. In other words, each truck tries to minimize its costs by choosing a logistics terminal from candidate nodes.

Fig. 2 indicates the structure of a mathematical model which has two levels of problems. The upper level problem describes the behavior of the planner for minimizing the total cost, which consists of both transportation costs and facility costs. The model simultaneously determines the optimal size and location of logistics terminals. The lower level problem describes the behavior of each company and each truck in choosing optimal logistics terminals and transportation routes. The assignment of pickup/delivery truck traffic is performed together with passenger car traffic. The mathematical formulation of the model is given below.

(upper level problem)

The following symbols are defined:

\[ x, y \] vector that represents the location pattern and the number of berths, respectively, of candidate logistics terminals.

\[ X, Y \] sets of vector \( x \) and \( y \), respectively.

\[ x_i = 1 \] if logistics terminal is located in candidate node \( i \);

\[ x_i = 0 \] otherwise.

\( C_i, C'_i \) total cost of pickup/delivery and line-haul trucks, respectively at logistics terminal \( i \) during the period \( T \) (yen).

\( c_t, c'_t \) transportation cost per hour for pickup/delivery and line-haul trucks, respectively (yen/hour/vehicle): given.

\( t_a, t_b \) travel time performance function on ordinary road network link \( a \) and expressway network link \( b \), respectively (hour): given.

\( V \) flow of all vehicles on link (vehicles/day).

\( V_a, V'_b \) flow of pickup/delivery trucks on ordinary road network link \( a \) and line-haul trucks on expressway network link \( b \), respectively (vehicles/day).

---

Fig. 2. Structure of mathematical model.
The upper level problem is formulated as:

\[
\begin{align*}
\text{minimize} & \quad TC(x, y) = \sum_i x_i C_i + \sum_a c_i t_a V_a + \sum_i x_i C_i' + \sum_b c'_i t_b V'_b \\
\text{subject to} & \quad C_i = c_{bi} y_i T + c_i n_{yi}(q_i) T \quad \forall i, \\
& \quad C_i' = c_{bi} y_i' T + c_i' n_{yi}(q_i') T \quad \forall i, \\
& \quad q_i = \sum_o q_{oi} + \sum_d q_{id} \quad \forall i, \\
& \quad q_i' = \sum_o q_{oi}' + \sum_d q_{id}' \quad \forall i, \\
& \quad V'_b = \sum_i \sum_o \delta_{bi} q_{oi}' + \sum_i \sum_d \delta_{bi} q_{id}' \quad \forall b, \\
& \quad \sum_o x q_{oi} = \sum_d x' q_{id} \quad \forall i, \\
& \quad \sum_d x q_{id} = \sum_o x' q_{oi} \quad \forall i.
\end{align*}
\]

(lower level problem)

The following symbols are defined:

- \( c_{bi} \) berth cost per hour at candidate node \( i \) (yen/hour/berth).
- \( y_i, y_i' \) number of berths for pickup/delivery and line-haul trucks, respectively at logistics terminal \( i \) (berths).
- \( T \) period in consideration (hours).
- \( q_i, q_i' \) total number of pickup/delivery and line-haul trucks, respectively using candidate node \( i \) (vehicles/day).
- \( n_{yi}(q_i) \) average number of trucks in logistics terminal that has the number of berths \( y_i \) during the period \( T \) (vehicles).
- \( q_{oi}, q_{oi}' \) flow between origin \( o \) and candidate node for logistics terminal \( i \) for pickup/delivery and line-haul trucks, respectively (vehicles/day).
- \( q_{id}, q_{id}' \) flow between candidate node for logistics terminal \( i \) and destination \( d \) for pickup/delivery and line-haul trucks, respectively (vehicles/day).
- \( \delta_{bi} = 1 \) if flow between origin \( o \) and candidate node for logistics terminal \( i \) or candidate node for logistics terminal \( i \) and destination \( d \) passes link \( b \);
- \( \delta_{bi} = 0 \) otherwise.
- \( x, x' \) load of pickup/delivery and line-haul trucks, respectively (ton/vehicle).
The lower level problem is formulated as:

