THE $Q$-MATRIX COMPLETION PROBLEM$^*$

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Abstract. A real $n \times n$ matrix is a $Q$-matrix if for every $k = 1, 2, \ldots, n$ the sum of all $k \times k$ principal minors is positive. A digraph $D$ is said to have $Q$-completion if every partial $Q$-matrix specifying $D$ can be completed to a $Q$-matrix. For the $Q$-completion problem, sufficient conditions for a digraph to have $Q$-completion are given, necessary conditions for a digraph to have $Q$-completion are provided, and those digraphs of order at most four that have $Q$-completion are characterized.

Key words. Partial matrix, Matrix completion, $Q$-matrix, $Q$-completion, Digraph.

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