LINEAR PRESERVERS OF LEFT MATRIX MAJORIZATION

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Abstract. For $X, Y \in M_{nm}(\mathbb{R}) (= M_{nm})$, we say that $Y$ is left (resp. right) matrix majorized by $X$ and write $Y \prec_{\ell} X$ (resp. $Y \prec_{r} X$) if $Y = RX$ (resp. $Y = XR$) for some row stochastic matrix $R$. A linear operator $T: M_{nm} \to M_{nm}$ is said to be a linear preserver of a given relation $\prec$ on $M_{nm}$ if $Y \prec X$ implies that $TY \prec TX$. The linear preservers of $\prec_{\ell}$ or $\prec_{r}$ are fully characterized by A.M. Hasani and M. Radjabalipour. Here, we launch an attempt to extend their results to the case where the domain and the codomain of $T$ are not necessarily identical. We begin by characterizing linear preservers $T: M_{p1} \to M_{n1}$ of $\prec_{\ell}$.

Key words. Row stochastic matrix, Doubly stochastic matrix, Matrix majorization, Weak matrix majorization, Left (right) multivariate majorization, Linear preserver.

AMS subject classifications. 15A04, 15A21, 15A51.