\[
\begin{align*}
\text{minimize} & \quad \sum_a \int_0^{V_a} t_a(V) dV - \sum_o \sum_d \int_0^{q_{od}} W_{od}(z) dz \\
\text{subject to} & \quad f_{r,od} = x_i f_{r,od} \quad \forall r, od, \\
& \quad \sum_r f_{r,od} = q_{od} \quad \forall od, \\
& \quad \sum_r f_{r,od}'' = q_{od}'' \quad \forall od, \\
& \quad V_a = \sum_o \sum_d \sum_r \delta_{r,a} f_{r,od} \quad \forall a, \\
& \quad \sum_d q_{od} = O_o \quad \forall o, \\
& \quad \sum_o q_{od} = D_d \quad \forall d, \\
& \quad f_{r,od} \geq 0 \quad \forall r, od, \\
& \quad f_{r,od}'' \geq 0 \quad \forall r, od.
\end{align*}
\]  

These equations represent nonlinear programming with two levels. The first and second terms of Eq. (1) are the costs associated with pickup/delivery trucks and the third and fourth terms are the costs associated with line-haul trucks. The first and third terms of Eq. (1) describe the facility costs which are composed of construction, maintenance, land, and truck operation costs within the terminal. These terms are related to the size of logistics terminals, which is represented by the number of berths. The facility costs \( C_i \) and \( C_i' \) can be calculated using queuing theory (Taniguchi et al., 1996) as follows. If Eq. (2) is divided by \( c_i T \), it yields.

\[
r_S = \frac{C_i}{c_i T} = \frac{c_{bi}}{c_i} y_i + n_{yi}(q_i) = r_{bi} y_i + n_{yi}(q_i),
\]  

where \( y_i \) is the flow of O-D pair \((o, d)\) for pickup/delivery trucks and passenger cars, respectively (vehicles/day), where \( o \) or \( d \) corresponds to candidate node for logistics terminal \( i \).
where $r_S$ is the ratio of total cost and transportation cost per truck at logistics terminal for the number of berth $y_i$ during the period $T$ and $r_{bt}$ the berth-truck cost ratio.

The value of $c_i T$ is known and hence the berth-truck cost ratio $r_{bt}$ could be used as the evaluation criteria for determining the optimal number of berths. In Eq. (18) $r_{bt}$ can be calculated by cost analysis performed separately, and the total cost ratio $r_S$ is only a function of the average number of existing trucks $n_{yi}(q_i)$ in the terminal if the number of berths $y_i$ is fixed. A relationship similar to Eq. (18) can also be defined for line-haul trucks.

If truck arrivals follow a Poisson distribution and service times have an Erlangian distribution with $k$ degrees of freedom (M/E$_k$/S($\infty$) in Kendall’s notation), the total cost ratio $r_S$ can be expressed in the following equation using Cosnetatos’s (1976) approximation applied to $n_{yi}(q_i)$ (see Appendix A).

$$
\hat{r}_S = r_{bt} y_i + \frac{a^{y_i+1}}{(y_i - 1)! (y_i - a)^2} \left\{ \sum_{n=0}^{y_i-1} \frac{n!}{n!} \frac{a^n}{(y_i - 1)! (y_i - a)} \right\}^{-1} \times \left\{ \frac{1 + \left(1/k\right)^2}{2} + \left(1 - \frac{1}{k}\right) \left(1 - \frac{a}{y_i}\right) \frac{\sqrt{4 + 5y_i - 2}}{32a} \right\} + a,
$$

where $a$ is the traffic intensity and $k$ the degrees of freedom in Erlangian distribution for service times.

The berth-truck cost ratio $r_{bt}$ then becomes only a function of the traffic density $a$, if the number of berths $S(= y_i)$ and degrees of freedom $k$ are given.

Eq. (9) describes the objective function of the combined distribution-assignment model (Beckman et al., 1956; Evans, 1976; Shefi, 1985), which incorporates the equal travel time principle for assignment and variable demand for distribution between a centroid for pickup/delivery trucks and a logistics terminal. In Eq. (9), $t_a(V)$ is referred to as a link performance function representing the travel time and traffic volume relationship. Here, the following equation developed by the Bureau of Public Roads (BPR) of the United States (Steenbrink, 1974) is used as a link performance function.

$$
t_a(V) = t_{a0} \left\{ 1 + 2.62 \left( \frac{V}{K_a} \right)^5 \right\},
$$

where $t_{a0}$ is the free travel time on link $a$ (hours) and $K_a$ the traffic capacity of link $a$ (vehicles/day).

