STABLE SUBNORMS ON FINITE-DIMENSIONAL POWER-ASSOCIATIVE ALGEBRAS

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Abstract. Let \( A \) be a finite-dimensional power-associative algebra over a field \( F \), either \( \mathbb{R} \) or \( \mathbb{C} \), and let \( S \), a subset of \( A \), be closed under scalar multiplication. A real-valued function \( f \) on \( S \) is called a subnorm if \( f(a) > 0 \) for all \( 0 \neq a \in S \), and \( f(\alpha a) = |\alpha|f(a) \) for all \( a \in S \) and \( \alpha \in F \). If in addition, \( S \) is closed under raising to powers, then a subnorm \( f \) is said to be stable if there exists a positive constant \( \sigma \) so that

\[
    f(a^k) \leq \sigma f(a)^k \quad \text{for all} \quad a \in S \quad \text{and} \quad k = 1, 2, 3, \ldots.
\]

The purpose of this paper is to provide an updated account of our study of stable subnorms on subsets of finite-dimensional power-associative algebras over \( F \). Our aim is to review and discuss some of the results in several previous papers, dealing with both continuous and discontinuous subnorms.

Key words. Finite-dimensional power-associative algebras, Norms, Subnorms, Submoduli, Stable subnorms, Minimal polynomial, Radius of an element in a finite-dimensional power-associative algebra.

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