This traffic flow and travel time relationship given by Eq. (20) is often used in practice. $W_{od}(z)$ in Eq. (9) is the inverse of a demand function for O-D flow and is assumed to be as follows:

$$
W_{od}(q_{od}) = t_{od} = -\frac{1}{\gamma} \ln q_{od},
$$

where $\gamma$ is the parameter and $t_{od}$ the travel time at O-D pair $(o,d)$ (hours).

The lower level problem (Eq. (9)) simultaneously deals with passenger car traffic and pickup/delivery truck traffic and both traffic modes satisfy the user equilibrium condition of the network. A unique solution for this lower level problem is assured since the set of feasible solutions is convex and the objective function is strictly convex (Shefi, 1985).

The upper level problem is a nonlinear optimization problem with discrete variables representing the location pattern of logistics terminals. To solve this type of problem exactly requires a
very long computation time and if there are many candidate logistics terminals, it is impossible in practice. Therefore, some approximate method is required and genetic algorithms have been applied here. Genetic algorithms provide an effective method to quickly obtain an approximate optimal solution.

Fig. 3 shows the calculation steps used by the genetic algorithm that was applied in this study. Step 1 assumes the genotype of a chromosome with some genes to display an individual corresponding to the location pattern of logistics terminals. If a gene is represented by 1, it means that a logistics terminal is built at this location, and if it is represented by 0, it means no logistics terminal is built there. In the calculation procedure, the evaluation value is defined as the inverse of total cost, shown in Eq. (1). The fitness of the individual is obtained by making the evaluation value linear normalized. Step 2 generates the first generation of certain numbers of individuals, and in Step 3 the fitness of each is calculated. Step 4 preserves the elite individuals with high scores of fitness to accelerate the calculation. Step 5 probabilistically performs selection and multiplication, and in Step 6 pairs are made from individuals and the crossover processing is executed for them to search for a better solution. Step 7 performs ‘mutation’ to avoid being caught in a local optimal solution. Some parameters shown in Fig. 3 were determined from trial-and-error procedures for small optimization problems with known strict solutions.

![Diagram of calculation by genetic algorithm.](image-url)

Fig. 3. Flow of calculation by genetic algorithm.
3. Application to an actual network

The model in Fig. 2 was applied to an actual road network in the Kyoto–Osaka area, as shown in Fig. 4, to determine the optimal size and location of logistics terminals. This network is planned for the year 2010, and 16 candidates for logistics terminals are specified along with several planned expressways. The network has two centroids for line-haul trucks in East and West Japan and 36 centroids for pickup/delivery trucks and passenger cars within the Kyoto–Osaka area. For passenger cars, 6 nodes outside the area are also included in the network. In Fig. 4, ordinary road links represent national highways and main local roads. Predicted O-D traffic volume levels in 2010 existed for passenger and freight traffic, and these were used in all subsequent calculations. But because there is no predicted goods movements between centroids in East/West Japan and those within the Kyoto–Osaka area in 2010, the present amount of goods was used in Case 1, and the amount of 1.5 times was used in Case 2. In Case 3 the capacity of ordinary roads between nodes 1 and 15 was increased from 4 to 6 lanes, with the other conditions being the same as that of

![Fig. 4. Road network and candidate nodes for logistics terminals in the Kyoto–Osaka area.](image)
Case 1. The land price is high for candidate nodes which are close to Osaka and Kyoto. It means that the construction costs of logistics terminals in these areas are higher, but the transportation costs are lower for pickup/delivery trucks between logistics terminals and customers. Thus the trade-offs can be taken into account between terminal costs and transportation costs.

Fig. 5 shows the results for Case 1. Three nodes (1, 5, and 15) were selected as optimal locations. Terminals 1 and 15 are at the junction of expressways and near large cities which have large demands for goods movement. Thus accessibility to large cities and interurban expressways is a significant factor in selecting logistics terminals. In Fig. 5 many ordinary roads indicate heavy congestion, especially near Osaka. This leads to an increase of transportation costs, and that is the reason why terminals close to large cities were chosen as the location for the optimal terminals. Fig. 5 also shows the influence area of each selected terminal, and Table 1 indicates the optimal number of berths in each of the selected logistics terminals. Terminal 1 processes a larger amount of goods generated and attracted in Osaka than in Kyoto, and so the required number of berths is greater than that of terminal 15, which is close to Kyoto, although the influence area of terminal 1 is smaller than that of terminal 15.

Fig. 5. Selected logistics terminals and degrees of congestion (= traffic volume/traffic capacity) in ordinary roads (Case 1).
Fig. 6 shows the costs of the optimal location and the non-optimal locations where terminal 2 or 3 was selected instead of terminal 1 and in which all terminals were selected. The comparison denotes that the cost incurred at a terminal has a relatively small difference in each solution, and the transportation cost of pickup/delivery trucks has a great difference in these instances. This implies that the congestion of ordinary roads greatly affects the optimal solution.

Fig. 7 shows the changes of costs and accumulative numbers of individuals at each generation in the genetic algorithm calculation and random search procedure. An individual corresponds to a location pattern of candidate logistics terminals. The random search adopts the same number of individuals chosen at random at each generation. This figure shows that as the accumulative number of individuals by genetic algorithms is smaller than that in random search procedure, hence genetic algorithms take less computation time than random search. In genetic algorithms, the minimum cost at each generation monotonically decreases as the number of generations increases, whereas within the random search it does not always decrease. Moreover, the minimum cost among 30 generations by genetic algorithms is smaller than that of the random search. It can be concluded that genetic algorithms applied in this research are more efficient than random search in searching for an optimal solution.

In Case 2 the total amount of goods movement was increased by 50 percent from that of Case 1, and the optimal location of logistics terminals became 1, 2, 5, and 15. Terminal 2 was added to

<table>
<thead>
<tr>
<th>Terminal number</th>
<th>Line-haul trucks</th>
<th>Pickup/delivery trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>297</td>
<td>1593</td>
</tr>
<tr>
<td>5</td>
<td>185</td>
<td>986</td>
</tr>
<tr>
<td>15</td>
<td>162</td>
<td>865</td>
</tr>
</tbody>
</table>

Table 1
Optimal number of berths for selected terminals (Case 1)
Fig. 7. Change in costs and numbers of individuals at each generation (Case 1). (a) genetic algorithms, (b) random search.
solution set of Case 1. Goods from the nodes around terminal 2 go to terminal 2 instead of terminal 1 because ordinary roads around terminal 1 suffer from severe congestion.

Fig. 8 shows the selected logistics terminals for Case 3 and links where travel times were reduced to less than 70% of that of Case 1. In Case 3 the capacity of ordinary roads between nodes 1 and 15 was increased, and terminal 7 was selected in addition to 1, 5, and 15 as an optimal solution. The improvement of ordinary roads reduced the travel time of links in the area between Kyoto and Osaka and decreased the transportation costs incurred by the pickup/delivery trucks. Terminal 7 emerged as part of the solution; it has lower terminal facility costs and a lower land price than the other candidate nodes. Also, the reduction of transportation costs by increasing the road capacity allowed the candidate logistics terminal that is away from large cities to be chosen such as the terminal 7.

4. Conclusions

This paper described a mathematical model developed for determining the optimal size and location of logistics terminals that explicitly takes into account the conditions on the road
network. The model incorporates queuing theory and nonlinear programming techniques and assumes user equilibrium with variable demand for the assignment of pickup/delivery trucks in urban areas. Large-scale nonlinear programming problems cannot be solved without the use of heuristic methods. Genetic algorithms were used to search for an approximate optimal solution. The following conclusions were derived from the development and application of the model described above.

(1) The model was successfully applied to an actual road network in the Kyoto–Osaka area. Genetic algorithms were found to be effective in obtaining an optimal solution for the size and location of logistics terminals. The optimal location of logistics terminals were generally at junctions of expressways and close to large cities, because of the heavy congestion on many ordinary roads which generates an increase in transportation costs.

(2) Improvements to the road network significantly affect the location of logistics terminals, even if the improvements were not near the terminals because improvements alleviate traffic congestion, which leads to decreased transportation cost of the pickup/delivery trucks.

Further investigation will be required to expand this model to deal with the multiple objective optimization problems, including the cost reduction along with less environmental impact and energy saving. This model can also be modified to investigate the location planning of the terminals for new freight transportation systems such have been proposed in Japan and The Netherlands (Koshi et al., 1992).

Appendix A. Approximation for M/E_k/S(∞) model

For M/E_k/S(∞) model, no theoretical formula has been derived concerning the average number of trucks in logistics terminals. Some approximation formulas have only been proposed that relate the average waiting time of trucks in M/E_k/S (∞) model to that in M/M/S (∞) model. Cosmetatos (1976) proposed the approximation formula given below.

\[ W_k = W_1 \left\{ \frac{(1 + v^2)}{2} + (1 - v^2)(1 - \rho)(S - 1) \frac{\sqrt{4 + 5S - 2}}{32\rho S} \right\}, \quad (A.1) \]

where \( W_k \) is the average waiting time of trucks in M/E_k/S (∞) model, \( W_1 \) the average waiting time of trucks in M/M/S (∞) model, \( v \) the coefficient of variation of trucks’ service time distribution and \( \rho = \frac{\lambda}{\mu} \), the utilization factor.

Note that M/M/S (∞) is the special case of M/E_k/S (∞), when \( k = 1 \).

Cosmetatos (1976) reported that any value of \( W_k \) given by Eq. (A.1) could be approximated with a relative percentage error less than ±2% for most practical purposes. When the service time of trucks obeys the Erlangian distribution with the degree of freedom \( k \), Eq. (A.2) holds.

\[ v = \frac{1}{\sqrt{k}}. \quad (A.2) \]

The following equation is obtained by substituting Eq. (A.2) into Eq. (A.1).

\[ W_k = W_1 \left\{ \frac{(1 + \frac{1}{k})}{2} + \left( 1 - \frac{1}{k} \right)(1 - \rho)(S - 1) \frac{\sqrt{4 + 5S - 2}}{32\rho S} \right\}. \quad (A.3) \]
The average waiting time $W_1$ in M/M/S ($\infty$) model is given below.

$$W_1 = \frac{a^S}{\mu(S-1)!(S-a)^2} \left\{ \sum_{n=0}^{S-1} \frac{a^n}{n!} + \frac{a^S}{(S-1)!(S-a)} \right\}^{-1},$$

(A.4)

where, $\mu$ is the service rate for trucks.

Let $n_{wk}$ be the average number of trucks waiting in the logistics terminal for berths in M/E$_k$/S($\infty$) model. Then, from Little’s formula,

$$n_{wk} = \lambda W_k,$$

(A.5)

where, $\lambda$ is the arrival rate of trucks.

Thus Eq. (A.6) is obtained by substituting Eq. (A.3) into Eq. (A.5).

$$n_{wk} = \lambda W_1 \left\{ \frac{(1+\frac{1}{k})}{2} + \left(1-\frac{1}{k}\right)(1-\rho)(S-1) \frac{\sqrt{4+5S}-2}{32\rho S} \right\}.$$

(A.6)

The average number of trucks in the logistics terminal $n_S$ is equal to the sum of the average number of trucks in waiting and in service as given below.

$$n_S = n_{wk} + a.$$

(A.7)

Substitute Eq. (A.6) into Eq. (A.7), then

$$n_S = \lambda W_1 \left\{ \frac{(1+\frac{1}{k})}{2} + \left(1-\frac{1}{k}\right)(1-\rho)(S-1) \frac{\sqrt{4+5S}-2}{32\rho S} \right\} + a.$$

(A.8)

Substitute Eq. (A.4) into Eq. (A.8), then

$$n_S = \frac{a^{S+1}}{(S-1)!(S-a)^2} \left\{ \sum_{n=0}^{S-1} \frac{a^n}{n!} + \frac{a^S}{(S-1)!(S-a)} \right\}^{-1}$$

$$\times \left\{ \frac{(1+\frac{1}{k})}{2} + \left(1-\frac{1}{k}\right)(1-\rho)(S-1) \frac{\sqrt{4+5S}-2}{32\rho S} \right\} + a.$$

(A.9)

Note that in deriving Eq. (A.9):

$$a = \frac{\lambda}{\mu}.$$

(A.10)

Let $S = y_i$, then Eq. (A.9) yields $n_{y_i}(q_i)$ as given in Eq. (19) in the main text.

References


